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A Semi Number-Theoretic approach and Proposed proof that CH is false.

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Abstract

We introduce number theoretic systems for use into the study of subsets associated with the reals. We will then show that such sets possess different cardinalities, whilst having cardinality greater than that of the naturals.

In mathematics, especially with new mathematics, there is a huge difficulty in drawing an entire picture of its usefulness in solving a particular problem or its effectiveness in the way of a theory in just a few lines, yet this is necessary, as very few will want to undertake the understanding of a new concept if one does not see its potential immediately!

So in the next few lines, we will aim to draw a picture of the usefulness of number theory in the resolution of CH. The time is ripe for many such pictures as was expressed in a Lecture by Matt Foreman, University of California, Irvine. Therein was described all mainstream set-theoretic attempts at resolving CH with concluding remarks surrounding other possible un-ventured possibilities which the writer writes "There are viable alternatives to the Woodin 'Solution' of the CH and these should be considered and explored before we rush to celebrate."

Many such attempts pioneered by W.H.Woodin involve a logical concept of models, as our approach is far from set theoretic we will not explore this concept beyond this mention.

Mathematic should not be just about finding an answer but more about understanding it. More specifically instead of some *reducto et absurdum* resolution to CH, a more appealing answer would express in conjunction with such an argument, (much like the original diagonalization) what specific property possessed by numbers disallows/allows a grouping of numbers so as to form a cardinality $\aleph_1 | \aleph_0 < \aleph_1 < 2^{\aleph_0}$.

INTRODUCTION.

At present, the only way it seems possible to form a set is physical inclusions where we physically choose individual numbers to include into a set or functional association, where we define a domain of values for the function and associate its outputs with a set. In each case the domain is in one to one correspondence with its functional output, so the cardinality of the output will have to be the same as that of its domain. The importance in knowing the ways and means of constructing such sets lies in the question, can one construct a set with cardi-

nality \aleph_1 ? to answer this, one needs ways of calling on all subsets of $\mathbb{R} = \mathbb{Q}^1$. To answer this we need to understand what it means for a number to belong to \mathbb{R} . One might simply say any number followed by numbers at infinitum forming its decimal portion must surely constitute an irrational number. There are two problems with this definition, for one there is no guarantee that such a number is surely irrational as certain rationals have this property. Secondly little contemplation shows such a definition has the exact shortcomings as that of the functional inclusion method, as we will physically have to include individual values to include into a set or make use of some functional argument that we associate with portions of decimal values associated with each such number, which renders the set inevitably countably finite.

Thus if we are to not form a set along the above lines, we require precision in our definition of $r \in \mathbb{R}$. A precise definition enables us to assign properties to the possible distributions associated with decimal portions of real numbers. This in turn might enable us to answer the question can one form a subset of the reals? If so what is its cardinality with respect to the reals? Or do all subsets of the reals have the same cardinality?

Finding such a definition might yet be simplified if we restrict ourselves decimal progressions solely in base two.

Thus by decimal progressions in base two we mean progressions of the form 11010001000101011101 forming the decimal portion of an arbitrary number. More specifically it is common to write base ten numbers in base two so long as the base ten numbers are associated with the naturals. One can just as easily extend this technique to cover base ten numbers belonging to the reals as well. Upon transforming such base ten numbers to base two it is the decimal portion of such transformed numbers that we will concern ourselves with.

For the sake of simplicity, we will denote such decimal portions of numbers by the symbol d and a collection i.e. set of d_i by the symbol \mathbb{D} . We will denote the number having the decimal portion d_i via use of the symbol n_i , thus for instance in 11.1001, $n_i = 11$, and we will denote the entire number simply by k_i .

Logic dictates that it is in the progression of 0,1 symbols associated with d_i that determines whether k_i is an element of the irrationals.

The defining characteristic of an irrational number is that it simply cant be expressed as a quotient of two integers. As simple as this sounds, one can obtain a vast amount of information from just this simple fact.

Let us start our analysis of this property by asking when is a number k expressible as a quotient $\frac{a}{b}$? The simple answer is that if $kb = a$ then it is so. Or alternatively if $\sum_k b = a$ then k is a rational number. The point we are to take

¹We needn't consider all subsets of R as any inclusion of elements of Q will be in one to one correspondence with Q

note of here is that k has an associated decimal portion whereas this is not the case of a . We may as such say that k belongs with the rationals if only if via a finite many summations of k to itself its decimal portion is reducible to zero. This is simple to see if d associated with k is finitely long, for every two summations, the decimal shifts a position to the right. So what then for rational numbers with infinitely long decimal numbers? We surely know of their existence as the simplest case is $\frac{1}{3}$ which in base two is expressed as $\frac{1}{11}$. A moment's deliberation of the base two fraction will show this to be true.

How then can such decimals be reducible to zero? As addition plays a key role in numbers associated with the rationals, we might attempt an understanding of what maybe required of the structure associated with such infinitely long decimal portions, in terms of addition, that renders itself as one of the rationals.

Following two simple rules, one can perform addition in base two, both of which pertain to the alignment of symbols associated with the numbers involved in the addition.

- a) (0) and (0) in alignment remains (0).
- a) (1) and (1) in alignment at some position p associated with the symbols leaves (0) at p and introduces (1) at $p - 1$.

This process is required (via finite additions) to reduce the decimal portions of the number to zero or alternatively $d_{one} = 1111111\dots$ at infinitum. The reason for the latter is simple to see if one understands why it is that $0.d_{one}$ is in fact equivalent to one see (Appendix 1) for a clear exposition of this.

To simplify our analysis, we will refer to an arbitrary number added to itself finitely many times as finite action on the number.

Consider now a decimal formed by a sequence of zeros and ones where every one symbol is spaced evenly apart by the same number of zero symbols as every other set of one symbols; as is in the following illustration.

$$d_u = (000010000100001) \tag{1}$$

It is easy to see that sufficient action on d_u (sufficient number of additions of d_u to itself) will transform the above sequence into:

$$(000100001000010)$$

and with further action into

$$(001000010000100)$$

With sufficiently many more all zeros inevitably begin to fill with one symbols

$$(110001100011000)$$

$$(111001110011100)$$

Until eventually.

(111111111111111)

This is due to the nature of the spacing being equal in measure of zero symbols between pairs of one symbols. As such logic dictates that forming a decimal with infinitely many such pairs of one symbols spaced **evenly** by finitely many zeros would surely render the chained decimal into d_{one} with finite action.

When then and for which schema of zero and ones is such a reduction impossible?

One logically intuitive idea seems to be the forming of a schema via the targeting of the measure of zero symbols alone that lie between pairs of one symbols. From the initial portion of this discourse, it seems clear that it is such spacing of one symbols that seems intuitively responsible for whether such reductions are possible or not. More clearly, the number of such periodically recursive intervals is irrelevant, as each interval is periodically equivalent to every other:

10001.....10001

and upon action (the number summed to itself), the result of the addition is that every respective interval involved in the addition process is transformed equivalently.

10001.....10001
10001.....10001
00010.....00010

If each such interval were to differ in the magnitude of zeros between pairs of one symbols one would easily find that with continuous action, the smaller such interval would initially fill out with one symbols before other larger such intervals. For instance given the following schema:

000100000100000001.....

the above with sufficient action would transform into:

111100111100001111.....

which illustrates how a smaller interval of zeros between ones would fill out with one symbols before other larger such intervals with sufficient action.

Naturally this argument can be generalized via induction. Specifically for ever growing such intervals, the smaller interval of any one set of growing intervals would naturally fill out in one symbols via finite action earlier than all others exceeding in measure of zero symbols, thus by this logic, if there is always one interval I_i exceeding in measure of zeros than every other, then by induction if it takes d summations to fill interval I_{i-1} then more summations than d is required to fill out I_i in this manner, since I is arbitrary, it is impossible to fill

every interval in this manner.

Let us formalize the means by which we define and describe such numbers and the sets to which these belong.

Definition 0.0 (Stream)

We define a **stream** to be an arbitrary series of non-terminating zeros and ones forming the decimal portion of a number.

An instance of this would be : 101.0001000101010001001....

We will from hereon denote such streams with the symbol \mathbb{S} .

Definition 1.0 (Interval)

A series of uninterrupted zero symbols in a stream we refer to as an **interval**.

We will denote these via use of the symbol \mathcal{I} .

Definition 2.0 (Action)

Given a stream \mathbb{S} the summing of a stream to itself we define to be an **action** on \mathbb{S} . We will denote such action via use of the symbol \mathcal{A}

Definition 3.0 (Packet)

Arbitrary portions of a stream we will refer to as packets. We will denote such packets via use of the symbol \mathcal{P}

For instance: 100101 in 1000000**100101**000000001, will be referred to as a packet.

Definition 4.0 (Packet/Interval-Measure)

We define a **packet/Interval-measure** to be the number of elements associated with any one packet/Interval. We will denote such measures via use of the symbol \mathcal{M} and $\mathcal{M}(\mathcal{I}), \mathcal{M}(\mathcal{P})$ will mean the measure associated with an interval, packet respectively.

In the above case the number (6) would be the measure of the packet.

Definition 5.0 (To Fill-Out a Packet)

We say a packet is filled out if via finite action, the positions associated with the contents of the packet with relation to the entire stream is filled with ones after sufficient action on the packet.

For instance if **1000000**100101000000001 is reduced to **1111111**.....0001, we say that the targeted packet has been filled out.

Theorem 1.0

Given a stream \mathbb{S} with filled-out packets spaced by ever-growing intervals, specifically each proceeding interval \mathcal{I}_i is greater in measure than its preceding interval \mathcal{I}_{i-1} , are irreducible to d_{one} via finite action on \mathbb{S} .

proof

If there is always one interval \mathcal{I}_i exceeding in measure of zeros than every other, then by induction if it takes d summations to fill interval \mathcal{I}_{i-1} then more sum-

mations than d is required to fill out \mathcal{I}_i in this manner, since \mathcal{I} is arbitrary, it is impossible to fill every interval in this manner. The nature of the packet following the interval is arbitrary in the way of affecting the nature of how fast such intervals are filled. \square

Sets of Packets.

As the nature of the packets are arbitrary in the way of affecting the nature of how fast such intervals are filled, we can firmly make the statement that the same is true of all arbitrary packets and not just those that are filled out.

In the way of grouping all such streams into a set, we may define the set in terms of the interval and Packet measures associated with all packet and interval measures associated with all elements of the set, and individual elements of the set will be those having arbitrary zeros and ones forming all associated packets of the stream forming the element.

To illustrate this, consider the number of streams one can form with the definition : $\forall t \in \mathbb{N}, \mathcal{M}(\mathcal{I}_t) = \mathcal{M}(\mathcal{I}_{t-1}) + 1$ and $\mathcal{M}(\mathcal{P}_i) = g|g \in \mathbb{N}$, for finitely countable $i \mapsto \mathbb{N}$.

If $g = 3$, we have the following few to be streams abiding by the same definition:

001 0 101 00 010 000..
101 0 111 00 110 000..
...

1-Spacing. With this new found ability a further endeavor would be to now try to construct a subset of such a set, or a set to which the above is a subset. Before this however we note some properties associated with numbers and their decimals.

Convention has it that given any one stream forming the decimal portion of a number, the nature of the positioning and spacing of one symbols determines the magnitude of the entire number. For instance: $11.01 > 11.001$, and $11.11011110; 11.11001110$. From these arbitrary examples one can easily see that a simple addition or removal of a single (one) at the right position can render the resultant number larger or smaller than the number in its initial form. In instances where sufficient intervals are available, it is easy to see that opportunities are always available in the way of adding or removing (ones) to effect change in the way of magnitude of the number from its initial form(See Fig 1.).

Subsets of Real Streams.

We shall define a real stream to be streams of the form:

Definition 6.0 (Real Stream)

$\mathbb{S}_R := \{\forall t \in \mathbb{N}, \mathcal{M}(\mathcal{I}_t) = \mathcal{M}(\mathcal{I}_{t-1}) + l$ and $\mathcal{M}(\mathcal{P}_i) = g|l, g \in \mathbb{N}\}$, for finitely countable $i \mapsto \mathbb{N}$.

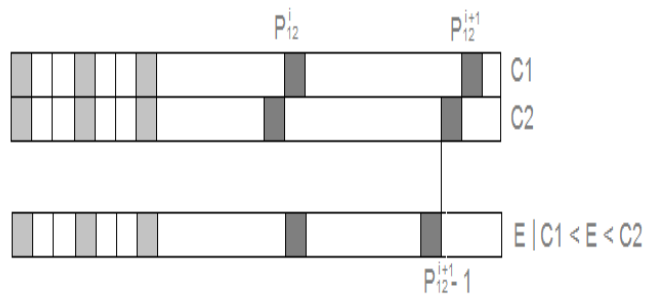


Figure 1: One symbols associated the same positions are highlighted in lite gray, while the darker gray highlights positions associated with one symbols that are different for c elements. The element E includes the symbols that are associated with the light gray positions, the position p_1 of the first pair P_{12}^1 and the position $P_{12}^2 - 1$. By doing this we form an element $E|_{c_1 < E < c_2}$.

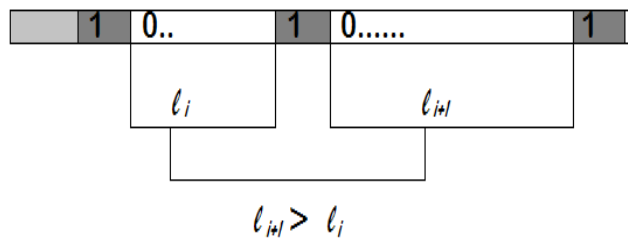


Figure 2: Real Stream: One symbols with growing number of zeros between them.

It is easy to see from the definition of real-streams that restricting positions available for the introduction of one symbols within each packet within the set definition, a means of constructing a sub set of such a set(See Fig 3.).

Restricted Sets.

We will from hereon refer to such sets as restricted real sets and we will use the symbol $/S$ to denote such sets.

An important junction has been reached here. One can take any two elements e_1, e_2 of S_R and by keeping some initial packet constant throughout all elements of $/S_R$ which is derived from either element of S_R , one can form an innumerous set of streams, all having elements whose magnitudes exist between e_1, e_2 , if all elements including e_1, e_2 were adjoined as a decimal of a single number.

With the aim of resolving the continuum hypothesis, we introduce the following useful terms.

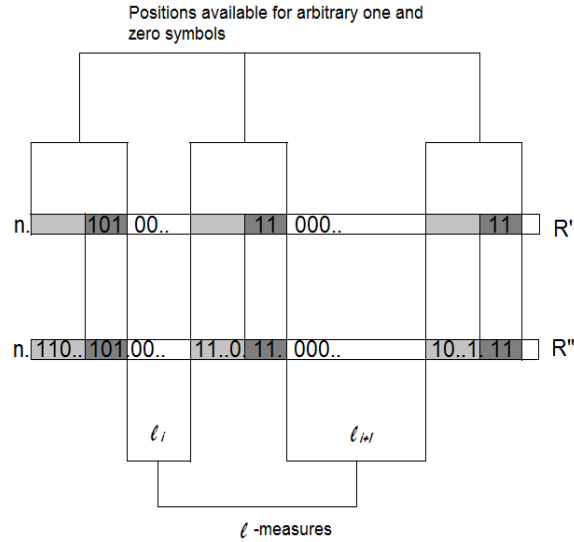


Figure 3:

Definition 6.0 (Stream/Interval/Packet-Positions)

We define the set of **positions** associated with a Stream/Interval/Packet, to be an ordered set, the elements of which are used in the identification of the location of specific zero/one elements within a Stream/Interval/Packet. These will be ordered from left to right.

For example in the packet associated with the following stream:

000000000000**101001** 000000000

the set : $\{p_1, p_2, p_3, p_4, p_5, p_6\}$ refers to the elements **101001** respectively. We will use the symbols p_1, p_2, \dots to denote such positions.

The Continuum Hypothesis

Introducing a new representation by shrinking positions of \mathbb{S}_R and \mathbb{S}_R that are in one to one correspondence down to a single representation $\alpha_1, \alpha_2, \dots$ for each element of \mathbb{S}_R , and by doing the same of \mathbb{S}_R and forming a stream of all zero/one elements not in one to one correspondence, i.e. of the elements left over in \mathbb{S}_R , we can rewrite the series in the following manner:

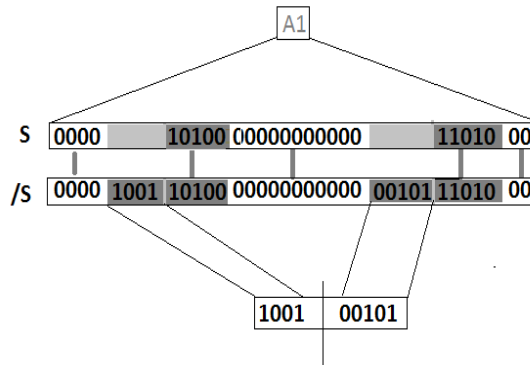


Figure 4:

\mathbb{S}_R	Unassociated packets of elements of $/\mathbb{S}_R$ made into a stream
α_1	[1000101001000101010...]
α_2	[1010100011010101010...]
α_3	[1010011000100100101...]

where in the above table, what is expressed to the right of the table is a stream formed with only packet entries associated with $/\mathbb{S}_R$ that can't be put into one to one correspondence with positions associated with the streams associated with \mathbb{S}_R .

What we see by the above process is that enumeration is again impossible as the characteristics of the elements associated with $\mathbb{S}_R, / \mathbb{S}_R$ is the same as that of \mathbb{N}, \mathbb{R} .

Specifically, we have formed two subsets of \mathbb{R} that have different cardinalities, both of which have cardinality greater than that of \mathbb{N} , as can be seen by forming packets of all streams associated with these sets and running through the diagonalization argument once again..

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