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# A tool to study azimuthal standing and spinning modes in annular combustors

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## Abstract

A methodology for the computation of azimuthal combustion instabilities which can occur in annular combustors is proposed in this work. A thermoacoustic numerical tool using the  $n - \tau$  model for the coupling of acoustic and combustion is required to solve the Helmholtz equation in reactive media. The methodology is based on the Independence Sector Assumption in Annular Combustor (ISAAC) which states that the heat release fluctuations in a given sector are driven only by the fluctuating mass flow rates due to the velocity perturbations through its own swirler. This assumption is first discussed with respect to a Large Eddy Simulation of an annular combustor. The methodology is then applied to an academic annular test case which exhibits amplified or damped, standing or rotating azimuthal eigenmodes depending on parameters  $n$  and  $\tau$ .

## 1 INTRODUCTION

Although thermo-acoustic instabilities have been known for a long time, they still are a challenging problem for gas turbine manufacturers. This phenomenon results from the coupling between the non-steady combustion process and one of the acoustic eigenmodes of the configuration. Large Eddy Simulations can capture those instabilities, but they still remain too expensive to be used in an industrial context. Moreover, aeronautical gas turbines use annular combustion chambers where azimuthal modes can appear at low

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frequencies: in many chambers the lowest eigenfrequency is often associated to the azimuthal eigenmode. This paper presents a simple thermoacoustic model tool to compute azimuthal thermoacoustic instabilities, which can occur in annular combustion chambers with multiple injectors. In the first part the thermoacoustic model presented in details in [6] is recalled. It solves the wave equation in reactive media but is limited to single flames and burners. The combustion/acoustic coupling is taken into account by using a  $n - \tau$  model. In order to extend the model to annular and multi-injectors combustion chambers an assumption has to be made. The Independence Sector Assumption in Annular Combustors (ISAAC) for parameters  $n$  and  $\tau$  is presented. This assumption is first tested thanks to a Large Eddy Simulation of a full annular combustion chamber [14]. Then the thermoacoustic model under ISAAC assumption is applied in an academic annular test case. This simple configuration shows standing or spinning eigenmodes, either amplified or damped, depending on the  $n$  and  $\tau$  parameters distribution. A sensitivity analysis of the azimuthal eigenmodes to the parameter  $\tau$  is also proposed.

## 2 THERMOACOUSTIC MODEL

The wave equation in reactive media used in the thermoacoustic model presented here is the following :

$$\frac{1}{\gamma p_0} \frac{\partial^2 p_1}{\partial t^2} - \nabla \cdot \frac{1}{\rho_0} \nabla p_1 = \frac{(\gamma - 1)}{\gamma p_0} \frac{\partial}{\partial t} q_1. \quad (1)$$

This equation is derived under the hypothesis of non viscous and zero Mach number flow. The fluctuating quantities (noted  $_1$ ) remains small compared to mean quantities (noted  $_0$ ). The derivation of Eq. (1) is described in details in [6] or [8]. Assuming harmonic motion for perturbed quantities :

$$p_1(\vec{x}, t) = \Re(\hat{p}(\vec{x}, t)e^{-i\omega t}), \quad (2)$$

$$q_1(\vec{x}, t) = \Re(\hat{q}(\vec{x}, t)e^{-i\omega t}), \quad (3)$$

one can obtain the Helmholtz equation in reactive media :

$$\nabla \cdot c_0^2 \vec{\nabla} \hat{p} + \omega^2 \hat{p} = i\omega(\gamma - 1)\hat{q} \text{ on } \Omega. \quad (4)$$

The associated boundary condition can be written :

$$\vec{\nabla} \hat{p} \cdot \vec{n} - i \frac{k}{Z(\omega)} \hat{p} = 0 \text{ on } \partial\Omega_R, \quad (5)$$

where  $Z(\omega)$  is the impedance of the boundary.  $Z(\omega) = \infty$  corresponds to walls and  $Z(\omega) = 0$  to constant pressure boundaries.

The coupling between acoustic and combustion is modeled using a  $n - \tau$  formulation introduced by Crocco in 1951 [1]. In its global formulation this model links the total fluctuating heat release  $Q_1(t) = \int_V q_1(x, t) dV$  of the combustion reaction to the fluctuating velocity at a reference point, noted  $x_{ref}$ . In a realistic configuration this reference point must be chosen in the injection zone where acoustic fluctuations can modify the flow rate or the equivalence ratio. It must be as close as possible of the combustion zone [16]. Under harmonic motion this model can be cast as :

$$Q_1 = \hat{Q}_1 e^{i\omega t} \text{ with } \frac{\hat{Q}_1}{Q_{tot}} = n(\omega) e^{i\omega\tau} \frac{\hat{U}_1(\vec{x}_{ref})}{U_{bulk}}. \quad (6)$$

The scaling by the total heat release  $Q_{tot} = \frac{1}{T} \int_T \int_V q(x, t) dV dt$  and the bulk velocity  $U_{bulk}$  leads to a dimension less value for  $n$ .  $\tau$  represents the time between a velocity perturbation and the heat release fluctuation it produces. This parameter can be influenced by the flow dynamics or by kinetics. It controls stability of the configuration according to the Rayleigh criterion [9] as it will be shown in the third part of this study.

In the present approach the initial global  $n - \tau$  model is extended to a local formulation where  $n_l(\vec{x})$  and  $\tau_l(\vec{x})$  depend on spacial location [6]. Eq. (6) takes the form :

$$\frac{\hat{q}_1(\vec{x})}{q_{tot}} = n_l(\vec{x}, \omega) e^{i\omega\tau_l(\vec{x})} \frac{\vec{u}_1(\vec{x}_{ref}) \cdot \vec{n}}{U_{bulk}}, \quad (7)$$

where  $q_{tot} = Q_{tot}/V$  and  $\vec{u}_1(\vec{x}_{ref}) \cdot \vec{n}$  is the longitudinal fluctuating velocity at the reference point. Injecting Eq. (6) in Eq. (4) and using a finite element approach leads to an algebraic non-linear eigenvalue problem :

$$(A - D(\omega))P + \omega B(\omega)P + \omega^2 CP = 0. \quad (8)$$

The flame transfer function is encapsulated in the  $D$  matrix. The  $A$ ,  $D$ ,  $B$  and  $C$  matrices are explicitly given in [6]. This eigenvalue problem is solved using iterative subspace methods [3, 13, 12] and point fix algorithm.

### 3 The ISAAC ASSUMPTION

#### 3.1 Description of the ISAAC methodology

Most aeronautical combustors present annular geometries with  $N$  injectors ( $N$  varies typically between 10 and 24) [4, 2]. Obviously, in multi-injection configurations, the coupling between the fluctuating heat release and the acoustic fluctuation can not be described by a flame transfer function involving only one reference point as it was presented above. The local formulation (Eq. (7)) can be extended as follows :

$$\frac{\hat{q}_1(\vec{x})}{q_{tot}} = \begin{cases} n_l(\vec{x}, \omega) e^{i\omega\tau_l(\vec{x})} \frac{\tilde{u}_1(\vec{x}_{ref_1}) \cdot \vec{n}_1}{U_{bulk}} & \text{for } \vec{x} \in \text{Sector 1,} \\ n_l(\vec{x}, \omega) e^{i\omega\tau_l(\vec{x})} \frac{\tilde{u}_1(\vec{x}_{ref_2}) \cdot \vec{n}_2}{U_{bulk}} & \text{for } x \in \text{Sector 2,} \\ \dots & \\ n_l(\vec{x}, \omega) e^{i\omega\tau_l(\vec{x})} \frac{\tilde{u}_1(\vec{x}_{ref_N}) \cdot \vec{n}_3}{U_{bulk}} & \text{for } x \in \text{Sector } N. \end{cases} \quad (9)$$

In this approach, the annular combustion chamber is split in  $N$  sectors. Here the Independence Sector Assumption in Annular Combustor is made.

*ISAAC assumption* : The heat release fluctuations in a given sector are driven only by the fluctuating mass flow rates due to the velocity perturbations through its own swirler.

#### 3.2 Verification of the ISAAC assumption using LES

The ISAAC assumption is implicitly used in most studies of annular combustors [10, 15, 5]. It is true only if flames issuing from neighboring injectors do not interact, a property which is known to be false in certain cases [7] but seems to be acceptable in most gas turbines. A recent confirmation of the ISAAC assumption was given in [14]. In this work a massively parallel Large Eddy Simulation of a full helicopter combustion chamber is performed. A self-excited azimuthal mode develops naturally. LES results show that these turning modes modulate the flow rate through the fifteen burners and the flames oscillate back and forth in front of each burner, leading to local heat release fluctuations. LES demonstrates also that the interaction between flames can be neglected and that the first effect of the turning modes is to induce longitudinal pulsations of the flow rates through individual burners. The transfer functions for each burner have been computed and the global parameters  $n$  and  $\tau$  plotted in Fig. (1). Amplitudes and delays are nearly the same for each sector. The frequency of the azimuthal instabilities observed in this LES is about 740 Hz ( $T = 1.35$  ms). The variability of  $\tau$  (about 0.2 ms) remains small compared to the period  $T$ . Probably, differences between sectors would decrease if the number of periods computed in the LES were increased. Those results confirm the validity of the ISAAC assumption for

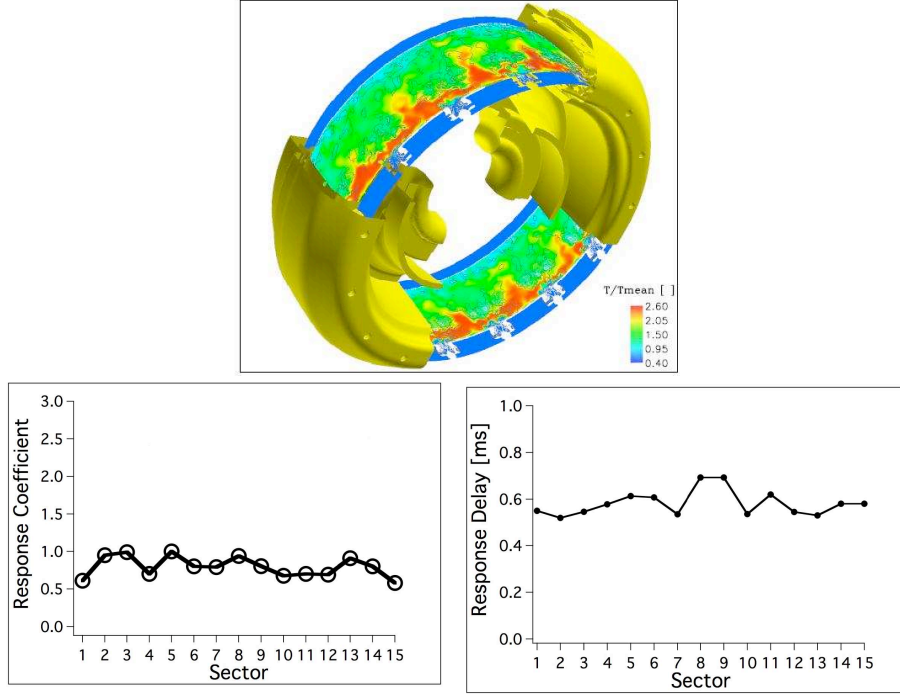


Figure 1: LES of a fifteen sector annular combustor.  $\circ$   $n$  (adimensionalised by  $n_{max}$ ) for each sector;  $\bullet$   $\tau$  (ms) for each sector.

the global parameters  $n$  and  $\tau$ . More LES post processing would be required to compare the local parameters  $n_l(\vec{x})$  and  $\tau_l(\vec{x})$  defined in Eq. (7). This will be done in further work.

## 4 ACADEMIC ANNULAR TEST CASE

### 4.1 Description of the test case

The thermoacoustic model presented above is now tested in a 3D annular configuration with a compact flame located at  $x = L/2$  ( $L$  is the chamber length). The ring is 0.5 m long, the inner radius is 0.4 m and the outer radius is 0.5 m. The sound speed is constant  $c = 347$  m/s. Wall boundary conditions ( $\hat{u}_1 = 0$ ) are set everywhere except at the outlet where a pressure imposed condition is set ( $\hat{p}_1 = 0$ ). The ring is split in five identical sectors. The flame transfer function is the following :

$$\frac{\hat{Q}_1(\vec{x})}{Q_{tot}} = \begin{cases} n_1 e^{i\omega\tau_1} \frac{\hat{u}_{1x}(\vec{x}_{ref1})}{U_{bulk}} & \text{for } \vec{x} \in \text{Sector 1,} \\ n_2 e^{i\omega\tau_2} \frac{\hat{u}_{1x}(\vec{x}_{ref2})}{U_{bulk}} & \text{for } \vec{x} \in \text{Sector 2,} \\ \dots & \\ n_5 e^{i\omega\tau_5} \frac{\hat{u}_{1x}(\vec{x}_{ref5})}{U_{bulk}} & \text{for } \vec{x} \in \text{Sector 5,} \end{cases} \quad (10)$$

where  $n_i$  and  $\tau_i$  are in their global formulation under the validated ISAAC assumption. The reference points  $\vec{x}_{ref_i}$  ( $x_{ref} = 0.22$  m) are placed in the middle of each sector very close (compared to the wave length considered) to the flame front located between  $x = 0.25$  m and  $x = 0.27$  m. The situation is illustrated in Fig. (2).

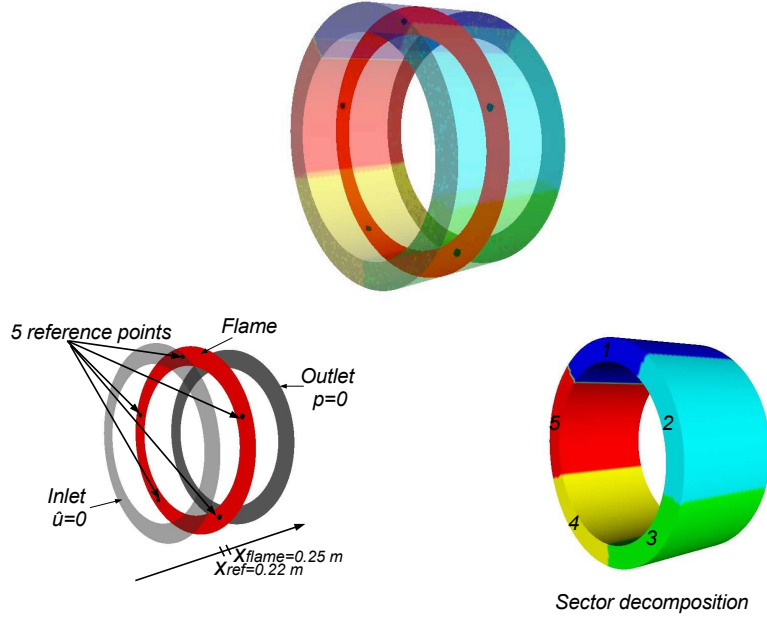


Figure 2: The academic annular test case with five sectors.

## 4.2 Parametric stability analysis of standing modes

In this section the flame transfer function is chosen to be homogeneous in  $\theta$ :

$$n_1 = n_2 = \dots = n_5 = n^* \text{ and } \tau_1 = \tau_2 = \dots = \tau_5 = \tau. \quad (11)$$

Fig. (3) displays the structure of the first azimuthal mode. This mode is coupled with an other azimuthal mode with the same frequency and the same amplification factor. These two modes are found spinning in opposite direction. A combination of these two modes gives a standing azimuthal mode amplified (or damped) depending on  $\tau$ . One can remark that an adequate combination of two orthogonal standing eigenmode sharing the same frequency could also lead to a rotating mode as done in [11]. According to the Rayleigh criterion the parameter  $\tau$  is known to control the stability of the configuration. In industrial applications, a usual way to damp azimuthal instabilities is to change the swirler. This influences the dynamics of the flow and hence the time delay  $\tau$  [4]. The stability of the first azimuthal mode

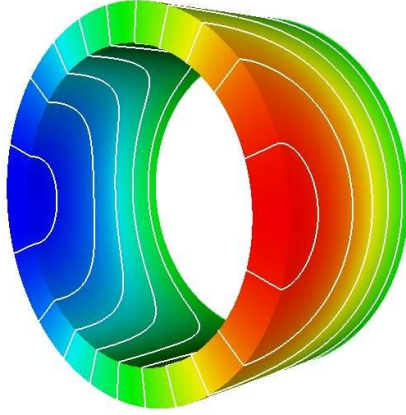


Figure 3: Pressure fluctuation of the first azimuthal mode.

is now investigated when the parameter  $\tau$  takes values between 0 and the period,  $T = 5$  ms, and two different values for  $n$  (Fig. (4)). The real part

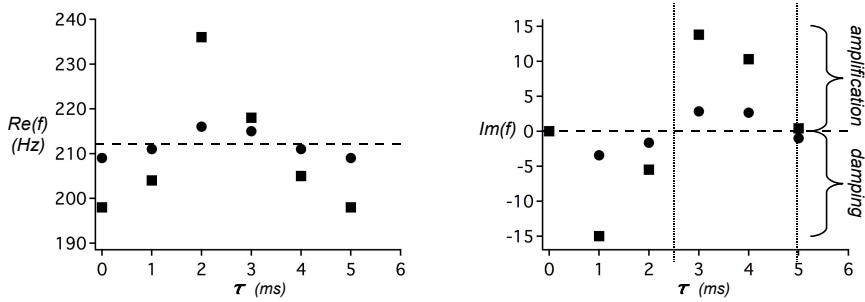


Figure 4: Influence of parameters  $n$  and  $\tau$  on the frequency and the stability of the first azimuthal mode. - -  $n=0$ ;  $\bullet$   $n = n^*$ ;  $\blacksquare$   $n = 5n^*$ .

of the frequency varies with both  $n$  and  $\tau$ , indicating that the mode shape is influenced by combustion effects. The time delay  $\tau$  controls the stability ranges and  $n$  emphasizes the instability effects. For  $n = 0$  all modes are neutral. When  $n > 0$ , for a given frequency of the first azimuthal mode, if  $\tau \in [0, T/2]$  the mode is found stable, and if  $\tau \in [T/2, T]$  the first azimuthal mode is found unstable. It has been checked that this observation was not dependent on the number of sectors. These simple results come from the fact that the flame was supposed to be compact, that the transfer function is simple and that the reference points are located at the flame position. Under these assumptions it is actually possible to verify analytically that the transition between stable and unstable states occurs when  $\tau = m \frac{T}{2}$ .



To do so one considers first a given sector. At the marginal state ( $\Im(\omega) = \omega_i = 0$ ), the Rayleigh criterion can be written as :

$$R(\tau) = \int_V \int_0^T p_1(\vec{x}, t) q(\vec{x}, t) dt dV, \quad (12)$$

$$= \int_V \int_0^T \Re(\hat{p}(x)e^{-i\omega t}) \Re(\hat{q}(x)e^{-i\omega t}) dt dV, \quad (13)$$

$$= \int_V \frac{1}{2} \Re(\hat{p}(x)\hat{q}^*(x)) dV. \quad (14)$$

It is important to notice that even if the Rayleigh criterion is considered in a single sector,  $\omega$  represents the azimuthal pulsation. The  $n - \tau$  model imposes :

$$\hat{q}(x) = n \frac{Q_{tot}}{U_{bulk}} \hat{u}(x_{ref}) e^{i\omega\tau}. \quad (15)$$

Since  $\omega$  is real, the acoustic pressure and the longitudinal velocity phase are in quadrature. It comes from the momentum equation and it is illustrated in Fig. (5) for the particular case  $n = 5n^*$ ,  $\tau = 5$  ms.

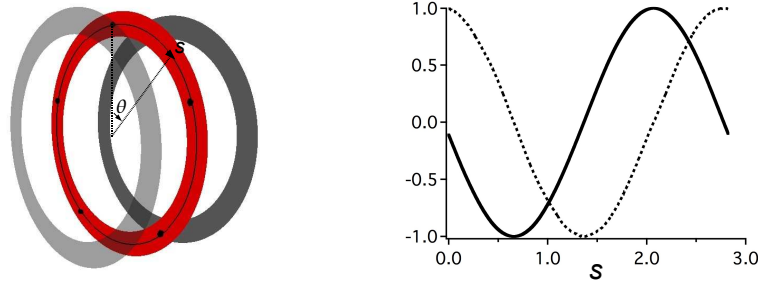


Figure 5: – Reduced pressure fluctuations  $\hat{p}(x_{ref})$ ; - - reduced longitudinal velocity fluctuations at the reference abscissa  $\hat{u}(x_{ref})$  for  $n = 5n^*$ ,  $\tau = 5$  ms.

It follows that:

$$\hat{q}_1(x) = n\alpha\hat{p}(x_{ref})e^{i(\omega\tau + \frac{\pi}{2})}, \alpha \in \mathbb{R}. \quad (16)$$

Using Eq. (16) and Eq. (14),  $R(\tau)$  takes the form :

$$R(\tau) = \frac{1}{2}\alpha n |\hat{p}_1(x_{ref})|^2 \cos(\omega\tau + \frac{\pi}{2}). \quad (17)$$

This equation applies to every sector, causing the stability ranges to be the same in all sectors. It confirms the results observed numerically in Fig. (4): the transition between stable and unstable states occurs when  $\tau = m\frac{T}{2}$ .

### 4.3 Spinning azimuthal modes

A basic question in the gas turbine community is the nature (standing or rotating) of the azimuthal mode. In [10] a theoretical non-linear model suggests that standing modes are observed for low amplitudes but that, at higher amplitudes, one of the two rotating modes eventually dominates. The present model shows that even in the linear regime, any change in the symmetry of the system can lead to the appearance of one rotating mode dominating the other one. In the example of Fig. (2) when the rotational symmetry of the configuration is broken (Fig. (6)), the standing azimuthal mode is changed into two counter rotating spinning azimuthal modes ( $1A^+$  and  $1A^-$ ) with different growth rates (Fig. (7)). Combination of modes cannot be used to produce a standing eigenmode. The particular case of the annular configuration presented below with parameters chosen as in Fig. (6) is computed.

$$\begin{aligned} n_1 &= n^*, & \tau_1 &= 1 \text{ ms}, \\ n_2 &= 2n^*, & \tau_2 &= 2 \text{ ms}, \\ & \dots & & \\ n_5 &= 5n^*, & \tau_5 &= 5 \text{ ms}. \end{aligned}$$

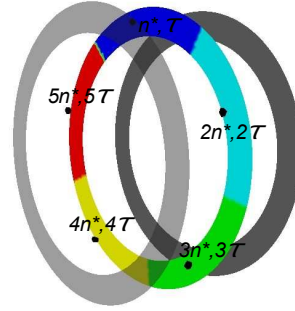


Figure 6: A case where characteristics of the flame are different in each sector.

The  $1A^+$  mode is found amplified ( $+4 \text{ s}^{-1}$ ), and the  $1A^-$  mode is found damped ( $-0.5 \text{ s}^{-1}$ ). A real configuration might not be perfectly symmet-

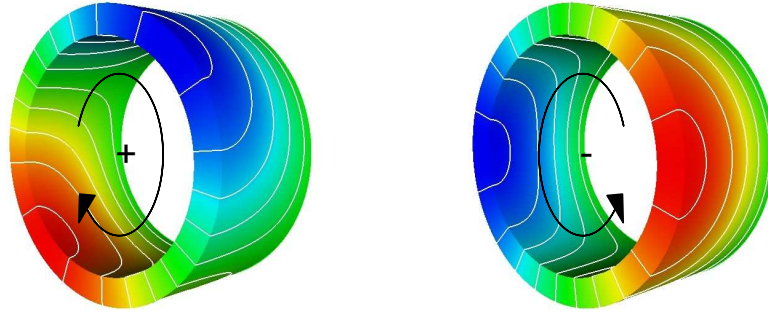
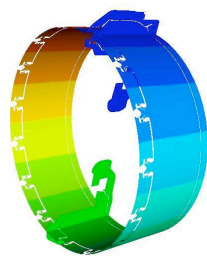


Figure 7: Azimuthal rotating eigenmodes.

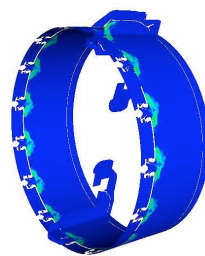
ric for many reasons. In annular combustion chambers there is a mean azimuthal flow because of the multi-perforated plates or compressor flow. Swirlers have also a common rotating direction, and turbulence effects can induce local inhomogeneities in the flow. Those arguments could explain why azimuthal spinning modes are often observed in industrial combustor [4]. Such modes have also recently been captured in LES calculations [14] and with network thermoacoustic models [10, 15, 5].

## 5 CONCLUSION

A methodology to compute azimuthal thermoacoustic instabilities has been presented. It assumes the Independence Sector Assumption in Annular Combustor (ISAAC) which states that *the heat release fluctuations in a given sector are driven only by the fluctuating mass flow rates due to the velocity perturbations through its own swirler*. This assumption has been validated for global parameters  $n$  and  $\tau$  thanks to a full annular Large Eddy Simulation of a real combustion chamber. A calculation of an academic test case has been performed in order to show the capability of the model to capture the azimuthal instabilities occurring when the flame delay  $\tau$  is varying. It has also been shown that when the rotational symmetry is broken, the azimuthal eigenmodes spin. Those conclusions confirm with results obtained with other models [10, 15, 5] and also with experimental observations [4]. Those ideas permit to propose a methodology which will be tested in further works. One can obtain the flame transfer function with a single sector unsteady reactive simulation using U-RANS or LES tools. Then, once the parameters function  $(n, \tau)$  are known they are duplicated in each sector (Fig. (8)) and the stability of the full azimuthal combustor can be studied.



The geometry is split in sectors.



The local  $n(x)$  field is replicated in each sector.

Figure 8: Application of the ISAAC methodology to a helicopter combustor.

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