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1 Recent Developments in the Study of Rapid Human Movements with the Kinematic Theory:
2 Applications to Handwriting and Signature Synthesis.

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13
14 **Abstract:** Human movement modeling can be of great interest for the design of pattern
15 recognition systems relying on the understanding of the fine motor control (such as on-
16 line handwriting recognition or signature verification) as well as for the development of
17 intelligent systems involving in a way or another the processing of human movements. In
18 this paper, we briefly list the different models that have been proposed in order to
19 characterize the handwriting process and focus on a representation involving a vectorial
20 summation of lognormal functions: the Sigma-Lognormal model. Then, from a practical
21 perspective, we describe a new stroke extraction algorithm suitable for the reverse
22 engineering of handwriting signals. In the following section it is shown how the resulting
23 representation can be used to study the writer and signer variability. We then report on
24 two joint projects dealing with the automatic generation of synthetic specimens for the
25 creation of large databases. The first application concerns the automatic generation of
26 totally synthetic signature specimens for the training and evaluation of verification
27 performances of automatic signature recognition systems. The second application deals

28 with the synthesis of handwritten gestures for speeding up the learning process in
29 customizable on-line recognition systems to be integrated in electronic pen pads.

30 **1. Introduction**

31 Human movement modeling can be of great interest for the design of pattern recognition
32 systems relying on the understanding of the fine motor control, like on-line handwriting
33 recognition and signature verification, as well as in the development of intelligent
34 systems involving in some way the analysis of human movements. Among other things,
35 this general approach aims at elaborating a theoretical background for any handwriting
36 processing application as well as providing some basic knowledge that can be integrated
37 in the development of automatic systems.

38 So far, many models have been proposed to study human movement production in
39 general and handwriting in particular : models relying on neural networks (Bullock and
40 Grossberg, 1988; Schomaker, 1991; Gangadhar et al., 2007; Kalveram, 1998),
41 equilibrium point models (Feldman, 1966; Feldman and Latash, 2005; Bizzi et al., 1978;
42 1992), behavioral models (Schmidt, 1999; Thomassen et al., 1983; van Galen and
43 Teulings, 1983), coupled oscillator models (Hollerbach, 1981; Kelso, 1995; Zazone et al.,
44 2005), kinematic models (Plamondon, 1995; Plamondon and Djioua, 2006), and models
45 exploiting minimization principles (Wada and Kawato, 1995; Engelbrecht, 2001) :
46 minimization of the acceleration (Neilson, 1993; Neilson and Neilson, 2005), of the
47 energy (Nelson, 1983), of the time (Tanaka et al., 2006; Enderle and Wolfe, 1987;
48 Hermes and LaSalle, 1969), of the jerk (Hogan, 1984; Flash and Hogan, 1985), of the
49 snap (Edelman and Flash, 1987), of the torque changes (Uno et al., 1989) and of the
50 sensory-motor noise (Harris and Wolpert, 1998). Finally, many models exploit the

51 properties of various functions to reproduce human movements: exponentials
52 (Plamondon and Lamarche, 1986), second order systems (Denier van der Gon and
53 Thuring, 1965; Dooijes, 1983), gaussians (Leclerc et al., 1992), beta functions (Alimi,
54 2003), splines (Morasso et al., 1983) and trigonometrical functions (Maarse, 1987).

55 Among the models which provide analytical representations, the Kinematic Theory of
56 rapid human movements (Plamondon, 1995a, 1995b; Plamondon and Djioua, 2006) and
57 its Delta- and Sigma-lognormal models have been used to explain most of the basic
58 phenomena reported in classical studies on human motor control (Plamondon and Alimi,
59 1997) and to study several factors involved in the fine motricity (Djioua and Plamondon,
60 2008; O'Reilly and Plamondon, 2010; Woch et al., 2010). Apart from these fundamental
61 studies, the theory has been used, directly or indirectly, in many practical applications
62 like the design of a signature verification system (Plamondon, 1994), the development of
63 tools to help children learning handwriting (Djeziri, Guerfali, Plamondon, and Robert,
64 2002), as well as of biomedical set ups to detect fine motor control problems associated
65 with brain strokes (O'Reilly and Plamondon, 2011, 2012).

66 In this paper, we report on two new and original case studies dealing with the automatic
67 generation of synthetic handwritten specimens for the creation of large databases. The
68 first application addresses the automatic generation of totally synthetic signature
69 specimens which may be used for the training and evaluation of the verification
70 performances of automatic recognition systems as well as for the quality assessment of
71 specimens. The second application regards the synthesis of handwritten gesture for
72 speeding up the learning process in customizable on-line recognition systems to be
73 integrated in electronic pen pads. Sections 5 and 6 reports detailed results about these two

74 genuine applications, which at the time of the ICFHR 2010 keynote address presented by
75 the first author, were the first trial of using the Kinematic Theory for the generation of
76 synthetic trajectories to be used in signature verification and gesture recognition
77 experiments.

78 To better understand these applications and estimate their potential interest, as well as
79 making the present paper self-consistent a brief survey of the Kinematic Theory is
80 presented in section 2, two algorithms used for sigma-lognormal parameter extraction are
81 outlined described in section 3 and the main results on previous studies of handwriting
82 variability are summarized in section 4. These sections present in a condensed and goal
83 oriented way, the main concepts and strategies that have been explored over the years and
84 that are necessary to understand the present applications, without coming back to these
85 complete and often more exhaustive studies.

86

87 **2. The Kinematic Theory of Rapid Human Movement and its Sigma-** 88 **Lognormal Model**

89 One key feature of the Kinematic Theory is that it relies on strong and robust
90 mathematical grounds. All the models that are used under this paradigm are based on the
91 lognormal function which has been proved to be the ideal curve for describing
92 asymptotically the impulse response of a neuromuscular network made up of a large
93 number of coupled subsystems controlling the velocity of a movement (Plamondon et al.,
94 2003). For simple reaching or pointing gestures, a target is specified and two of these
95 networks are needed to control a trajectory, an agonist network which is acting in the

96 target direction and one antagonist, acting in the opposite direction. Overall, the speed
 97 profile is then described by a Delta-lognormal equation, a weighted difference of two
 98 lognormals (Plamondon, 1995a, 1995b). When more complex trajectories have to be
 99 generated, like in handwriting or in signing, a sequence of targets has to be reached and,
 100 globally, the trajectory of the pen tip can then be described by a vectorial summation of
 101 lognormals, hereinafter called sigma-lognormal equations, which takes into account the
 102 various changes of direction.

103 In this vectorial summation context, the production of a word or of a signature requires
 104 the definition beforehand of an action plan that is made up of virtual targets, which are
 105 linked in pairs with an arc of circle. This map of paired target points represents a
 106 sequence of discontinuous strokes. This plan triggers a motor command generator that
 107 produces a series of impulses activating the neuromuscular systems characterized by their
 108 lognormal impulse response (Plamondon and Privitera, 1995). For each impulse, a
 109 lognormal velocity profile is generated at the pen tip and the time superimposition of
 110 these strokes results in a smooth and well controlled trajectory. According to this
 111 representation, the original strokes are thus hidden in the signal.

112 Mathematically, the Sigma-Lognormal model considers the velocity of the pen tip, $\vec{v}(t)$, as
 113 described by a vectorial summation of N lognormal primitives:

$$114 \quad \vec{v}(t) = \sum_{i=1}^N \vec{v}_i(t) = \sum_{i=1}^N \vec{D}_i(t) \Lambda_i(t, t_{0i}, \mu_i, \sigma_i^2); N \geq 2 \quad (1)$$

115 Each lognormal in this summation defines a stroke scaled in amplitude by a command
 116 parameter (\vec{D}) and time-shifted by the time occurrence of this command (t_0), any
 117 individual stroke pattern being described by a lognormal time function:

118
$$\Lambda(t; t_{0i}, \mu_i, \sigma_i^2) = \frac{1}{\sigma_i \sqrt{2\pi} (t - t_{0i})} \exp\left(\frac{-[\ln(t - t_{0i}) - \mu_i]^2}{2\sigma_i^2}\right) \quad (2)$$

119 Each of these primitives is also assumed to occur around a pivot, and the evolution of the
 120 angular position of the trajectory can be calculated using an error function (*erf*):

121
$$\theta_i(t) = \theta_{si} + \frac{(\theta_{ei} - \theta_{si})}{2} \left[1 + \operatorname{erf}\left(\frac{\ln(t - t_{0i}) - \mu_i}{\sigma_i \sqrt{2}}\right) \right] \quad (3)$$

122

123 where θ_{si} and θ_{ei} refer, respectively, to the starting and ending angular direction of each
 124 stroke. In equations (2) and (3), μ_i and σ_i represent correspondingly the logtime delay
 125 and the logresponse time of the neuromuscular system as it reacts to the i^{th} command
 126 (Plamondon and Djioua, 2006).

127 Under these conditions, the synergy produced by the interaction and coupling of many of
 128 these neuromuscular systems results in the sequential generation of a complex
 129 handwriting sample or a signature pattern.

130

131 **3. Sigma-lognormal Parameter Extraction**

132 To use the Sigma-Lognormal model for analyzing human movements, it is necessary to
 133 have an algorithm to solve the inverse problem in a fully automatic fashion, that is, to
 134 extract the lognormal parameters that most adequately fit the experimental data. The
 135 Sigma-Lognormal parameters are considered to be well estimated and fitted for statistical
 136 analysis if the SNR, defined in (4), is over 20dB.

$$SNR = 10 \log \left(\frac{\int v_{x_n}^2 + v_{y_n}^2 dt}{\int (v_{x_n} - v_{x_\Sigma})^2 + (v_{y_n} - v_{y_\Sigma})^2 dt} \right) \quad (4)$$

137 In this equation, (v_{x_n}, v_{y_n}) are the experimental (numerical) velocity signals and
 138 $(v_{x_\Sigma}, v_{y_\Sigma})$ are the velocity signals of the sigma-lognormal reconstruction.

139 In the last years, two complementary algorithms have been proposed to solve this
 140 nonlinear regression problem, the Robust Xzero based algorithm and the prototype based
 141 algorithm. The next subsections briefly overview the state-of-the-art regarding these
 142 parameter extractors.

143

144 **3.1. The Robust Xzero based extractor**

145 The Robust Xzero (RX0) based extractor is a powerful algorithm that provides an
 146 accurate set of sigma-lognormal parameters describing the end-effector trajectory (e.g.
 147 the pen tip trajectory in handwriting studies) of arbitrarily complex motions without any
 148 *a priori* knowledge regarding the nature of the movement. In the following text, an
 149 outline of the algorithm is presented. A more comprehensive description can be found in
 150 (O'Reilly and Plamondon, 2009; O'Reilly, 2012).

151 To implement this algorithm, sequences of five characteristic points (t_{i_n}, v_{ti_n})
 152 ($i=1,2,\dots,5$) must be located in the original speed signal v_t . Following a time occurrence
 153 order, these points are: a local minimum, an inflexion point, a local maximum, an
 154 inflexion point and a local minimum. The sigma-lognormal representation of these points
 155 can be written as in equations (5-6) with the parameters α_i defined in (7).

$$t_{i_\Sigma} = t_0 + e^\mu e^{-\alpha_i} \quad i \in \{1,2, \dots, 5\} \quad (5)$$

$$v_{ti,\Sigma} = \frac{D}{\sqrt{2\pi}} e^{-\mu} \sigma^{-1} e^{\left(a_i - \frac{a_i^2}{2\sigma^2}\right)} \quad i \in \{1, 2, \dots, 5\} \quad (6)$$

$$\begin{cases} a_1 = 3\sigma \\ a_2 = 1.5\sigma^2 + \sigma\sqrt{0.25\sigma^2 + 1} \\ a_3 = \sigma^2 \\ a_4 = 1.5\sigma^2 - \sigma\sqrt{0.25\sigma^2 + 1} \\ a_5 = -3\sigma \end{cases} \quad (7)$$

156 Nine estimators can be obtained for the values of the kinematic parameters $(t_{0i}, D_i, \mu_i, \sigma_i)$
 157 corresponding to the nine different combinations $(t_{j_n}, t_{k_n}, v_{tl_n}, v_{tm_n})$ (with
 158 $j, k, l, m \in \{2, 3, 4\}, l \neq m, j \neq k$) in the equations (8)-(11).

$$t_0 = t_{j_n} - e^\mu e^{-a_j} \quad (8)$$

$$D = \sqrt{2\pi} v_{tl_n} e^\mu \sigma e^{\left(\frac{a_l^2}{2\sigma^2} - a_l\right)} \quad (9)$$

$$\mu = \ln \left\{ \frac{t_{j_n} - t_{k_n}}{e^{-a_j} - e^{-a_k}} \right\} \quad (10)$$

$$\sigma = \begin{cases} \sqrt{-2 - 2 \ln \left(\frac{v_{tl_n}}{v_{tm_n}} \right) - \frac{1}{2 \ln \left(\frac{v_{tl_n}}{v_{tm_n}} \right)}} & l \in \{2, 4\}, m = 3 \\ \sqrt{2 \sqrt{1 + \ln^2 \left(\left(\frac{v_{tl_n}}{v_{tm_n}} \right) \right)} - 2} & l = 4, m = 2 \end{cases} \quad (11)$$

159 The angular parameters (θ_s, θ_e) associated with each estimation of the kinematic
 160 parameter set $(t_{0i}, D_i, \mu_i, \sigma_i)$ is obtained using (12)-(13), where
 161 $l_3 = \int_{t_0}^{t_s} D \Lambda(t - t_0; \mu, \sigma) dt = \frac{D}{2} \operatorname{erfc} \left(\frac{\sigma}{\sqrt{2}} \right)$ (with erfc being the complementary error
 162 function defined as $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$) and $\phi(t)$ is direction angle of the trajectory
 163 with respect to time.

$$\theta_s = \phi(t_{3_n}) - \frac{d\phi(t_{3_n})}{dt} l_3 \quad (12)$$

$$\theta_s = \phi(t_{3_n}) + \frac{d\phi(t_{3_n})}{dt} (D - l_3) \quad (13)$$

164 A choice can be made between the nine estimations of the six sigma-lognormal
 165 parameters by keeping the solution minimizing the error function (14).

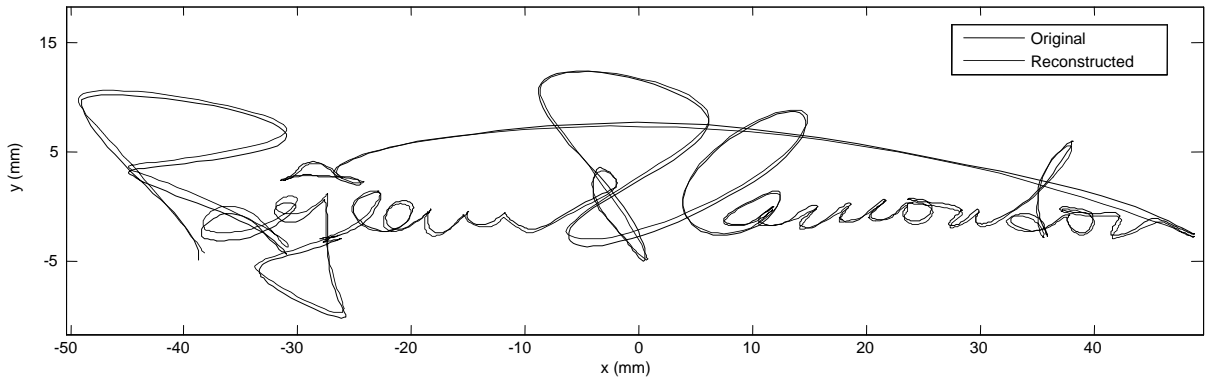
$$\int_{t_{i_n}}^{t_{s_n}} \left((v_t(t) - \widehat{D}\Lambda(t - \widehat{t}_0; \widehat{\mu}, \widehat{\sigma}))\Lambda(t - \widehat{t}_0; \widehat{\mu}, \widehat{\sigma}) \right)^2 dt \quad (14)$$

166 It should be noticed that, before using the values (t_{i_n}, v_{ti_n}) in the previous expressions,
 167 some preprocessing should be applied to get proper estimations¹. To extract the
 168 parameters of a whole velocity signal, good results have been obtained by, first,
 169 extracting sequentially (i.e. in increasing order of their time occurrence) the lognormal
 170 components. For that matter, each lognormal is extracted and subtracted from the original
 171 signal before proceeding to the next component. Then, a global non-linear optimization
 172 process can be applied to improve the estimated values. If this approach results in an
 173 unsatisfactory reconstruction SNR, more lognormal components can be extracted by
 174 processing them by decreasing importance of their impact (assessed here by their relative
 175 size) on the signal.

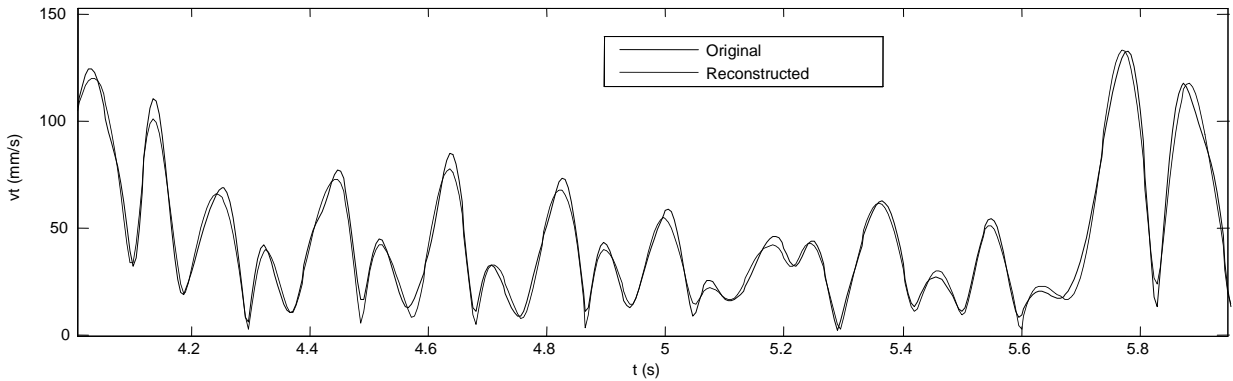
176 The latest improvements included in this extraction system have resulted in a significant
 177 increase of the SNR fitting accuracy. For example, on a 683 signatures database

¹ The details of these preliminary computations are presented in (O'Reilly & Plamondon, 2009; O'Reilly, 2012).

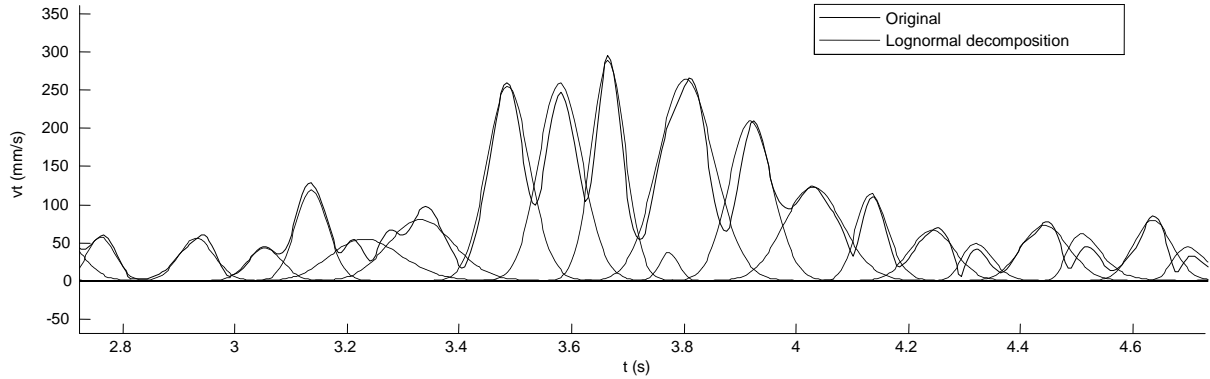
178 comprising 124 subjects, an average increase of 7.9 dB has been obtained, passing from
179 17.4 dB in (O'Reilly & Plamondon, 2009) to 25.3 dB. Fig. 1 gives an example of a
180 complex movement, in this case a signature, fitted using the proposed RX0 approach
181 (SNR=22.2dB).



(a)



(b)



(c)

182 Fig. 1. Example of a signature reconstruction (SNR=22.2dB) following the proposed
 183 RX0 approach: (a) the trajectory, (b) the speed profile and (c) the lognormal
 184 decomposition of the speed profile. In (b) and (c), only a small part of the actual signals
 185 are shown to better allow the reader to appreciate the curve fitting and its lognormal
 186 decomposition.

187

188 3.2. The prototype based extractor

189 Although the system based on the Robust Xzero estimator gives very satisfying
 190 extraction results on complex movements, an alternative extraction strategy may be of
 191 great use under certain experimental scenarios. For example, this is the case of the
 192 analysis of stereotypical movements such as those often involved in psychophysical
 193 experiments. This type of experiments present some specific characteristics that can make
 194 an alternative extraction method better fitted for the task than RX0. First, a lot of *a priori*
 195 information on the nature of the movement is available which may be very helpful during
 196 the extraction process (the RX0 algorithm is not designed to take advantage of this
 197 knowledge). Second, in this kind of experimental framework, researchers may want to

198 perform statistical testing of hypotheses on the value of local parameters. This may be
199 difficult with the solution obtained by an extractor such as the one based on RX0
200 because, in this case, there is no clear correspondence between the parameters extracted
201 from various movements.

202 These reasons supported the development of the prototype based extractor presented in
203 (O'Reilly and Plamondon, 2010). The advantages of such an extractor can be seen, for
204 example, in (O'Reilly and Plamondon, 2011) where it has been used to assess the
205 neuromuscular health of subjects on the basis of the sigma-lognormal parameters of their
206 movements.

207 This extractor applies a three step methodology: 1) synthesis of a sigma-lognormal
208 prototype of the stereotypical movements, 2) time scaling and offsetting of the prototype
209 to make it more closely correspond to the experimental movement, and 3) global
210 nonlinear optimization of the scaled and offset prototype to improve the fitting. These
211 three steps can be briefly described as follow:

- 212 - **Step 1: Synthesis of the prototype.** The initial prototype can be built from i) the
213 results of the RX0 extractor in order to find what is the expected sigma-lognormal
214 decomposition of the stereotypical movement or ii) from a sigma-lognormal
215 synthesizer such as SimScript (O'Reilly and Plamondon, 2007).
- 216 - **Step 2: Movement scaling and offsetting.** It is performed by finding the value of
217 the scaling (C_s) and the offsetting (t_s) factors that maximize the reconstruction
218 SNR between the experimental data and the prototype signals modified in such a
219 way that the original μ_i and t_{0i} parameters are changed for the scaled and shifted
220 parameters μ_{ia} and t_{0ia} according to (15)-(16).

$$t_{0i\alpha} = C_s t_{0i} + t_s \quad (15)$$

$$\mu_{i\alpha} = \mu_i + \ln(C_s) \quad (16)$$

221 - **Step 3: Non-linear optimization.** It can be performed according to any suitable
222 optimization algorithm. For our work, we used a custom implementation of a
223 direct search optimization (O'Reilly, 2012) which monotonically increases the
224 SNR without risk of divergence or of finding solutions that are too far away from
225 the original prototypes. This preserves the psychophysical correspondence of the
226 lognormal components among movements.

227 Using this algorithm on a database of 1440 triangular movements performed by 120
228 subjects (see (O'Reilly & Plamondon, 2011) for a more complete description of this
229 dataset), we obtained a mean SNR of 20.8 dB for the prototype based extractor compared
230 to a 22.1 dB for the RX0 based system. Although its SNR is slightly lower, the prototype
231 based extractor has the clear advantage of producing solutions with fixed number of
232 lognormals which enables the comparison among movements with any standard tool of
233 statistical analysis of variance.

234

235 **4. Automatic Generation of Trajectories: Variability Issues**

236 As we have seen, according to the Kinematic Theory, a complex movement results from
237 the superposition of a set of elementary movements (corresponding to single curved
238 strokes), localized both in time and space. So, the large variability observed in
239 handwriting patterns can be interpreted as caused both by the intrinsic variability of the
240 individual strokes and by fluctuations occurring in the time plan of the superimposition
241 process as controlled by the central nervous system.

242 The local variability observed in handwriting and signing can thus come from various
243 sources. At the central level, a movement is represented by an action plan, a sequence of
244 virtual targets describing a piece-wise discontinuous trajectory. In parameter terms, this
245 plan is a timed sequence of arcs described by their length, directions and time of
246 activation. This discontinuous pattern, once instantiated, stimulates a set of
247 neuromuscular networks that react to each of these fundamental primitives with specific
248 time delays and response times.

249 In this context, at least three basic sources of variability can be identified:

- 250 1- a time variability associated with the temporal information contained in the
251 activation sequence of the different commands,
- 252 2- a spatial variability associated with the geometrical information contained in
253 commands themselves (the magnitude D_i and direction (θ_{bi} and θ_{ei}) of each
254 stroke), and
- 255 3- a neuromuscular variability reflected in the timing properties μ_i, σ_i of the
256 neuromuscular networks reacting to these commands.

257 Those sources of variability have been investigated using a semi-automatic sigma-
258 lognormal parameter extraction methodology somewhat similar to the one described for
259 the prototype based extraction. Especially, we have shown that the Sigma-Lognormal
260 model, can explain the great variability of individual stroke trajectories and their
261 corresponding velocity profiles (Plamondon and Djoua 2005, 2006). We have also
262 applied this paradigm to study the possible sources of handwriting deformations caused
263 both by the disruptions in the motor control and the neuromuscular systems (Djoua and
264 Plamondon 2007, 2009). Particularly, we have shown that, without altering the rest of the

265 factors involved in handwriting, the distortion of the shapes of a handwritten word is very
266 sensitive to slight changes of the time plan, represented by the sequence of command
267 time occurrences $\{t_{0i}\}$. This stresses out the fact that, to write a readable word or to
268 generate a consistent signature, the production of strokes composing that word or
269 signature must be planned in advance in order to keep almost constant the timing of the
270 learned original plan. In contrast, the command parameters, that affect the direction (θ_{bi}
271 and θ_{ei}) and the amplitude D_i of the strokes, seem to be less critical. And, finally, the
272 changes induced by the neuromuscular timing parameters μ_i, σ_i seem to have an even
273 smaller influence on the final variability of the trajectory.

274 Overall, these studies have shown the existence of a direct relationship between the
275 fluctuations of the sigma-lognormal parameters and the space and time warping of a pen
276 tip trajectory, which suggests the feasibility of using this model as a new tool for the
277 design of synthetic human like movements. This will be shown in the next two sections,
278 where the relative importance of the three sources of variability will be critical for the
279 successful design of large synthetic databases.

280

281 **5. Application 1: Automatic On-line Signature Database Generation**

282 One of the main obstacles that the biometric technology has found, and still finds, to
283 become one of the leading solutions in the security market, is the lack of large real
284 biometric databases which may serve as common benchmarks for the development of this
285 thriving technology. Two main reasons may explain such a scarcity of biometric data. On
286 the one hand, biometric database collection is not at all an easy job, involving a lot of
287 effort in terms of time and resources in order to reflect the variability present in biometric

288 traits (both inter- and intra-class). This process includes a number of pre-acquisition and
289 post-acquisition demanding tasks such as the recruitment of donors, the supervision of
290 collected data, error correction or labeling (Flynn, 2008). On the other hand, biometric
291 traits are classified as personal data, and as such they are subdued to the different
292 personal data protection laws existing in each country, which makes the acquisition
293 (donor's consent), and later storage and distribution (licensing) of these data very
294 difficult.

295 Such a complex context has promoted over recent years the apparition of new algorithms
296 for the generation of synthetic biometric databases (Cappelli, 2003; Zuo, 2007). These
297 synthetically produced datasets are not affected by the acquisition and legal issues
298 mentioned before: *i)* first, once the appropriate generation method has been developed,
299 they are effortless to be produced, avoiding this way the arduous acquisition campaigns,
300 and *ii)* second, the synthetic samples which conform these databases cannot be
301 considered personal data as they have not been produced by a person, and so they may be
302 freely distributed in order to be used as common evaluation benchmarks. These desirable
303 characteristics make synthetic databases very powerful tools for the performance
304 assessment of biometric recognition systems, and have already been used in international
305 competitive evaluation campaigns (Cappelli, 2006).

306 In spite of presenting some very interesting features, the use of synthetic biometric
307 databases is not yet generalized as the production of realistic synthetic samples still
308 remains a challenging problem: modeling the information contained in a certain
309 biometric trait as well as the inter-class and intra-class variation found in real databases
310 (i.e., variation between samples of different subjects, and variation between samples of

311 the same subject). Accepted solutions have been proposed for fingerprint (Cappelli,
312 2003) or iris (Zuo, 2007; Shah, 2006), but still no consistent method has been given for
313 the generation of synthetic handwritten signature databases.

314 As presented in previous sections, the Kinematic Theory of rapid human movements
315 provides a powerful theoretical framework which models in a precise and compact
316 manner the kinematic information involved in most of human writing processes,
317 including signature. Thus, the Kinematic Theory and its associated Sigma-Lognormal
318 model constitute a very high potential instrument for many different applications and
319 have been applied in the present work to the development of an algorithm for the
320 generation of fully synthetic on-line signatures.

321 **5.1. The generation method**

322 Two main parameters are involved in the design of real biometric databases and, hence,
323 should also be critical in the generation of synthetically produced datasets: *i*) number of
324 users comprised in the database, and *ii*) number of samples per user to be acquired. As
325 can be seen in Fig. 2, the generation method of synthetic signature databases proposed in
326 the present work is constituted of two different algorithms in order to produce: *i*) the first
327 sample of fully synthetic individuals (i.e., it allows to control the number of users in the
328 database), and *ii*) different samples derived from that original master signature (i.e.,
329 permitting to fix the number of samples per user).

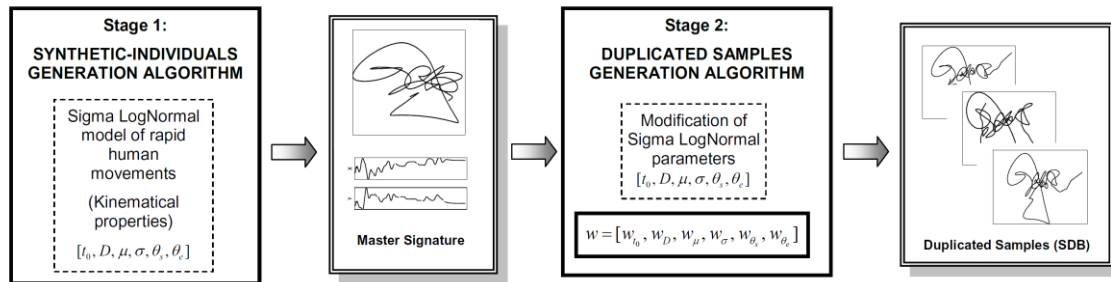


Fig. 2. General architecture of the algorithm proposed for the generation of synthetic signature databases based on the Kinematic Theory of rapid human movements.

330 5.1.1. Generating the master signature

331 Although other signals such as the azimuth and elevation angles of the input pen or the
 332 pressure applied during the signing process might be taken into account, in this work we
 333 consider that an online signature is defined by two time sequences $[x[n] \ y[n]]$ specifying
 334 respectively the x and y coordinates, at the time instants $n = 1, \dots, N$.

335 The objective of this initial stage of the generation algorithm is to produce samples from
 336 different synthetic signers (i.e., this algorithm is responsible for controlling the number of
 337 users in the database and for the inter-class variability present in the dataset).

338 In a first approach, a signature-like graphic is generated following the spectral approach
 339 described in (Galbally, 2009). Although this first specimen has approximately the
 340 appearance of a genuine signature, it does not possess many of the humanly produced
 341 kinematic characteristics of real writing. Thus, in order to confer this preliminary master
 342 signature with the velocity and acceleration properties of human strokes, it is processed
 343 using the Sigma-Lognormal model in two consecutive stages, as shown in Fig. 3:

- 344 - **Stage 1:** Extraction of the sigma-lognormal parameters using the RX0 system. In this
 345 phase, the velocity function of the initial synthetic master signature (v_I) is decomposed

346 in singular strokes and the sigma-lognormal parameters $(t_0, D, \mu, \sigma, \theta_s, \theta_e)$ which best
347 fit each of the individual strokes.

348 - **Stage 2:** Reconstruction of the velocity function of the definitive synthetic master
349 signature according to the previously computed parameters. The new coordinate
350 signals are then obtained from the reconstructed velocity function (v_D) where we can
351 observe that certain abnormal artifacts such as the very high velocity peaks at the
352 starting and ending parts of the velocity profile have been corrected (see Fig. 3)

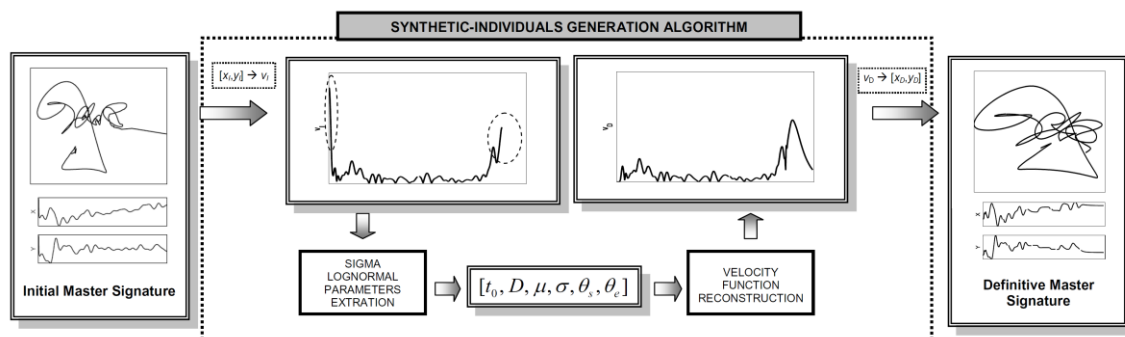


Fig. 3. General diagram of the generation process of a synthetic on-line master signature with the kinematic properties of a human-produced sample, based on the sigma-lognormal parameters.

353

354 5.1.2. Generating the duplicated samples

355 Once the time sequences $[x[n] y[n]]$ defining the master signature of a synthetic user have
356 been produced following the method described in Sect. 5.1.1, the next phase for the
357 automatic generation of synthetic on-line signature databases is the creation of duplicated
358 samples starting from that master sample (as is shown in Fig. 2).

359 Therefore, the objective of this part of the proposed method is to produce different
 360 samples of one same synthetic individual following the intra-variability found in real
 361 signatures (i.e., existing variability among signatures produced by the same user).
 362 For this purpose, the velocity function v of the master signature is decomposed into single
 363 strokes following the Sigma-Lognormal model where each stroke is defined by the set of
 364 features $p=[t_0, D, \mu, \sigma, \theta_s, \theta_e]$. The duplicated samples are then generated adding to each
 365 of the single strokes a certain amount of noise which is modeled by a vector $w=[w_{t_0}, w_D,$
 366 $w_\mu, w_\sigma, w_{\theta_s}, w_{\theta_e}]$ where w_{t_0} is extracted from a uniform distribution $[-w_{t_0}^{\max}, w_{t_0}^{\max}]$ which
 367 is estimated according to the intra-user variability found in the development database
 368 BiosecurID (Fierrez, 2010) (analogously for the rest of distortion elements comprised in
 369 the vector w). After the distortion stage, the new velocity function v_n is computed, and in
 370 a subsequent step the new coordinates x_n and y_n are recovered from that velocity
 371 information (see Fig. 4).

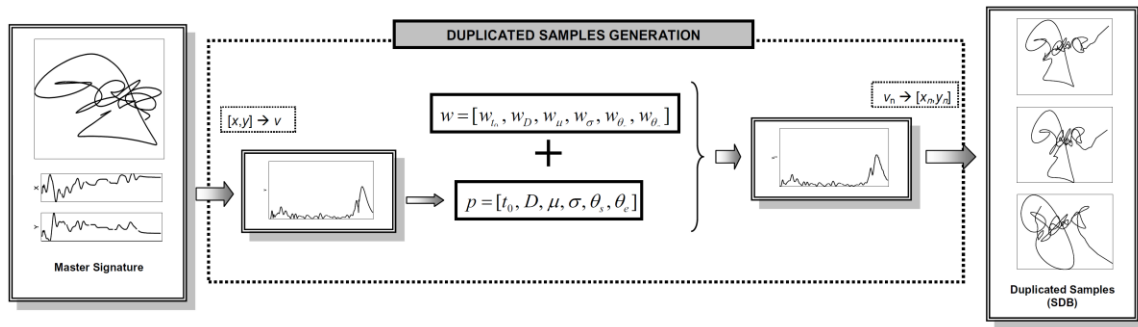
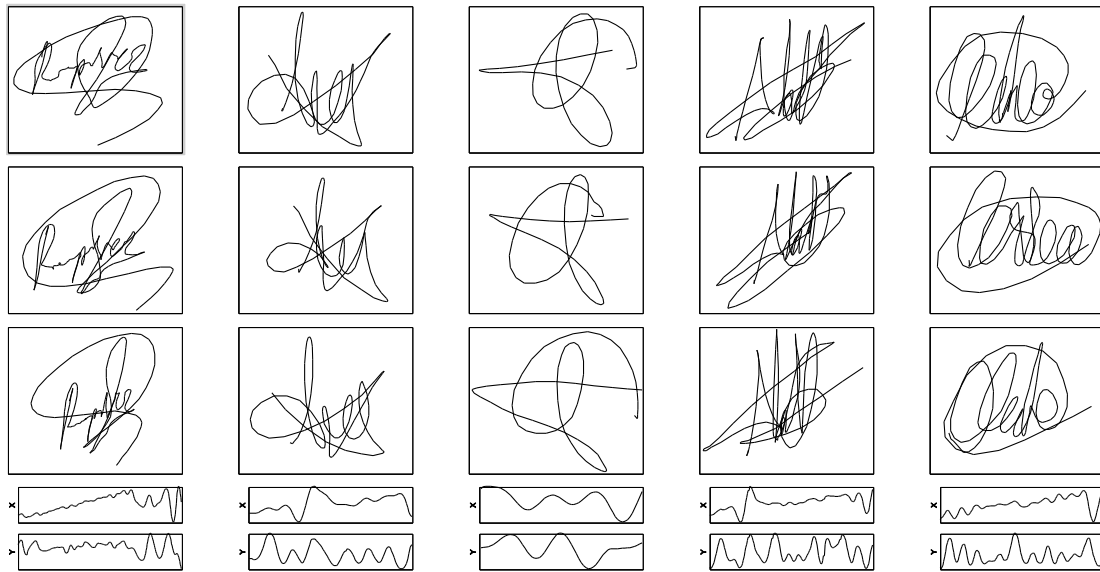


Fig. 4. General diagram of the generation process of duplicated samples starting from a fully synthetic specimen, based on the sigma-lognormal parameters.

372

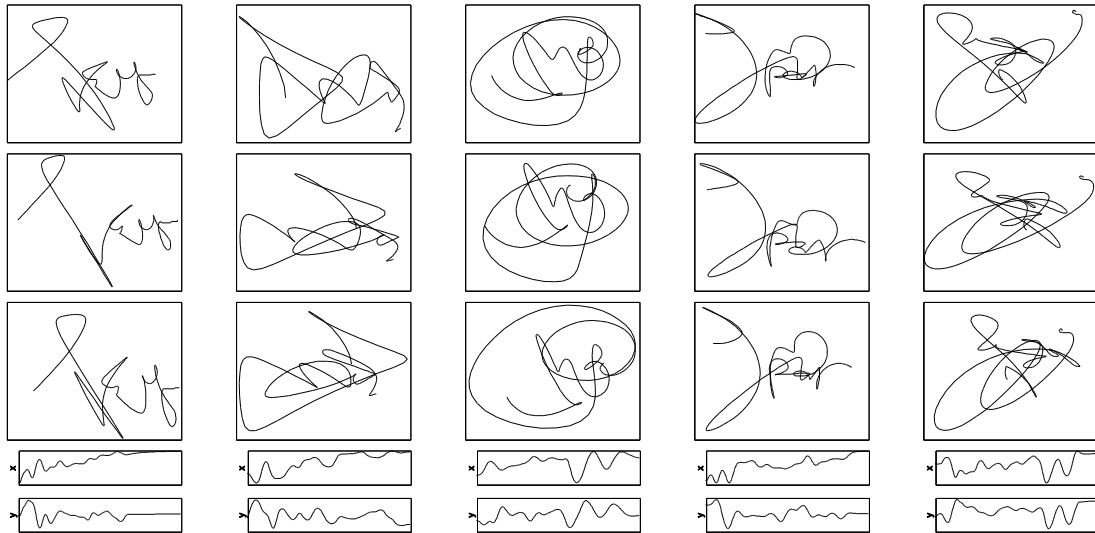
373 In Fig. 5 some examples of synthetic signatures generated following the described
 374 approach are shown, together with real samples extracted from the BiosecurID database

375 (Fierrez, 2010) which was used as development set in order to compute the values of the
376 different parameters involved in the generation method. As it can be observed from a
377 general visual comparison, the synthetic signatures, and especially their time functions,
378 present a very realistic appearance² in terms of: 1) smoothness of the strokes; 2) growing



a) Real signatures extracted from the BiosecurID database.

² In this context, one concern that might be raised is dealing with the potential use of this methodology to fabricate forged signatures. This might be an issue except that for making realistic forgeries, the forger would need to have access to the complete on-line information about the target signature and if he does, there would be no need to use a complex methodology like the one described in this paper. The addition of some noise might be sufficient. However, the complete on-line information is generally not available in real life systems since it is generally not stored in the reference database.



b) Synthetic signatures produced with the proposed generation algorithm

Fig. 5. Examples of real (a) and synthetic (b) signatures extracted from BiosecurID and SDB. Three samples of 5 different real and synthetic signers are shown together with their time sequences $x[n]$ and $y[n]$ corresponding to the first sample.

379 tendency of the function x (as it corresponds to left-to-right occidental signatures); 3)
 380 large fluctuation at the end of the x and y signals in some of the signatures (corresponding
 381 to some sort of round-like flourish); 4) degree of correlation between some of the most
 382 relevant maxima and minima points in the x and y directions. Furthermore, even though it
 383 is a model-based approach, some recognizable characters may be distinguished in the
 384 synthetic samples.

385

386 5.2. Validation experiments

387 The experimental validation of the proposed generation method is aimed at determining if
 388 the performance of signature verification systems is similar when it is evaluated on real
 389 and synthetic databases. If the error rates presented by signature-based recognition
 390 applications is comparable in both scenarios (i.e., performance evaluation with real and

391 synthetic signatures), it would mean that, from a computer-based perspective, the
392 synthetic signatures present a very similar behavior to that of real samples and that they
393 can be used to obtain a fair estimation of a system performance, avoiding this way the
394 different problems linked to real databases (i.e., high resource-consuming acquisition
395 campaigns and legal issues regarding their acquisition and distribution).

396 In order to achieve this objective, the performance of two signature verification systems,
397 using totally different feature sets and matchers, has been evaluated on the MCYT real
398 database (Ortega, 2003) and on a Synthetic DB (SDB) produced using the proposed
399 synthetic generation method.

400 The MCYT dataset has been selected as real test set since it has no overlap with the
401 BiosecurID database used as development set for the estimation of the generation method
402 parameters. This way we ensure to obtain totally unbiased results. The SDB has been
403 created with the same number of users (300) and samples per user (25) as MCYT in order
404 to permit the use of the same evaluation protocol for both scenarios.

405 The two on-line verification systems evaluated in the experiments are:

- 406 • **System A: function based + HMM** (Fierrez, 2007). This function-based verification
407 system applies a regional approach using a statistical model built using Hidden
408 Markov Models (HMMs) to a set of 10 time sequences selected applying the SFFS
409 feature selection algorithm (Pudil, 1994) to the total set of 34 functions defined in
410 (Martinez-Diaz, 2009a).
- 411 • **System B: function-based + DTW** (Martinez-Diaz, 2009b). In this function-based
412 local approach, a subset of nine time functions (selected using the SFFS from the total

413 34 feature set as in the case of system B) are directly matched using the elastic
 414 technique Dynamic Time Warping (DTW) (Kholmatov, 2005).

415 The performance results (Detection Error Trade-off, DET, curves) obtained for both
 416 verification systems are shown in Fig. 6. We can observe that the curves of the two
 417 systems present a very high degree of resemblance, both from a quantitative (EERs) and
 418 qualitative (general behavior) point of view, for the case of real and synthetic signatures.

419 The results derived from this validation experiment confirm the great potential of the
 420 Kinematic Theory of Rapid Human Movements applied to the generation of synthetic on-
 421 line signature databases, and the suitability of such datasets to obtain reliable estimations
 422 of the performance of signature verification systems.

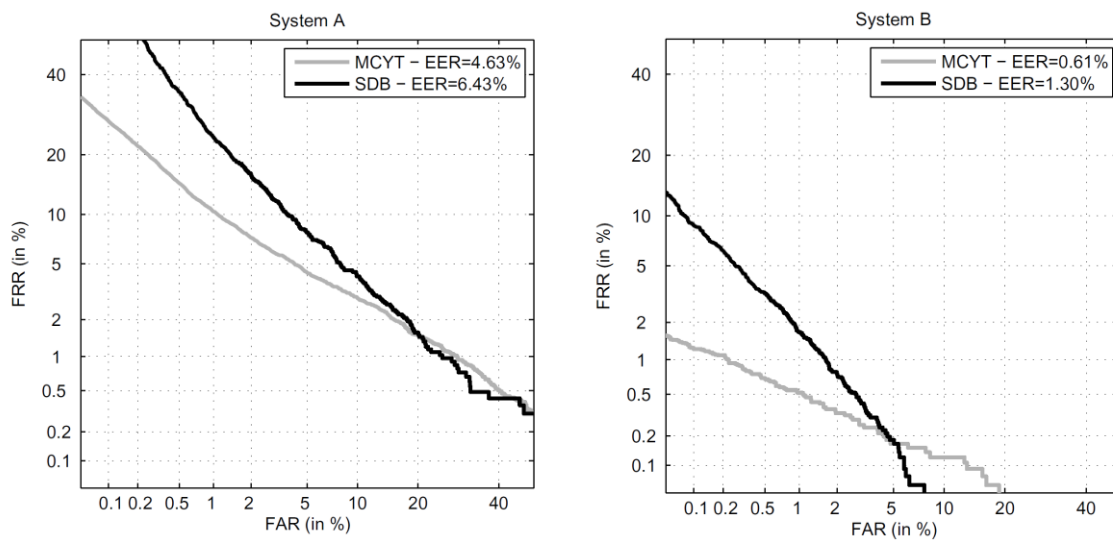


Fig. 6. Performance evaluation of systems A and B, on a real (MCYT, grey DET curve) and synthetic database (SDB, black DET curve). The EER is indicated in each plot.

423

424 **6. Application 2: Synthetic Gesture Generation for Evolving**
 425 **Handwriting Classifiers**

426 Motivated by the increasing spread of many types of devices equipped with pen-based
427 interfaces, such as PDAs, e-book, Tablet PCs, Whiteboards, etc., more emphasis is placed
428 on the development of efficient recognition systems that can correctly interpret the
429 gestures sketched by the user and then translate them either into computerized text or into
430 some specific commands. Nowadays, the recognition systems in use are always pre-
431 trained on a fixed, predefined and a limited group of gestures, which usually contains the
432 Latin letters and a few specific gestures. These systems do not allow users to add gestures
433 in order to assign them to new commands or shortcuts, or to replace default gestures
434 mapped to existing commands. In order to meet this important functionality, the static
435 handwriting recognition systems that have been used so far must be replaced by novel
436 dynamic ones where the knowledge base can constantly evolve during the use of the
437 system. The evolving nature comes from the fact that the system must be able to integrate
438 at any moment a new class (gesture in our context), and must also continue its adaptation
439 to the existing classes using the new available data. Although the dynamic nature of
440 evolving classifiers offers many important advantages, the operation of these systems
441 suffers from the lack of learning data. The training process is done directly by the final
442 user in an online and interactive manner, so that the quantity of teaching samples is
443 limited because it is impractical to ask the user to enter a large number of samples in
444 order to obtain a functional classifier. Therefore, the main challenge in the conception of
445 incremental learning algorithms of evolving classification systems consists in reaching
446 high recognition performance as fast as possible; i.e. with the minimum number of
447 samples. Besides the beginning of the incremental learning from scratch, the problem of
448 lack of data samples appears again during the adaptation process when new classes are

449 added to the classifier. The evolving system is supposed to be able to learn these new
450 classes without forgetting the old ones. However, it is difficult to completely avoid
451 perturbations on the global performance of the classifier when adding new classes and the
452 efforts must be focused on reducing as much as possible these perturbations.

453 In addition to the efforts of improving the classification systems and the training
454 algorithms, the incremental learning process can be further accelerated and enhanced by
455 generating artificial data based on some knowledge related to the application domain. For
456 handwritten gesture recognition problems, this idea can be implemented by generating
457 synthetic gestures from the available real ones after applying on them some deformations
458 in a realistic and significant manner. Thus, when a new class of gestures is introduced to
459 the system with few samples provided by the user, many artificial samples can be
460 generated. Geometric distortions are usually applied on real handwritten symbols in order
461 to generate synthetic ones (Mitoma, 2005; Wang, 2005; Lin, 2007; Mouchère, 2007).
462 These deformations can be either based on class-dependent models of gesture variability
463 and require a learning phase, or on class-independent general strategies without specific
464 deformation models.

465 In this work, we incorporate a handwriting generation technique using class-independent
466 lognormal-based deformations in the incremental learning of evolving handwritten
467 gesture classifiers. Thanks to the RX0 based extractor, the $\Sigma\Lambda$ parameter extraction and
468 the data generation is performed automatically as a part of the adaptation process. Motion
469 pattern variability rooted in the motor representation space of the handwritten gestures is
470 regarded to be more realistic than geometric distortions and thus more valuable in the
471 training process. In addition to the great advantage of integrating the lognormal-based

472 handwriting generation technique in our evolving handwriting classifier, an objective and
473 numerical evaluation of the quality of generated data is provided for the first time, to the
474 best of our knowledge. The generated handwritten samples are considered realistic as
475 much as they help the classifier to predict future real samples from the same class of
476 gestures. The capacity of prediction is translated by the improvement of recognition
477 performance of the evolving classifier.

478

479 **6.1. Acceleration of the learning process using synthetic data**

480 As aforementioned, we believe that distortions obtained by applying some variations on
481 lognormal parameters are more realistic than those obtained using direct geometrical
482 distortions. The idea is to extract the sigma-lognormal profiles of a real handwritten
483 gesture provided by the user. Then, we apply some variation on the extracted parameters
484 within some specific ranges, and we regenerate artificial gestures using the modified
485 profiles. The resemblance between the synthetic and the real gestures is controlled by the
486 variation intervals. Thus, a suitable setting of these intervals is required in order to avoid
487 over-deformed gestures. We show in Fig. 7 some examples of the artificial gestures that
488 can be generated by applying modifications on the sigma-lognormal parameters. We can
489 note that the real gestures can be almost predicted from the synthetic ones.

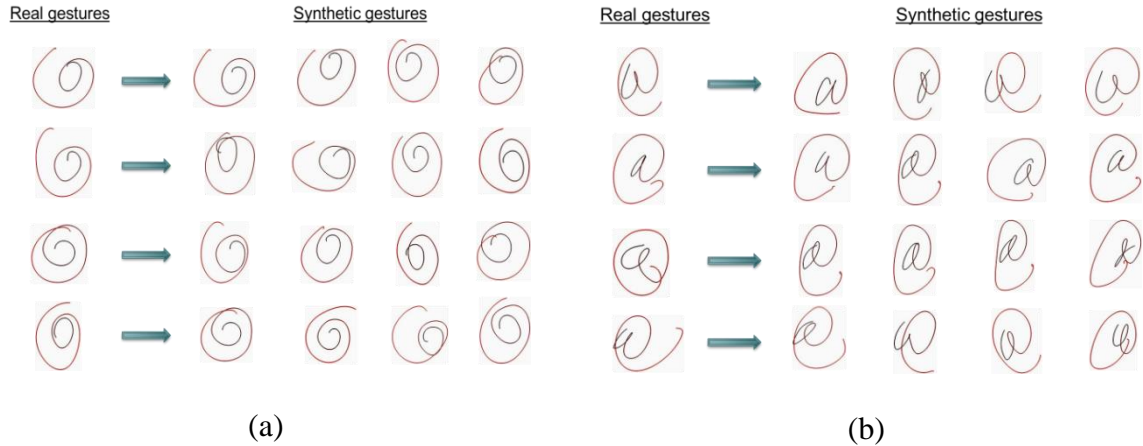


Fig. 7. Some examples of generated gestures using sigma-lognormal based deformations.

490

491 In the context of incremental learning of evolving systems, one can overcome the
 492 problem of lack of samples at the beginning of the inclusion of a new class by generating,
 493 in an adequate manner, a number of synthetic samples. For an evolving handwriting
 494 classifier, the abovementioned sigma-lognormal based technique for synthetic gesture
 495 generation can be incorporated into the incremental training process. The handwriting
 496 generation is automatically performed transparently, with no user intervention. Fig. 8
 497 shows the different steps of the generation method. First, the sigma-lognormal parameters
 498 of each incoming sample are first extracted. These parameters are then modified and a
 499 number of synthetic samples are generated. The original sample and the synthetic ones
 500 are then introduced in the incremental learning algorithm.

501 Using SimScript (O'Reilly and Plamondon, 2007), a visualization interface developed by
 502 the Scribens laboratory that allows an interactive modification of the sigma-lognormal
 503 parameters of a given handwritten gesture, we have experimentally studied the valid
 504 variation intervals of the six parameters within which the generated gesture is generally
 505 considered similar (from a human viewpoint) to the original one.

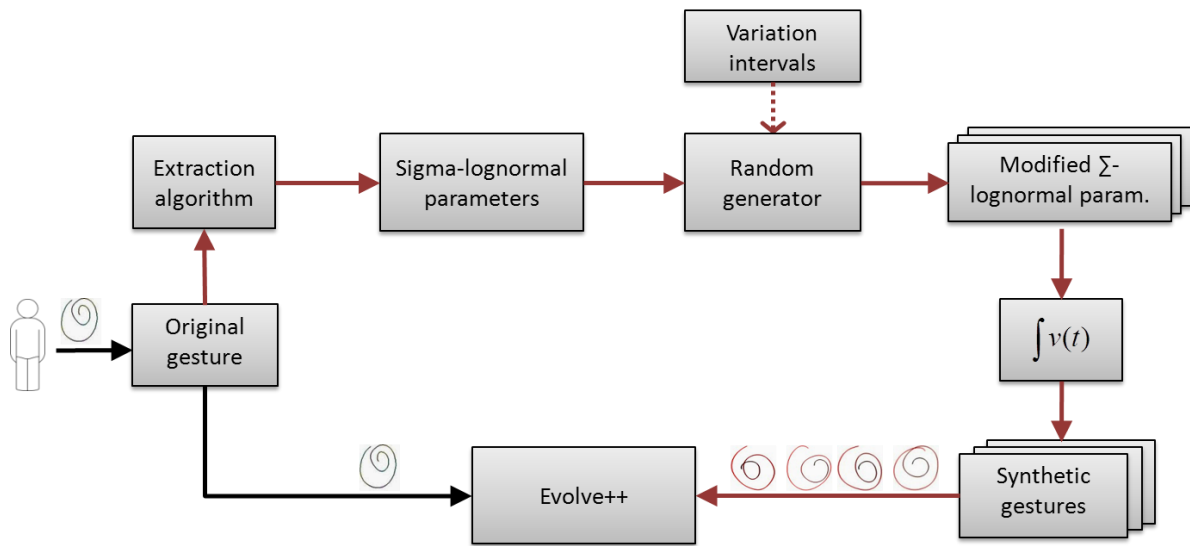


Fig. 8. Incorporating lognormal-based data generation into the learning stage of an evolving handwriting classifier.

506

507 6.2. Experimental results

508 Experiments have been performed on a dataset of on-line handwritten gestures. It was
 509 composed of 11 different gestures drawn by 7 writers on a Tablet PC. Each writer has
 510 drawn 100 samples of each gesture, i.e. 1,100 gestures in each writer specific dataset.
 511 Each gesture was described by a set of 10 features. The presented results are the average
 512 of 7 different tests for the 7 writers. In order to avoid the data order inducing a bias into
 513 the outcome, we repeated 40 times the experiment for each writer with different random
 514 data orders and only the averages are reported. We have used about 40% of the dataset
 515 for the incremental training and the rest is used to estimate the evolution of the
 516 performance during the learning process. We generated 10 synthetic samples (gestures)
 517 for each real sample. The evolving classifier used in these experiments is based on a first-

518 order Takagi-Sugeno (TS) fuzzy inference system, and taught with our original
519 incremental learning approach “Evolve++” (Almaksour, 2011).

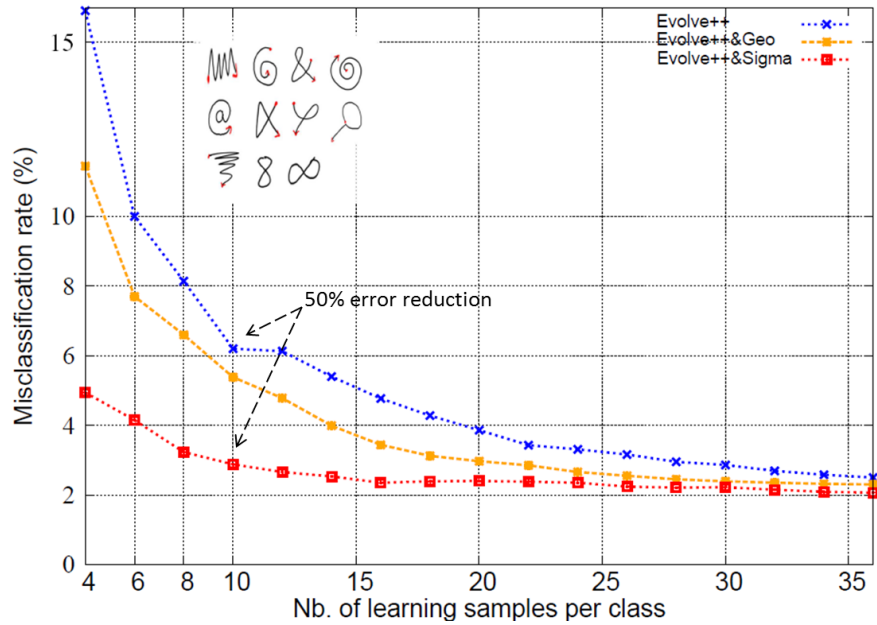
520 We compared the lognormal-based handwriting generation method to the geometric
521 distortions explained in (Mouchère, 2007). Therefore, three performance curves are
522 presented in the figures:

523 I. **Evolve++**: our evolving classification approach with Evolve++ algorithm presented
524 in (Almaksour, 2011). Only real samples are considered (no synthetic data);

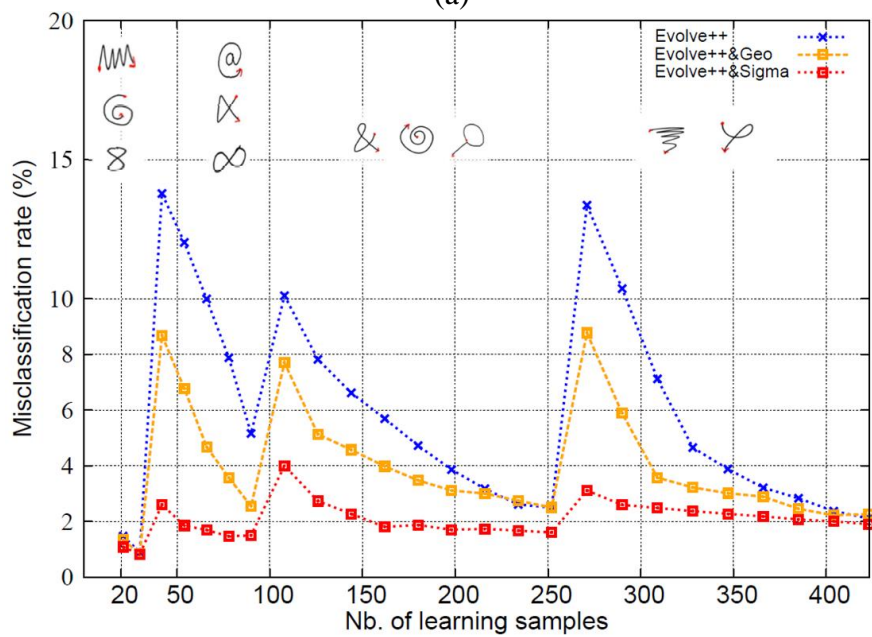
525 II. **Evolve++&Geo**: synthetic samples are generated by applying geometric distortions
526 and used along with real ones to train Evolve++ system;

527 III. **Evolve++&Sigma**: synthetic samples are generated using the lognormal-based
528 method.

529 The results are presented for two different experimental scenarios: the 11 gestures were
530 introduced together in the first case, while the gestures were progressively introduced in
531 the second one. We measure in the former the impact of the synthetic samples at the
532 beginning of the learning process from scratch, while the latter scenario aims at showing
533 the impact of these synthetic samples when introducing new gestures. The results of the
534 first set-up are presented in Fig.9.a and those of the second in Fig.9.b.



(a)



(b)

Fig. 9 (a) Performance improvement using synthetic data generation.

(b) Impact of synthetic samples when adding new gestures.

535

536 As it can be seen in Fig. 9.a, there is an important impact of the synthetic gestures at the

537 beginning of the incremental learning process. For example, the misclassification rate is

538 reduced by 50% for 10 real samples per class when the training is enriched with synthetic
539 samples. It must also be noted from the same figure that distortions applied on the sigma-
540 lognormal parameters produces a more realistic variability in the synthetic gestures as
541 compared to direct geometrical distortions. Thanks to the realistic human-like distortions,
542 the synthetic samples present a significant ability to predict the appearance of future real
543 samples, which significantly accelerates the adaptation process. Fig. 9.b shows that using
544 synthetic samples, the classifier resists much better when introducing new classes. It is
545 able to rapidly re-estimate all its parameters and to improve the recognition performance
546 for the old and the new gestures. Again, the superiority of lognormal-based deformations
547 over the traditional geometrical ones is quite apparent.

548 These experimental results show that sigma-lognormal based synthetic samples play an
549 important role in improving the classification performance and accelerating the learning
550 process both when it starts from scratch and also when new gestures are introduced. One
551 interest of the present methodology is also that it does not depend on the way reference
552 gestures are defined and collected. For example, gestures that are recorded on hand held
553 mobile devices while the user is standing up, writing vertically, or while he is sitting in a
554 moving vehicle could also be used to train the system without really affecting the results.

555

556 **7. Conclusion**

557 In the last few years, great advances have been done on the problem of parameter
558 extraction of the Sigma-Lognormal model. New algorithms have been designed which
559 now allow an automatic representation of complex human motions such as those involved
560 in on-line signature and handwriting. The availability of these new systems and a better

561 understanding of the variability of the sigma-lognormal parameters have paved the way
562 for the use of this model in the context of automatic generation of synthetic databases of
563 human movements. This paper has summarized the promising results of two different
564 investigations on that topic.

565 Needless to say that further research is still needed on the various topic addressed in this
566 paper, but as can already be seen, the use of the Sigma-Lognormal model for the
567 generation of human like movements offers very interesting perspectives for the field of
568 pattern recognition and the development of verification and recognition systems based on
569 human movements.

570

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