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TARGETED ENERGY TRANSFER WITH SEVERAL NES IN PARALLEL: THEORY AND EXPERIMENTS

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ABSTRACT

The crucial point in the field of seismic engineering is to diminish the induced vibration energy as much as possible in a fast and almost irreversible manner. Recently the concept of Nonlinear Energy Sink (NES) has been developed such that the imposed energy to a linear single Degree of Freedom (DoF) substructure is transferred to one or series of strongly nonlinear light attachments; the mechanism is based on a 1:1 resonance capture. Nonlinear attachments can be designed to passively vibrate with any frequency; hence the system is efficient for both of transient and periodic excitations. Some drawbacks of these systems are as follows: they cannot kill the first peak of oscillation in the free time response that is linked to the energy activation of NES; moreover, the transformation of energy vanishes in time due to decrease of the strength of energy pumping. Using NES in series even cannot accelerate the phenomenon of energy pumping and some strange behavior due to the delay in the cooperation of NES in series is noticed. In this study, the transient dynamic behavior of multiple DoF systems with trees of parallel NES at each DoF is investigated, then experimental and numerical results of a four DoF structure with two parallel NES at the top floor are demonstrated and commented upon.

1 INTRODUCTION

Classical seismic mitigation devices mainly are linear and require the addition of significant mass to the structure. Novel designs of structures commonly seek to reduce the total structural mass. Thus, it is important to develop new absorption devices that reduce both stationary and transient responses as much as possible while adding extra mass to the structure as less as possible; moreover these devices should be able to absorb seismic effects for a broad band of frequencies. To this end, the concept of NES has been developed that the imposed energy to a linear single DOF system is transferred to one or

series of strongly nonlinear light attachments in an irreversible manner; the mechanism is based on a 1:1 resonance capture. Theoretical background of these systems is quite well developed [1-7]. Manevitch et. al. [8] presented new analytical approach to the problem of energy pumping in strongly non homogeneous nonlinear 2DOF systems with single anchor spring under condition of initial impact. Their approach was based on the application of Laplace transformation to the principal asymptotic approximation of the equations of motion in complex form and using the power expansion of the solution in terms of time. Schmidt and Lamarque [9, 10] studied the phenomenon of energy pumping between an elasto-plastic oscillator coupled to an essentially nonlinear oscillator. They illustrated that under free excitation, a NES designed for underlying linear elastic oscillator is still efficient for the elasto-plastic case and leads to reduced oscillations in comparison with the behavior without coupled NES. They show that under periodic solicitation and the same design, efficiency of energy pumping can be reduced in the whole system, but that it is possible to improve the design by a numerical parametric study to get correct results. Experimental studies about the energy pumping phenomenon are quite limited. McFarland et. al [11] and Gourdon et. al [12, 13] experimentally verified theoretical effects of energy pumping with a single NES coupled with single and four DoF systems, respectively. They experimentally proved the efficiency of energy pumping systems with respect to classical tuned mass dampers. The phenomenon of energy pumping in structures with parallel NES never has been studied. To this end, this paper focuses on series of vibration tests that were carried out on a 4 DoF structure with two attached NES in parallel at the DGCB lab of the ENTPE, France.

2 ACADEMIC MODEL OF A FOUR STOREY STRUCTURE WITH TWO NES IN PARALLEL

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The system under study, illustrated on Fig. 1, is a 4 DoF structure. M_i , C_i , K_i and X_i are mass, damping coefficient, stiffness and displacements of the i^{th} DoF, $i=1, 2, \dots, 4$. Two parallel NES are attached to the 4th DoF. $\mu_{4,j}$, $c_{4,j}$, $k_{4,j}$ and $x_{4,j}$ are mass, damping coefficient, cubic nonlinear stiffness and displacements of the j^{th} NES, $j=1, 2$.

By endowing the Hamiltonian principle, equations of motion for the mentioned system are as follows:

$$M_1 \ddot{X}_1 + C_1 \dot{X}_1 + C_2 (\dot{X}_1 - \dot{X}_2) + K_1 X_1 + K_2 (X_1 - X_2) = F(t) \quad (1)$$

$$M_2 \ddot{X}_2 + C_2 (\dot{X}_2 - \dot{X}_1) + C_3 (\dot{X}_2 - \dot{X}_3) + K_2 (X_2 - X_1) + K_3 (X_2 - X_3) = 0 \quad (2)$$

$$M_3 \ddot{X}_3 + C_3 (\dot{X}_3 - \dot{X}_2) + C_4 (\dot{X}_3 - \dot{X}_4) + K_3 (X_3 - X_2) + K_4 (X_3 - X_4) = 0 \quad (3)$$

$$M_4 \ddot{X}_4 + C_4 (\dot{X}_4 - \dot{X}_3) + K_4 (X_4 - X_3) + \sum_{j=1}^2 \mu_{4,j} \ddot{x}_{4,j} = 0 \quad (4)$$

$$\mu_{4,j} \ddot{x}_{4,j} + c_{4,j} (\dot{x}_{4,j} - \dot{X}_4) + k_{4,j} (x_{4,j} - X_4)^3 = 0 \quad j = 1, 2 \quad (5)$$

The matrix forms of the above mentioned equations are:

$$\begin{bmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 \\ 0 & 0 & M_3 & 0 \\ 0 & 0 & 0 & M_4 \end{bmatrix} \begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \\ \ddot{X}_3 \\ \ddot{X}_4 \end{bmatrix} + \begin{bmatrix} C_1 + C_2 & -C_2 & 0 & 0 \\ -C_2 & C_2 + C_3 & -C_3 & 0 \\ 0 & -C_3 & C_3 + C_4 & -C_4 \\ 0 & 0 & -C_4 & C_4 \end{bmatrix} \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} + \begin{bmatrix} K_1 + K_2 & -K_2 & 0 & 0 \\ -K_2 & K_2 + K_3 & -K_3 & 0 \\ 0 & -K_3 & K_3 + K_4 & -K_4 \\ 0 & 0 & -K_4 & K_4 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \sum_{j=1}^2 \left(\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_{4,j} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \ddot{x}_{4,j} \end{bmatrix} \right) = \begin{bmatrix} F(t) \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (6)$$

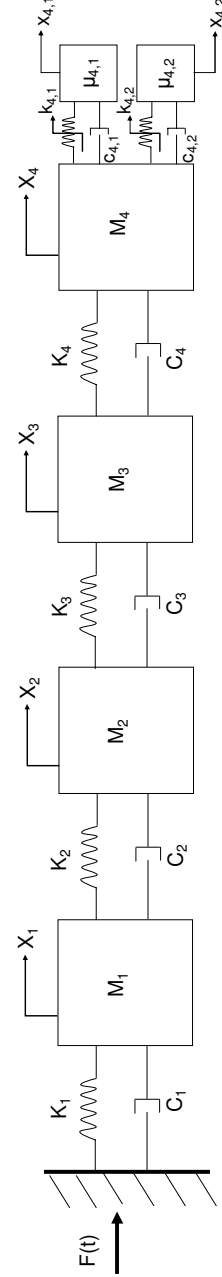


Figure 1. The 4DoF system with two nonlinear parallel NES coupled with the 4th DoF.

$$\begin{bmatrix} \mu_{4,1} & 0 \\ 0 & \mu_{4,2} \end{bmatrix} \begin{bmatrix} \ddot{x}_{4,1} \\ \ddot{x}_{4,2} \end{bmatrix} + \begin{bmatrix} c_{4,1} & 0 \\ 0 & c_{4,2} \end{bmatrix} \begin{bmatrix} \dot{x}_{4,1} - \dot{X}_4 \\ \dot{x}_{4,2} - \dot{X}_4 \end{bmatrix} + \begin{bmatrix} k_{4,1} & 0 \\ 0 & k_{4,2} \end{bmatrix} \begin{bmatrix} (x_{4,1} - X_4)^3 \\ (x_{4,2} - X_4)^3 \end{bmatrix} = \mathbf{0} \quad (7)$$

And in a compact form, system of equations will read:

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} + \mathbf{L}(\ddot{\mathbf{x}}) = \mathbf{F}(\mathbf{t}) \quad (8)$$

$$\mu\ddot{\mathbf{x}} + \mathbf{c}(\dot{\mathbf{x}} - \dot{X}_4\mathbf{1}) + \mathbf{k}(\mathbf{x} - X_4\mathbf{1})^3 = \mathbf{0} \quad (9)$$

We assume that the nonlinear behavior of NES won't change the linear dynamics of the main system; this leads us to shift from physical domain to modal domain by changing variable with the $\mathbf{X} = \boldsymbol{\phi}\mathbf{q}$ system. $\boldsymbol{\phi}$ and \mathbf{q} are mode shape matrix and modal coordinate vector, respectively:

$$\boldsymbol{\phi} = [\boldsymbol{\phi}_1 \quad \boldsymbol{\phi}_2 \quad \boldsymbol{\phi}_3 \quad \boldsymbol{\phi}_4] = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} \\ \phi_{21} & \phi_{22} & \phi_{23} & \phi_{24} \\ \phi_{31} & \phi_{32} & \phi_{33} & \phi_{34} \\ \phi_{41} & \phi_{42} & \phi_{43} & \phi_{44} \end{bmatrix} \quad (10)$$

$$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \quad (11)$$

By applying the mentioned definition to Eq. (8), the new discrete system of equations will appear:

$$\mathbf{M}^*\ddot{\mathbf{q}} + \mathbf{C}^*\dot{\mathbf{q}} + \mathbf{K}^*\mathbf{q} + \boldsymbol{\phi}^T\mathbf{L}(\ddot{\mathbf{x}}) = \boldsymbol{\phi}^T\mathbf{F}(\mathbf{t}) \quad (12)$$

\mathbf{M}^* , \mathbf{C}^* and \mathbf{K}^* are modal mass, modal damping and modal stiffness of the main system, respectively.

The energy pumping or the resonance capture phenomenon in the non conservative system under consideration can be understood and explained by studying the energy dependence of the nonlinear free periodic solutions of the corresponding conservative system that is obtained when all damping forces are eliminated [14]. So, the forcing term in Eq. (12) can be eliminated in the system under consideration. Let's suppose that we are interested to pump the energy of the first mode, so Eq. (12) will read:

$$M_1^*\ddot{q}_1 + C_1^*\dot{q}_1 + K_1^*q_1 + \phi_{41} \sum_{j=1}^2 \mu_{4,j} \ddot{x}_{4,j} = 0 \quad (13)$$

Since $X_4 = \phi_{41}q_1$,

$$\mu\ddot{\mathbf{x}} + \mathbf{c}(\dot{\mathbf{x}} - \phi_{41}\dot{q}_1\mathbf{1}) + \mathbf{k}(\mathbf{x} - \phi_{41}q_1\mathbf{1})^3 = \mathbf{0} \quad (14)$$

By introducing $\frac{K_1^*}{M_1^*} = \omega_1^2$, $\frac{k_{4,j}}{\mu_{4,j}} = \omega_{4,j}^2$, $\frac{c_{4,j}}{\mu_{4,j}} = \lambda_{4,j}$,

$\frac{\mu_{4,j}}{M_1^*} = \varepsilon_{4,j}$, $\sum_{j=1}^2 \varepsilon_{4,j} = \varepsilon_1$, $\frac{C_1^*}{\sum_{j=1}^2 \mu_{4,j}} = \lambda_1$ and $\frac{C_1^*}{M_1^*} = \lambda_1\varepsilon_1$, Eq. (13) and Eq. (14) will lead to:

$$\ddot{q}_1 + \lambda_1\varepsilon_1\dot{q}_1 + \omega_1^2q_1 + \phi_{41} \sum_{j=1}^2 \varepsilon_{4,j} \ddot{x}_{4,j} = 0 \quad (15)$$

$$\begin{aligned} \ddot{x}_{4,j} + \lambda_{4,j}(\dot{x}_{4,j} - \phi_{41}\dot{q}_1) + \\ \omega_{4,j}^2(x_{4,j} - \phi_{41}q_1)^3 = 0 \quad j = 1, 2 \end{aligned} \quad (16)$$

Analytical developments are currently achieving to analyse the behaviour of such reduced systems.

3 IDENTIFICATION OF THE MODAL PROPERTIES OF THE TEST STRUCTURE

The prototype test structure as shown in Fig. 2, is a 4DoF steel structure. Two parallel NES with a very small mass compared to the main structural mass, are mounted on top of the structure as depicted in Fig. 3.

The strongly cubic nonlinear attached oscillator is composed of two mobile parts in parallel; each mobile part is anchored by two springs. Optimal stiffness values of the nonlinear springs of the NES are obtained by experimental testing. Further theoretical and optimization developments are going on for evaluating the optimal stiffness values of nonlinear springs of the NES. Characteristics of the system are summarized in Table 1.

Table 1. Main characteristics of the test structure

Mass (g)			Springs of the NES	
Main Structure	Nes 1	Nes 2	Number	Stiffness (N/m)
2357	30	30	4	480

As illustrated in Fig. 2, five ICP accelerometers are attached to the structure: three of them to the ground, second and last floors and two of them on top of the moving parts of each NES.

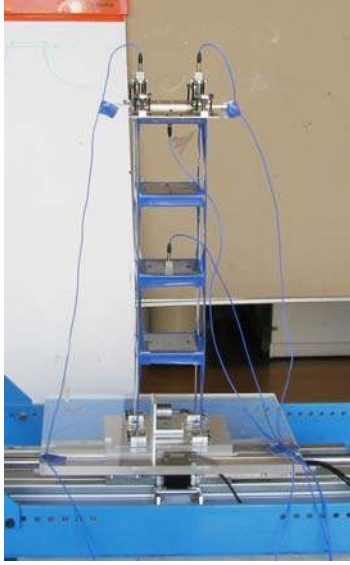


Figure 2. Experimental test set up.

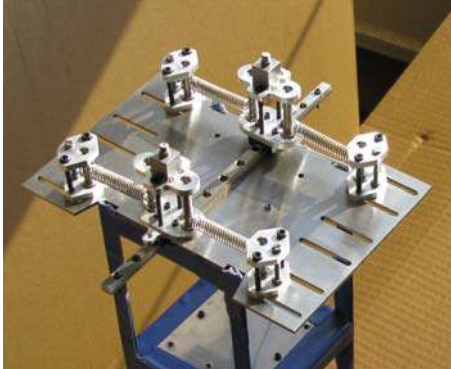


Figure 3. Two parallel NES that were attached to the 4th DoF.

In order to identify modal properties of the structure for feeding the nonlinear model with parallel NES, some series of vibration tests including shock hammer test, Chirp and sinusoidal ramp excitations, are repeated. The first two resonance frequencies of the structure are gathered in Table 2., while the Frequency Response Function (FRF) of the second and the last floors during the shock hammer test are depicted in Fig. 4.

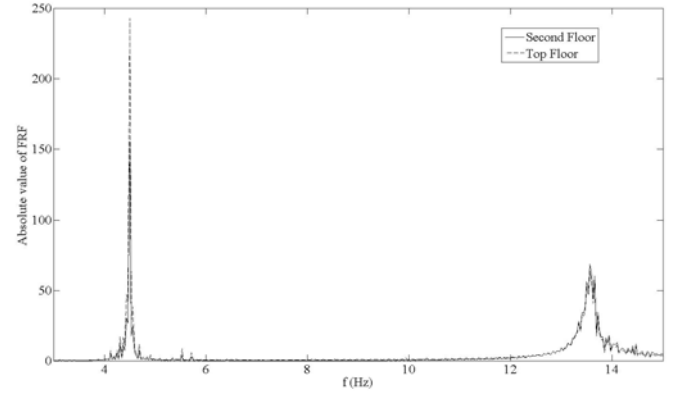


Figure 4. FRF of the second and the top floors during the shock hammer test.

Damping of the structure is estimated by endowing the Hilbert transform [15] and the logarithmic decrement of the impulse response function during the sinusoidal ramp excitation in the decay part of the vibration in the displacements domain and the Bandwidth method in the FRF domain during the Chirp and the shock hammer tests [16]. A discrete-time analytic signal via the Hilbert transform can be evaluated from the impulse response function; its magnitude is the envelope of the original signal; so the rapid oscillations from original signal can be removed to produce a direct representation of the envelope alone. This envelope represents a straight line in the logarithmic domain which slope is proportional to the damping. Two different kinds of sinusoidal ramp excitation with frequencies equal to the first and second modes, i.e. $f=4.44$ Hz and 13.55 Hz, respectively are repeated. For endowing sinusoidal ramp excitation a linear modulus ramp should apply onto the sinusoidal oscillation to ensure that the payload is at rest at the beginning of the test. Figures 5-10 summarize response of the structure, Power Spectrum Density (PSD) for envisioning the frequency contents of the structure and the envelope of the selected decaying part of the response obtained via the Hilbert transform, during two above mentioned excitations.

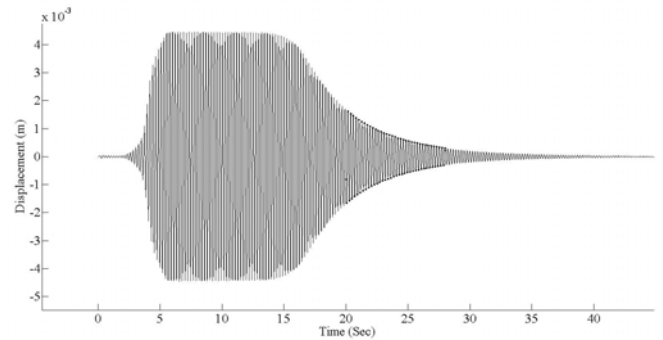


Figure 5. Response of the structure during sinusoidal ramp excitation with $f=4.44$ Hz.

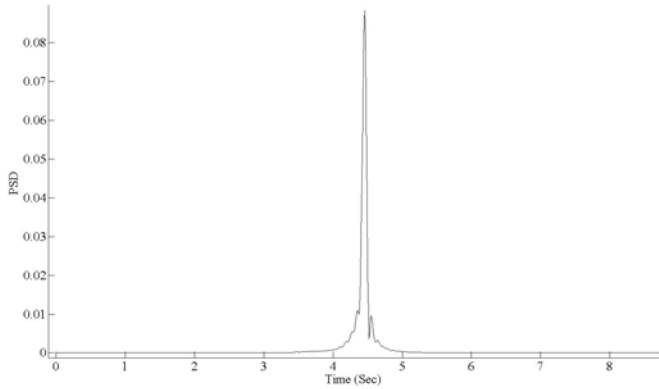


Figure 6. PSD of the response during the sinusoidal ramp excitation with $f=4.45$ Hz.

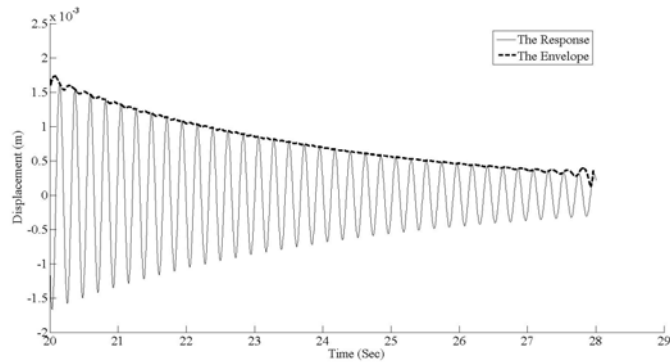


Figure 7. The envelope of the selected decaying part of the response of the structure during sinusoidal ramp excitation with $f=4.44$ Hz.

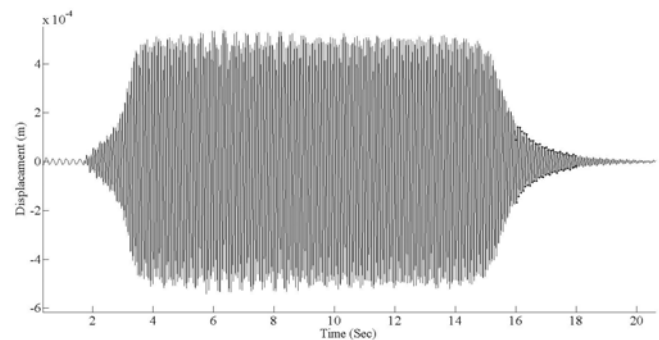


Figure 8. Response of the structure during sinusoidal ramp excitation with $f=13.55$ Hz.

To evaluate the expected value of all identified damping values that are obtained from different tests with different techniques for the first and the second mode, the assumption of the Normal Distribution for evaluated data are retained. In order to justify this assumption, the Normal probability plots

are used to assess whether data comes from a normal distribution. Normal probability plots for the evaluated damping of the first and the second modes are illustrated in Fig. 11 and Fig. 12, respectively.

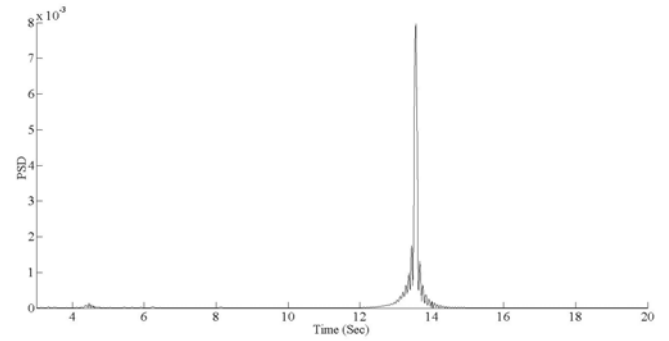


Figure 9. PSD of the response during the sinusoidal ramp excitation with $f=13.55$ Hz.

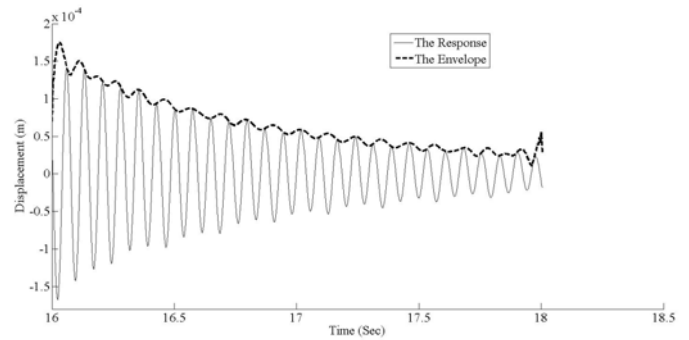


Figure 10. The envelope of the selected decaying part of the response of the structure during sinusoidal ramp excitation with $f=13.55$ Hz.

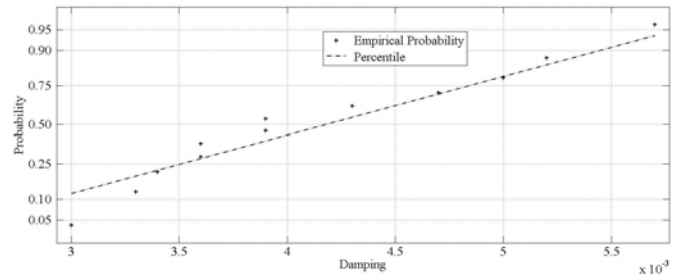


Figure 11. The Normal probability plot for evaluated damping values of the first mode.

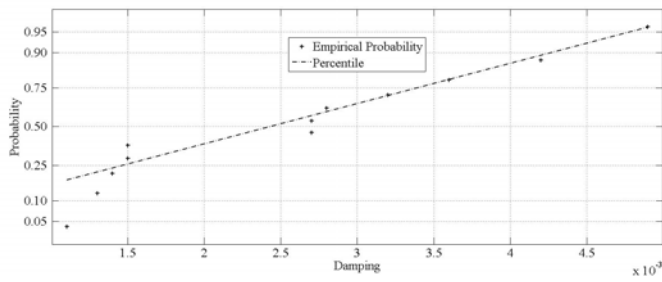


Figure 12. The Normal probability plot for evaluated damping values of the second mode.

All the data points in normal probability plots fall near the lines; therefore an assumption of normality is reasonable. The normal Cumulative Distribution Functions (CDF) for evaluated damping are depicted in Fig. 13 and Fig. 14.

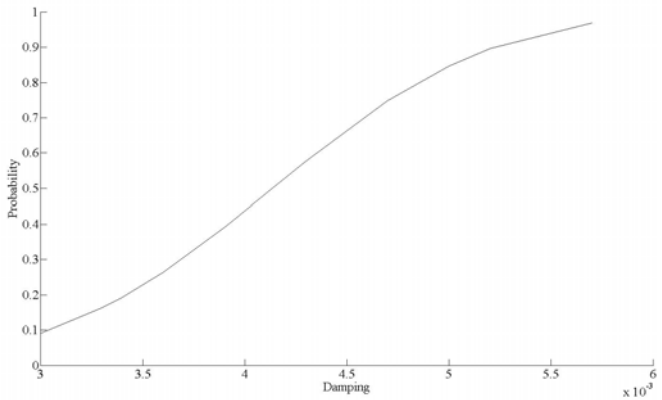


Figure 13. The CDF of the evaluated dampings related to the first mode.

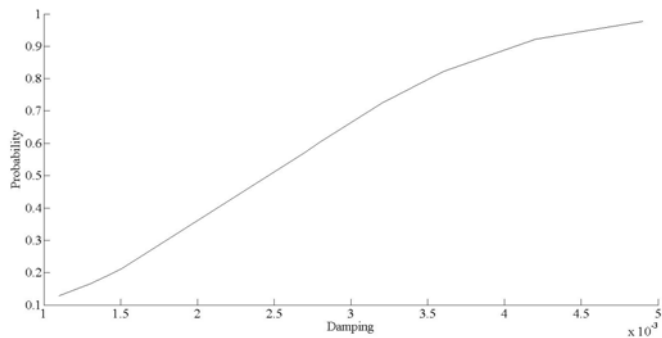


Figure 14. The CDF of the evaluated dampings related to the second mode.

With the aforementioned assumption for data, the expected values of damping for the first and the second modes are collected in Table 2.

Table 2. Identified frequencies and damping of the structure

	Frequency (Hz)	Damping (%)
Mode I	4.44	0.41
Mode II	13.55	0.26

4 EXPLORATION OF NONLINEAR DYNAMICS OF THE TEST STRUCTURE

The aim of this study is to transfer the energy of the first mode of the main structure to two parallel NES systems by a 1:1 resonance capture. The structure is excited by a Chirp signal with sweeping frequencies between 3-5 Hz. The absolute value FRF of the top floor for linear case without energy pumping and for nonlinear case with energy pumping are illustrated in Fig. 15. This figure proves the capability of parallel NES in absorbing most of the energy of the first mode. Figures 16-19 summarize absolute and phase values of FRF of each NES during the linear and nonlinear behavior of the structure under chirp excitations.

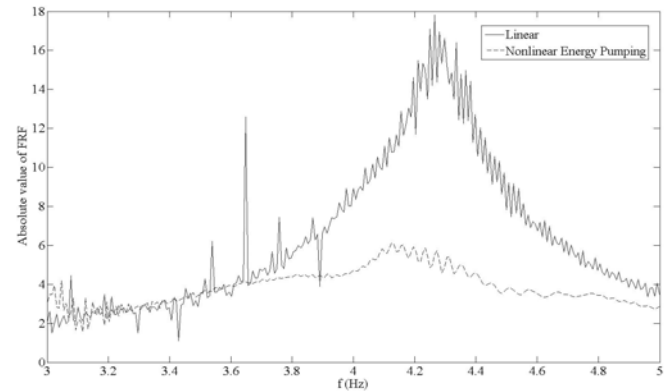


Figure 15. The FRF of the last floor of the structure during the chirp excitation.

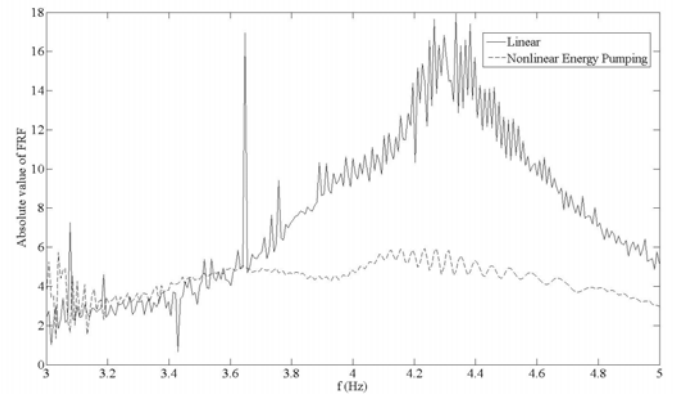


Figure 16. The absolute values of the FRF of the NES 1.

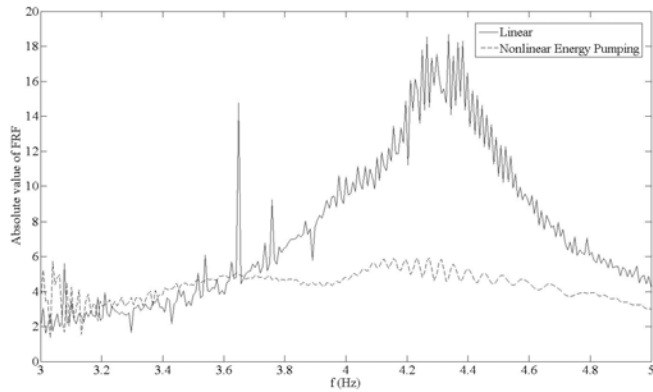


Figure 17. The absolute values of the FRF of the NES 2.

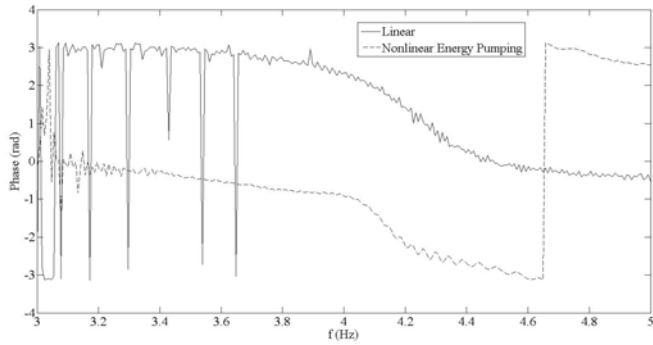


Figure 18. The phase values of the FRF of the NES 1.

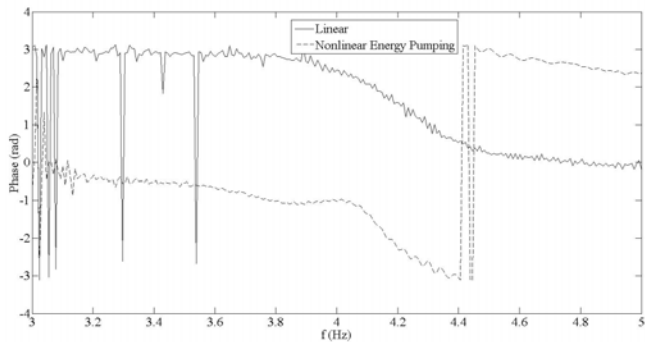


Figure 19. The phase values of the FRF of the NES 2.

The absolute and phase values of parallel NES during the pumping phenomenon are depicted in Fig. 20 and Fig. 21. Both parallel NES move in-phase during the energy pumping phenomenon with almost equal accelerations. This behavior shows a good cooperation of parallel NES in absorbing irreversible energy during the excitation.

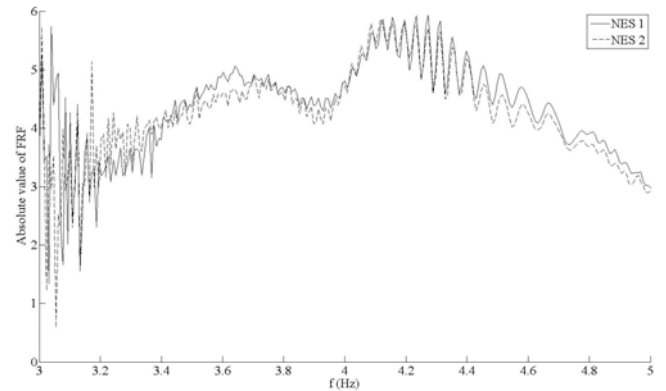


Figure 20. The absolute values of the FRF of the NES system during the energy pumping phenomenon.

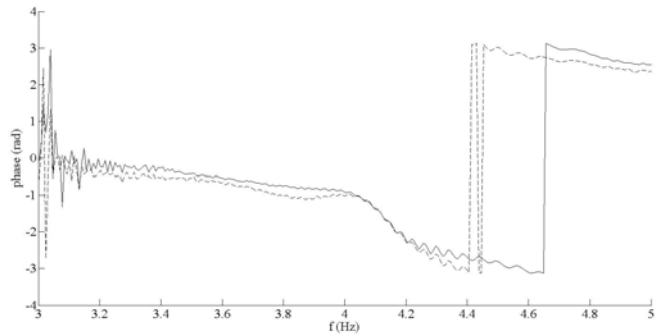


Figure 21. The scaled phase values of the FRF of the NES system during the energy pumping phenomenon.

5 CONCLUSIONS

Governing equations of a four degrees of freedom structure with two parallel NES at the top storey has been demonstrated. Experimental results showed the efficiency of parallel NES system in absorbing most of the seismic energy by cooperating with each other. More detailed theoretical developments are going on in order to find the optimal stiffness of parallel NES system and to compare experimental results with theoretical ones.

7 ACKNOWLEDGMENTS

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