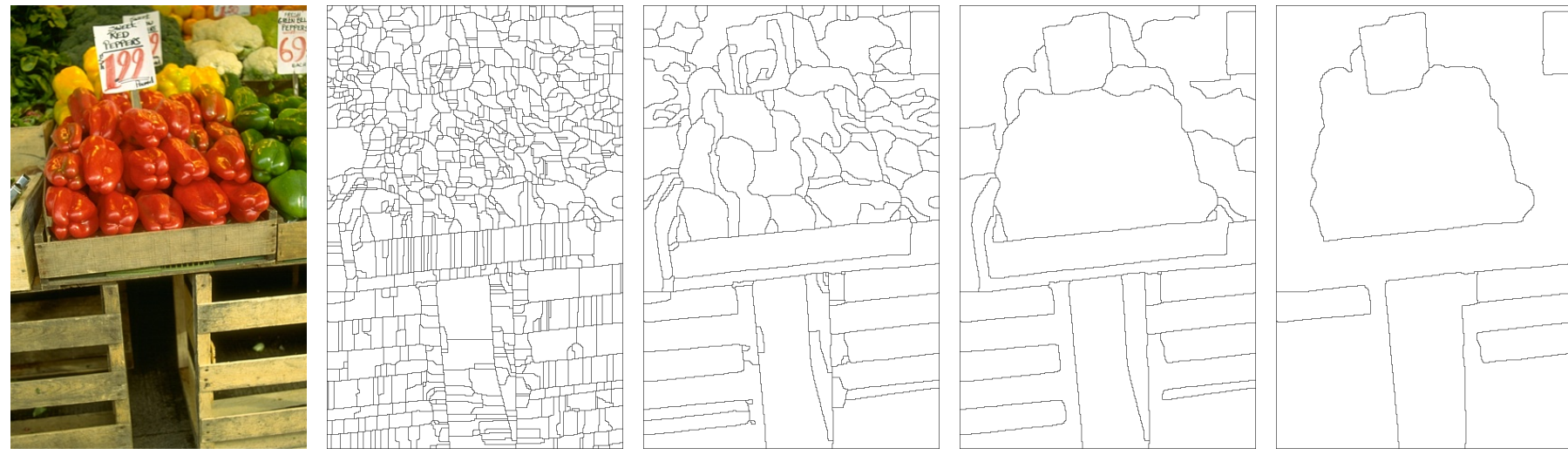


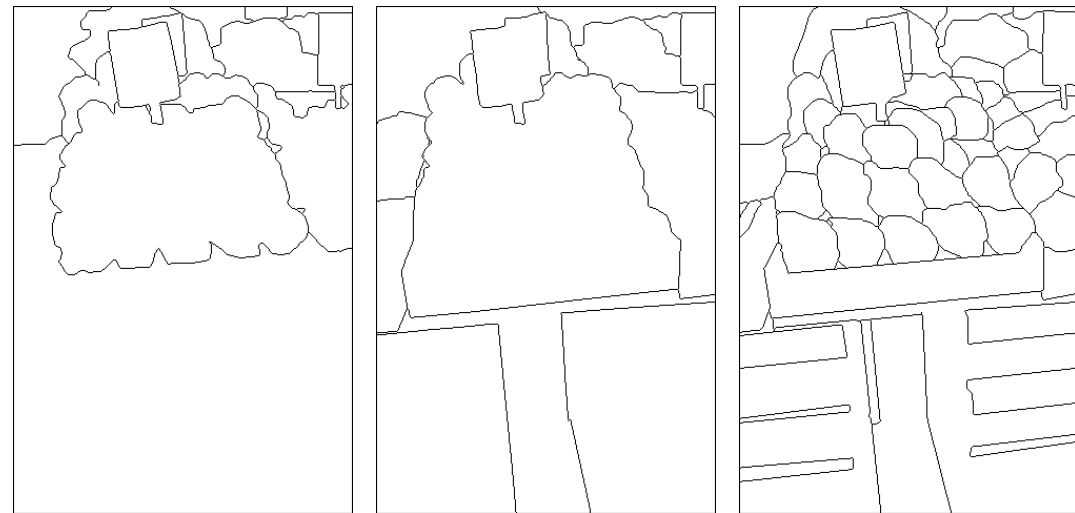


## PROBLEMS

A hierarchy of segmentations, is a set of progressively simplifying partitions.



Classically ground truths are used to evaluate the hierarchy of segmentations.



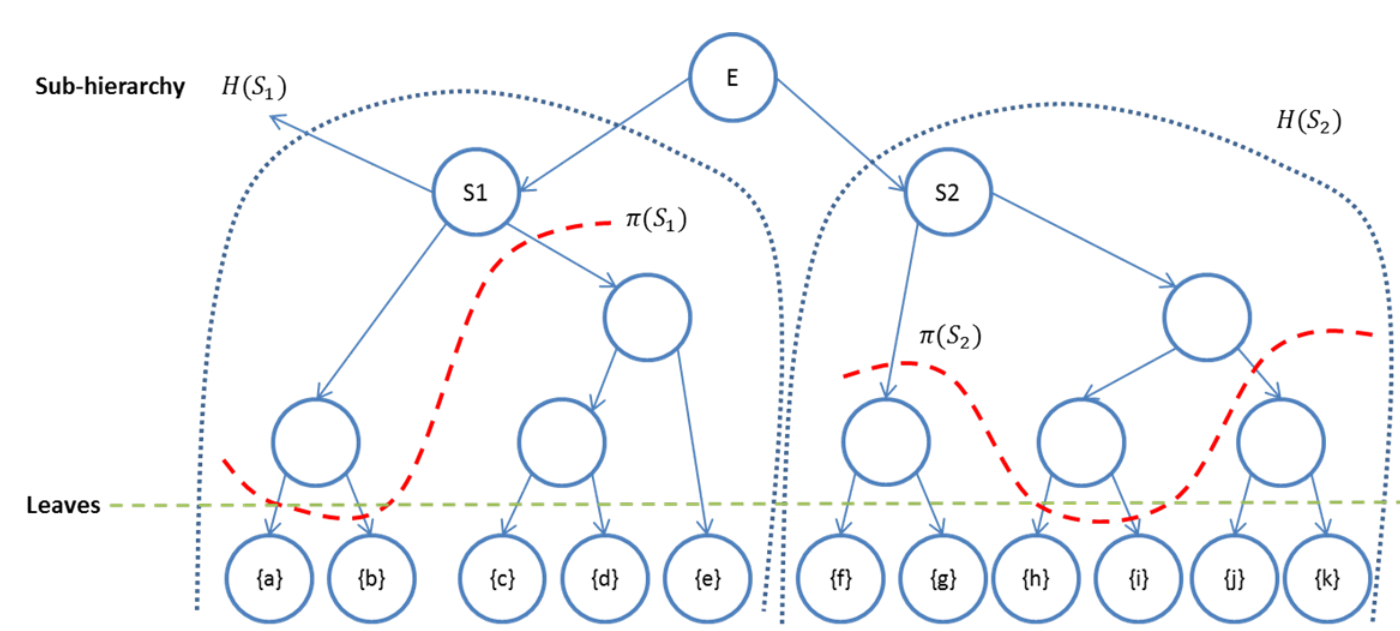
But conversely could the ground truth be used to modify and improve the hierarchy itself? How to generate a transformed hierarchy based on the proximity to the ground truth?

## NOTATIONS

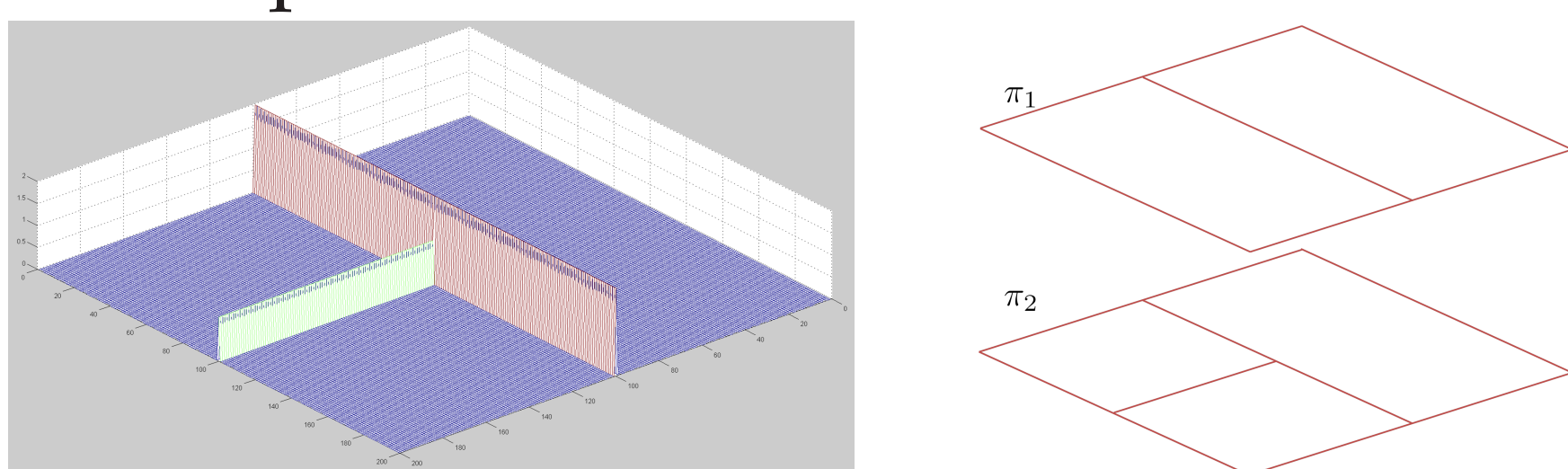
A hierarchy of partitions is a chain of increasing partitions of some finite set  $E$ .

$$H = \{\pi_i, 0 \leq i \leq n \mid i \leq k \leq n \Rightarrow \pi_i \leq \pi_k\},$$

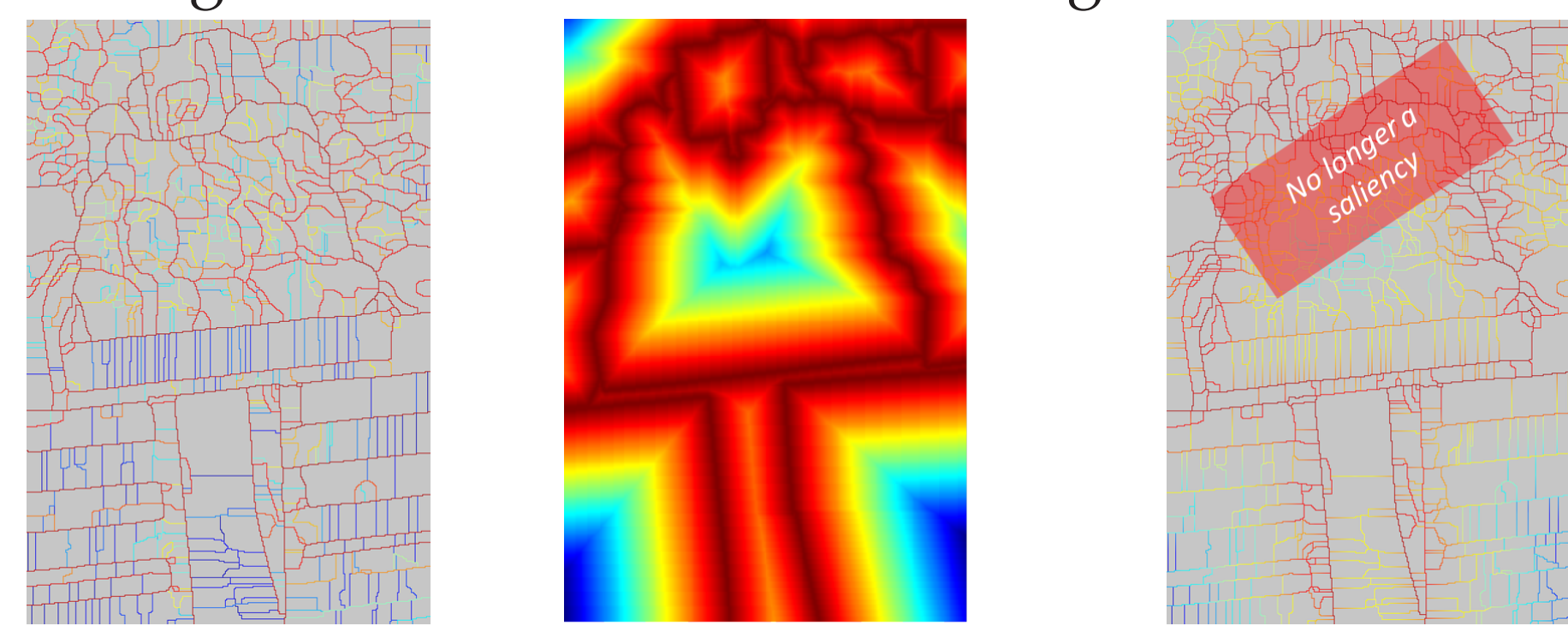
There many ways to represent the hierarchy, the most common being the dendrogram



The other less know but useful representation is the saliency function. Its function with a constant weight on each simple arc/edge and represents the persistency or frequency of occurrence of this arc in the sequence of partitions in  $H$ .



Saliency function for partitions shown on the right. The color corresponds to a heat map: blue lowest, red highest and will be used for images further on



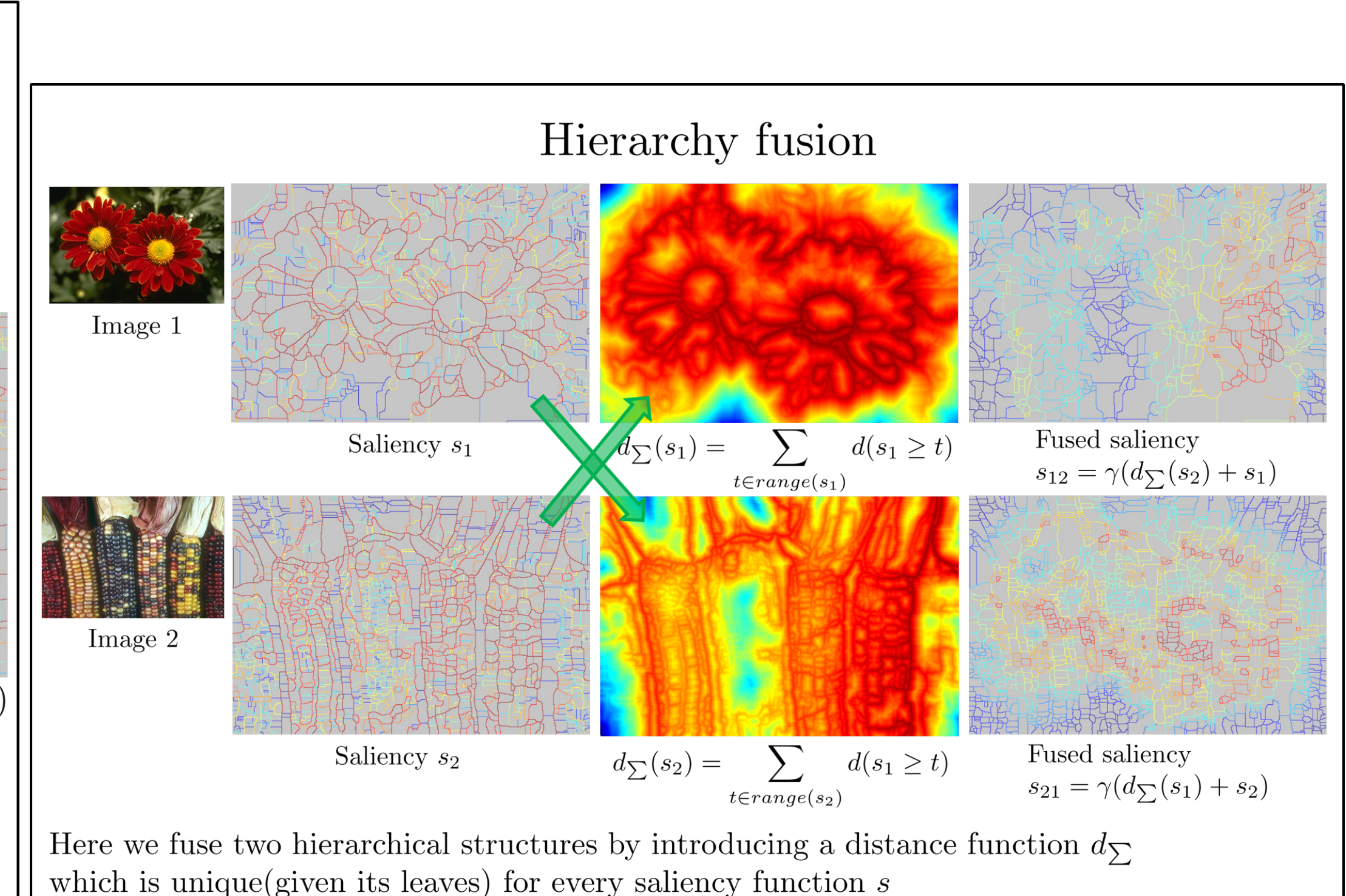
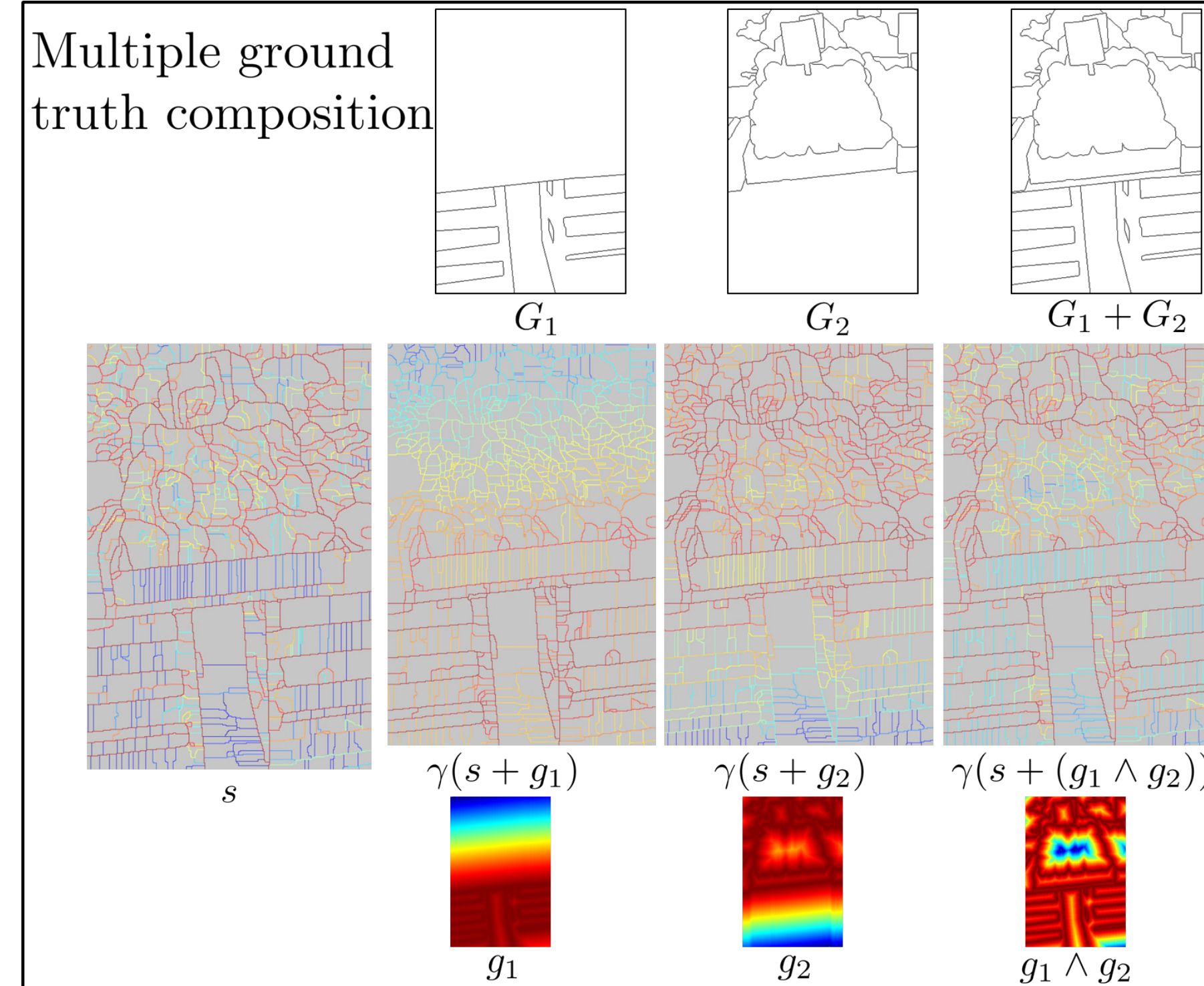
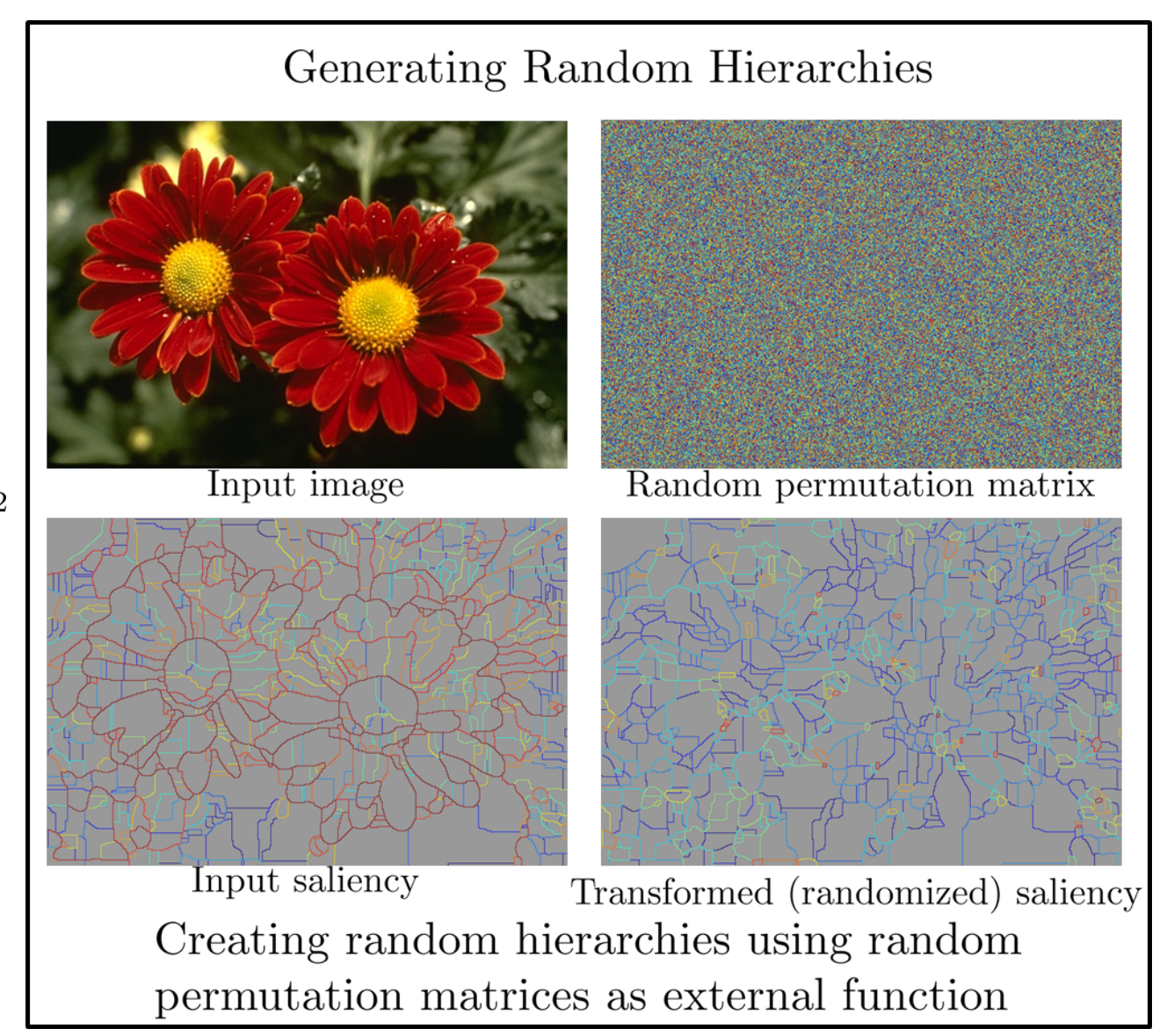
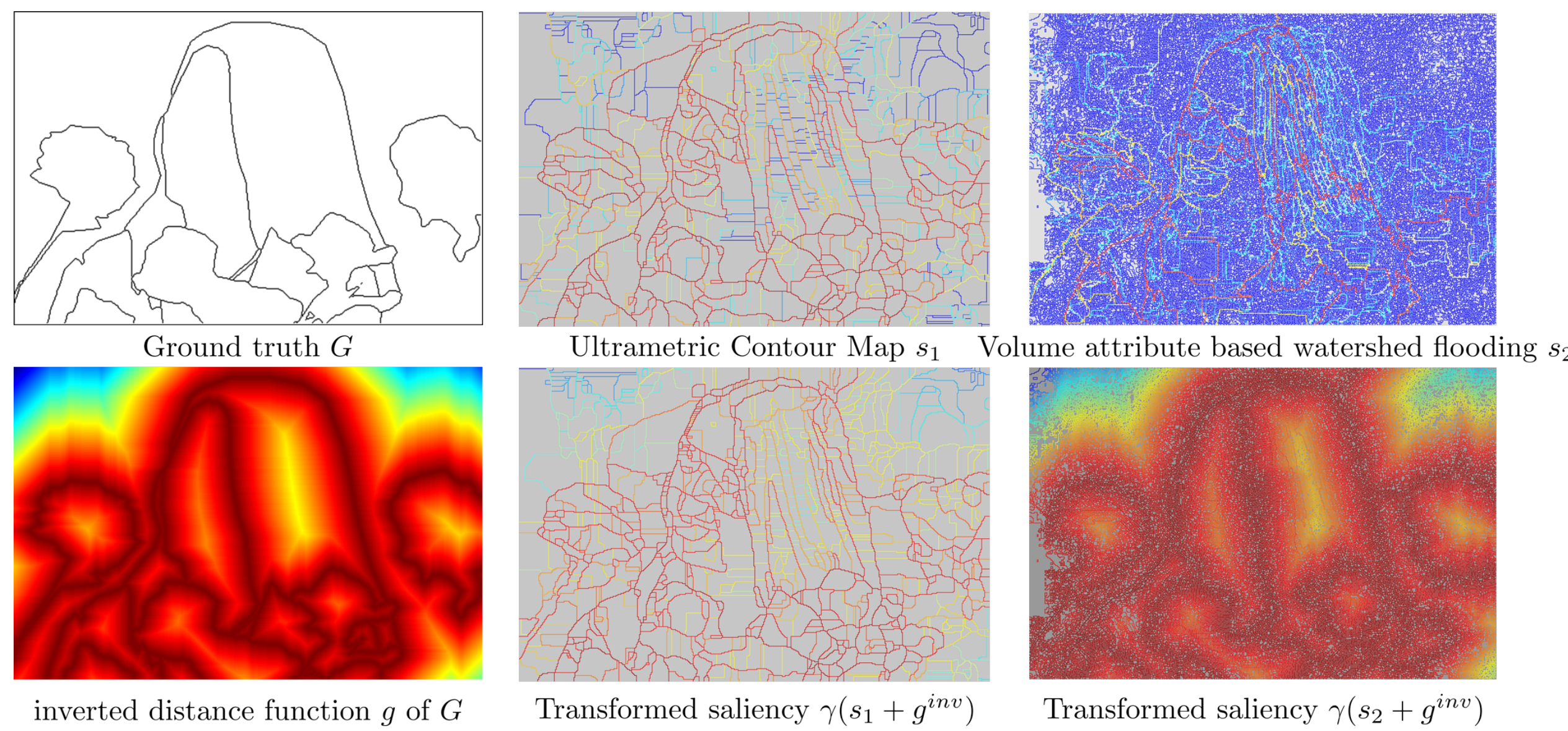
To transform the contours of the partitions in the hierarchy we use the ground truth inverse distance function. We introduce a similarity function which is defined as:

$$\varphi = s + g \quad (1)$$

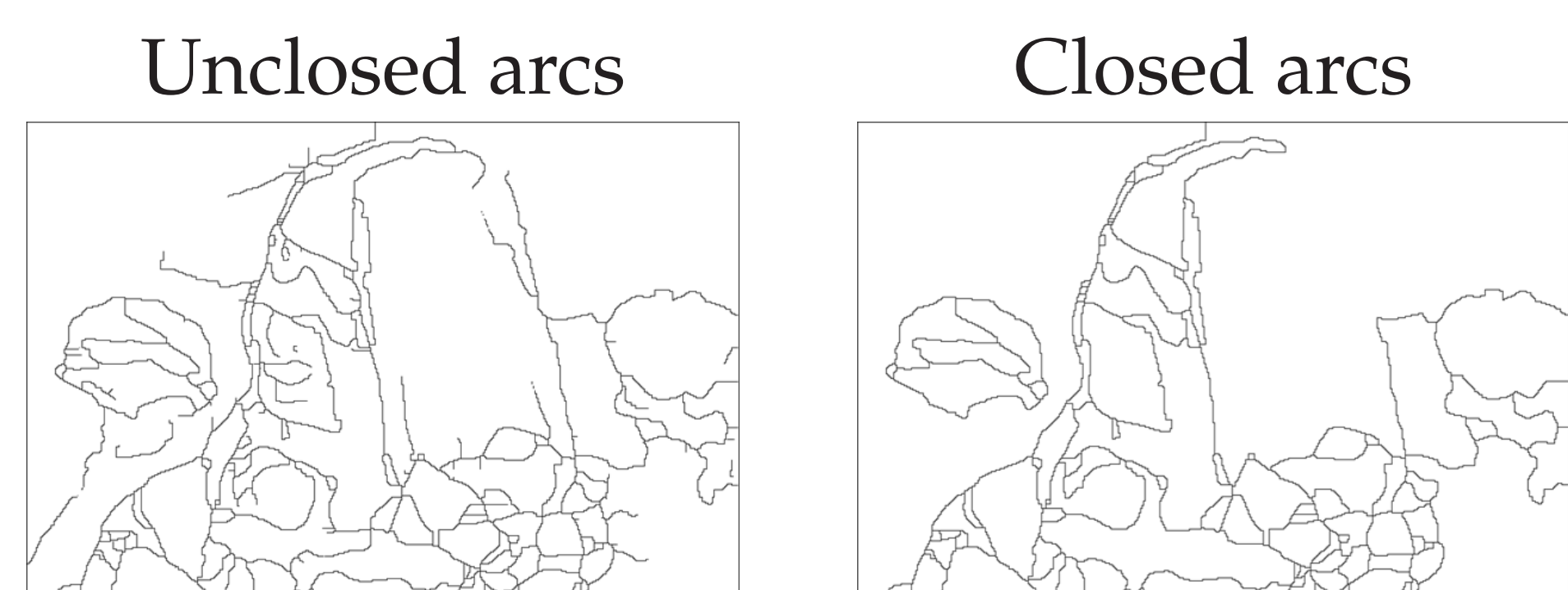
## REFERENCES

- [1] Arbelaez, P., Maire, M., Fowlkes, C., Malik, J.: Contour Detection and Hierarchical Image Segmentation. IEEE PAMI 33 (2011)
- [2] Jordi Pont-Tuset and Ferran Marqués, Supervised Assessment of Segmentation Hierarchies - In ECCV 2012 5-46

## RESULTS



## CLASS OPENING



Given a finite set of simple arcs  $\mathcal{P}(E_0)$  in 2D space  $E$ , we define

$$\gamma : \mathcal{P}(E_0) \rightarrow \mathcal{P}(E_0)$$

$\gamma(X)$  reduces each set of arcs  $X \in \mathcal{P}(E_0)$  to the closed contours it may produce.

**Theorem:** the operation  $\gamma : \mathcal{P}(E_0) \rightarrow \mathcal{P}(E_0)$  is an opening.

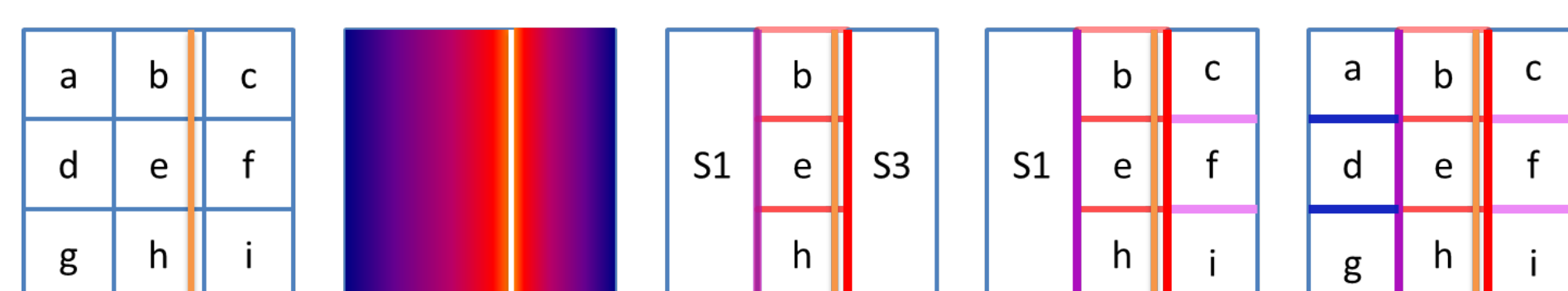
The numerical extension of  $\gamma$  the class opening, holds now on a numerical function  $\varphi$  on the edges of the leaves.

$$X_t(\varphi) = \varphi \geq t \quad (2)$$

the numerical opening  $\gamma(\varphi)$  by its level sets

$$X_t[\gamma(\varphi)] = \gamma[X_t(\varphi)], \quad t > 0 \quad (3)$$

When  $\varphi$  spans the class of all positive functions, then  $\gamma(\varphi)$  produces all possible saliencies.

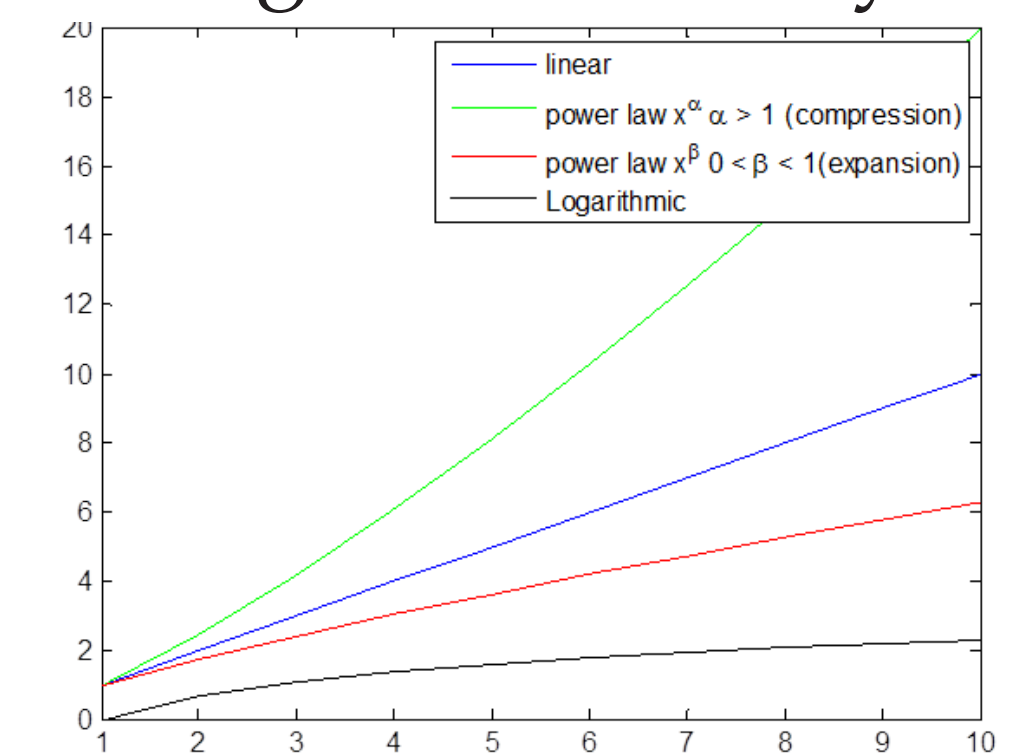


Toy Example: From left to right: Leaves partition (small letters) and ground truth partition contour in orange, Inverted normalized distance function  $(1-g)$  presented as heat map (red high value, pink intermediate value, blue low value), first partition with its contours weighted with distance function, second partition, final partition with all weights shown. The corresponding hierarchy is given by

$$\{\{a\}, \{d\}, \{g\}\} \subseteq S1, \{\{b\}, \{e\}, \{h\}\} \subseteq S2, \{\{c\}, \{f\}, \{i\}\} \subseteq S3, \{\{S1\}, \{S2\}, \{S3\}\} \subseteq E$$

## PROPERTIES

Class opening  $\gamma(\varphi)$  orders  $\varphi$  to obtain a saliency, which gives hierarchy  $H_\varphi$ .



**Degeneracy:** Any strictly increasing mapping of the grey levels  $\varphi' = \alpha(\varphi)$ , e.g. square root, log, etc., yields a  $\gamma(\varphi')$  that generates the same hierarchy  $H_{\varphi'}$  as  $\gamma(\varphi)$  does.

**Operations:** If  $g_1 \otimes g_2$  denotes an operation from  $\mathcal{G} \times \mathcal{G} \rightarrow \mathcal{G}$ , such as  $+$ ,  $-$ ,  $\times$ ,  $\div$ ,  $\vee$ , or  $\wedge$ , then  $\gamma(g_1 \otimes g_2)$  is the largest saliency under  $g_1 \otimes g_2$  and in particular,

## FUTURE DIRECTIONS

**Overview:**

1. Generation of family of saliencies using the Class opening  $\gamma(s)$  by composition with external function  $g$ . Results for ground truth distance function.
2. Composition of multiple external functions.
3. Fuse two or more hierarchies (saliencies).

**Future directions:**

1. Develop the converse approach where we interchange the roles of saliency and the ground truth.
2. Introduce conditional saliency transform based on attributes like volume, area, dynamic.
3. Use the approach for time varying hierarchies.

