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Abstract: Human movement modeling can be of great interest for the design of pattern recognition systems relying on the understanding of the fine motor control (such as on-line handwriting recognition or signature verification) as well as for the development of intelligent systems involving in a way or another the processing of human movements. In this paper, we briefly list the different models that have been proposed in order to characterize the handwriting process and focus on a representation involving a vectorial summation of lognormal functions: the Sigma-Lognormal model. Then, from a practical perspective, we describe a new stroke extraction algorithm suitable for the reverse engineering of handwriting signals. In the following section it is shown how the resulting representation can be used to study the writer and signer variability. We then report on two joint projects dealing with the automatic generation of synthetic specimens for the creation of large databases. The first application concerns the automatic generation of totally synthetic signature specimens for the training and evaluation of verification performances of automatic signature recognition systems. The second application deals

with the synthesis of handwritten gestures for speeding up the learning process in customizable on-line recognition systems to be integrated in electronic pen pads.

1. Introduction

Human movement modeling can be of great interest for the design of pattern recognition systems relying on the understanding of the fine motor control, like on-line handwriting recognition and signature verification, as well as in the development of intelligent systems involving in some way the analysis of human movements. Among other things, this general approach aims at elaborating a theoretical background for any handwriting processing application as well as providing some basic knowledge that can be integrated in the development of automatic systems.

So far, many models have been proposed to study human movement production in general and handwriting in particular : models relying on neural networks (Bullock and Grossberg, 1988; Schomaker, 1991; Gangadhar et al., 2007; Kalveram, 1998), equilibrium point models (Feldman, 1966; Feldman and Latash, 2005; Bizzi et al., 1978; 1992), behavioral models (Schmidt, 1999; Thomassen et al., 1983; van Galen and Teulings, 1983), coupled oscillator models (Hollerbach, 1981; Kelso, 1995; Zazone et al., 2005), kinematic models (Plamondon, 1995; Plamondon and Djioua, 2006), and models exploiting minimization principles (Wada and Kawato, 1995; Engelbrecht, 2001) : minimization of the acceleration (Neilson, 1993; Neilson and Neilson, 2005), of the energy (Nelson, 1983), of the time (Tanaka et al., 2006; Enderle and Wolfe, 1987; Hermes and LaSalle, 1969), of the jerk (Hogan, 1984; Flash and Hogan, 1985), of the snap (Edelman and Flash, 1987), of the torque changes (Uno et al., 1989) and of the sensory-motor noise (Harris and Wolpert, 1998). Finally, many models exploit the properties of various functions to reproduce human movements: exponentials (Plamondon and Lamarche, 1986), second order systems (Denier van der Gon and Thuring, 1965; Dooijes, 1983), gaussians (Leclerc et al., 1992), beta functions (Alimi, 2003), splines (Morasso et al., 1983) and trigonometrical functions (Maarse, 1987).

Among the models which provide analytical representations, the Kinematic Theory of rapid human movements (Plamondon, 1995a, 1995b; Plamondon and Djioua, 2006) and its Delta- and Sigma-lognormal models have been used to explain most of the basic phenomena reported in classical studies on human motor control (Plamondon and Alimi, 1997) and to study several factors involved in the fine motricity (Djioua and Plamondon, 2008; O'Reilly and Plamondon, 2010; Woch et al., 2010). Apart from these fundamental studies, the theory has been used, directly or indirectly, in many practical applications like the design of a signature verification system (Plamondon, 1994), the development of tools to help children learning handwriting (Djeziri, Guerfali, Plamondon, and Robert, 2002), as well as of biomedical set ups to detect fine motor control problems associated with brain strokes (O'Reilly and Plamondon, 2011, 2012).

In this paper, we report on two new and original case studies dealing with the automatic generation of synthetic handwritten specimens for the creation of large databases. The first application addresses the automatic generation of totally synthetic signature specimens which may be used for the training and evaluation of the verification performances of automatic recognition systems as well as for the quality assessment of specimens. The second application regards the synthesis of handwritten gesture for speeding up the learning process in customizable on-line recognition systems to be integrated in electronic pen pads. Sections 5 and 6 reports detailed results about these two genuine applications, which at the time of the ICFHR 2010 keynote address presented by the first author, were the first trial of using the Kinematic Theory for the generation of synthetic trajectories to be used in signature verification and gesture recognition experiments.

To better understand these applications and estimate their potential interest, as well as making the present paper self-consistent a brief survey of the Kinematic Theory is presented in section 2, two algorithms used for sigma-lognormal parameter extraction are outlined described in section 3 and the main results on previous studies of handwriting variability are summarized in section 4. These sections present in a condensed and goal oriented way, the main concepts and strategies that have been explored over the years and that are necessary to understand the present applications, without coming back to these complete and often more exhaustive studies.

2. The Kinematic Theory of Rapid Human Movement and its Sigma-Lognormal Model

One key feature of the Kinematic Theory is that it relies on strong and robust mathematical grounds. All the models that are used under this paradigm are based on the lognormal function which has been proved to be the ideal curve for describing asymptotically the impulse response of a neuromuscular network made up of a large number of coupled subsystems controlling the velocity of a movement (Plamondon et al., 2003). For simple reaching or pointing gestures, a target is specified and two of these networks are needed to control a trajectory, an agonist network which is acting in the target direction and one antagonist, acting in the opposite direction. Overall, the speed profile is then described by a Delta-lognormal equation, a weighted difference of two lognormals (Plamondon, 1995a, 1995b). When more complex trajectories have to be generated, like in handwriting or in signing, a sequence of targets has to be reached and, globally, the trajectory of the pen tip can then be described by a vectorial summation of lognormals, hereinafter called sigma-lognormal equations, which takes into account the various changes of direction.

In this vectorial summation context, the production of a word or of a signature requires the definition beforehand of an action plan that is made up of virtual targets, which are linked in pairs with an arc of circle. This map of paired target points represents a sequence of discontinuous strokes. This plan triggers a motor command generator that produces a series of impulses activating the neuromuscular systems characterized by their lognormal impulse response (Plamondon and Privitera, 1995). For each impulse, a lognormal velocity profile is generated at the pen tip and the time superimposition of these strokes results in a smooth and well controlled trajectory. According to this representation, the original strokes are thus hidden in the signal.

Mathematically, the Sigma-Lognormal model considers the velocity of the pen tip, $\vec{v}(t)$, as described by a vectorial summation of N lognormal primitives:

$$\vec{v}(t) = \sum_{i=1}^{N} \vec{v}_i(t) = \sum_{i=1}^{N} \vec{D}_i(t)\Lambda_i(t, t_{0i}, \mu_i, \sigma_i^2); N \geq 2 \quad (1)$$

Each lognormal in this summation defines a stroke scaled in amplitude by a command parameter (D) and time-shifted by the time occurrence of this command (t_0), any individual stroke pattern being described by a lognormal time function:

$$\Lambda(t;t_{0i},\mu_i,\sigma_i^2)=\frac{1}{\sigma_i\sqrt{2\pi}(t-t_{0i})}\exp\left(\frac{-\left[\ln(t-t_{0i})-\mu_i\right]^2}{2\sigma_i^2}\right) \quad (2)$$

Each of these primitives is also assumed to occur around a pivot, and the evolution of the angular position of the trajectory can be calculated using an error function (*erf*):

$$\theta_i(t)=\theta_{si}+\frac{(\theta_{ei}-\theta_{si})}{2}\left[1+erf\left(\frac{\ln(t-t_{0i})-\mu_i}{\sigma_i\sqrt{2}}\right)\right] \quad (3)$$

where θ_{si} and θ_{ei} refer, respectively, to the starting and ending angular direction of each stroke. In equations (2) and (3), μ_i and σ_i represent correspondingly the logtime delay and the logresponse time of the neuromuscular system as it reacts to the i^{th} command (Plamondon and Djioua, 2006).

Under these conditions, the synergy produced by the interaction and coupling of many of these neuromuscular systems results in the sequential generation of a complex handwriting sample or a signature pattern.

3. Sigma-lognormal Parameter Extraction

To use the Sigma-Lognormal model for analyzing human movements, it is necessary to have an algorithm to solve the inverse problem in a fully automatic fashion, that is, to extract the lognormal parameters that most adequately fit the experimental data. The Sigma-Lognormal parameters are considered to be well estimated and fitted for statistical analysis if the SNR, defined in (4), is over 20dB.

$$SNR = 10log\left(\frac{\int v_{x\_n}^2 + v_{y\_n}^2 dt}{\int (v_{x\_n} - v_{x\_\Sigma})^2 + (v_{y\_n} - v_{y\_\Sigma})^2 dt}\right) \tag{4}$$

In this equation, (v_{x_n}, v_{y_n}) are the experimental (numerical) velocity signals and $(v_{x\_\Sigma}, v_{y\_\Sigma})$ are the velocity signals of the sigma-lognormal reconstruction.

In the last years, two complementary algorithms have been proposed to solve this nonlinear regression problem, the Robust Xzero based algorithm and the prototype based algorithm. The next subsections briefly overview the state-of-the-art regarding these parameter extractors.

3.1. The Robust Xzero based extractor

The Robust Xzero (RX0) based extractor is a powerful algorithm that provides an accurate set of sigma-lognormal parameters describing the end-effector trajectory (e.g. the pen tip trajectory in handwriting studies) of arbitrarily complex motions without any *a priori* knowledge regarding the nature of the movement. In the following text, an outline of the algorithm is presented. A more comprehensive description can be found in (O'Reilly and Plamondon, 2009; O'Reilly, 2012).

To implement this algorithm, sequences of five characteristic points (t_{i_n}, v_{ti_n}) (*i=1,2,..,5*) must be located in the original speed signal v_t. Following a time occurrence order, these points are: a local minimum, an inflexion point, a local maximum, an inflexion point and a local minimum. The sigma-lognormal representation of these points can be written as in equations (5-6) with the parameters a_i defined in (7).

$$t_{i\_\Sigma} = t_0 + e^{\mu} e^{-a_i} \qquad i \in \{1,2,\ldots,5\} \tag{5}$$

$$v_{ti\_\Sigma} = \frac{D}{\sqrt{2\pi}} e^{-\mu} \sigma^{-1} e^{\left(a_i - \frac{a_i^2}{2\sigma^2}\right)} \qquad i \in \{1,2,\ldots,5\} \tag{6}$$

$$\begin{cases} a_1 = 3\sigma \\ a_2 = 1.5\sigma^2 + \sigma\sqrt{0.25\sigma^2 + 1} \\ a_3 = \sigma^2 \\ a_4 = 1.5\sigma^2 - \sigma\sqrt{0.25\sigma^2 + 1} \\ a_5 = -3\sigma \end{cases} \tag{7}$$

Nine estimators can be obtained for the values of the kinematic parameters $(t_{0i}, D_i, \mu_i, \sigma_i)$ corresponding to the nine different combinations $(t_{j\_n}, t_{k\_n}, v_{tl\_n}, v_{tm\_n})$ (with $j, k, l, m \in \{2,3,4\}$, $l \neq m$, $j \neq k$) in the equations (8)-(11).

$$t_0 = t_{j\_n} - e^{\mu} e^{-a_j} \tag{8}$$

$$D = \sqrt{2\pi} v_{tl\_n} e^{\mu} \sigma e^{\left(\frac{a_l^2}{2\sigma^2} - a_l\right)} \tag{9}$$

$$\mu = ln\left\{\frac{t_{j\_n} - t_{k\_n}}{e^{-a_j} - e^{-a_k}}\right\} \tag{10}$$

$$\sigma = \begin{cases} \sqrt{-2 - 2ln\left(\frac{v_{tl\_n}}{v_{tm\_n}}\right) - \frac{1}{2ln\left(\frac{v_{tl\_n}}{v_{tm\_n}}\right)}} & l \in \{2,4\}, m = 3 \\ \sqrt{2\sqrt{1 + ln^2\left(\left(\frac{v_{tl\_n}}{v_{tm\_n}}\right)\right)} - 2} & l = 4, m = 2 \end{cases} \tag{11}$$

The angular parameters (θ_s, θ_e) associated with each estimation of the kinematic parameter set $(t_{0i}, D_i, \mu_i, \sigma_i)$ is obtained using (12)-(13), where $l_3 = \int_{t_0}^{t_3} D\Lambda(t - t_0; \mu, \sigma)dt = \frac{D}{2} erfc\left(\frac{\sigma}{\sqrt{2}}\right)$ (with erfc being the complementary error function defined as $erfc(x) = 1 - erf(x)$) and $\phi(t)$ is direction angle of the trajectory with respect to time.

$$\theta_s = \phi\left(t_{3\_n}\right) - \frac{d\phi\left(t_{3\_n}\right)}{dt}\, l_3 \tag{12}$$

$$\theta_e = \phi\left(t_{3\_n}\right) + \frac{d\phi\left(t_{3\_n}\right)}{dt}\left(\mathrm{D} - l_3\right) \tag{13}$$

A choice can be made between the nine estimations of the six sigma-lognormal parameters by keeping the solution minimizing the error function (14).

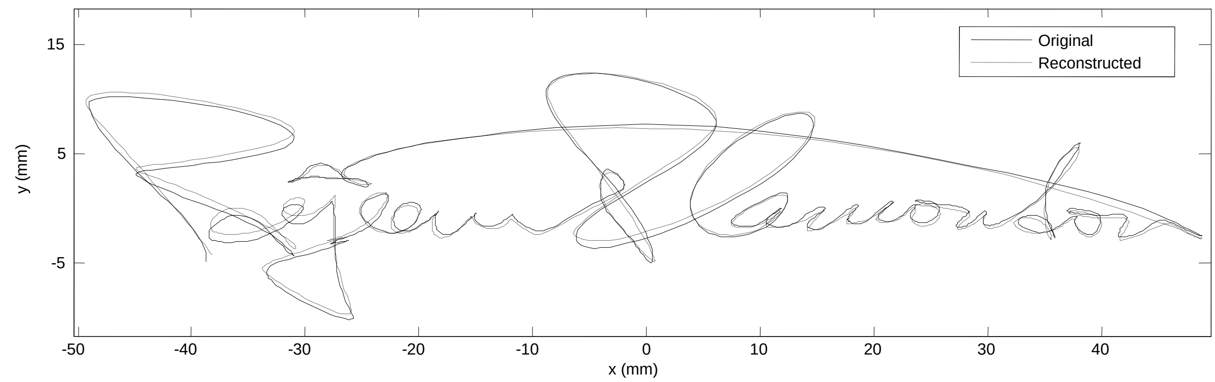
$$\int_{t_{1\_n}}^{t_{5\_n}} \left(\left(v_t(t) - \hat{D}\Lambda(t - \hat{t_0}; \hat{\mu}, \hat{\sigma})\right)\Lambda(t - \hat{t_0}; \hat{\mu}, \hat{\sigma})\right)^2 dt \tag{14}$$

It should be noticed that, before using the values $\left(t_{i\_n},\ v_{ti\_n}\right)$ in the previous expressions, some preprocessing should be applied to get proper estimations[1]. To extract the parameters of a whole velocity signal, good results have been obtained by, first, extracting sequentially (i.e. in increasing order of their time occurrence) the lognormal components. For that matter, each lognormal is extracted and subtracted from the original signal before proceeding to the next component. Then, a global non-linear optimization process can be applied to improve the estimated values. If this approach results in an unsatisfactory reconstruction SNR, more lognormal components can be extracted by processing them by decreasing importance of their impact (assessed here by their relative size) on the signal.

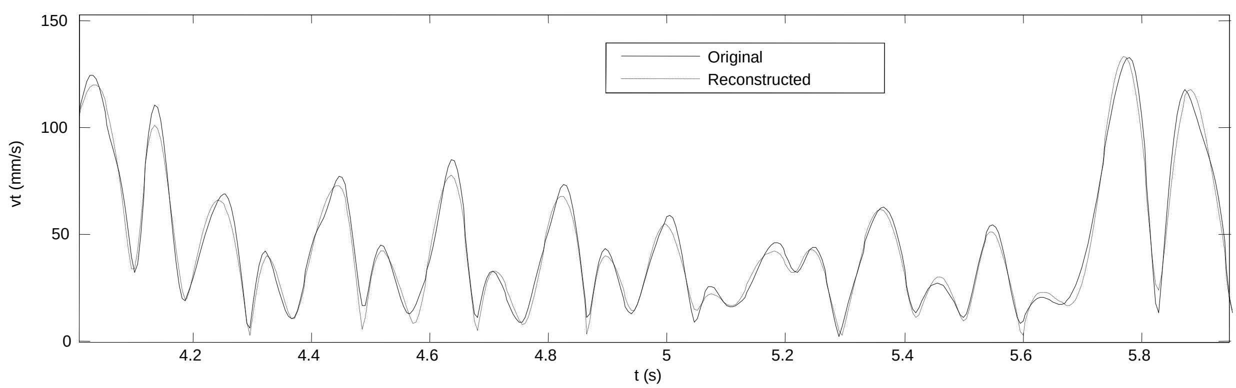
The latest improvements included in this extraction system have resulted in a significant increase of the SNR fitting accuracy. For example, on a 683 signatures database

[1] The details of these preliminary computations are presented in (O’Reilly & Plamondon, 2009; O’Reilly, 2012).

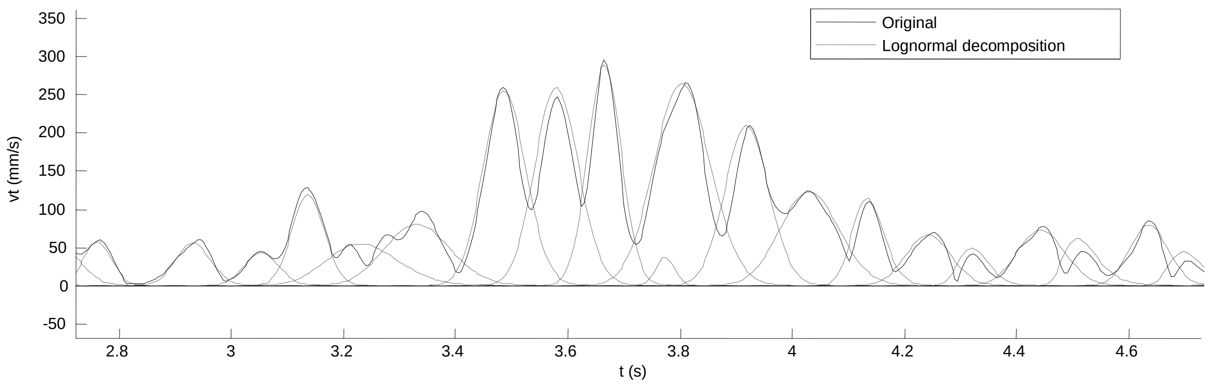
comprising 124 subjects, an average increase of 7.9 dB has been obtained, passing from 17.4 dB in (O'Reilly & Plamondon, 2009) to 25.3 dB. Fig. 1 gives an example of a complex movement, in this case a signature, fitted using the proposed RX0 approach (SNR=22.2dB).



(a)



(b)



(c)

Fig. 1. Example of a signature reconstruction (SNR=22.2dB) following the proposed RX0 approach: (a) the trajectory, (b) the speed profile and (c) the lognormal decomposition of the speed profile. In (b) and (c), only a small part of the actual signals are shown to better allow the reader to appreciate the curve fitting and its lognormal decomposition.

3.2. The prototype based extractor

Although the system based on the Robust Xzero estimator gives very satisfying extraction results on complex movements, an alternative extraction strategy may be of great use under certain experimental scenarios. For example, this is the case of the analysis of stereotypical movements such as those often involved in psychophysical experiments. This type of experiments present some specific characteristics that can make an alternative extraction method better fitted for the task than RX0. First, a lot of *a priori* information on the nature of the movement is available which may be very helpful during the extraction process (the RX0 algorithm is not designed to take advantage of this knowledge). Second, in this kind of experimental framework, researchers may want to perform statistical testing of hypotheses on the value of local parameters. This may be difficult with the solution obtained by an extractor such as the one based on RX0 because, in this case, there is no clear correspondence between the parameters extracted from various movements.

These reasons supported the development of the prototype based extractor presented in (O'Reilly and Plamondon, 2010). The advantages of such an extractor can be seen, for example, in (O'Reilly and Plamondon, 2011) where it has been used to assess the neuromuscular health of subjects on the basis of the sigma-lognormal parameters of their movements.

This extractor applies a three step methodology: 1) synthesis of a sigma-lognormal prototype of the stereotypical movements, 2) time scaling and offsetting of the prototype to make it more closely correspond to the experimental movement, and 3) global nonlinear optimization of the scaled and offset prototype to improve the fitting. These three steps can be briefly described as follow:

- **Step 1: Synthesis of the prototype.** The initial prototype can be built from i) the results of the RX0 extractor in order to find what is the expected sigma-lognormal decomposition of the stereotypical movement or ii) from a sigma-lognormal synthesizer such as SimScript (O'Reilly and Plamondon, 2007).
- **Step 2: Movement scaling and offsetting.** It is performed by finding the value of the scaling (C_s) and the offsetting (t_s) factors that maximize the reconstruction SNR between the experimental data and the prototype signals modified in such a way that the original μ_i and t_{0i} parameters are changed for the scaled and shifted parameters μ_{ia} and t_{0ia} according to (15)-(16).

$$t_{0ia} = C_s t_{0i} + t_s \tag{15}$$

$$\mu_{ia} = \mu_i + \ln(C_s) \tag{16}$$

- **Step 3: Non-linear optimization.** It can be performed according to any suitable optimization algorithm. For our work, we used a custom implementation of a direct search optimization (O'Reilly, 2012) which monotonically increases the SNR without risk of divergence or of finding solutions that are too far away from the original prototypes. This preserves the psychophysical correspondence of the lognormal components among movements.

Using this algorithm on a database of 1440 triangular movements performed by 120 subjects (see (O'Reilly & Plamondon, 2011) for a more complete description of this dataset), we obtained a mean SNR of 20.8 dB for the prototype based extractor compared to a 22.1 dB for the RX0 based system. Although its SNR is slightly lower, the prototype based extractor has the clear advantage of producing solutions with fixed number of lognormals which enables the comparison among movements with any standard tool of statistical analysis of variance.

4. Automatic Generation of Trajectories: Variability Issues

As we have seen, according to the Kinematic Theory, a complex movement results from the superposition of a set of elementary movements (corresponding to single curved strokes), localized both in time and space. So, the large variability observed in handwriting patterns can be interpreted as caused both by the intrinsic variability of the individual strokes and by fluctuations occurring in the time plan of the superimposition process as controlled by the central nervous system.

The local variability observed in handwriting and signing can thus come from various sources. At the central level, a movement is represented by an action plan, a sequence of virtual targets describing a piece-wise discontinuous trajectory. In parameter terms, this plan is a timed sequence of arcs described by their length, directions and time of activation. This discontinuous pattern, once instantiated, stimulates a set of neuromuscular networks that react to each of these fundamental primitives with specific time delays and response times.

In this context, at least three basic sources of variability can be identified:

1- a time variability associated with the temporal information contained in the activation sequence of the different commands,

2- a spatial variability associated with the geometrical information contained in commands themselves (the magnitude D_i and direction (θ_{bi} and θ_{ei}) of each stroke), and

3- a neuromuscular variability reflected in the timing properties μ_i, σ_i of the neuromuscular networks reacting to these commands.

Those sources of variability have been investigated using a semi-automatic sigma-lognormal parameter extraction methodology somewhat similar to the one described for the prototype based extraction. Especially, we have shown that the Sigma-Lognormal model, can explain the great variability of individual stroke trajectories and their corresponding velocity profiles (Plamondon and Djioua 2005, 2006). We have also applied this paradigm to study the possible sources of handwriting deformations caused both by the disruptions in the motor control and the neuromuscular systems (Djioua and Plamondon 2007, 2009). Particularly, we have shown that, without altering the rest of the factors involved in handwriting, the distortion of the shapes of a handwritten word is very sensitive to slight changes of the time plan, represented by the sequence of command time occurrences $\{t_{0i}\}$. This stresses out the fact that, to write a readable word or to generate a consistent signature, the production of strokes composing that word or signature must be planned in advance in order to keep almost constant the timing of the learned original plan. In contrast, the command parameters, that affect the direction (θ_{bi} and θ_{ei}) and the amplitude D_i of the strokes, seem to be less critical. And, finally, the changes induced by the neuromuscular timing parameters μ_i, σ_i seem to have an even smaller influence on the final variability of the trajectory.

Overall, these studies have shown the existence of a direct relationship between the fluctuations of the sigma-lognormal parameters and the space and time warping of a pen tip trajectory, which suggests the feasibility of using this model as a new tool for the design of synthetic human like movements. This will be shown in the next two sections, where the relative importance of the three sources of variability will be critical for the successful design of large synthetic databases.

5. Application 1: Automatic On-line Signature Database Generation

One of the main obstacles that the biometric technology has found, and still finds, to become one of the leading solutions in the security market, is the lack of large real biometric databases which may serve as common benchmarks for the development of this thriving technology. Two main reasons may explain such a scarcity of biometric data. On the one hand, biometric database collection is not at all an easy job, involving a lot of effort in terms of time and resources in order to reflect the variability present in biometric traits (both inter- and intra-class). This process includes a number of pre-acquisition and post-acquisition demanding tasks such as the recruitment of donors, the supervision of collected data, error correction or labeling (Flynn, 2008). On the other hand, biometric traits are classified as personal data, and as such they are subdued to the different personal data protection laws existing in each country, which makes the acquisition (donor's consent), and later storage and distribution (licensing) of these data very difficult.

Such a complex context has promoted over recent years the apparition of new algorithms for the generation of synthetic biometric databases (Cappelli, 2003; Zuo, 2007). These synthetically produced datasets are not affected by the acquisition and legal issues mentioned before: *i*) first, once the appropriate generation method has been developed, they are effortless to be produced, avoiding this way the arduous acquisition campaigns, and *ii*) second, the synthetic samples which conform these databases cannot be considered personal data as they have not been produced by a person, and so they may be freely distributed in order to be used as common evaluation benchmarks. These desirable characteristics make synthetic databases very powerful tools for the performance assessment of biometric recognition systems, and have already been used in international competitive evaluation campaigns (Cappelli, 2006).

In spite of presenting some very interesting features, the use of synthetic biometric databases is not yet generalized as the production of realistic synthetic samples still remains a challenging problem: modeling the information contained in a certain biometric trait as well as the inter-class and intra-class variation found in real databases (i.e., variation between samples of different subjects, and variation between samples of the same subject). Accepted solutions have been proposed for fingerprint (Cappelli, 2003) or iris (Zuo, 2007; Shah, 2006), but still no consistent method has been given for the generation of synthetic handwritten signature databases.

As presented in previous sections, the Kinematic Theory of rapid human movements provides a powerful theoretical framework which models in a precise and compact manner the kinematic information involved in most of human writing processes, including signature. Thus, the Kinematic Theory and its associated Sigma-Lognormal model constitute a very high potential instrument for many different applications and have been applied in the present work to the development of an algorithm for the generation of fully synthetic on-line signatures.

5.1. The generation method

Two main parameters are involved in the design of real biometric databases and, hence, should also be critical in the generation of synthetically produced datasets: *i*) number of users comprised in the database, and *ii*) number of samples per user to be acquired. As can be seen in Fig. 2, the generation method of synthetic signature databases proposed in the present work is constituted of two different algorithms in order to produce: *i*) the first sample of fully synthetic individuals (i.e., it allows to control the number of users in the database), and *ii*) different samples derived from that original master signature (i.e., permitting to fix the number of samples per user).

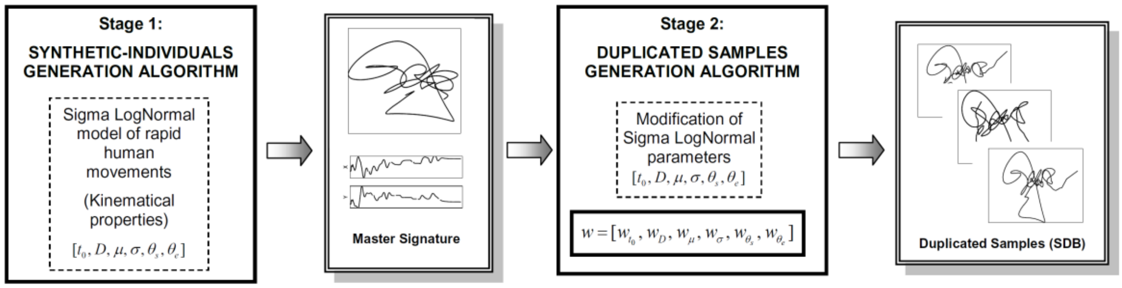


Fig. 2. General architecture of the algorithm proposed for the generation of synthetic signature databases based on the Kinematic Theory of rapid human movements.

5.1.1. Generating the master signature

Although other signals such as the azimuth and elevation angles of the input pen or the pressure applied during the signing process might be taken into account, in this work we consider that an online signature is defined by two time sequences $[x[n]\ y[n]]$ specifying respectively the x and y coordinates, at the time instants $n = 1,\ldots,N$.

The objective of this initial stage of the generation algorithm is to produce samples from different synthetic signers (i.e., this algorithm is responsible for controlling the number of users in the database and for the inter-class variability present in the dataset).

In a first approach, a signature-like graphic is generated following the spectral approach described in (Galbally, 2009). Although this first specimen has approximately the appearance of a genuine signature, it does not possess many of the humanly produced kinematic characteristics of real writing. Thus, in order to confer this preliminary master signature with the velocity and acceleration properties of human strokes, it is processed using the Sigma-Lognormal model in two consecutive stages, as shown in Fig. 3:

- **Stage 1:** Extraction of the sigma-lognormal parameters using the RX0 system. In this phase, the velocity function of the initial synthetic master signature (v_I) is decomposed in singular strokes and the sigma-lognormal parameters (t_0, D, μ, σ, θ_s, θ_e) which best fit each of the individual strokes.

- **Stage 2:** Reconstruction of the velocity function of the definitive synthetic master signature according to the previously computed parameters. The new coordinate signals are then obtained from the reconstructed velocity function (v_D) where we can observe that certain abnormal artifacts such as the very high velocity peaks at the starting and ending parts of the velocity profile have been corrected (see Fig. 3)

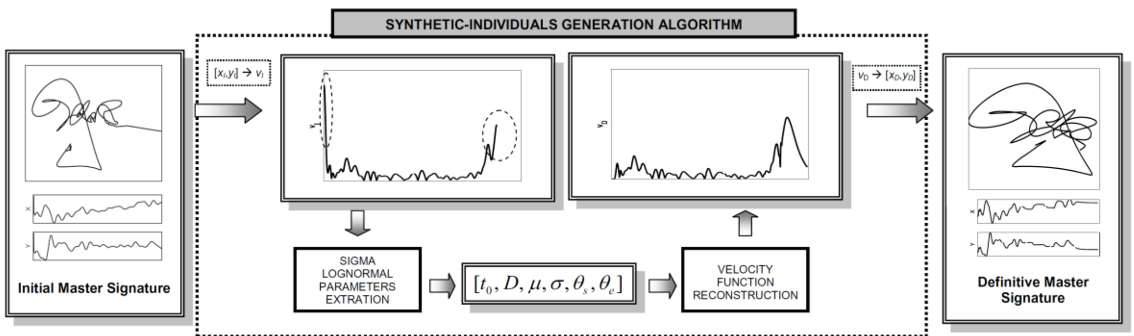


Fig. 3. General diagram of the generation process of a synthetic on-line master signature with the kinematic properties of a human-produced sample, based on the sigma-lognormal parameters.

5.1.2. Generating the duplicated samples

Once the time sequences [x[n] y[n]] defining the master signature of a synthetic user have been produced following the method described in Sect. 5.1.1, the next phase for the automatic generation of synthetic on-line signature databases is the creation of duplicated samples starting from that master sample (as is shown in Fig. 2).

Therefore, the objective of this part of the proposed method is to produce different samples of one same synthetic individual following the intra-variability found in real signatures (i.e., existing variability among signatures produced by the same user).

For this purpose, the velocity function v of the master signature is decomposed into single strokes following the Sigma-Lognormal model where each stroke is defined by the set of features $\boldsymbol{p}$=[t_0, D, μ, σ, θ_s, θ_e]. The duplicated samples are then generated adding to each of the single strokes a certain amount of noise which is modeled by a vector $\boldsymbol{w}$=[w_{t0}, w_D, w_μ, w_σ, $w_{\theta s}$, $w_{\theta e}$] where w_{t0} is extracted from a uniform distribution $\left[-w_{t_0}^{\max}\ w_{t_0}^{\max}\right]$ which is estimated according to the intra-user variability found in the development database BiosecurID (Fierrez, 2010) (analogously for the rest of distortion elements comprised in the vector $\boldsymbol{w}$). After the distortion stage, the new velocity function v_n is computed, and in a subsequent step the new coordinates x_n and y_n are recovered from that velocity information (see Fig. 4).

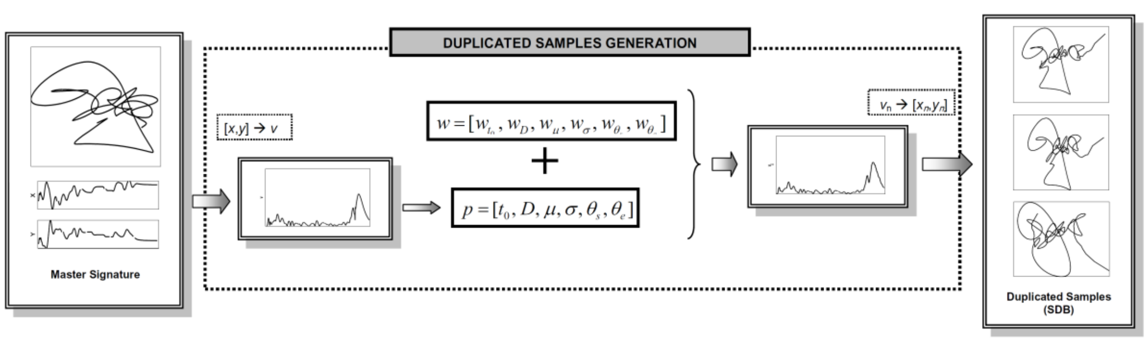
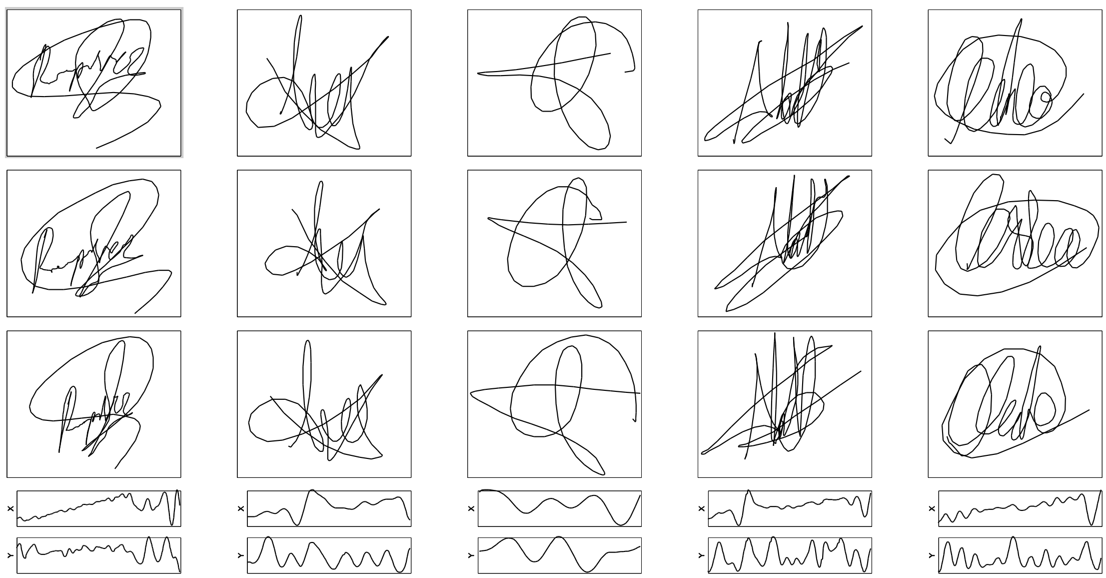


Fig. 4. General diagram of the generation process of duplicated samples starting from a fully synthetic specimen, based on the sigma-lognormal parameters.

In Fig. 5 some examples of synthetic signatures generated following the described approach are shown, together with real samples extracted from the BiosecurID database

(Fierrez, 2010) which was used as development set in order to compute the values of the different parameters involved in the generation method. As it can be observed from a general visual comparison, the synthetic signatures, and especially their time functions, present a very realistic appearance[2] in terms of: 1) smoothness of the strokes; 2) growing



a) Real signatures extracted from the BiosecurID database.

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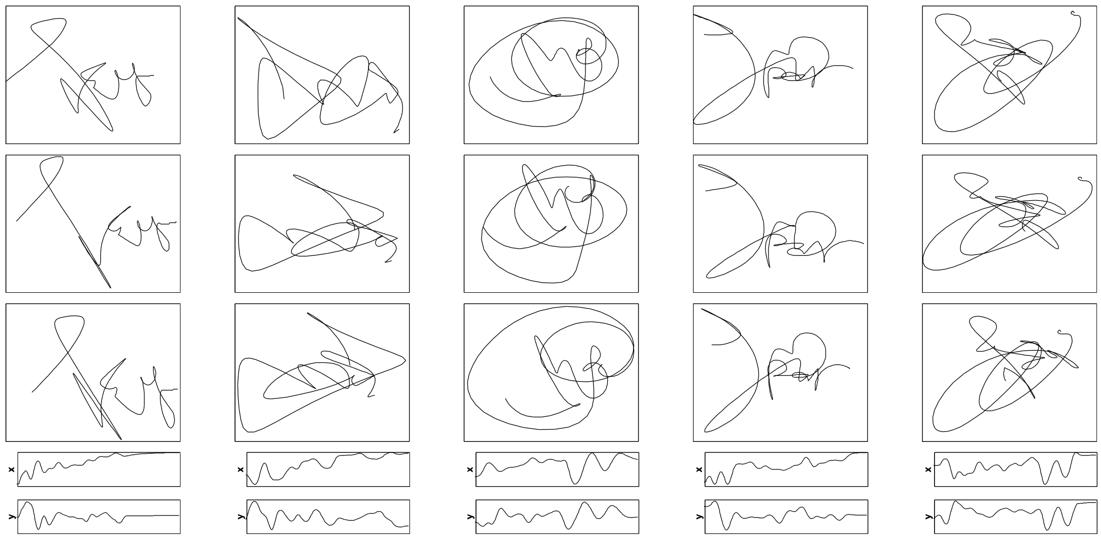
[2] In this context, one concern that might be raised is dealing with the potential use of this methodology to fabricate forged signatures. This might be an issue except that for making realistic forgeries, the forger would need to have access to the complete on-line information about the target signature and if he does, there would be no need to use a complex methodology like the one described in this paper. The addition of some noise might be sufficient. However, the complete on-line information is generally not available in real life systems since it is generally not stored in the reference database.



b) Synthetic signatures produced with the proposed generation algorithm

Fig. 5. Examples of real (a) and synthetic (b) signatures extracted from BiosecurID and SDB. Three samples of 5 different real and synthetic signers are shown together with their time sequences $x[n]$ and $y[n]$ corresponding to the first sample.

tendency of the function x (as it corresponds to left-to-right occidental signatures); 3) large fluctuation at the end of the x and y signals in some of the signatures (corresponding to some sort of round-like flourish); 4) degree of correlation between some of the most relevant maxima and minima points in the x and y directions. Furthermore, even though it is a model-based approach, some recognizable characters may be distinguished in the synthetic samples.

5.2. Validation experiments

The experimental validation of the proposed generation method is aimed at determining if the performance of signature verification systems is similar when it is evaluated on real and synthetic databases. If the error rates presented by signature-based recognition applications is comparable in both scenarios (i.e., performance evaluation with real and synthetic signatures), it would mean that, from a computer-based perspective, the synthetic signatures present a very similar behavior to that of real samples and that they can be used to obtain a fair estimation of a system performance, avoiding this way the different problems linked to real databases (i.e., high resource-consuming acquisition campaigns and legal issues regarding their acquisition and distribution).

In order to achieve this objective, the performance of two signature verification systems, using totally different feature sets and matchers, has been evaluated on the MCYT real database (Ortega, 2003) and on a Synthetic DB (SDB) produced using the proposed synthetic generation method.

The MCYT dataset has been selected as real test set since it has no overlap with the BiosecurID database used as development set for the estimation of the generation method parameters. This way we ensure to obtain totally unbiased results. The SDB has been created with the same number of users (300) and samples per user (25) as MCYT in order to permit the use of the same evaluation protocol for both scenarios.

The two on-line verification systems evaluated in the experiments are:

- **System A**: **function based + HMM** (Fierrez, 2007). This function-based verification system applies a regional approach using a statistical model built using Hidden Markov Models (HMMs) to a set of 10 time sequences selected applying the SFFS feature selection algorithm (Pudil, 1994) to the total set of 34 functions defined in (Martinez-Diaz, 2009a).
- **System B: function-based + DTW** (Martinez-Diaz, 2009b). In this function-based local approach, a subset of nine time functions (selected using the SFFS from the total

34 feature set as in the case of system B) are directly matched using the elastic technique Dynamic Time Warping (DTW) (Kholmatov, 2005).

The performance results (Detection Error Trade-off, DET, curves) obtained for both verification systems are shown in Fig. 6. We can observe that the curves of the two systems present a very high degree of resemblance, both from a quantitative (EERs) and qualitative (general behavior) point of view, for the case of real and synthetic signatures. The results derived from this validation experiment confirm the great potential of the Kinematic Theory of Rapid Human Movements applied to the generation of synthetic on-line signature databases, and the suitability of such datasets to obtain reliable estimations of the performance of signature verification systems.

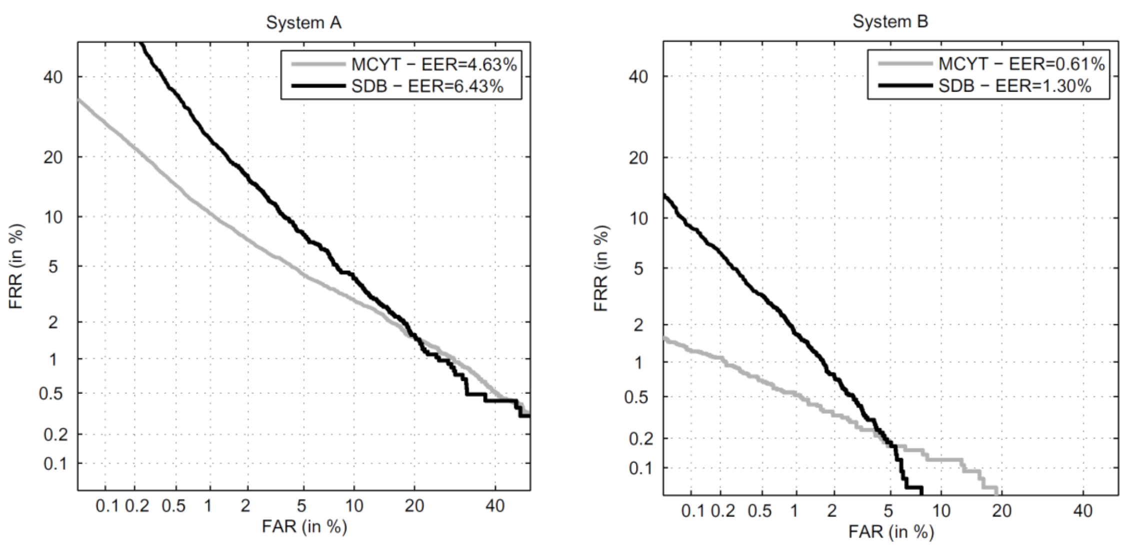


Fig. 6. Performance evaluation of systems A and B, on a real (MCYT, grey DET curve) and synthetic database (SDB, black DET curve). The EER is indicated in each plot.

6. Application 2: Synthetic Gesture Generation for Evolving Handwriting Classifiers

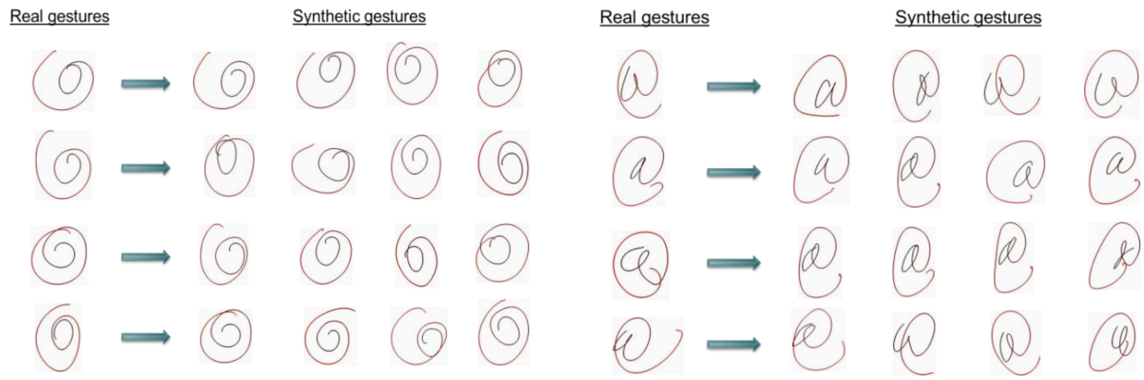
Motivated by the increasing spread of many types of devices equipped with pen-based interfaces, such as PDAs, e-book, Tablet PCs, Whiteboards, etc., more emphasis is placed on the development of efficient recognition systems that can correctly interpret the gestures sketched by the user and then translate them either into computerized text or into some specific commands. Nowadays, the recognition systems in use are always pre-trained on a fixed, predefined and a limited group of gestures, which usually contains the Latin letters and a few specific gestures. These systems do not allow users to add gestures in order to assign them to new commands or shortcuts, or to replace default gestures mapped to existing commands. In order to meet this important functionality, the static handwriting recognition systems that have been used so far must be replaced by novel dynamic ones where the knowledge base can constantly evolve during the use of the system. The evolving nature comes from the fact that the system must be able to integrate at any moment a new class (gesture in our context), and must also continue its adaptation to the existing classes using the new available data. Although the dynamic nature of evolving classifiers offers many important advantages, the operation of these systems suffers from the lack of learning data. The training process is done directly by the final user in an online and interactive manner, so that the quantity of teaching samples is limited because it is impractical to ask the user to enter a large number of samples in order to obtain a functional classifier. Therefore, the main challenge in the conception of incremental learning algorithms of evolving classification systems consists in reaching high recognition performance as fast as possible; i.e. with the minimum number of samples. Besides the beginning of the incremental learning from scratch, the problem of lack of data samples appears again during the adaptation process when new classes are added to the classifier. The evolving system is supposed to be able to learn these new classes without forgetting the old ones. However, it is difficult to completely avoid perturbations on the global performance of the classifier when adding new classes and the efforts must be focused on reducing as much as possible these perturbations.

In addition to the efforts of improving the classification systems and the training algorithms, the incremental learning process can be further accelerated and enhanced by generating artificial data based on some knowledge related to the application domain. For handwritten gesture recognition problems, this idea can be implemented by generating synthetic gestures from the available real ones after applying on them some deformations in a realistic and significant manner. Thus, when a new class of gestures is introduced to the system with few samples provided by the user, many artificial samples can be generated. Geometric distortions are usually applied on real handwritten symbols in order to generate synthetic ones (Mitoma, 2005; Wang, 2005; Lin, 2007; Mouchère, 2007). These deformations can be either based on class-dependent models of gesture variability and require a learning phase, or on class-independent general strategies without specific deformation models.

In this work, we incorporate a handwriting generation technique using class-independent lognormal-based deformations in the incremental learning of evolving handwritten gesture classifiers. Thanks to the RX0 based extractor, the $\Sigma\Lambda$ parameter extraction and the data generation is performed automatically as a part of the adaptation process. Motion pattern variability rooted in the motor representation space of the handwritten gestures is regarded to be more realistic than geometric distortions and thus more valuable in the training process. In addition to the great advantage of integrating the lognormal-based handwriting generation technique in our evolving handwriting classifier, an objective and numerical evaluation of the quality of generated data is provided for the first time, to the best of our knowledge. The generated handwritten samples are considered realistic as much as they help the classifier to predict future real samples from the same class of gestures. The capacity of prediction is translated by the improvement of recognition performance of the evolving classifier.

6.1. Acceleration of the learning process using synthetic data

As aforementioned, we believe that distortions obtained by applying some variations on lognormal parameters are more realistic that those obtained using direct geometrical distortions. The idea is to extract the sigma-lognormal profiles of a real handwritten gesture provided by the user. Then, we apply some variation on the extracted parameters within some specific ranges, and we regenerate artificial gestures using the modified profiles. The resemblance between the synthetic and the real gestures is controlled by the variation intervals. Thus, a suitable setting of these intervals is required in order to avoid over-deformed gestures. We show in Fig. 7 some examples of the artificial gestures that can be generated by applying modifications on the sigma-lognormal parameters. We can note that the real gestures can be almost predicted from the synthetic ones.



(a) (b)

Fig. 7. Some examples of generated gestures using sigma-lognormal based deformations.

In the context of incremental learning of evolving systems, one can overcome the problem of lack of samples at the beginning of the inclusion of a new class by generating, in an adequate manner, a number of synthetic samples. For an evolving handwriting classifier, the abovementioned sigma-lognormal based technique for synthetic gesture generation can be incorporated into the incremental training process. The handwriting generation is automatically performed transparently, with no user intervention. Fig. 8 shows the different steps of the generation method. Frist, the sigma-lognormal parameters of each incoming sample are first extracted. These parameters are then modified and a number of synthetic samples are generated. The original sample and the synthetic ones are then introduced in the incremental learning algorithm.

Using SimScript (O'Reilly and Plamondon, 2007), a visualization interface developed by the Scribens laboratory that allows an interactive modification of the sigma-lognormal parameters of a given handwritten gesture, we have experimentally studied the valid variation intervals of the six parameters within which the generated gesture is generally considered similar (from a human viewpoint) to the original one.

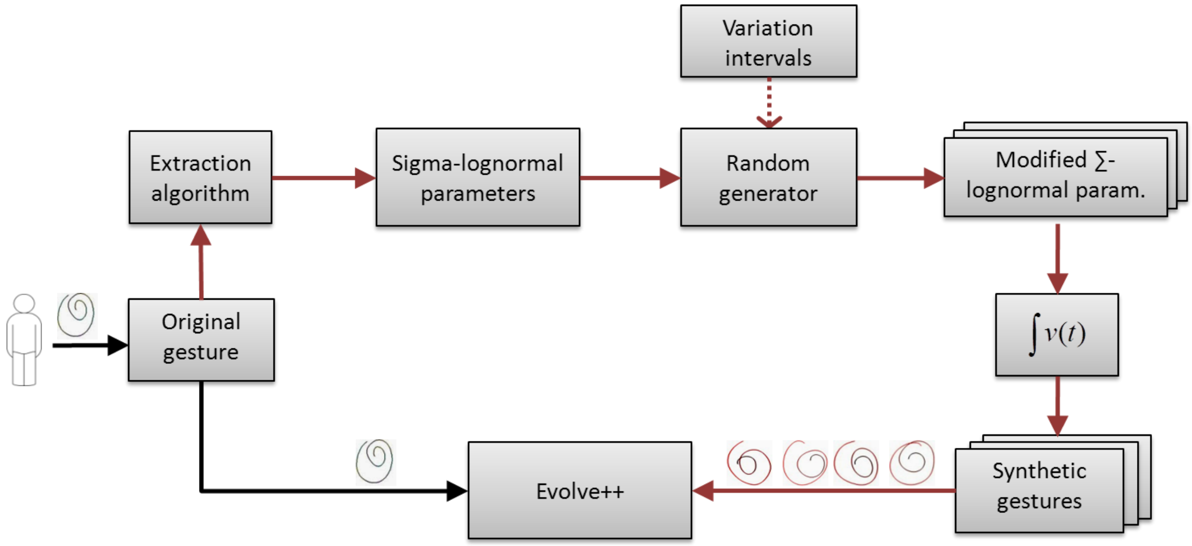


Fig. 8. Incorporating lognormal-based data generation into the learning stage of an evolving handwriting classifier.

6.2. Experimental results

Experiments have been performed on a dataset of on-line handwritten gestures. It was composed of 11 different gestures drawn by 7 writers on a Tablet PC. Each writer has drawn 100 samples of each gesture, i.e. 1,100 gestures in each writer specific dataset. Each gesture was described by a set of 10 features. The presented results are the average of 7 different tests for the 7 writers. In order to avoid the data order inducing a bias into the outcome, we repeated 40 times the experiment for each writer with different random data orders and only the averages are reported. We have used about 40% of the dataset for the incremental training and the rest is used to estimate the evolution of the performance during the learning process. We generated 10 synthetic samples (gestures) for each real sample. The evolving classifier used in these experiments is based on a first-order Takagi-Sugeno (TS) fuzzy inference system, and taught with our original incremental learning approach "Evolve++" (Almaksour, 2011).

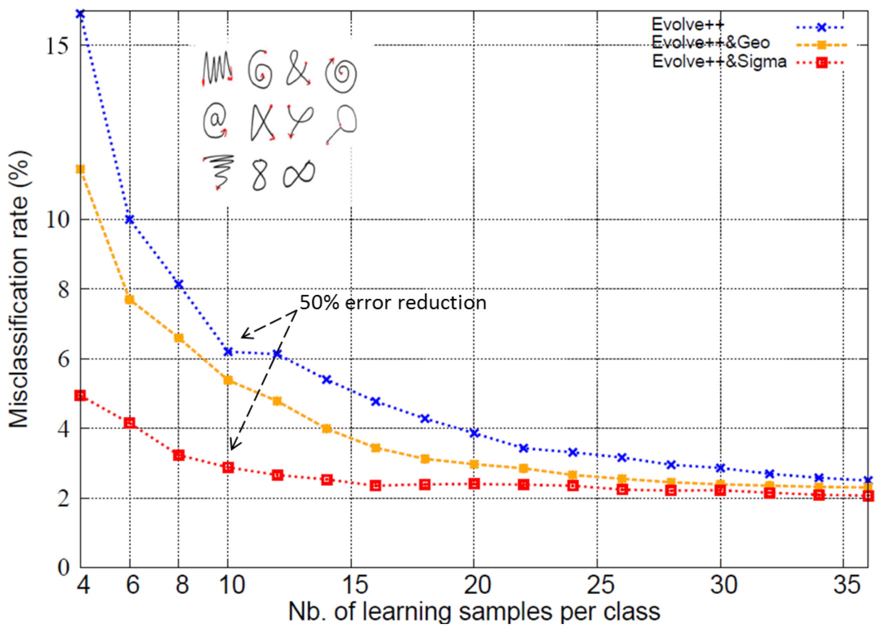
We compared the lognormal-based handwriting generation method to the geometric distortions explained in (Mouchère, 2007). Therefore, three performance curves are presented in the figures:

I. **Evolve++**: our evolving classification approach with Evolve++ algorithm presented in (Almaksour, 2011). Only real samples are considered (no synthetic data);

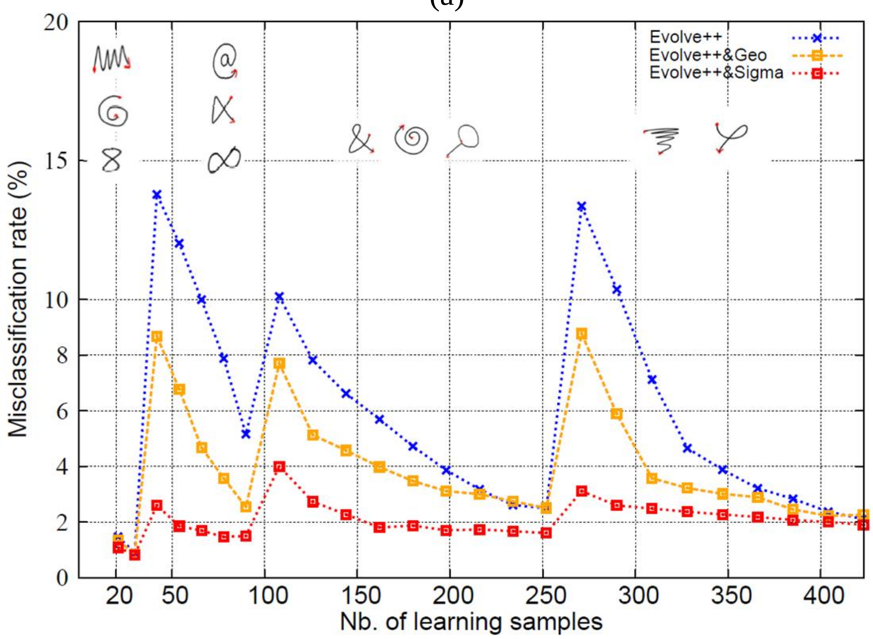
II. **Evolve++&Geo**: synthetic samples are generated by applying geometric distortions and used along with real ones to train Evolve++ system;

III. **Evolve++&Sigma**: synthetic samples are generated using the lognormal-based method.

The results are presented for two different experimental scenarios: the 11 gestures were introduced together in the first case, while the gestures were progressively introduced in the second one. We measure in the former the impact of the synthetic samples at the beginning of the learning process from scratch, while the latter scenario aims at showing the impact of these synthetic samples when introducing new gestures. The results of the first set-up are presented in Fig.9.a and those of the second in Fig.9.b.



(a)



(b)

Fig. 9 (a) Performance improvement using synthetic data generation.

(b) Impact of synthetic samples when adding new gestures.

As it can be seen in Fig. 9.a, there is an important impact of the synthetic gestures at the beginning of the incremental learning process. For example, the misclassification rate is reduced by 50% for 10 real samples per class when the training is enriched with synthetic samples. It must also be noted from the same figure that distortions applied on the sigma-lognormal parameters produces a more realistic variability in the synthetic gestures as compared to direct geometrical distortions. Thanks to the realistic human-like distortions, the synthetic samples present a significant ability to predict the appearance of future real samples, which significantly accelerates the adaptation process. Fig. 9.b shows that using synthetic samples, the classifier resists much better when introducing new classes. It is able to rapidly re-estimate all its parameters and to improve the recognition performance for the old and the new gestures. Again, the superiority of lognormal-based deformations over the traditional geometrical ones is quite apparent.

These experimental results show that sigma-lognormal based synthetic samples play an important role in improving the classification performance and accelerating the learning process both when it starts from scratch and also when new gestures are introduced. One interest of the present methodology is also that it does not depend on the way reference gestures are defined and collected. For example, gestures that are recorded on hand held mobile devices while the user is standing up, writing vertically, or while he is sitting in a moving vehicle could also be used to train the system without really affecting the results.

7. Conclusion

In the last few years, great advances have been done on the problem of parameter extraction of the Sigma-Lognormal model. New algorithms have been designed which now allow an automatic representation of complex human motions such as those involved in on-line signature and handwriting. The availability of these new systems and a better understanding of the variability of the sigma-lognormal parameters have paved the way for the use of this model in the context of automatic generation of synthetic databases of human movements. This paper has summarized the promising results of two different investigations on that topic.

Needless to say that further research is still needed on the various topic addressed in this paper, but as can already be seen, the use of the Sigma-Lognormal model for the generation of human like movements offers very interesting perspectives for the field of pattern recognition and the development of verification and recognition systems based on human movements.

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