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# Mastering the Processing of Preferences by Using Symbolic Priorities in Possibilistic Logic

Souhila Kaci and Henri Prade<sup>1</sup>

**Abstract.** The paper proposes a new approach to the handling of preferences expressed in a compact way under the form of conditional statements. These conditional statements are translated into classical logic formulas associated with symbolic levels. Ranking two alternatives then leads to compare their respective amount of violation with respect to the set of formulas expressing the preferences. These symbolic violation amounts, which can be computed in a possibilistic logic manner, can be partially ordered lexicographically once put in a vector form. This approach is compared to the *ceteris paribus*-based CP-net approach, which is the main existing artificial intelligence approach to the compact processing of preferences. It is shown that the partial order obtained with the CP-net approach fully agrees with the one obtained with the proposed approach, but generally includes further strict preferences between alternatives (considered as being not comparable by the symbolic level logic-based approach). These additional strict preferences are in fact debatable, since they are not the reflection of explicit user's preferences but the result of the application of the *ceteris paribus* principle that implicitly, and quite arbitrarily, favors father node preferences in the graphical structure associated with conditional preferences. Adding constraints between symbolic levels for expressing that the violation of father nodes is less allowed than the one of children nodes, it is shown that it is possible to recover the CP-net-induced partial order. Due to existing results in possibilistic logic with symbolic levels, the proposed approach is computationally tractable.

Key words: preference, priority, partial order, CP-net, possibilistic logic.

## 1 Introduction

The compact representation of preferences has raised a vast interest in artificial intelligence in the last decade [5, 9, 18, 14, 10]. Indeed, it has been early recognized that, since value functions cannot be explicitly defined in case of a great number of alternatives described by means of attributes, preferences should be handled in a compact way, starting from non completely explicit preferences expressed by a user. In particular, conditional statements are often used for describing preferences in a local, contextualized manner. Moreover some generic principle is often used for completing the preferences [5, 14]. The CP-net approach [6] has emerged in the last decade as the preeminent and prominent method for processing preferences in artificial intelligence, due to its intuitive appeal. The CP-net approach directly exploits sets of conditional preferences and their associated graphical structures, assuming an apparently natural and innocuous *ceteris*

*paribus* principle that expresses that conditional preferences, which in general refer to two incompletely described alternatives, still hold when the specification of the two alternatives are completed in the same way. However, the CP-net approach may be computationally costly for dominance queries, which ask whether a ranking for two alternatives holds in any preference ordering that satisfies the CP-net requirements, rather than just asking if it holds in at least one of these preference orderings. This has led to look for tractable approximations of CP-nets [10, 18, 16].

Generally speaking, conditional statements express, in a given context, preferences about what are the most plausible states of the world according to pieces of default knowledge, or what are the most satisfactory states when expressing desires. It has been shown that conditional statements can be expressed under the form of constraints that may be turned into sets of prioritized logical formulas [17, 2, 1]. However, although the case might be encountered in practice, the available approaches for handling preferences do not usually allow for the simultaneous expression of general preferences and of more specific ones that are reversed with respect to the general tendency. In this latter case, the various levels of specificity of the conditionals induce a complete preorder on the logical formulas encoding the defaults. But, in the case of a set of (monotonic) conditional preference statements, we have not necessarily indications about their respective levels of importance. It is why in the following we encode the conditional preferences statements by means of classical logical formulas associated with symbolic priorities (since no a priori ordering between them is known), as already done in the approximation of CP-nets recently proposed [16].

Then the respective amount of preference violation of an alternative with respect to the set of formulas encoding the preferences, can be computed in a possibilistic logic manner [13, 3], and results in a conjunctive combination of symbolic levels. Such combinations of symbolic levels can be partially ordered lexicographically, once they are put in a vector form. After introducing the basic definitions in Section 2, this is explained in Section 3 on a motivating example taken from the CP-net literature. In Section 4, after a refresher on the CP-net approach, it is shown that the partial order obtained with the CP-net approach fully agrees with the one obtained with the symbolic priorities approach, but generally includes further strict preferences. A discussion shows that it is due to a debatable use of *ceteris paribus* principle on pairs of alternatives for which there is no inclusion relation between the two sets of preferences that they violate. Section 5 shows how the CP-net partial order can be recovered by adding constraints between symbolic levels for expressing that the violation of father nodes is less allowed than the one of children nodes. Such a representation framework, where logical formulas are associated with symbolic priority levels between which further con-

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straints may be added, is akin to the one presented in [3] (for handling multiple sources information) for which tractable computational procedures exist.

## 2 Definitions and notations

Let  $V = \{X_1, \dots, X_l\}$  be a set of  $l$  variables. Each variable  $X_i$  takes its values in a domain denoted  $Dom(X_i) = \{x_1^i, \dots, x_{m_i}^i\}$ . Let  $V'$  be a subset of  $V$ . An assignment of  $V'$  is the result of giving a value in  $Dom(X_i)$  to each variable  $X_i$  in  $V'$ .  $Asst(V')$  is the set of all possible assignments to variables in  $V'$ . In particular  $Asst(V)$ , denoted  $\Omega$ , is the set of all possible assignments of the variables in  $V$ . Each element in  $\Omega$ , denoted  $\omega$ , is called an alternative.

When dealing with binary variables, formulas of propositional logic are denoted  $a, b, c, \dots$ .

Let  $\succeq$  (resp.  $\succ$ ) be a binary relation on a finite set  $A = \{x, y, z, \dots\}$  such that  $x \succeq y$  (resp.  $x \succ y$ ) means that  $x$  is at least as preferred as (resp. strictly preferred to)  $y$ .  $x = y$  means that both  $x \succeq y$  and  $y \succeq x$  hold, i.e.  $x$  and  $y$  are equally preferred. Lastly  $x \sim y$  means that neither  $x \succeq y$  nor  $y \succeq x$  holds, i.e.  $x$  and  $y$  are incomparable.  $\succeq$  is a partial preorder on  $A$  if and only if  $\succeq$  is reflexive ( $x \succeq x$ ) and transitive (if  $x \succeq y$  and  $y \succeq z$  then  $x \succeq z$ ).  $\succ$  is a partial order on  $A$  if and only if  $\succ$  is irreflexive ( $x \succ x$  does not hold) and transitive. A partial order  $\succ$  may be defined from a partial preorder  $\succeq$  as  $x \succ y$  if  $x \succeq y$  holds but  $y \succeq x$  does not. A (pre-)order is asymmetric if and only if  $\forall x, y \in A$ , if  $x \succ y$  holds then  $y \succ x$  does not. A preorder  $\succeq$  on  $A$  is complete if and only if all pairs are comparable i.e.  $\forall x, y \in A$ , we have  $x \succeq y$  or  $y \succeq x$ .

## 3 Motivating example and preference encoding

We first motivate the proposed approach on an example inspired from [11] about how to be dressed for an evening party.

**Example 1** Let  $V$  (vest),  $P$  (pants),  $S$  (shirt) and  $C$  (shoes) be four binary variables taking their values in  $\{V_b, V_w\}$ ,  $\{P_b, P_w\}$ ,  $\{S_r, S_w\}$  and  $\{C_r, C_w\}$  respectively, where  $b, w$  and  $r$  stand respectively for black, white and red. Clearly there are sixteen possible evening dress  $\Omega = \{V_b P_b S_r C_r, V_b P_b S_w C_r, V_b P_w S_r C_r, V_b P_w S_w C_r, V_w P_b S_r C_r, V_w P_b S_w C_r, V_w P_w S_r C_r, V_w P_w S_w C_r, V_b P_b S_r C_w, V_b P_b S_w C_w, V_b P_w S_r C_w, V_b P_w S_w C_w, V_w P_b S_r C_w, V_w P_b S_w C_w, V_w P_w S_r C_w, V_w P_w S_w C_w\}$ . Assume that when choosing his evening dress, Peter is not able to compare the sixteen possible choices but expresses the following partial preferences:

- ( $P_1$ ): he prefers black vest to white vest,
- ( $P_2$ ): he prefers black pants to white pants,
- ( $P_3$ ): when vest and pants have the same color, he prefers red shirt to white shirt otherwise he prefers white shirt, and
- ( $P_4$ ): when the shirt is red then he prefers red shoes otherwise he prefers white shoes.

The problem now is how to rank-order the sixteen possible choices according to Peter's preferences.

The above preferences are conditionals of the form "in context  $c$ ,  $a$  is preferred to  $b$ ", where  $c$  may be a tautology. Such a preference can be modelled as a pair of prioritized goals  $\{(-c \vee a \vee b, 1), (-c \vee a, 1 - \alpha)\}$ , which stand for "when  $c$  is true, one should have  $a$  or  $b$  (the choice is only between  $a$  and  $b$ ) and in context  $c$ , it is somewhat imperative to have  $a$  true". These pairs of propositional formulas associated with a level are known as

possibilistic formulas [13]. Indeed, e.g.  $(-c \vee a, 1 - \alpha)$  encodes a constraint of the form  $\Pi(c \wedge \neg a) \leq \alpha$  ( $\equiv N(-c \vee a) \geq 1 - \alpha$ ), where  $\Pi, N$  are dual possibilistic measures ( $1 - \Pi(\neg p) = N(p)$ ). This expresses that the satisfaction level when the constraint is violated is upper bounded by  $\alpha$ .

Note that when  $b \equiv \neg a$ , the clause  $(-c \vee a \vee b, 1)$  becomes a tautology, and thus does not need to be written. Indeed the clause  $(-c \vee a, 1 - \alpha)$  expresses a preference for  $a$  over  $\neg a$  in context  $c$ . The clause  $(-c \vee a \vee b, 1)$  is only needed if  $a \vee b$  does not cover all the possible choices. Assume  $a \vee b \equiv \neg d$  (where  $\neg d$  is not a tautology), then it makes sense to understand the preference for  $a$  over  $b$  in context  $c$ , as the fact that in context  $c$ ,  $b$  is a default choice if  $a$  is not available. If one wants to open the door to remaining choices, it is always possible to use  $(-c \vee a \vee b, 1 - \alpha')$  with  $1 - \alpha' > 1 - \alpha$ , instead of  $(-c \vee a \vee b, 1)$ . Thus, the approach would easily extend to non binary choices.

**Example 2** (Example 1 continued) Thus  $P_1$  and  $P_2$  are encoded by means of (i) :  $\{(V_b, 1 - \alpha)\}$  and (ii) :  $\{(P_b, 1 - \beta)\}$  respectively.  $P_3$  is encoded by (iii) :  $\{(\neg V_b \vee \neg P_b \vee S_r, 1 - \gamma)\}$ , (iv) :  $\{(\neg V_w \vee \neg P_w \vee S_r, 1 - \eta)\}$ , (v) :  $\{(\neg V_w \vee \neg P_b \vee S_w, 1 - \delta)\}$  and (vi) :  $\{(\neg V_b \vee \neg P_w \vee S_w, 1 - \varepsilon)\}$ . Lastly  $P_4$  is encoded by (vii) :  $\{(\neg S_r \vee C_r, 1 - \theta)\}$  and (viii) :  $\{(\neg S_w \vee C_w, 1 - \rho)\}$ .

Note that we have chosen here, in order to be as general as possible, to give distinct symbolic priority levels for the formulas associated to the different contexts covered by a preference  $P_i$ .

Since one does not know precisely how imperative the preferences are, the weights will be handled in a symbolic manner. However, they are assumed to belong to a linearly ordered scale (the strict order will be denoted by  $>$  on this scale), with a top element (denoted 1) and a bottom element (denoted 0). Thus,  $1 - (\cdot)$  should be regarded here just as denoting an order-reversing map on this scale (without having a numerical flavor necessarily), with  $1 - (0) = 1$ , and  $1 - (1) = 0$ . On this scale, one has  $1 > 1 - \alpha$ , as soon as  $\alpha \neq 0$ . The order-reversing map exchanges two scales: the one graded in terms of necessity degrees, or if we prefer here in terms of imperativeness, and the one graded in terms of possibility degrees, i.e. here, in terms of satisfaction levels. Thus, the level of priority  $1 - \alpha$  for satisfying a preference is changed by the involutive mapping  $1 - (\cdot)$  into a satisfaction level  $\alpha < 1$  when this preference is violated.

Since in the example the values of the weights  $1 - \alpha, 1 - \beta, 1 - \gamma, 1 - \eta, 1 - \delta, 1 - \varepsilon, 1 - \theta$  and  $1 - \rho$  are unknown, no particular ordering is assumed between them. Table 1 gives the satisfaction levels of the above clauses and the sixteen possible choices. The last column gives the vector of the global satisfaction, exhibiting symbolic satisfaction levels that are different from 1, each time a formula is violated. In practice, this violation amounts can be syntactically computed using the approach proposed in [3].

Even if the values of the weights are unknown, a partial order between the sixteen choices can be naturally induced. For example  $V_b P_b S_r C_r$  is preferred to all remaining alternatives since it is the only alternative that satisfies all Peter's preferences. Also,  $V_w P_b S_w C_w$  is preferred to  $V_w P_w S_r C_r$  since the former falsifies  $(V_b, 1 - \alpha)$  while the latter falsifies both  $(V_b, 1 - \alpha)$  and  $(P_b, 1 - \beta)$ . This partial order is depicted in Figure 1. An edge from  $\omega$  to  $\omega'$  means that  $\omega'$  is preferred to  $\omega$ .

Indeed an alternative  $\omega$  is naturally preferred to an alternative  $\omega'$  when the set of clauses falsified by  $\omega$  is included in the set of clauses

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	satisfaction levels
$V_b P_b S_r C_r$	1	1	1	1	1	1	1	1	(1, 1, 1, 1, 1, 1, 1, 1)
$V_b P_b S_w C_r$	1	1	$\gamma$	1	1	1	1	$\rho$	(1, 1, $\gamma$ , 1, 1, 1, 1, $\rho$ )
$V_b P_w S_r C_r$	1	$\beta$	1	1	1	$\varepsilon$	1	1	(1, $\beta$ , 1, 1, 1, 1, $\varepsilon$ , 1, 1)
$V_b P_w S_w C_r$	1	$\beta$	1	1	1	1	1	$\rho$	(1, $\beta$ , 1, 1, 1, 1, 1, $\rho$ )
$V_w P_b S_r C_r$	$\alpha$	1	1	1	$\delta$	1	1	1	( $\alpha$ , 1, 1, 1, $\delta$ , 1, 1, 1)
$V_w P_b S_w C_r$	$\alpha$	1	1	1	1	1	1	$\rho$	( $\alpha$ , 1, 1, 1, 1, 1, 1, $\rho$ )
$V_w P_w S_r C_r$	$\alpha$	$\beta$	1	1	1	1	1	1	( $\alpha$ , $\beta$ , 1, 1, 1, 1, 1, 1)
$V_w P_w S_w C_r$	$\alpha$	$\beta$	1	$\eta$	1	1	1	$\rho$	( $\alpha$ , $\beta$ , 1, $\eta$ , 1, 1, 1, $\rho$ )
$V_b P_b S_r C_w$	1	1	1	1	1	1	$\theta$	1	(1, 1, 1, 1, 1, 1, $\theta$ , 1)
$V_b P_b S_w C_w$	1	1	$\gamma$	1	1	1	1	1	(1, 1, $\gamma$ , 1, 1, 1, 1, 1)
$V_b P_w S_r C_w$	1	$\beta$	1	1	1	$\varepsilon$	$\theta$	1	(1, $\beta$ , 1, 1, 1, 1, $\varepsilon$ , $\theta$ , 1)
$V_b P_w S_w C_w$	1	$\beta$	1	1	1	1	1	1	(1, $\beta$ , 1, 1, 1, 1, 1, 1)
$V_w P_b S_r C_w$	$\alpha$	1	1	1	$\delta$	1	$\theta$	1	( $\alpha$ , 1, 1, 1, $\delta$ , 1, $\theta$ , 1)
$V_w P_b S_w C_w$	$\alpha$	1	1	1	1	1	1	1	( $\alpha$ , 1, 1, 1, 1, 1, 1, 1)
$V_w P_w S_r C_w$	$\alpha$	$\beta$	1	1	1	1	$\theta$	1	( $\alpha$ , $\beta$ , 1, 1, 1, 1, 1, $\theta$ , 1)
$V_w P_w S_w C_w$	$\alpha$	$\beta$	1	$\eta$	1	1	1	1	( $\alpha$ , $\beta$ , 1, $\eta$ , 1, 1, 1, 1)

Table 1. Satisfaction levels.

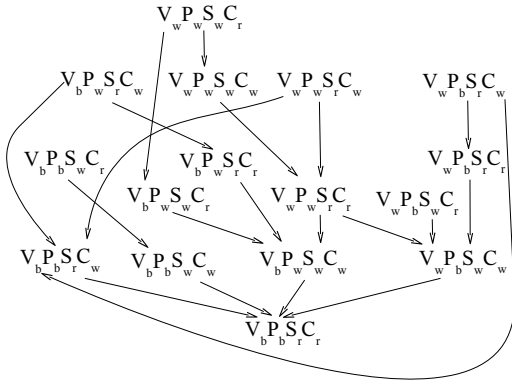


Figure 1. Basic partial order.

falsified by  $\omega'$ .

**Definition 1 (Basic preference relation)** Let  $\Sigma = \{(a_i, \alpha_i)\}$  be a set of formulas associated with symbolic weights. Let  $\omega$  and  $\omega'$  be two alternatives and  $\mathcal{F}_\omega$  and  $\mathcal{F}_{\omega'}$  be the sets of  $\Sigma$  falsified by  $\omega$  and  $\omega'$  respectively.  $\omega$  is basically preferred to  $\omega'$ , denoted  $\omega \succ_{b,\Sigma} \omega'$ , iff  $\mathcal{F}_\omega \subset \mathcal{F}_{\omega'}$ .

Thus  $\omega$  is preferred to  $\omega'$  only when the components of its associated satisfaction vector are equal to 1 for those components that are different in the two satisfaction vectors associated to  $\omega$  and  $\omega'$ . Formally we describe the basic preference relation as follows. Let  $v = (v_1, \dots, v_n)$  and  $v' = (v'_1, \dots, v'_n)$  be two vectors of satisfaction levels. These satisfaction levels are ordered according to the order in which we consider the formulas. In our example from (i) to (viii). Discrimin criterion [12] is defined by ignoring the values that are the same in both  $v$  and  $v'$  for a given vector's component pertaining to the same formula. For example the two vectors  $v = (\alpha, \beta, 1, 1, 1, 1, 1, 1)$  and  $v' = (1, \beta, 1, 1, 1, 1, \varepsilon, 1, 1)$  reduce to  $d(v) = (\alpha, 1)$  and  $d(v') = (1, \varepsilon)$  respectively. For further comparing the reduced vectors, we define the following preference re-

lation (called “ordered Pareto” and denoted  $\succ_{OP}$ ) that exploits the available information about the relative values of the symbolic levels. Then  $v$  is preferred to  $v'$ , denoted  $v \succ_{OP} v'$ , if there is a reordering of each vector of symbolic levels such that  $\forall i, d_o(v_i) \geq d_o(v'_i)$  and  $\exists j, d_o(v_j) > d_o(v'_j)$  according to the current knowledge about the ordering between symbolic levels, where  $d_o(v)$  is the reordered vector associated to  $d(v)$ .

Initially the only available knowledge about the ordering between the symbolic levels is  $\alpha < 1$  when  $\alpha \neq 1$  and  $1 \leq 1$ . Then, for example  $d(v) = (\alpha, 1)$  and  $d(v') = (1, \varepsilon)$  are incomparable. Now if we also know that  $\alpha > \varepsilon$  then  $v \succ_{OP} v'$  (i.e.  $(\alpha, \beta, 1, 1, 1, 1, 1, 1) \succ_{OP} (1, \beta, 1, 1, 1, 1, \varepsilon, 1, 1)$ ) since  $d_o(v) = (\alpha, 1)$  and  $d_o(v') = (\varepsilon, 1)$  are now Pareto comparable.

**Proposition 1** Let  $\Sigma = \{(a_i, \alpha_i)\}$  be a set of formulas. Let  $\omega$  and  $\omega'$  be two alternatives. Let  $\mathcal{F}_\omega$  and  $\mathcal{F}_{\omega'}$  be the sets of formulas of  $\Sigma$  falsified by  $\omega$  and  $\omega'$  respectively. Let  $v$  and  $v'$  be the satisfaction levels of  $\omega$  and  $\omega'$  respectively. Then,

$$\omega \succ_{b,\Sigma} \omega' \text{ iff } v \succ_{OP} v'.$$

Each additional preference between two alternatives should be the consequence of an explicit constraint between symbolic weights. For example  $V_b P_b S_w C_w$  and  $V_b P_b S_r C_w$  are incomparable since  $\gamma$  and  $\theta$  are incomparable. Now if we state that  $\theta > \gamma$  then  $V_b P_b S_r C_w$  would be preferred to  $V_b P_b S_w C_w$ .

#### 4 Conditional Preference Networks (CP-nets)

Conditional preference networks (CP-nets for short) [5] encode comparative conditional statements and are based on ceteris paribus principle. More precisely, a CP-net is a directed graphical representation of conditional preferences, where nodes represent variables and edges express preference links between variables. When there exists a link from  $X$  to  $Y$ ,  $X$  is called a parent of  $Y$ .  $Pa(X)$  denotes the set of parents of a given node  $X$ . It determines the user's preferences over possible values of  $X$ . For the sake of simplicity, we suppose that variables are binary. Preferences are expressed at each node by means of a conditional preference table (CPT for short) such that:

- for root nodes  $X_i$ , the conditional preference table, denoted  $CPT(X_i)$ , provides the strict preference<sup>2</sup> over  $x^i$  and its negation  $\neg x^i$ , other things being equal, i.e.  $\forall y \in Asst(Y), x^i y \succ \neg x^i y$  where  $Y = V \setminus \{X_i\}$ . This is the ceteris paribus principle.
- For other nodes  $X_j$ ,  $CPT(X_j)$  describes the preferences over  $x^j$  and  $\neg x^j$  other things being equal given any assignment of  $Pa(X_j)$ , i.e.  $x^j z y \succ \neg x^j z y, \forall z \in Asst(Pa(X_j))$  and  $\forall y \in Asst(Y)$  where  $Y = V \setminus (\{X_j\} \cup Pa(X_j))$ . For each assignment  $z$  of  $Pa(X_j)$  we write for short a statement of the form  $z : x^j \succ \neg x^j$ . Note that this is a parent-dependent specification.

**Definition 2** A complete preorder  $\succeq$  on  $\Omega$ , called also preference ranking, satisfies a CP-net  $N$  if and only if it satisfies each conditional preference expressed in  $N$ . In this case, we say that the preference ranking  $\succeq$  is consistent with  $N$ .

A CP-net  $N$  is consistent when there exists an asymmetric preference ranking that is consistent with  $N$ . We focus in this paper on acyclic CP-nets in order to ensure their consistency.

**Definition 3 (Preference entailment)** Let  $N$  be a CP-net over a set of variables  $V$ , and  $\omega, \omega' \in \Omega$ .  $N$  entails that  $\omega$  is strictly preferred to  $\omega'$ , denoted  $\omega \succ_N \omega'$ , if and only if  $\omega \succ \omega'$  holds in every preference ranking  $\succeq$  that satisfies  $N$ .

Indeed  $\succ_N$  is the intersection of all preference rankings consistent with  $N$ . When  $\omega \succ_N \omega'$  holds, we say that  $\omega$  dominates  $\omega'$ . The preferential comparison in CP-nets is based on the notion of *worsening flip*. A worsening flip is a change of the assignment of a variable to an assignment that is less preferred following the conditional preference table of that variable, and under ceteris paribus assumption, w.r.t. the CP-net  $N$ . Then  $\omega$  is preferred to  $\omega'$  w.r.t.  $N$  iff there is a chain of worsening flips from  $\omega$  to  $\omega'$ .

**Example 3 (Example 1 continued)** Peter's preferences can be represented by the CP-net depicted in Figure 2. As one would expect, the CP-net fully agrees with basic preference relation.

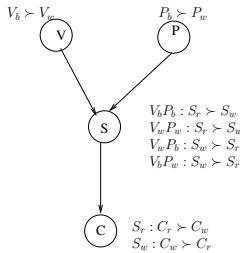


Figure 2. A CP-net and its associated order.

**Proposition 2** Let  $N$  be a CP-net. Let  $\Sigma = \{(\neg u_i \vee x, \alpha_i)\}$  where  $u_i : x \succ \neg x$  are unconditional/conditional local preferences expressed in  $N$ . Then,  $\forall \omega, \omega' \in \Omega$ ,

$$\text{if } \omega \succ_{b, \Sigma} \omega' \text{ then } \omega \succ_N \omega'.$$

For example  $V_w P_w S_r C_r$  falsifies  $(V_b, 1 - \alpha)$ ,  $(P_b, 1 - \beta)$  and  $V_w P_w S_w C_r$  falsifies  $(V_b, 1 - \alpha)$ ,  $(P_b, 1 - \beta)$ ,

<sup>2</sup> We restrict ourselves to a complete order over  $x^i$  and  $\neg x^i$  as it is the case with CP-nets in general. However this can be easily extended to a preorder.

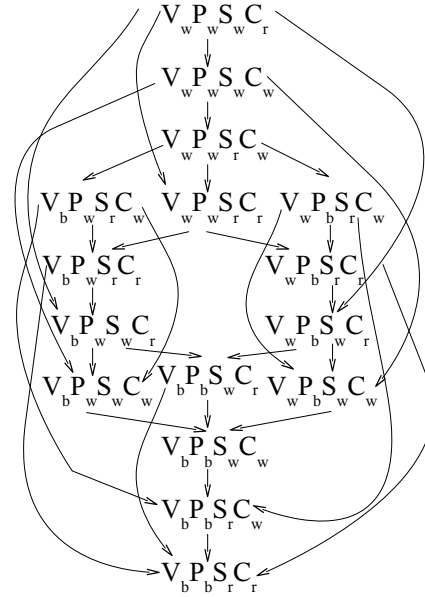


Figure 3. A CP-net and its associated order.

$(\neg V_w \vee \neg P_w \vee S_r, 1 - \eta)$  and we have  $V_w P_w S_r C_r \succ_N V_w P_w S_w C_r$ .

However as we can check in Figure 2, the partial order associated to the CP-net is more refine than the basic preference relation, i.e. some incomparabilities in the latter have been turned into strict comparabilities in the former. For example  $V_w P_b S_r C_w$  is preferred to  $V_w P_w S_r C_w$  w.r.t. the CP-net while they are incomparable w.r.t.  $\succ_{b, \Sigma}$  since  $V_w P_b S_r C_w$  falsifies  $(V_b, 1 - \alpha)$ ,  $(\neg V_w \vee \neg P_b \vee S_w, 1 - \delta)$  and  $(\neg S_r \vee C_r, 1 - \theta)$  while  $V_w P_w S_r C_w$  falsifies  $(V_b, 1 - \alpha)$ ,  $(P_b, 1 - \beta)$  and  $(\neg S_r \vee C_r, 1 - \theta)$ .

These additional strict preferences are due to the fact that preferences in CP-nets depend on the structure of the graph. More precisely, since preferences over the values of a variable are conditioned on the values of its parents, the application of ceteris paribus principle implicitly gives priority to father nodes. For example  $V_w P_b S_r C_w \succ_N V_w P_w S_r C_w$  due to  $P_b \succ P_w$ . Indeed  $V_w P_w S_r C_w$  is less preferred than  $V_w P_b S_r C_w$  since the former falsifies  $(P_b, 1 - \beta)$  while the latter falsifies  $(\neg V_w \vee \neg P_b \vee S_w, 1 - \delta)$  (they both falsify  $(V_b, 1 - \alpha)$  and  $(\neg S_r \vee C_r, 1 - \theta)$ ). Indeed when two alternatives  $\omega$  and  $\omega'$  differ on the value of one variable only,  $\omega$  is preferred to  $\omega'$  w.r.t. a CP-net if and only if

- either  $\mathcal{F}_\omega \subset \mathcal{F}_{\omega'}$ , (cf. Definition 1)
- or  $\omega'$  falsifies a father node preference while  $\omega$  falsifies a child node preference.

## 5 Encoding CP-nets

We show in this section that the partial order associated to a CP-net can be retrieved in our approach using additional constraints on symbolic levels. This encoding follows three steps:

- Let  $X$  be a node in the CP-net  $N$  and  $CPT(X)$  be its associated conditional preference table. For each local preference

$u_i : x \succ \neg x$  in  $CPT(X)$  we associate a base made of one formula  $\neg u_i \vee x$  as follows  $\Sigma_{X,u_i} = \{(\neg u_i \vee x, 1 - \alpha_i)\}$ . We do not add  $(\neg u_i \vee x \vee \neg x, 1)$  since we are dealing with binary variables.

- For each node  $X$  in the CP-net  $\mathcal{N}$ , build  $\Sigma_X = \bigcup_i \Sigma_{X,u_i}$  where the bases  $\Sigma_{X,u_i}$  have been obtained at the previous step. Then  $\Sigma = \bigcup_X \Sigma_X$  is the partially ordered base associated to  $\mathcal{N}$ .
- For each formula  $(\neg u_i \vee x, 1 - \alpha_i)$  in  $\Sigma_X$  and each formula  $(\neg u_j \vee y, 1 - \alpha_j)$  in  $\Sigma_Y$  such that  $X$  is a father of  $Y$  and we are in the same context, i.e.  $\neg u_j = \neg x \vee \neg u_k$ , we put  $1 - \alpha_i > 1 - \alpha_j$ .

**Example 4** (Example 1 cont'd) We have  $\Sigma = \{(V_b, 1 - \alpha), (P_b, 1 - \beta), (\neg V_b \vee \neg P_b \vee S_r, 1 - \gamma), (\neg V_w \vee \neg P_w \vee S_r, 1 - \eta), (\neg V_w \vee \neg P_b \vee S_w, 1 - \delta), (\neg V_b \vee \neg P_w \vee S_w, 1 - \varepsilon), (\neg S_r \vee C_r, 1 - \theta), (\neg S_w \vee C_w, 1 - \rho)\}$ .

We define the following constraints between symbolic weights, which express that constraints associated with father nodes have priority w.r.t. the ones associated with their child nodes:

$1 - \alpha > 1 - \gamma, 1 - \alpha > 1 - \varepsilon, 1 - \beta > 1 - \gamma, 1 - \beta > 1 - \delta, 1 - \gamma > 1 - \theta, 1 - \eta > 1 - \theta, 1 - \delta > 1 - \rho, 1 - \varepsilon > 1 - \rho$  which are equivalent to  $\alpha < \gamma < \theta, \alpha < \varepsilon < \rho, \beta < \delta < \rho, \beta < \gamma$  and  $\eta < \theta$ .

Then we have the following general result:

**Proposition 3** Let  $\mathcal{N}$  be a CP-net and  $\Sigma$  be its associated formulas base as described above and its associated partial order  $>$  on symbolic levels. Then  $\forall \omega, \omega' \in \Omega$ ,

$$v \succ_{OP} v' \text{ iff } \omega \succ_N \omega',$$

where  $v$  (resp.  $v'$ ) is the vector of satisfaction levels associated to  $\omega$  (resp.  $\omega'$ ).

**Example 5** (Example 1 continued) Let us consider again the two alternatives  $\omega : V_w P_b S_r C_w$  and  $\omega' : V_w P_w S_r C_w$ . We have  $v_\omega = (\alpha, 1, 1, 1, \delta, 1, \theta, 1)$  and  $v_{\omega'} = (\alpha, \beta, 1, 1, 1, 1, \theta, 1)$ . Then  $v_\omega \succ_{OP} v_{\omega'}$  since  $v_\omega$  and  $v_{\omega'}$  reduce to  $(1, \delta)$  and  $(\beta, 1)$  following discrimin criterion, i.e.  $d(v_\omega) = (1, \delta)$  and  $d(v_{\omega'}) = (\beta, 1)$ . Now since  $\delta > \beta$ ,  $(1, \delta)$  and  $(\beta, 1)$  can be reordered into  $d_o(v_\omega) = (\delta, 1)$  and  $d_o(v_{\omega'}) = (\beta, 1)$  such that we have  $\delta > \beta$  and  $1 \geq 1$ . We can check that  $V_w P_b S_r C_w \succ_N V_w P_w S_r C_w$ .

Generally speaking, the proposed approach allows us to add any further constraint between priority levels, which may privilege a particular child node if it is desirable, or express, as in TCP-nets [7], a conditional relative importance of the satisfaction of a particular requirement over another. Indeed a contextual preference in favor of a variable attached to a node can be expressed in our framework by means of additional constraints between symbolic levels.

## 6 Conclusion

The paper has proposed an encoding of conditional preferences by means of classical logic formulas associated with symbolic priority levels, in a possibilistic logic manner. It has led to the definition of a natural partial order that is always more cautious than the corresponding partial order obtained with a CP-net approach. Moreover adding constraints between symbolic priority levels has enabled us to recover the CP-net partial order exactly (although as explained in the paper, the strict preferences found in the CP-net approach, but not with our approach, are debatable). The approach can benefit from the

existence of a computationally tractable inference procedure in possibilistic logic with partially ordered symbolic levels [3].

Besides, it is worth noticing that the representation obtained looks similar to an hybrid possibilistic Bayesian-like network [4] since each node of the graphical structure reflecting the conditional preferences is associated with a set of constraints encoded by possibilistic logic-like formulas. The precise linkage between the representation presented in this paper and hybrid possibilistic networks is a topic of a further research.

Lastly the proposed approach might be applied to the management of preference queries addressed to a database for rank-ordering the answers according to their amounts violation of conditional preferences associated to the queries, and thus contributes to an active database research trend [8, 15].

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