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Six-parameters deformations of fourth order Peregrine breather solutions of the NLS equation.

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Abstract

We construct solutions of the focusing NLS equation as a quotient of two determinants. This formulation gives in the case of the order 4, new deformations of the Peregrine breather with 6 real parameters. We construct families of quasi-rational solutions of the NLS equation and describe the apparition of multi rogue waves. With this method, we construct the analytical expressions of deformations of the Peregrine breather of order 4 with 6 real parameters and plot different types of rogue waves.

1 Introduction

The nonlinear Schrödinger equation was first solved in 1972 by Zakharov and Shabat, using the inverse scattering method. The first quasi-rational solutions of NLS equation were constructed in 1983 by Peregrine. In 1986 Eleonski, Akhmediev and Kulagin obtained the two-phase almost periodic solution to the NLS equation and obtained the first higher order analogue of the Peregrine breather[3]. In a series of articles [1, 2], other families of higher order were constructed by Akhmediev et al., using Darboux transformations. In 2010, it has been proved that rational solutions of the NLS equation can be written as a quotient of two Wronskians in [8]. Recently, a new representation of the solutions of the NLS equation has been constructed in [11], in

terms of a ratio of two Wronskians determinants of even order $2N$ composed of elementary functions; these solutions of NLS are called of order N . By the passage to the limit when some parameter tended to 0, it has been possible to obtain quasi-rational solutions of the NLS equation.

It can be compared with the results obtained recently by Akhmediev et al. in [5] with Darboux dressing method and numerical approach.

In 2012, an other representation of the solutions of the focusing NLS equation, as a ratio of two determinants has been given in [14] using generalized Darboux transform.

Ohta and Yang [18] have given a new approach where solutions of the focusing NLS equation by means of a determinant representation, obtained from Hirota bilinear method.

At the beginning of the year 2012, one obtained a representation in terms of determinants which does not involve limits [13].

But, these first two formulations given in [11, 13] did depend in fact only on two parameters; this remark was first pointed out by V.B. Matveev for [11]. Then we found for the order N (for determinants of order $2N$), solutions depending on $2N - 2$ real parameters.

With this new method, we construct new deformations at order 4 with 6 real parameters, which are similar to those first found by V.B Matveev and P. Dubard recently published [9].

2 Degenerate determinant representation of solutions of NLS equation

2.1 Quasi-rational solutions of NLS equation in terms of a limit of wronskians

We consider the focusing NLS equation

$$iv_t + v_{xx} + 2|v|^2v = 0. \quad (1)$$

We recall briefly the results obtained in [11].

We consider the $2N$ parameters λ_ν , $\nu = 1, \dots, 2N$ satisfying the relations

$$0 < \lambda_j < 1, \quad \lambda_{N+j} = -\lambda_j, \quad 1 \leq j \leq N. \quad (2)$$

We consider the terms κ_ν , δ_ν , γ_ν functions of the parameters λ_ν given by the following equations,

$$\kappa_\nu = 2\sqrt{1 - \lambda_\nu^2}, \quad \delta_\nu = \kappa_\nu \lambda_\nu, \quad \gamma_\nu = \sqrt{\frac{1 - \lambda_\nu}{1 + \lambda_\nu}}, \quad (3)$$

and

$$\kappa_{N+j} = \kappa_j, \quad \delta_{N+j} = -\delta_j, \quad \gamma_{N+j} = 1/\gamma_j, \quad j = 1 \dots N. \quad (4)$$

The terms $x_{r,\nu}$ ($r = 3, 1$) are defined by

$$x_{r,\nu} = (r - 1) \ln \frac{\gamma_\nu - i}{\gamma_\nu + i}, \quad 1 \leq j \leq 2N. \quad (5)$$

The parameters e_ν are defined by

$$e_j = ia_j - b_j, \quad e_{N+j} = ia_j + b_j, \quad 1 \leq j \leq N, \quad (6)$$

where a_j and b_j , for $1 \leq j \leq N$ are arbitrary real numbers.

We use the following notations :

$$\Theta_{r,\nu} = \kappa_\nu x/2 + i\delta_\nu t - ix_{r,\nu}/2 + \gamma_\nu y - ie_\nu, \quad 1 \leq \nu \leq 2N.$$

We consider the functions

$$\begin{aligned} \phi_{r,\nu}(y) &= \sin \Theta_{r,\nu}, & 1 \leq \nu \leq N, \\ \phi_{r,\nu}(y) &= \cos \Theta_{r,\nu}, & N + 1 \leq \nu \leq 2N. \end{aligned} \quad (7)$$

$W_r(y) = W(\phi_{r,1}, \dots, \phi_{r,2N})$ is the wronskian

$$W_r(y) = \det[(\partial_y^{\mu-1} \phi_{r,\nu})_{\nu, \mu \in [1, \dots, 2N]}]. \quad (8)$$

Then we have the following result [11].

Theorem 2.1 *The function v defined by*

$$v(x, t) = \frac{W_3(0)}{W_1(0)} \exp(2it - i\varphi). \quad (9)$$

is solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2 v = 0.$$

Then, we take the limit when the parameters $\lambda_j \rightarrow 1$ for $1 \leq j \leq N$ and $\lambda_j \rightarrow -1$ for $N + 1 \leq j \leq 2N$.

We consider the parameter λ_j written in the form

$$\lambda_j = 1 - 2j^2\epsilon^2, \quad 1 \leq j \leq N. \quad (10)$$

When ϵ goes to 0, then we get quasi-rational solutions of the NLS equation given by [11] :

Theorem 2.2 *The function v defined by*

$$v(x, t) = \exp(2it - i\varphi) \lim_{\epsilon \rightarrow 0} \frac{W_3(0)}{W_1(0)}, \quad (11)$$

is a quasi-rational solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2v = 0.$$

2.2 Expression of solutions of NLS equation in terms of a ratio of two determinants without limit

We give the construction of the solutions of the NLS equation expressed as a quotient of two determinants which does not involve a passage to the limit. For this we need the following notations :

$$\begin{aligned} A_\nu &= \kappa_\nu x/2 + i\delta_\nu t - ix_{3,\nu}/2 - ie_\nu/2, \\ B_\nu &= \kappa_\nu x/2 + i\delta_\nu t - ix_{1,\nu}/2 - ie_\nu/2, \end{aligned}$$

for $1 \leq \nu \leq 2N$, with κ_ν , δ_ν , $x_{r,\nu}$ defined in (3), (4) and (5). The parameters e_ν are defined by (6).

Below we use the following functions :

$$\begin{aligned} f_{4j+1,k} &= \gamma_k^{4j-1} \sin A_k, & f_{4j+2,k} &= \gamma_k^{4j} \cos A_k, \\ f_{4j+3,k} &= -\gamma_k^{4j+1} \sin A_k, & f_{4j+4,k} &= -\gamma_k^{4j+2} \cos A_k, \end{aligned} \quad (12)$$

for $1 \leq k \leq N$, and

$$\begin{aligned} f_{4j+1,k} &= \gamma_k^{2N-4j-2} \cos A_k, & f_{4j+2,k} &= -\gamma_k^{2N-4j-3} \sin A_k, \\ f_{4j+3,k} &= -\gamma_k^{2N-4j-4} \cos A_k, & f_{4j+4,k} &= \gamma_k^{2N-4j-5} \sin A_k, \end{aligned} \quad (13)$$

for $N + 1 \leq k \leq 2N$.

We define the functions $g_{j,k}$ for $1 \leq j \leq 2N$, $1 \leq k \leq 2N$ in the same way, we replace only the term A_k by B_k .

$$\begin{aligned} g_{4j+1,k} &= \gamma_k^{4j-1} \sin B_k, & g_{4j+2,k} &= \gamma_k^{4j} \cos B_k, \\ g_{4j+3,k} &= -\gamma_k^{4j+1} \sin B_k, & g_{4j+4,k} &= -\gamma_k^{4j+2} \cos B_k, \end{aligned} \quad (14)$$

for $1 \leq k \leq N$, and

$$\begin{aligned} g_{4j+1,k} &= \gamma_k^{2N-4j-2} \cos B_k, & g_{4j+2,k} &= -\gamma_k^{2N-4j-3} \sin B_k, \\ g_{4j+3,k} &= -\gamma_k^{2N-4j-4} \cos B_k, & g_{4j+4,k} &= \gamma_k^{2N-4j-5} \sin B_k, \end{aligned} \quad (15)$$

for $N + 1 \leq k \leq 2N$.

Then we get the following result :

Theorem 2.3 *The function v defined by*

$$v(x, t) = \exp(2it - i\varphi) \times \frac{\det((n_{jk})_{j,k \in [1,2N]})}{\det((d_{jk})_{j,k \in [1,2N]})} \quad (16)$$

is a quasi-rational solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2v = 0,$$

where

$$n_{j1} = f_{j,1}(x, t, 0), \quad 1 \leq j \leq 2N \quad n_{jk} = \frac{\partial^{2k-2} f_{j,1}}{\partial \epsilon^{2k-2}}(x, t, 0), \quad 2 \leq k \leq N, \quad 1 \leq j \leq 2N$$

$$n_{jN+1} = f_{j,N+1}(x, t, 0), \quad 1 \leq j \leq 2N \quad n_{jN+k} = \frac{\partial^{2k-2} f_{j,N+1}}{\partial \epsilon^{2k-2}}(x, t, 0), \quad 2 \leq k \leq N, \quad 1 \leq j \leq 2N$$

$$d_{j1} = g_{j,1}(x, t, 0), \quad 1 \leq j \leq 2N \quad d_{jk} = \frac{\partial^{2k-2} g_{j,1}}{\partial \epsilon^{2k-2}}(x, t, 0), \quad 2 \leq k \leq N, \quad 1 \leq j \leq 2N$$

$$d_{jN+1} = g_{j,N+1}(x, t, 0), \quad 1 \leq j \leq 2N \quad d_{jN+k} = \frac{\partial^{2k-2} g_{j,N+1}}{\partial \epsilon^{2k-2}}(x, t, 0), \quad 2 \leq k \leq N, \quad 1 \leq j \leq 2N$$

The functions f and g are defined in (12), (13), (14), (15).

We don't give here the proof of this result. The ideas are the same as exposed in [13]. We will give the proof in a forthcoming paper.

The solutions of the NLS equation can also be written in the form :

$$v(x, t) = \exp(2it - i\varphi) \times Q(x, t)$$

where $Q(x, t)$ is defined by :

$$Q(x, t) := \left| \begin{array}{cccccc} f_{1,1}[0] & \dots & f_{1,1}[N-1] & f_{1,N+1}[0] & \dots & f_{1,N+1}[N-1] \\ f_{2,1}[0] & \dots & f_{2,1}[N-1] & f_{2,N+1}[0] & \dots & f_{2,N+1}[N-1] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_{2N,1}[0] & \dots & f_{2N,1}[N-1] & f_{2N,N+1}[0] & \dots & f_{2N,N+1}[N-1] \\ \hline g_{1,1}[0] & \dots & g_{1,1}[N-1] & g_{1,N+1}[0] & \dots & g_{1,N+1}[N-1] \\ g_{2,1}[0] & \dots & g_{2,1}[N-1] & g_{2,N+1}[0] & \dots & g_{2,N+1}[N-1] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ g_{2N,1}[0] & \dots & g_{2N,1}[N-1] & g_{2N,N+1}[0] & \dots & g_{2N,N+1}[N-1] \end{array} \right| \quad (17)$$

3 Quasi-rational solutions of order 4 with six parameters

We have already constructed in [11, 12] Peregrine breathers for the cases $N = 1$ until 6, and in [13] solutions with two parameters for $N = 1$ until 6. In this construction, I was persuaded to obtain solutions depending on eight parameters. But, in fact, there were links between these parameters, which led to solutions depending only on six parameters. After various transformations, one can reduce these solutions to only 6 parameters. Precisely, we use the following transformations :

1. $X \rightarrow X + \tilde{a}_1, T \rightarrow T - \tilde{b}_1.$
2. $\tilde{a}_1 \rightarrow -\frac{1}{768} \tilde{a}_1, \tilde{a}_2 \rightarrow -\frac{1}{1536} \tilde{a}_2, \tilde{a}_3 \rightarrow \frac{1}{7680} \tilde{a}_3, \tilde{a}_4 \rightarrow -\frac{1}{53760} \tilde{a}_4, \tilde{b}_1 \rightarrow -\frac{1}{3840} \tilde{b}_1, \tilde{b}_2 \rightarrow -\frac{1}{1536} \tilde{b}_2, \tilde{b}_3 \rightarrow \frac{1}{7680} \tilde{b}_3, \tilde{b}_4 \rightarrow -\frac{1}{53760} \tilde{b}_4.$
3. $\tilde{a}_1 \rightarrow \tilde{a}_1 - \tilde{a}_2, \tilde{b}_1 \rightarrow \tilde{b}_1 - \tilde{b}_2, \tilde{a}_4 \rightarrow \tilde{a}_4 - \frac{665}{8} \tilde{a}_1 + \frac{35}{2} \tilde{a}_2 + \frac{21}{2} \tilde{a}_3, \tilde{b}_4 \rightarrow \tilde{b}_4 - \frac{114373}{8} \tilde{b}_1 + 371 \tilde{b}_2 - \frac{427}{2} \tilde{b}_3.$
4. $\tilde{a}_3 \rightarrow \tilde{a}_3 + \frac{325}{28} \tilde{a}_1 - \frac{5}{4} \tilde{a}_2, \tilde{b}_3 \rightarrow \tilde{b}_3 - \frac{4359}{28} \tilde{b}_1 + \frac{7}{4} \tilde{b}_2.$

Then coefficients \tilde{a}_2 and \tilde{b}_2 are deleted.

5. $\tilde{a}_3 \rightarrow \tilde{a}_2, \tilde{b}_3 \rightarrow \tilde{b}_2, \tilde{a}_4 \rightarrow \tilde{a}_3, \tilde{b}_4 \rightarrow \tilde{b}_3.$

Thus we obtain solutions depending only on 6 parameters. In the following, the study will be done starting from these 6 parameters $\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{b}_1, \tilde{b}_2, \tilde{b}_3.$

3.1 Analytical expressions of the solutions of NLS equation with six parameters

Because of the length of the expression v of the solution of NLS equation with six parameters, we only give in the appendix.

3.2 Plots of the solutions of NLS equation with six parameters

Conversely to the study with two parameters given in preceding works [11, 13], we get other type of patterns in the plots in the (x, t) plane. We give some examples of this fact in the following.

3.2.1 Peregrine breather of order 4

If we choose $\tilde{a}_1 = \tilde{b}_1 = \tilde{a}_2 = \tilde{b}_2 = \tilde{a}_3 = \tilde{b}_3 = 0$, we obtain the classical Peregrine breather

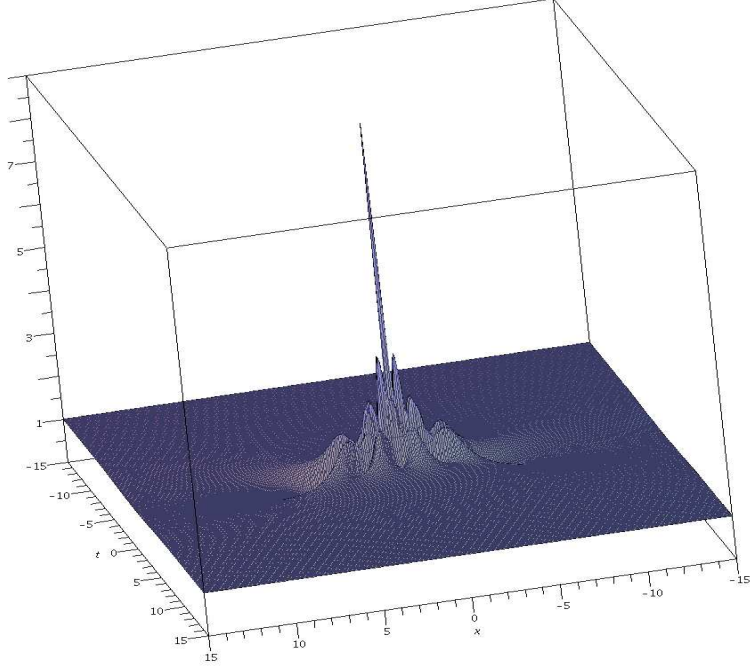


Figure 1: Solution of NLS, $N=4$, $\tilde{a}_1 = \tilde{b}_1 = \tilde{a}_2 = \tilde{b}_2 = \tilde{a}_3 = \tilde{b}_3 = 0$.

With other choices of parameters, we obtain all types of configurations : triangles, rings and concentric rings configurations with a maximum of 10 peaks.

3.2.2 Variation of one parameter

We obtain triangles in the cases $\tilde{a}_1 \neq 0$ or $\tilde{b}_1 \neq 0$, concentric ring configurations in the cases $\tilde{a}_2 \neq 0$ or $\tilde{b}_2 \neq 0$ without central peak, ring configurations in the cases $\tilde{a}_3 \neq 0$ or $\tilde{b}_3 \neq 0$ with the apparition in the center of the Peregrine breather of order 2.

If we choose $\tilde{a}_1 = 10^3$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$, we obtain :

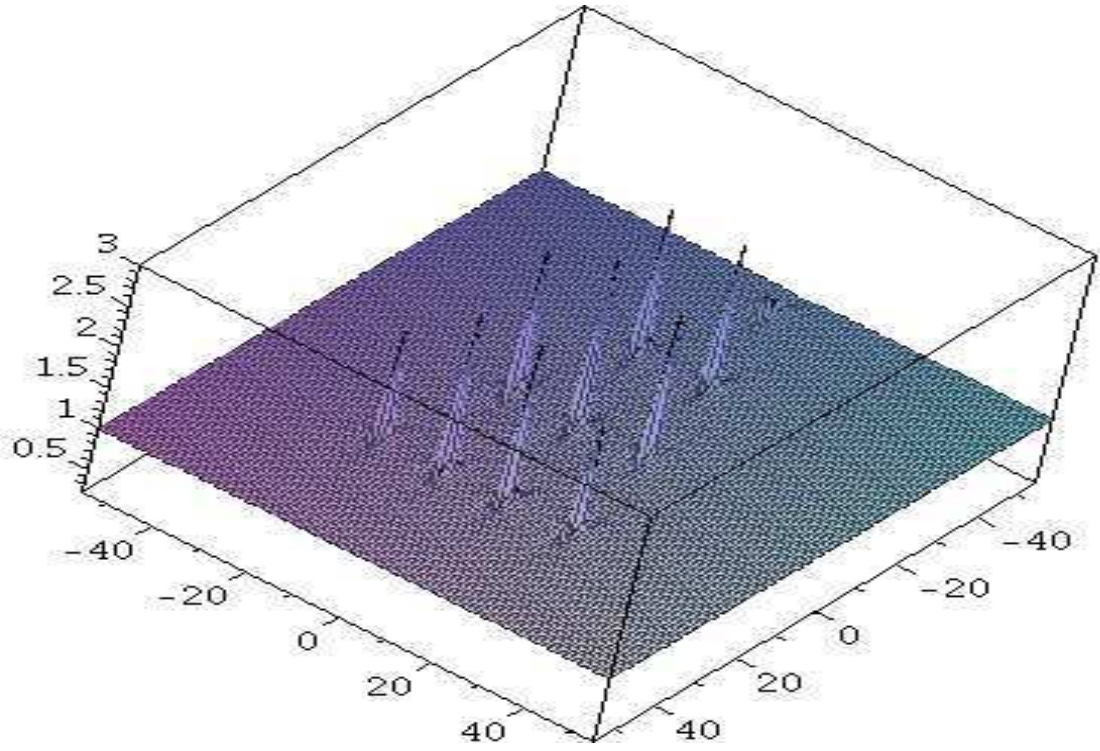


Figure 2: Solution of NLS, $N=4$, $\tilde{a}_1 = 10^3$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{a}_2 = 10^5$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$, we obtain :

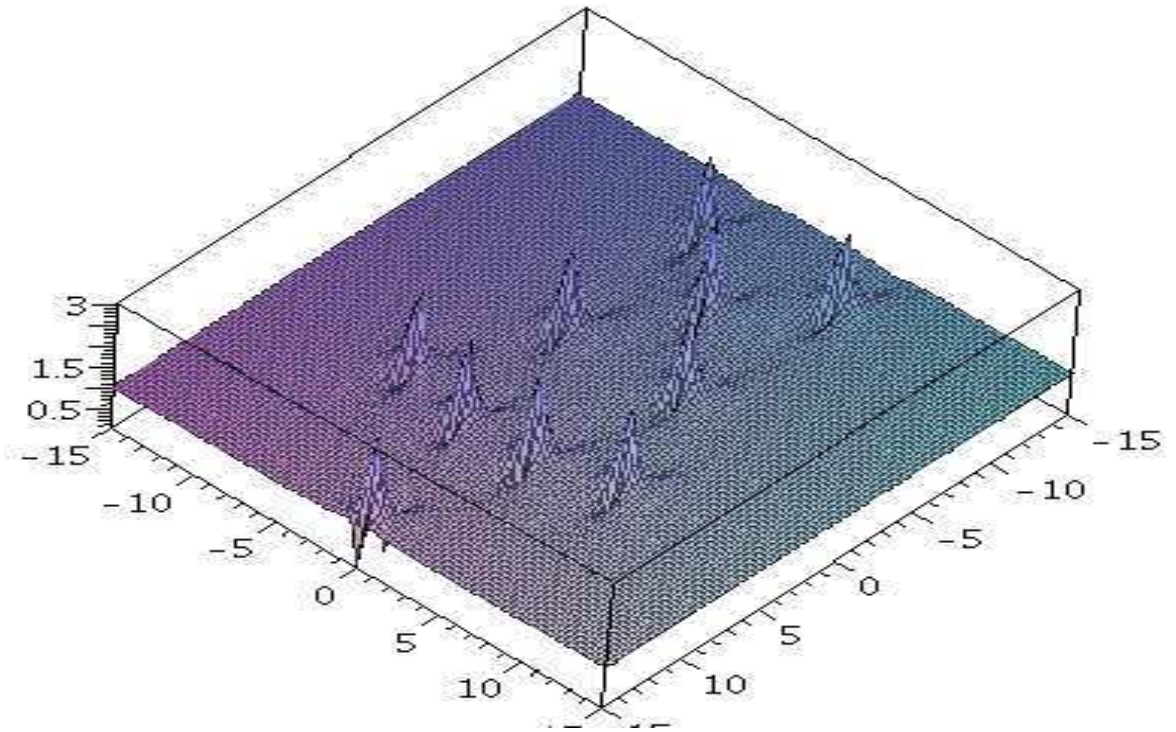


Figure 3: Solution of NLS, $N=4$, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 10^5$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 10^8$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$, we obtain :

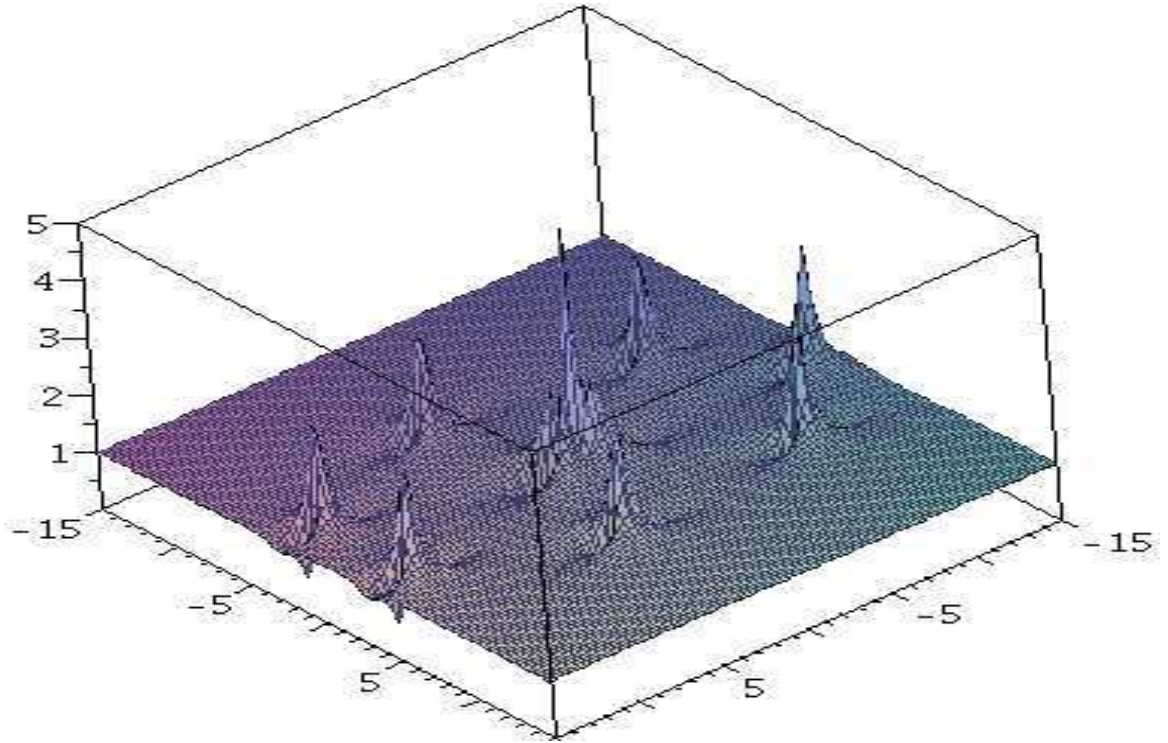


Figure 4: Solution of NLS, $N=4$, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 10^8$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 10^5$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$, we obtain :

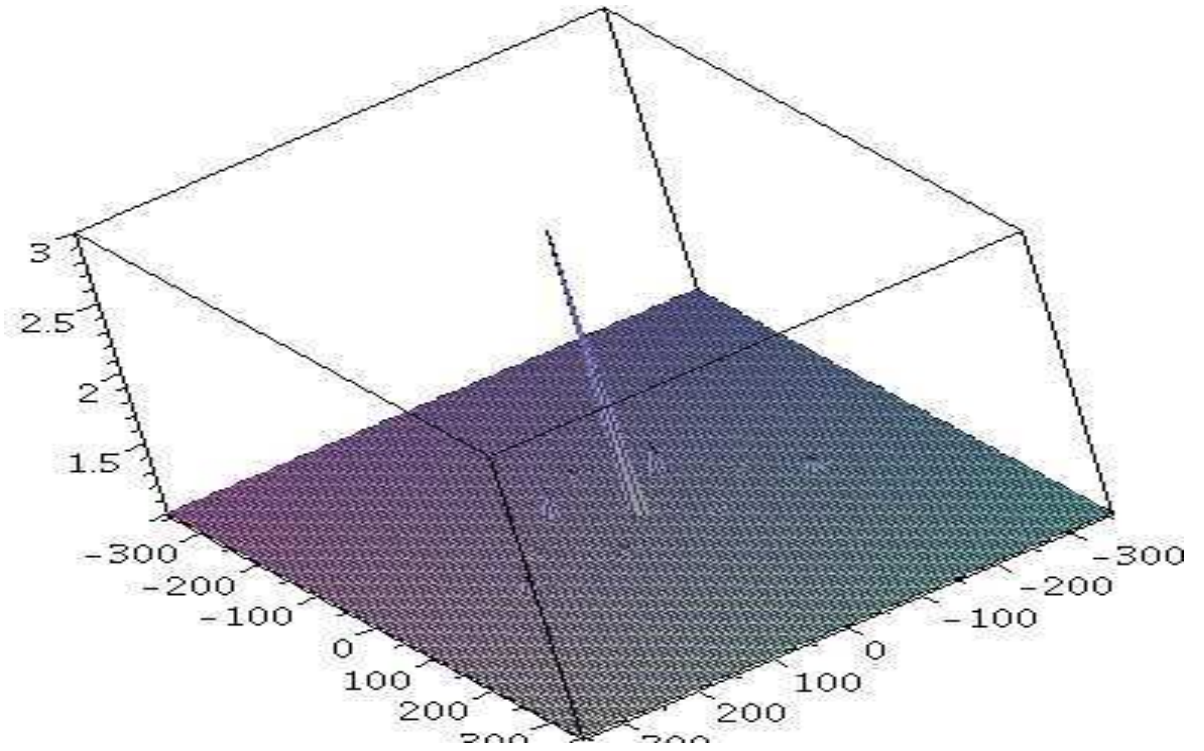


Figure 5: Solution of NLS, $N=4$, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 10^5$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 10^5$, $\tilde{b}_3 = 0$, we obtain :

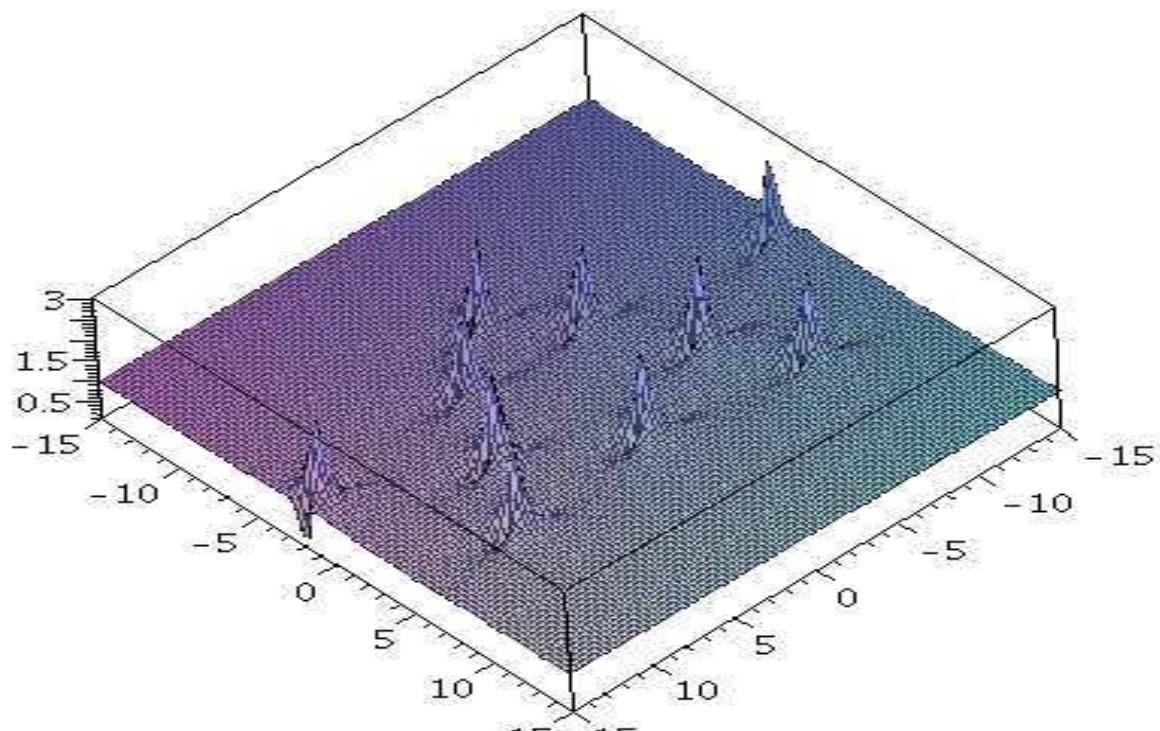


Figure 6: Solution of NLS, $N=4$, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 10^5$, $\tilde{b}_3 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 10^6$, we obtain :

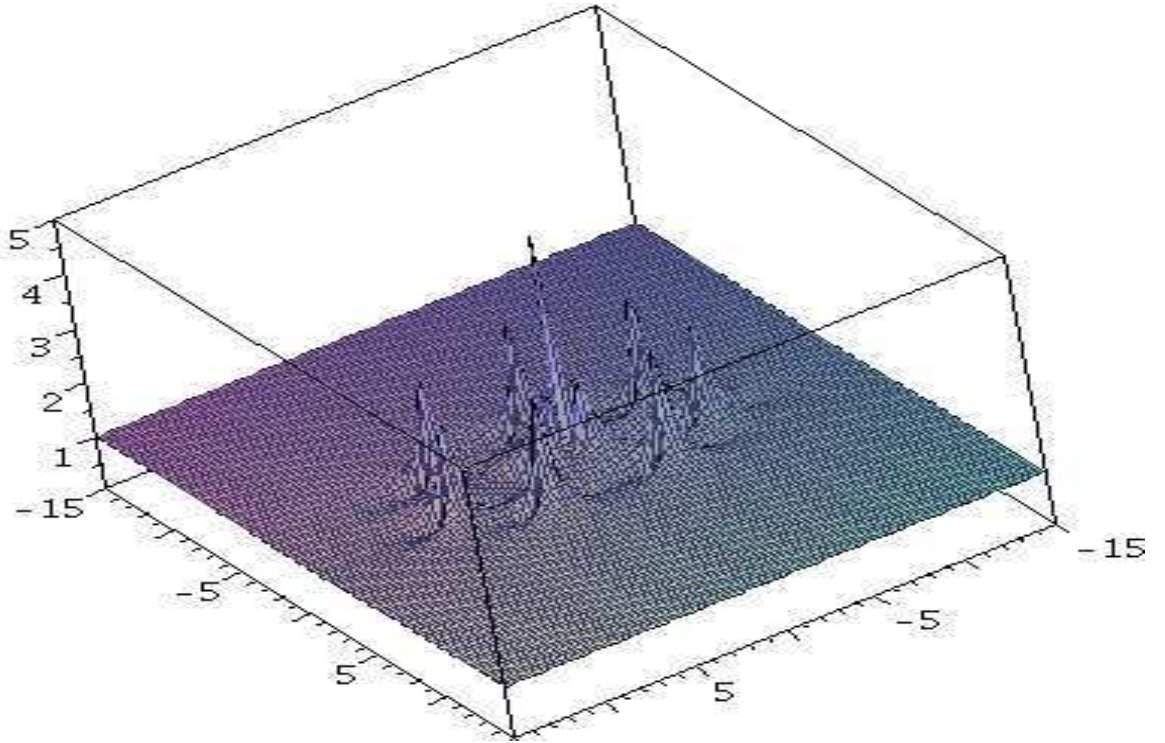


Figure 7: Solution of NLS, $N=4$, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 10^6$.

3.2.3 Ring, triangular and concentric rings configurations

In general we obtain regular forms as ring, triangular and concentric rings configurations. We present here some examples.

If we choose $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 10^3$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$, we obtain :

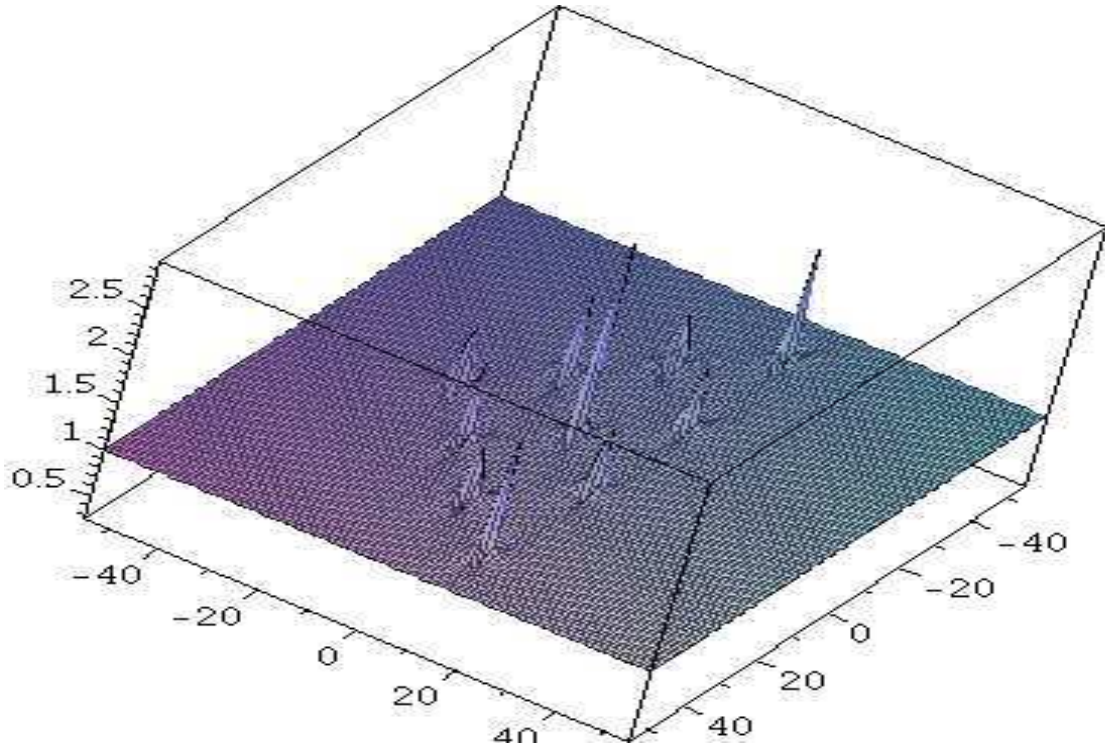


Figure 8: Solution of NLS, $N=4$, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 10^3$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{a}_2 = 10^6$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$, we obtain :

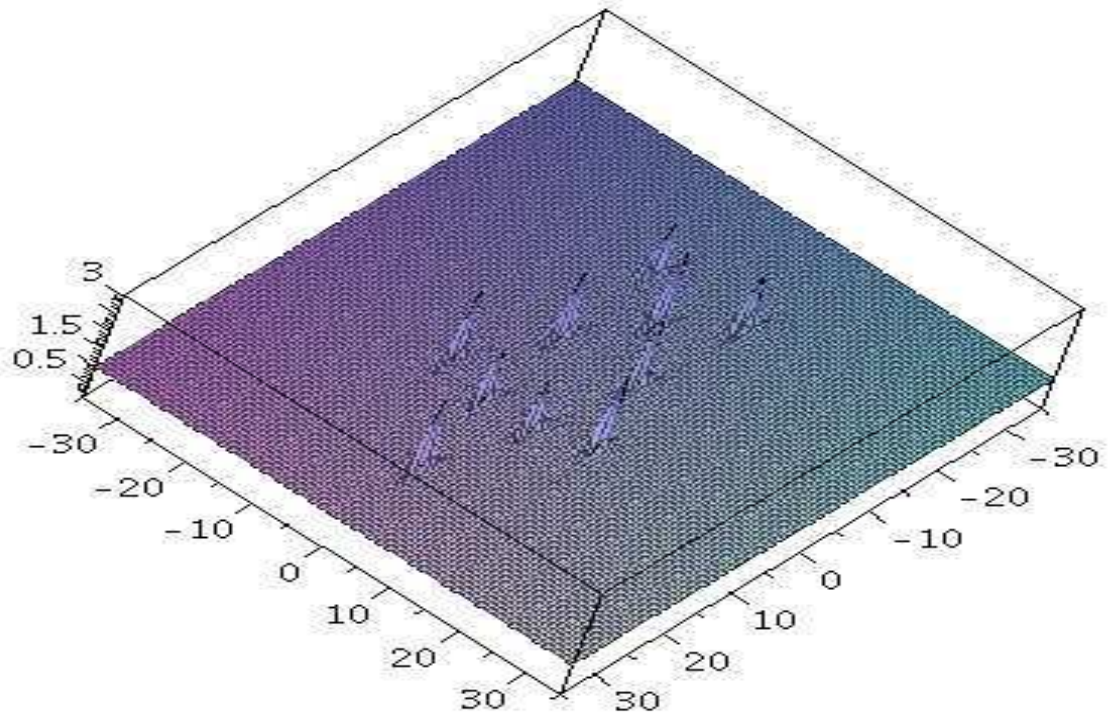


Figure 9: Solution of NLS, $N=4$, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 10^6$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 10^7$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$, we obtain :

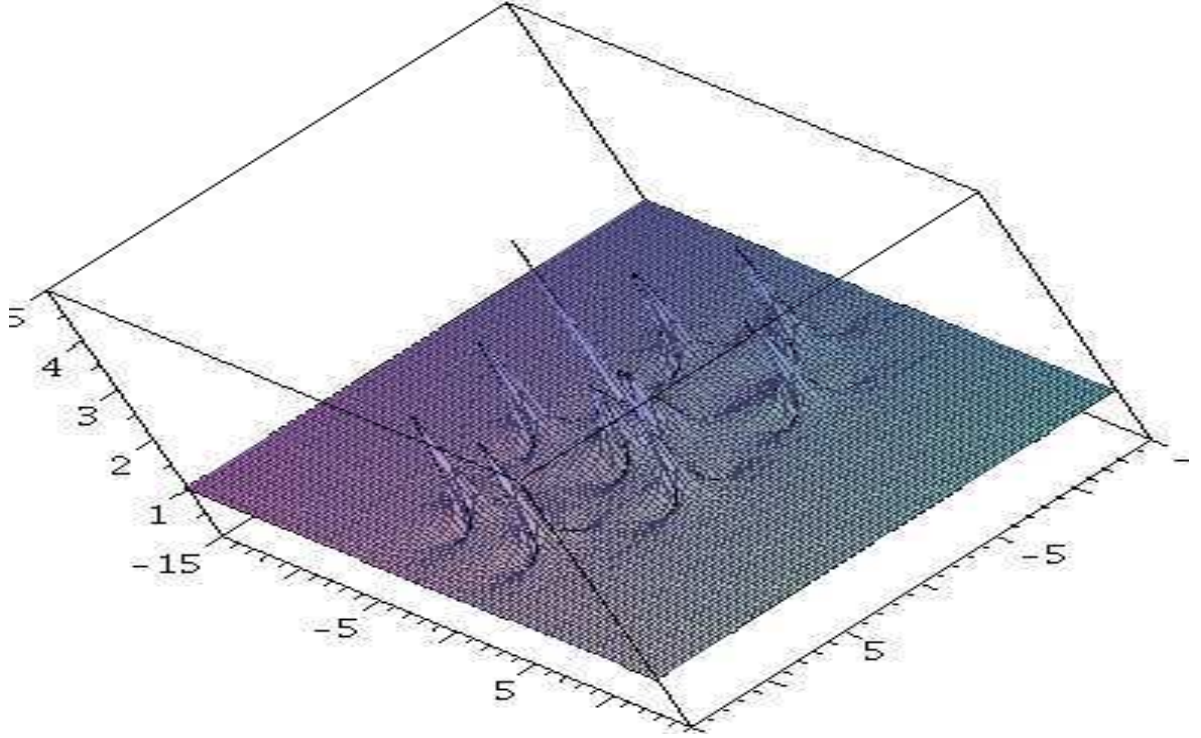


Figure 10: Solution of NLS, $N=4$, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 10^7$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$.

4 Conclusion

The method described in the present paper provides a new tool to get explicitly solutions of the NLS equation of order N with $2N - 2$ real parameters. By different choices of these parameters, we obtained new patterns in the $(x; t)$ plane; we recognized rings as already observed in the case of deformations depending on two parameters [11, 13]. We get new triangular shapes and multiple concentric rings.

In a recent work of P. Dubard and V.B. Matveev [9], based on [10], solutions of the NLS equation depending on 6 parameters has also been found. The exact link between these solutions and those given in this paper reads as follows :

$$\begin{aligned} \tilde{a}_1 &= -\frac{1}{28} \alpha_1, \quad \tilde{b}_1 = -\frac{1}{28} \beta_1, \quad \tilde{a}_2 = \frac{45}{14} \alpha_1 - \frac{1}{4} \alpha_2, \quad \tilde{b}_2 = -\frac{265}{7} \beta_1 - \frac{1}{4} \beta_2, \\ \tilde{a}_3 &= -\frac{77}{3} \alpha_1 + \frac{49}{12} \alpha_2 - \frac{1}{6} \alpha_3, \quad \tilde{b}_3 = -\frac{10052}{3} \beta_1 - \frac{623}{12} \beta_2 - \frac{1}{6} \beta_3. \end{aligned}$$

The method given in [8, 9] is however badly adapted to extract the Peregrine breathers and heavier for calculations. The method presented in this article seems more powerful since it is very easy to isolate Peregrine breathers, and it makes it possible to continue the study for higher orders than 5. We will give these presentations in other publications not to overload this text. It would be very interesting to carry out experiments to observe these phenomena.

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Appendix

The solution of NLS equation takes the form

$$v_N(x, t) = \frac{n(x, t)}{d(x, t)} \exp(2it) = \left(1 - \alpha_N \frac{G_N(2x, 4t) + iH_N(2x, 4t)}{Q_N(2x, 4t)}\right) e^{2it}$$

with

$$G_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{g}_k(T) X^k$$

$$H_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{h}_k(T) X^k$$

$$Q_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{q}_k(T) X^k$$

Here, we present the expressions of the polynomials G_N , H_N and Q_N , for $N = 4$.

By simplicity, we chose to remove all the \sim with the parameters in the presentation of the polynomials.

$$\begin{aligned}
& \alpha_4 = 40, \quad \mathbf{g}_{20} = 0, \quad \mathbf{g}_{19} = 0, \quad \mathbf{g}_{18} = 1, \quad \mathbf{g}_{17} = 0, \quad \mathbf{g}_{16} = 27T^2 + 27, \quad \mathbf{g}_{15} = 672a_1, \quad \mathbf{g}_{14} = 180T^4 - 360T^2 \\
& -10080Tb_1 + 900, \quad \mathbf{g}_{13} = 3360a_1 + 168a_2, \quad \mathbf{g}_{12} = 588T^6 - 5460T^4 - 94080T^3b_1 - 1260T^2 + 493920a_1^2 \\
& + 493920b_1^2 + (-1767360b_1 + 1512b_2)T + 18900, \quad \mathbf{g}_{11} = -70560T^4a_1 + 7056T^2a_2 + 131040a_1 + 10080a_2 - 432a_3, \\
& \mathbf{g}_{10} = 1134T^8 - 17640T^6 - 296352T^5b_1 + 18900T^4 + (114240b_1 + 336b_2)T^3 + (8890560a_1^2 + 8890560b_1^2 - 37800)T^2 \\
& + 1740480a_1^2 - 357504a_1a_2 - 151798080b_1^2 + 131712b_1b_2 + (145867680b_1 - 227808b_2 + 1008b_3)T - 107730, \\
& \mathbf{g}_9 = -329280T^6a_1 + (-991200a_1 + 32760a_2)T^4 - 215129600a_1^3 + 645388800a_1b_1^2 + (-2721600a_1 + 110880a_2 \\
& - 3600a_3)T^2 + (-2180774400a_1b_1 + 1975680a_1b_2 + 282240a_2b_1)T + 15573600a_1 + 37800a_2 - 3600a_3, \quad \mathbf{g}_8 = 1386T^{10} \\
& - 20790T^8 - 423360T^7b_1 + 235620T^6 + (39795840b_1 - 31752b_2)T^5 + (37044000a_1^2 + 37044000b_1^2 + 170100)T^4 \\
& + (2569795200b_1 - 3739680b_2 + 15120b_3)T^3 + (370440000a_1^2 + 1691323200b_1^2 - 1693440b_1b_2 - 5698350)T^2 \\
& - 231890400a_1^2 + 1895040a_1a_2 - 120960a_1a_3 + 26208a_2^2 + 69703200b_1^2 - 2298240b_1b_2 + 2016b_2^2 \\
& + (4148928000a_1^2b_1 - 1382976000b_1^3 + 1898164800b_1 - 3167640b_2 + 15120b_3)T - 481950, \quad \mathbf{g}_7 = -635040T^8a_1 \\
& + (-5080320a_1 + 42336a_2)T^6 + (-94852800a_1 + 20160a_2 + 12960a_3)T^4 + (-13163673600a_1b_1 + 10160640a_1b_2 \\
& - 1128960a_2b_1)T^3 + 2054707200a_1^3 - 63221760a_1^2a_2 + 2054707200a_1b_1^2 - 63221760a_2b_1^2 + (3319142400a_1^3 \\
& - 9957427200a_1b_1^2 + 187488000a_1 - 151200a_2 + 95040a_3)T^2 + (-5222246400a_1b_1 + 6289920a_1b_2 + 206277120a_2b_1 \\
& - 193536a_2b_2 + 967680a_3b_1)T - 9828000a_1 - 967680a_2 - 56160a_3, \quad \mathbf{g}_6 = 1092T^{12} - 1848T^{10} - 258720T^9b_1 \\
& + 696780T^8 + (99146880b_1 - 78624b_2)T^7 + (69148800a_1^2 + 69148800b_1^2 + 5609520)T^6 + (4759554240b_1 \\
& - 5842368b_2 + 18144b_3)T^5 + (-108662400a_1^2 + 11854080a_1a_2 + 14432342400b_1^2 - 11854080b_1b_2 - 34265700)T^4 \\
& - 72283545600a_1^4 + 72283545600b_1^4 + (-7744665600a_1^2b_1 + 2581555200b_1^3 - 30793795200b_1 + 48192480b_2 \\
& - 221760b_3)T^3 + (613872000a_1^2 + 16934400a_1a_2 - 169344a_2^2 + 158668272000b_1^2 - 389491200b_1b_2 - 3386880b_1b_3 \\
& + 508032b_2^2 + 19845000)T^2 + 814262400a_1^2 + 7338240a_1a_2 - 161280a_1a_3 - 112896a_2^2 - 8064a_2a_3 + 1196721590400b_1^2 \\
& - 3043111680b_1b_2 + 8547840b_1b_3 + 1806336b_2^2 - 8064b_2b_3 + (-7744665600a_1^2b_1 + 12907776000a_1b_1^3 + 20732997600b_1 \\
& - 38102400b_2 + 211680b_3)T - 1190700, \quad \mathbf{g}_5 = -620928T^{10}a_1 + (-9344160a_1 - 1512a_2)T^8 + (-161441280a_1 \\
& - 2342592a_2 + 42336a_3)T^6 + (-5263211520a_1b_1 + 2370816a_1b_2 - 21337344a_2b_1)T^5 + (3872332800a_1^3 - 11616998400a_1b_1^2 \\
& - 31752000a_1 - 10478160a_2 - 90720a_3)T^4 + (-553935513600a_1b_1 + 817367040a_1b_2 - 3386880a_1b_3 - 1688924160a_2b_1 \\
& + 1354752a_2b_2 - 3386880a_3b_1)T^3 - 34534886400a_1^3 - 244984320a_1^2a_2 - 23708160a_1^2a_3 + 6322176a_1a_2^2 \\
& - 3269908454400a_1b_1^2 + 6448619520a_1b_1b_2 - 3161088a_1b_2^2 - 10328855040a_2b_1^2 + 9483264a_2b_1b_2 - 23708160a_3b_1^2 \\
& + (25446758400a_1^3 - 663828480a_1^2a_2 + 56425420800a_1b_1^2 - 663828480a_2b_1^2 + 1740009600a_1 + 20321280a_2 \\
& + 272160a_3)T^2 + (433701273600a_1^3b_1 + 433701273600a_1b_1^3 - 239621760000a_1b_1 + 408119040a_1b_2 - 1935360a_1b_3 \\
& + 8739843840a_2b_1 - 14563584a_2b_2 + 72576a_2b_3 + 76930560a_3b_1 - 72576a_3b_2)T - 3175200a_1 - 4604040a_2 - 272160a_3, \\
& \mathbf{g}_4 = 540T^{14} + 16380T^{12} + 903420T^{10} + (86318400b_1 - 64680b_2)T^9 + (66679200a_1^2 + 66679200b_1^2 + 10791900)T^8 \\
& + (-2009347200b_1 + 4213440b_2 - 21600b_3)T^7 + (-2538748800a_1^2 + 7902720a_1a_2 + 3941481600b_1^2 - 61519500)T^6 \\
& + (-34850995200a_1^2b_1 + 11616998400b_1^3 - 35401363200b_1 + 45405360b_2 - 151200b_3)T^5 + (-5295528000a_1^2 \\
& + 149587200a_1a_2 - 8467200a_1a_3 + 1270080a_2^2 - 761247144000b_1^2 + 1154361600b_1b_2 - 423360b_2^2 + 17860500)T^4 \\
& + 103262208000a_1^4 + 5163110400a_1^3a_2 - 1781273088000a_1^2b_1^2 + 15489331200a_1^2b_1b_2 - 15489331200a_1a_2b_1^2 \\
& + 5834314752000b_1^4 - 5163110400b_1^3b_2 + (-1366380288000a_1^2b_1 + 1106380800a_1^2b_2 - 1211486976000b_1^3 \\
& + 1106380800b_1^2b_2 + 70256592000b_1 - 105840000b_2 + 453600b_3)T^3 + (22670928000a_1^2 + 474163200a_1a_2 \\
& - 16934400a_1a_3 + 5927040a_2^2 + 1491899472000b_1^2 - 1998259200b_1b_2 + 16934400b_1b_3 - 846720b_2^2 + 65488500)T^2 \\
& - 18357948000a_1^2 - 372556800a_1a_2 - 9676800a_1a_3 - 1905120a_2^2 - 60480a_2a_3 + 4320a_3^2 + 96854199876000b_1^2 \\
& - 298299456000b_1b_2 + 1302739200b_1b_3 + 229249440b_2^2 - 1995840b_2b_3 + 4320b_3^2 + (17240969088000a_1^2b_1 \\
& - 26671680000a_1^2b_2 + 118540800a_1^2b_3 + 54370713600a_1a_2b_1 - 47416320a_1a_2b_2 + 47416320a_2^2b_1 + 17422731648000b_1^3 \\
& - 26829734400b_1^2b_2 + 118540800b_1^2b_3 - 63408744000b_1 + 100812600b_2 - 453600b_3)T + 17860500, \quad \mathbf{g}_3 = -305760T^{12}a_1 \\
& + (-7392000a_1 - 33264a_2)T^{10} + (303660000a_1 - 2933280a_2 + 15120a_3)T^8 + (10860595200a_1b_1 - 3386880a_1b_2 \\
& - 23708160a_2b_1)T^7 + (-12907776000a_1^3 + 38723328000a_1b_1^2 + 1879718400a_1 - 8255520a_2 + 141120a_3)T^6 \\
& + (1141265664000a_1b_1 - 1615541760a_1b_2 + 6773760a_1b_3 + 1364912640a_2b_1 - 1354752a_2b_2)T^5 + (64538880000a_1^3 \\
& + 580849920000a_1b_1^2 - 862596000a_1 + 25401600a_2 + 2268000a_3)T^4 + (1445670912000a_1^3b_1 + 1445670912000a_1b_1^3 \\
& + 4847415552000a_1b_1 - 6982617600a_1b_2 + 27417600a_1b_3 + 48833164800a_2b_1 - 66608640a_2b_2 + 241920a_2b_3 \\
& + 312883200a_3b_1 - 241920a_3b_2)T^3 - 165618432000a_1^3 - 3465907200a_1^2a_2 + 327398400a_1^2a_3 - 58705920a_1a_2^2 \\
& + 4515840a_1a_2a_3 - 701892834048000a_1b_1^2 + 1743836774400a_1b_1b_2 - 4786790400a_1b_1b_3 - 1020579840a_1b_2^2 \\
& + 4515840a_1b_2b_3 - 451584a_2^3 - 542566886400a_2b_1^2 + 988968960a_2b_1b_2 - 451584a_2b_2^2 - 79027200a_3b_1^2 \\
& + (-71914752000a_1^3 - 1659571200a_1^2a_2 + 237081600a_1^2a_3 + 123163100928000a_1b_1^2 - 216218419200a_1b_1b_2 \\
& + 94832640a_1b_2^2 + 105501312000a_2b_1^2 - 94832640a_2b_1b_2 + 237081600a_3b_1^2 + 1905120000a_1 - 49215600a_2 \\
& - 4536000a_3)T^2 + (1445670912000a_1^3b_1 + 1445670912000a_1b_1^3 + 4607172864000a_1b_1 - 7772889600a_1b_2 + 38707200a_1b_3 \\
& + 30735936000a_2b_1 - 50803200a_2b_2 + 241920a_2b_3 + 222566400a_3b_1 - 241920a_3b_2)T - 1190700000a_1 - 9525600a_2 \\
& - 226800a_3, \quad \mathbf{g}_2 = 153T^{16} + 13320T^{14} + 70560T^{13}b_1 + 550620T^{12} + (23204160b_1 - 13104b_2)T^{11} + (32598720a_1^2 \\
& + 32598720b_1^2 - 11680200)T^{10} + (-5343861600b_1 + 6844320b_2 - 23760b_3)T^9 + (2652350400a_1^2 - 5927040a_1a_2 \\
& - 19972008000b_1^2 + 16087680b_1b_2 - 173133450)T^8 + (23233996800a_1^2b_1 - 7744665600b_1^3 - 118724054400b_1
\end{aligned}$$

$$\begin{aligned}
& +159516000 b_2 - 613440 b_3)T^7 + (6038524800 a_1^2 - 53061120 a_1 a_2 + 56448 a_2^2 - 103560912000 b_1^2 - 473034240 b_1 b_2 \\
& - 3386880 b_1 b_3 + 733824 b_2^2 + 253222200)T^6 + (-1337614387200 a_1^2 b_1 + 1327656960 a_1^2 b_2 - 1957187635200 b_1^3 \\
& + 1327656960 b_1^2 b_2 - 457714605600 b_1 + 734106240 b_2 - 3356640 b_3)T^5 + (1084253184000 a_1^4 - 1084253184000 b_1^4 \\
& - 59587920000 a_1^2 - 1024531200 a_1 a_2 + 2419200 a_1 a_3 - 3386880 a_2^2 + 120960 a_2 a_3 - 17358754896000 b_1^2 \\
& + 42158188800 b_1 b_2 - 195955200 b_1 b_3 - 18627840 b_2^2 + 120960 b_2 b_3 - 89302500)T^4 + 1880847360000 a_1^4 \\
& + 48680755200 a_1^3 a_2 + 442552320 a_1^2 a_2^2 + 533054269440000 a_1^2 b_1^2 - 969189580800 a_1^2 b_1 b_2 + 442552320 a_1^2 b_2^2 \\
& + 48680755200 a_1 a_2 b_1^2 + 442552320 a_2^2 b_1^2 + 531173422080000 b_1^4 - 969189580800 b_1^3 b_2 + 442552320 b_1^2 b_2^2 \\
& + (-52468528896000 a_1^2 b_1 + 68832691200 a_1^2 b_2 - 237081600 a_1^2 b_3 - 123914649600 a_1 a_2 b_1 + 94832640 a_1 a_2 b_2 \\
& - 31610880 a_2^2 b_1 + 48227929344000 b_1^3 - 91434470400 b_1^2 b_2 - 237081600 b_1^2 b_3 + 63221760 b_1 b_2^2 + 1068296040000 b_1 \\
& - 1732071600 b_2 + 8164800 b_3)T^3 + (2788079616000 a_1^4 + 30978662400 a_1^3 a_2 - 106876385280000 a_1^2 b_1^2 \\
& + 92935987200 a_1^2 b_1 b_2 - 92935987200 a_1 a_2 b_1^2 + 32837382144000 b_1^4 - 30978662400 b_1^3 b_2 + 116434584000 a_1^2 \\
& + 1930521600 a_1 a_2 - 21772800 a_1 a_3 + 3810240 a_2^2 - 1088640 a_2 a_3 + 25920 a_3^2 + 718266405528000 b_1^2 \\
& - 2110364928000 b_1 b_2 + 8738150400 b_1 b_3 + 1543147200 b_2^2 - 12700800 b_2 b_3 + 25920 b_3^2 + 464373000)T^2 + 15209208000 a_1^2 \\
& + 342921600 a_1 a_2 + 14515200 a_1 a_3 + 6350400 a_2^2 + 725760 a_2 a_3 + 25920 a_3^2 + 429662951928000 b_1^2 - 1401901603200 b_1 b_2 \\
& + 6662476800 b_1 b_3 + 1144342080 b_2^2 - 10886400 b_2 b_3 + 25920 b_3^2 + (-33967358208000 a_1^2 b_1 + 53885260800 a_1^2 b_2 \\
& - 237081600 a_1^2 b_3 - 144687513600 a_1 a_2 b_1 + 149022720 a_1 a_2 b_2 - 270950400 a_1 a_3 b_1 - 1436037120 a_2^2 b_1 + 1354752 a_2^2 b_2 \\
& - 13547520 a_2 a_3 b_1 + 442921602816000 b_1^3 - 552230784000 b_1^2 b_2 + 15071616000 b_1^2 b_3 - 1368299520 b_1 b_2^2 - 13547520 b_1 b_2 b_3 \\
& + 1354752 b_2^3 - 103971924000 b_1 + 161935200 b_2 - 680400 b_3)T + 22325625, \quad \mathbf{g}_1 = -60480 T^{14} a_1 + (-2140320 a_1 \\
& - 15288 a_2)T^{12} + (-104287680 a_1 - 155232 a_2 - 11088 a_3)T^{10} + (-9646963200 a_1 b_1 + 7244160 a_1 b_2 + 1034880 a_2 b_1)T^9 \\
& + (2489356800 a_1^3 - 7468070400 a_1 b_1^2 - 935776800 a_1 + 13645800 a_2 - 498960 a_3)T^8 + (225685555200 a_1 b_1 - 468034560 a_1 b_2 \\
& + 2419200 a_1 b_3 + 31933440 a_2 b_1 + 193536 a_2 b_2 - 3386880 a_3 b_1)T^7 + (-78553036800 a_1^3 + 1106380800 a_1^2 a_2 \\
& - 419318323200 a_1 b_1^2 + 1106380800 a_2 b_1^2 + 8211067200 a_1 + 66044160 a_2 - 2207520 a_3)T^6 + (-1301103820800 a_1^3 b_1 \\
& - 1301103820800 a_1 b_1^3 + 4359490329600 a_1 b_1 - 5728907520 a_1 b_2 + 20321280 a_1 b_3 + 19726882560 a_2 b_1 - 32175360 a_2 b_2 \\
& + 169344 a_2 b_3 + 145635840 a_3 b_1 - 169344 a_3 b_2)T^5 + 5782683648000 a_1^5 + 289134182400 a_1^4 a_2 - 653443252224000 a_1^3 b_1^2 \\
& + 578268364800 a_1^3 b_1 b_2 - 659225935872000 a_1 b_1^4 + 578268364800 a_1 b_1^3 b_2 - 289134182400 a_2 b_1^4 + (-346534272000 a_1^3 \\
& - 908812800 a_1^2 a_2 - 118540800 a_1^2 a_3 + 94832640 a_1 a_2^2 + 84536581248000 a_1 b_1^2 - 128340172800 a_1 b_1 b_2 + 47416320 a_1 b_2^2 \\
& - 36154944000 a_2 b_1^2 + 47416320 a_2 b_1 b_2 - 118540800 a_3 b_1^2 - 13177080000 a_1 + 34133400 a_2 + 7711200 a_3)T^4 \\
& + (-53489823744000 a_1^3 b_1 + 41304883200 a_1^3 b_2 - 123914649600 a_1^2 a_2 b_1 + 136512638976000 a_1 b_1^3 - 123914649600 a_1 b_1^2 b_2 \\
& + 41304883200 a_2 b_1^3 - 2917797120000 a_1 b_1 + 4199731200 a_1 b_2 - 16934400 a_1 b_3 + 247547059200 a_2 b_1 - 382717440 a_2 b_2 \\
& + 1693440 a_2 b_3 + 1981324800 a_3 b_1 - 1693440 a_3 b_2)T^3 - 318790080000 a_1^3 + 7806758400 a_1^2 a_2 + 1003968000 a_1^2 a_3 \\
& + 111767040 a_1 a_2^2 + 10644480 a_1 a_2 a_3 - 483840 a_1 a_3^2 - 11421770592960000 a_1 b_1^2 + 34855736832000 a_1 b_1 b_2 \\
& - 150009753600 a_1 b_1 b_3 - 26529431040 a_1 b_2^2 + 227404800 a_1 b_2 b_3 - 483840 a_1 b_3^2 + 677376 a_2^3 + 96768 a_2^2 a_3 \\
& - 28705010342400 a_2 b_1^2 + 72309888000 a_2 b_1 b_2 - 205148160 a_2 b_1 b_3 - 42674688 a_2 b_2^2 + 193536 a_2 b_2 b_3 - 108847065600 a_3 b_1^2 \\
& + 205148160 a_3 b_1 b_2 - 96768 a_3 b_2^2 + (899894016000 a_1^5 + 20829312000 a_1^2 a_2 - 1185408000 a_1^2 a_3 + 270950400 a_1 a_2^2 \\
& - 95562512640000 a_1 b_1^2 + 103774003200 a_1 b_1 b_2 - 1625702400 a_1 b_1 b_3 + 121927680 a_1 b_2^2 + 1354752 a_2^3 + 3576511411200 a_2 b_1^2 \\
& - 6001551360 a_2 b_1 b_2 + 13547520 a_2 b_1 b_3 + 1354752 a_2 b_2^2 + 16019942400 a_3 b_1^2 - 13547520 a_3 b_1 b_2 - 7144200000 a_1 \\
& + 9525600 a_2 + 2041200 a_3)T^2 + (603154556928000 a_1^3 b_1 - 960338534400 a_1^3 b_2 + 4425523200 a_1^3 b_3 - 30978662400 a_1^2 a_2 b_1 \\
& - 13276569600 a_1^2 a_3 b_1 - 1951965517824000 a_1 b_1^3 + 3004930252800 a_1 b_1^2 b_2 - 13276569600 a_1 b_1^2 b_3 - 30978662400 a_2 b_1^3 \\
& + 4425523200 a_3 b_1^3 + 8241041088000 a_1 b_1 - 13018320000 a_1 b_2 + 58060800 a_1 b_3 + 56963088000 a_2 b_1 - 86365440 a_2 b_2 \\
& + 362880 a_2 b_3 + 384652800 a_3 b_1 - 362880 a_3 b_2)T - 2524284000 a_1 - 26195400 a_2 - 680400 a_3, \quad \mathbf{g}_0 = 19 T^{18} + 3315 T^{16} \\
& + 22848 T^{15} b_1 + 129420 T^{14} + (-1881600 b_1 + 3528 b_2)T^{13} + (6420960 a_1^2 + 6420960 b_1^2 + 328860)T^{12} + (72011520 b_1 \\
& - 215712 b_2 + 1872 b_3)T^{11} + (89517120 a_1^2 + 2483712 a_1 a_2 - 1160617920 b_1^2 + 1241856 b_1 b_2 + 4862970)T^{10} \\
& + (-1014182400 a_1^2 b_1 + 338060800 b_1^3 + 22518417600 b_1 - 31935960 b_2 + 130320 b_3)T^9 + (877312800 a_1^2 + 36892800 a_1 a_2 \\
& + 846720 a_1 a_3 + 139104 a_2^2 + 346301978400 b_1^2 - 520490880 b_1 b_2 + 967680 b_1 b_3 + 114912 b_2^2 - 201540150)T^8 \\
& + (309312460800 a_1^2 b_1 - 284497920 a_1^2 b_2 + 381596006400 b_1^3 - 284497920 b_1^2 b_2 + 58491720000 b_1 - 10775360 b_2 \\
& + 548640 b_3)T^7 + (-144567091200 a_1^4 + 144567091200 b_1^4 + 14855702400 a_1^2 + 314979840 a_1 a_2 + 10160640 a_1 a_3 \\
& - 2201472 a_2^2 + 112896 a_2 a_3 - 21040564406400 b_1^2 + 49648273920 b_1 b_2 - 114024960 b_1 b_3 - 28619136 b_2^2 + 112896 b_2 b_3 \\
& - 184558500)T^6 + 94450499584000 a_1^6 + 283351498752000 a_1^4 b_1^2 + 283351498752000 a_1^2 b_1^4 + 94450499584000 b_1^6 \\
& + (11235217996800 a_1^2 b_1 - 11972620800 a_1^2 b_2 + 23708160 a_1^2 b_3 + 18207866880 a_1 a_2 b_1 - 9483264 a_1 a_2 b_2 - 28449792 a_2^2 b_1 \\
& - 47328678835200 b_1^3 + 82575521280 b_1^2 b_2 + 23708160 b_1^2 b_3 - 37933056 b_1 b_2^2 + 726828681600 b_1 - 1032733800 b_2 \\
& + 3900960 b_3)T^5 + (-1032622080000 a_1^4 - 15489331200 a_1^3 a_2 + 53438192640000 a_1^2 b_1^2 - 46467993600 a_1^2 b_1 b_2 \\
& + 46467993600 a_1 a_2 b_1^2 - 16780108800000 b_1^4 + 15489331200 b_1^3 b_2 - 153092268000 a_1^2 - 1236211200 a_1 a_2 + 116121600 a_1 a_3 \\
& + 9102240 a_2^2 - 544320 a_2 a_3 + 21600 a_3^2 + 572373908820000 b_1^2 - 1666074009600 b_1 b_2 + 6973344000 b_1 b_3 + 1215678240 b_2^2 \\
& - 10221120 b_2 b_3 + 21600 b_3^2 + 1458607500)T^4 + 10102837248000 a_1^4 + 225385574400 a_1^3 a_2 - 5689958400 a_1^3 a_3 \\
& + 2528870400 a_1^2 a_2^2 - 63221760 a_1^2 a_2 a_3 + 9768409943040000 a_1^2 b_1^2 - 24493058150400 a_1^2 b_1 b_2 + 67015065600 a_1^2 b_1 b_3 \\
& + 14414561280 a_1^2 b_2^2 - 63221760 a_1^2 b_2 b_3 + 12644352 a_1 a_2^3 + 11712779366400 a_1 a_2 b_1^2 - 24530042880 a_1 a_2 b_1 b_2
\end{aligned}$$

$$\begin{aligned}
& +12644352 a_1 a_2 b_2^2 - 5689958400 a_1 a_3 b_1^2 - 14288117760 a_2^2 b_1^2 + 12644352 a_2^2 b_1 b_2 - 63221760 a_2 a_3 b_1^2 \\
& - 6462183748608000 b_1^4 + 20438646681600 b_1^3 b_2 + 67015065600 b_1^3 b_3 - 26932469760 b_1^2 b_2^2 - 63221760 b_1^2 b_2 b_3 \\
& + 12644352 b_1 b_2^3 + (-108367402752000 a_1^2 b_1 + 166961894400 a_1^2 b_2 - 756403200 a_1^2 b_3 + 1985976115200 a_1 a_2 b_1 \\
& - 3084318720 a_1 a_2 b_2 + 13547520 a_1 a_2 b_3 + 17340825600 a_1 a_3 b_1 - 13547520 a_1 a_3 b_2 + 699955200 a_2^2 b_1 - 451584 a_2^2 b_2 \\
& - 9031680 a_2 a_3 b_1 + 2535603178752000 b_1^3 - 5977199692800 b_1^2 b_2 + 11391206400 b_1^2 b_3 + 3712020480 b_1 b_2^2 \\
& - 9031680 b_1 b_2 b_3 - 451584 b_2^3 - 2216480112000 b_1 + 3584800800 b_2 - 16556400 b_3) T^3 + (-4115736576000 a_1^4 \\
& - 70808371200 a_1^3 a_2 - 4425523200 a_1^3 a_3 + 1327656960 a_1^2 a_2^2 - 232295712768000 a_1^2 b_1^2 - 26553139200 a_1^2 b_1 b_2 \\
& - 13276569600 a_1^2 b_1 b_3 + 1327656960 a_1^2 b_2^2 - 318637670400 a_1 a_2 b_1^2 + 13276569600 a_1 a_3 b_1^2 + 1327656960 a_2^2 b_1^2 \\
& + 2611943792640000 b_1^4 - 4239651225600 b_1^3 b_2 + 4425523200 b_1^3 b_3 + 1327656960 b_1^2 b_2^2 - 19146456000 a_1^2 \\
& - 4115059200 a_1 a_2 - 261273600 a_1 a_3 - 13970880 a_2^2 - 362880 a_2 a_3 + 77760 a_3^2 + 1634378221224000 b_1^2 \\
& - 5102139974400 b_1 b_2 + 22527590400 b_1 b_3 + 3984240960 b_2^2 - 35199360 b_2 b_3 + 77760 b_3^2 + 209860875) T^2 - 5540724000 a_1^2 \\
& - 520732800 a_1 a_2 - 23587200 a_1 a_3 - 6350400 a_2^2 - 544320 a_2 a_3 - 12960 a_3^2 - 190866558708000 b_1^2 + 638634326400 b_1 b_2 \\
& - 3138912000 b_1 b_3 - 534703680 b_2^2 + 5261760 b_2 b_3 - 12960 b_3^2 + (-994621587456000 a_1^4 b_1 + 867402547200 a_1^4 b_2 \\
& - 1734805094400 a_1^3 a_2 b_1 + 5782683648000 a_1^2 b_1^3 - 1734805094400 a_1 a_2 b_1^3 + 1000404271104000 b_1^5 - 867402547200 b_1^4 b_2 \\
& + 89949182400000 a_1^2 b_1 - 164517696000 a_1^2 b_2 + 902361600 a_1^2 b_3 + 2091872563200 a_1 a_2 b_1 - 3542676480 a_1 a_2 b_2 \\
& + 17418240 a_1 a_2 b_3 + 16431206400 a_1 a_3 b_1 - 17418240 a_1 a_3 b_2 + 13036101120 a_2^2 b_1 - 20998656 a_2^2 b_2 + 96768 a_2^2 b_3 \\
& + 198374400 a_2 a_3 b_1 - 193536 a_2 a_3 b_2 + 483840 a_3^2 b_1 - 5551805448000000 b_1^3 + 22625594611200 b_1^2 b_2 + 36822643200 b_1^2 b_3 \\
& - 36256550400 b_1 b_2^2 - 18385920 b_1 b_2 b_3 + 483840 b_1 b_3^2 + 22353408 b_2^3 - 96768 b_2^2 b_3 + 66393432000 b_1 - 121451400 b_2 \\
& + 680400 b_3) T - 4465125, \\
\mathbf{h}_{20} = 0, \quad \mathbf{h}_{19} = 0, \quad \mathbf{h}_{18} = T, \quad \mathbf{h}_{17} = 0, \quad \mathbf{h}_{16} = 9T^3 - 27T - 252b_1, \quad \mathbf{h}_{15} = 672Ta_1, \quad \mathbf{h}_{14} = 36T^5 - 600T^3 \\
& - 5040T^2b_1 - 540T - 95760b_1 + 84b_2, \quad \mathbf{h}_{13} = (3360a_1 + 168a_2)T, \quad \mathbf{h}_{12} = 84T^7 - 2940T^5 - 23520T^4b_1 + 1260T^3 \\
& + (-719040b_1 + 756b_2)T^2 + (493920a_1^2 + 493920b_1^2 - 13860)T - 13263600b_1 + 19824b_2 - 84b_3, \quad \mathbf{h}_{11} = -14112T^5a_1 \\
& + (94080a_1 + 2352a_2)T^3 + 9972480a_1b_1 - 9408a_1b_2 - 65856a_2b_1 + (131040a_1 - 4032a_2 - 432a_3)T, \quad \mathbf{h}_{10} = 126T^9 \\
& - 6552T^7 - 49392T^6b_1 + 25956T^5 + (734160b_1 + 84b_2)T^4 + (2963520a_1^2 + 2963520b_1^2 - 52920)T^3 - 82978560a_1^2b_1 \\
& + 27659520b_1^3 + (64305360b_1 - 104832b_2 + 504b_3)T^2 + (-16040640a_1^2 - 357504a_1a_2 - 145871040b_1^2 + 131712b_1b_2 \\
& - 198450)T - 83628720b_1 + 123228b_2 - 504b_3, \quad \mathbf{h}_9 = -47040T^7a_1 + (460320a_1 + 6552a_2)T^5 + (4939200a_1 - 1200a_3)T^3 \\
& + (-998188800a_1b_1 + 987840a_1b_2 + 141120a_2b_1)T^2 - 6861792000a_1b_1 + 10590720a_1b_2 - 47040a_1b_3 - 3601920a_2b_1 \\
& + 2688a_2b_2 + 33600a_3b_1 + (-215129600a_1^3 + 645388800a_1b_1^2 + 2469600a_1 - 123480a_2 + 3600a_3)T, \quad \mathbf{h}_8 = 126T^{11} \\
& - 7770T^9 - 52920T^8b_1 + 71820T^7 + (7973280b_1 - 5292b_2)T^6 + (7408800a_1^2 + 7408800b_1^2 - 532980)T^5 + (592905600b_1 \\
& - 902160b_2 + 3780b_3)T^4 + (172872000a_1^2 + 415598400b_1^2 - 564480b_1b_2 - 614250)T^3 + 21633696000a_1^2b_1 \\
& - 19756800a_1^2b_2 + 22924473600b_1^3 - 19756800b_1^2b_2 + (2074464000a_1^2b_1 - 691488000b_1^3 - 1260403200b_1 + 1863540b_2 \\
& - 7560b_3)T^2 + (-837144000a_1^2 + 1895040a_1a_2 - 120960a_1a_3 + 26208a_2^2 + 5281264800b_1^2 - 6814080b_1b_2 + 2016b_2^2 \\
& + 2976750)T - 1697711400b_1 + 2623320b_2 - 11340b_3, \quad \mathbf{h}_7 = -70560T^9a_1 + (967680a_1 + 6048a_2)T^7 + (-25905600a_1 \\
& - 88704a_2 + 2592a_3)T^5 + (-3528000000a_1b_1 + 2540160a_1b_2 - 282240a_2b_1)T^4 - 30978662400a_1^3b_1 - 30978662400a_1b_1^3 \\
& + (1106380800a_1^3 - 3319142400a_1b_1^2 + 73382400a_1 - 776160a_2 + 37440a_3)T^3 + (8001100800a_1b_1 - 7015680a_1b_2 \\
& + 95235840a_2b_1 - 96768a_2b_2 + 483840a_3b_1)T^2 + 18478656000a_1b_1 - 24877440a_1b_2 + 80640a_1b_3 + 537828480a_2b_1 \\
& - 862848a_2b_2 + 4032a_2b_3 + 4354560a_3b_1 - 4032a_3b_2 + (-4583577600a_1^3 - 63221760a_1^2a_2 - 4583577600a_1b_1^2 \\
& - 63221760a_2b_1^2 - 75146400a_1 - 423360a_2 - 38880a_3)T, \quad \mathbf{h}_6 = 84T^{13} - 4872T^{11} - 25872T^{10}b_1 + 71820T^9 \\
& + (13522320b_1 - 9828b_2)T^8 + (9878400a_1^2 + 9878400b_1^2 + 297360)T^7 + (650069280b_1 - 846720b_2 + 3024b_3)T^6 \\
& + (-215349120a_1^2 + 2370816a_1a_2 + 2803489920b_1^2 - 2370816b_1b_2 - 7276500)T^5 + (-1936166400a_1^2b_1 + 645388800b_1^3 \\
& - 20296231200b_1 + 30111480b_2 - 126000b_3)T^4 + (-388080000a_1^2 - 10160640a_1a_2 - 56448a_2^2 + 37703030400b_1^2 \\
& - 114024960b_1b_2 - 1128960b_1b_3 + 169344b_2^2 + 43659000)T^3 - 701405913600a_1^2b_1 + 999694080a_1^2b_2 - 3951360a_1^2b_3 \\
& - 1833431040a_1a_2b_1 + 1580544a_1a_2b_2 + 1580544a_2^2b_1 + 3497309222400b_1^3 - 6286613760b_1^2b_2 - 3951360b_1^2b_3 \\
& + 3161088b_1b_2^2 + (3872332800a_1^2b_1 + 24524774400b_1^3 + 4998294000b_1 - 8255520b_2 + 45360b_3)T^2 + (-72283545600a_1^4 \\
& + 72283545600b_1^4 - 2073052800a_1^2 - 26530560a_1a_2 - 161280a_1a_3 + 225792a_2^2 - 8064a_2a_3 + 1546961673600b_1^2 \\
& - 3590657280b_1b_2 + 9676800b_1b_3 + 1919232b_2^2 - 8064b_2b_3 + 14685300)T - 24654898800b_1 + 40774860b_2 - 196560b_3, \\
\mathbf{h}_5 = -56448T^{11}a_1 + (1031520a_1 - 168a_2)T^9 + (403200a_1 - 403200a_2 + 6048a_3)T^7 + (-987840000a_1b_1 + 395136a_1b_2 \\
& - 3556224a_2b_1)T^6 + (774466560a_1^3 - 2323399680a_1b_1^2 + 383140800a_1 - 402192a_2 - 127008a_3)T^5 + (-127262016000a_1b_1 \\
& + 200390400a_1b_2 - 846720a_1b_3 - 402474240a_2b_1 + 338688a_2b_2 - 846720a_3b_1)T^4 + 2385357004800a_1^3b_1 \\
& - 2065244160a_1^3b_2 + 6195732480a_1^2a_2b_1 - 7114766131200a_1b_1^3 + 6195732480a_1b_1^2b_2 - 2065244160a_2b_1^3 \\
& + (13645363200a_1^3 - 221276160a_1^2a_2 + 65276467200a_1b_1^2 - 221276160a_2b_1^2 + 402192000a_1 + 9737280a_2 - 272160a_3)T^3 \\
& + (216850636800a_1^3b_1 + 216850636800a_1b_1^3 + 326275084800a_1b_1 - 520732800a_1b_2 + 2419200a_1b_3 + 5159064960a_2b_1 \\
& - 7959168a_2b_2 + 36288a_2b_3 + 38465280a_3b_1 - 36288a_3b_2)T^2 + 290678976000a_1b_1 - 440294400a_1b_2 + 1814400a_1b_3 \\
& - 4723004160a_2b_1 + 7620480a_2b_2 - 36288a_2b_3 - 41005440a_3b_1 + 36288a_3b_2 + (7507584000a_1^3 + 1082672640a_1^2a_2 \\
& - 23708160a_1^2a_3 + 6322176a_1a_2^2 - 5263606656000a_1b_1^2 + 8218828800a_1b_1b_2 - 3161088a_1b_2^2 - 10771407360a_2b_1^2 \\
& + 9483264a_2b_1b_2 - 23708160a_3b_1^2 - 1146247200a_1 - 17304840a_2 - 816480a_3)T, \quad \mathbf{h}_4 = 36T^{15} - 1260T^{13} + 13860T^{11}
\end{aligned}$$

$$\begin{aligned}
& + (8843520 b_1 - 6468 b_2) T^{10} + (7408800 a_1^2 + 7408800 b_1^2 - 585900) T^9 + (-479430000 b_1 + 720720 b_2 - 2700 b_3) T^8 \\
& + (-659030400 a_1^2 + 1128960 a_1 a_2 + 503798400 b_1^2 - 32829300) T^7 + (-5808499200 a_1^2 b_1 + 1936166400 b_1^3 - 3418632000 b_1 \\
& - 52920 b_2 + 25200 b_3) T^6 + (699249600 a_1^2 + 14112000 a_1 a_2 - 1693440 a_1 a_3 + 254016 a_2^2 - 176332968000 b_1^2 \\
& + 246677760 b_1 b_2 - 84672 b_2^2 + 107559900) T^5 + (-283510080000 a_1^2 b_1 + 276595200 a_1^2 b_2 - 322233408000 b_1^3 \\
& + 276595200 b_1^2 b_2 + 136877580000 b_1 - 203742000 b_2 + 869400 b_3) T^4 + (515088000 a_1^2 - 101606400 a_1 a_2 + 5644800 a_1 a_3 \\
& + 1411200 a_2^2 + 2346098832000 b_1^2 - 3482841600 b_1 b_2 + 11289600 b_1 b_3 + 282240 b_2^2 + 77395500) T^3 - 5909795136000 a_1^2 b_1 \\
& + 8746617600 a_1^2 b_2 - 36691200 a_1^2 b_3 + 157941504000 a_1 a_2 b_1 - 249500160 a_1 a_2 b_2 + 1128960 a_1 a_2 b_3 + 1354752000 a_1 a_3 b_1 \\
& - 1128960 a_1 a_3 b_2 + 111767040 a_2^2 b_1 - 112896 a_2^2 b_2 + 155814289344000 b_1^3 - 419174380800 b_1^2 b_2 + 19756800 b_1^2 b_3 \\
& + 374814720 b_1 b_2^2 - 112896 b_2^3 + (8692399296000 a_1^2 b_1 - 13335840000 a_1^2 b_2 + 59270400 a_1^2 b_3 + 24972595200 a_1 a_2 b_1 \\
& - 23708160 a_1 a_2 b_2 + 23708160 a_2^2 b_1 + 11173063104000 b_1^3 - 15627628800 b_1^2 b_2 + 59270400 b_1^2 b_3 + 143741304000 b_1 \\
& - 235361700 b_2 + 1134000 b_3) T^2 + (103262208000 a_1^4 + 5163110400 a_1^3 a_2 - 17812730880000 a_1^2 b_1^2 + 15489331200 a_1^2 b_1 b_2 \\
& - 15489331200 a_1 a_2 b_1^2 + 5834314752000 b_1^4 - 5163110400 b_1^3 b_2 - 37705500000 a_1^2 - 440294400 a_1 a_2 + 24192000 a_1 a_3 \\
& - 3598560 a_2^2 - 60480 a_2 a_3 + 4320 a_3^2 + 101065234788000 b_1^2 - 305564313600 b_1 b_2 + 1319673600 b_1 b_3 + 230942880 b_2^2 \\
& - 1995840 b_2 b_3 + 4320 b_3^2 + 53581500) T - 33744438000 b_1 + 57153600 b_2 - 283500 b_3, \quad \mathbf{h}_3 = -23520 T^{13} a_1 + (551040 a_1 \\
& - 3024 a_2) T^{11} + (84722400 a_1 - 262080 a_2 + 1680 a_3) T^9 + (1555142400 a_1 b_1 - 423360 a_1 b_2 - 2963520 a_2 b_1) T^8 \\
& + (-1843968000 a_1^3 + 5531904000 a_1 b_1^2 + 323769600 a_1 + 5675040 a_2 - 20160 a_3) T^7 + (175564569600 a_1 b_1 - 261354240 a_1 b_2 \\
& + 1128960 a_1 b_3 + 282804480 a_2 b_1 - 225792 a_2 b_2) T^6 + (49049548800 a_1^3 + 69701990400 a_1 b_1^2 - 206388000 a_1 + 35985600 a_2 \\
& + 574560 a_3) T^5 + (361417728000 a_1^3 b_1 + 361417728000 a_1 b_1^3 - 524204352000 a_1 b_1 + 845308800 a_1 b_2 - 4435200 a_1 b_3 \\
& + 11327702400 a_2 b_1 - 15523200 a_2 b_2 + 60480 a_2 b_3 + 83865600 a_3 b_1 - 60480 a_3 b_2) T^4 + 124276067328000 a_1^3 b_1 \\
& - 180708864000 a_1^3 b_2 + 737587200 a_1^3 b_3 + 15489331200 a_1^2 a_2 b_1 - 2212761600 a_1^2 a_3 b_1 - 301577278464000 a_1 b_1^3 \\
& + 480169267200 a_1 b_1^2 b_2 - 2212761600 a_1 b_1^2 b_3 + 15489331200 a_2 b_1^3 + 737587200 a_3 b_1^3 + (27659520000 a_1^3 \\
& - 553190400 a_1^2 a_2 + 79027200 a_1^2 a_3 + 40486424832000 a_1 b_1^2 - 72072806400 a_1 b_1 b_2 + 31610880 a_1 b_2^2 + 35167104000 a_2 b_1^2 \\
& - 31610880 a_2 b_1 b_2 + 79027200 a_3 b_1^2 + 5905872000 a_1 + 17463600 a_2 - 2116800 a_3) T^3 + (-722835456000 a_1^3 b_1 \\
& - 722835456000 a_1 b_1^3 - 8401663872000 a_1 b_1 + 12692332800 a_1 b_2 - 53222400 a_1 b_3 - 29728339200 a_2 b_1 + 38949120 a_2 b_2 \\
& - 120960 a_2 b_3 - 145152000 a_3 b_1 + 120960 a_3 b_2) T^2 - 2514208032000 a_1 b_1 + 4189147200 a_1 b_2 - 20563200 a_1 b_3 \\
& - 25009992000 a_2 b_1 + 39795840 a_2 b_2 - 181440 a_2 b_3 - 183859200 a_3 b_1 + 181440 a_3 b_2 + (287997696000 a_1^3 - 146764800 a_1^2 a_2 \\
& - 146764800 a_1^2 a_3 - 58705920 a_1 a_2^2 + 4515840 a_1 a_2 a_3 - 869760831744000 a_1 b_1^2 + 2025173606400 a_1 b_1 b_2 \\
& - 5419008000 a_1 b_1 b_3 - 1083801600 a_1 b_2^2 + 4515840 a_1 b_2 b_3 - 451584 a_2^3 - 606262809600 a_2 b_1^2 + 1052190720 a_2 b_1 b_2 \\
& - 451584 a_2 b_2^2 + 79027200 a_3 b_1^2 - 2968812000 a_1 + 12700800 a_2 + 1587600 a_3) T, \quad \mathbf{h}_2 = 9 T^{17} + 120 T^{15} + 5040 T^{14} b_1 \\
& - 18900 T^{13} + (1698480 b_1 - 1092 b_2) T^{12} + (2963520 a_1^2 + 2963520 b_1^2 - 4014360) T^{11} + (-655991280 b_1 + 758016 b_2 \\
& - 2376 b_3) T^{10} + (232142400 a_1^2 - 658560 a_1 a_2 - 2373873600 b_1^2 + 1787520 b_1 b_2 - 30778650) T^9 + (2904249600 a_1^2 b_1 \\
& - 968083200 b_1^3 - 3465428400 b_1 + 3488940 b_2 - 14040 b_3) T^8 + (-5261961600 a_1^2 + 8225280 a_1 a_2 + 8064 a_2^2 \\
& + 20513808000 b_1^2 - 101445120 b_1 b_2 - 483840 b_1 b_3 + 104832 b_2^2 + 208542600) T^7 + (-277148390400 a_1^2 b_1 + 21276160 a_1^2 b_2 \\
& - 328779494400 b_1^3 + 221276160 b_1^2 b_2 + 148696203600 b_1 - 183526560 b_2 + 589680 b_3) T^6 + (216850636800 a_1^4 \\
& - 216850636800 b_1^4 - 44118345600 a_1^2 - 265870080 a_1 a_2 + 483840 a_1 a_3 - 1016064 a_2^2 + 24192 a_2 a_3 - 3118423190400 b_1^2 \\
& + 8997246720 b_1 b_2 - 35804160 b_1 b_3 - 4741632 b_2^2 + 24192 b_2 b_3 - 446512500) T^5 - 179263193088000 a_1^4 b_1 \\
& + 144567091200 a_1^4 b_2 - 289134182400 a_1^3 a_2 b_1 - 26022076416000 a_1^2 b_1^3 - 289134182400 a_1 a_2 b_1^3 + 153241116672000 b_1^5 \\
& - 144567091200 b_1^4 b_2 + (-12337133760000 a_1^2 b_1 + 16101792000 a_1^2 b_2 - 59270400 a_1^2 b_3 - 33191424000 a_1 a_2 b_1 \\
& + 23708160 a_1 a_2 b_2 - 7902720 a_2^2 b_1 + 16311016512000 b_1^3 - 26177760000 b_1^2 b_2 - 59270400 b_1^2 b_3 + 15805440 b_1 b_2^2 \\
& + 1309206402000 b_1 - 2026968300 b_2 + 8845200 b_3) T^4 + (-516311040000 a_1^4 + 103262208000 a_1^3 a_2 - 35625461760000 a_1^2 b_1^2 \\
& + 30978662400 a_1^2 b_1 b_2 - 30978662400 a_1 a_2 b_1^2 + 12391464960000 b_1^4 - 10326220800 b_1^3 b_2 + 17220168000 a_1^2 \\
& + 1151539200 a_1 a_2 - 16934400 a_1 a_3 + 8043840 a_2^2 - 846720 a_2 a_3 + 8640 a_3^2 + 321900537672000 b_1^2 - 897997363200 b_1 b_2 \\
& + 3527193600 b_1 b_3 + 615988800 b_2^2 - 4717440 b_2 b_3 + 8640 b_3^2 + 440559000) T^3 + 24567369120000 a_1^2 b_1 \\
& - 37467360000 a_1^2 b_2 + 158457600 a_1^2 b_3 - 192442521600 a_1 a_2 b_1 + 348848640 a_1 a_2 b_2 - 1935360 a_1 a_2 b_3 - 2119219200 a_1 a_3 b_1 \\
& + 1935360 a_1 a_3 b_2 - 6612883200 a_2^2 b_1 + 10499328 a_2^2 b_2 - 48384 a_2^2 b_3 - 105960960 a_2 a_3 b_1 + 96768 a_2 a_3 b_2 - 241920 a_3^2 b_1 \\
& + 3742845956640000 b_1^3 - 13791011270400 b_1^2 b_2 - 11705299200 b_1^2 b_3 + 19645597440 b_1 b_2^2 + 2419200 b_1 b_2 b_3 - 241920 b_1 b_3^2 \\
& - 11176704 b_2^3 + 48384 b_2^2 b_3 + (31018232448000 a_1^2 b_1 - 39677299200 a_1^2 b_2 + 118540800 a_1^2 b_3 + 31972147200 a_1 a_2 b_1 \\
& - 20321280 a_1 a_2 b_2 - 135475200 a_1 a_3 b_1 - 623185920 a_2^2 b_1 + 677376 a_2^2 b_2 - 6773760 a_2 a_3 b_1 + 274915589760000 b_1^3 \\
& - 347476953600 b_1^2 b_2 + 7772889600 b_1^2 b_3 - 684149760 b_1 b_2^2 - 6773760 b_1 b_2 b_3 + 677376 b_2^3 - 384619914000 b_1 + 638215200 b_2 \\
& - 3061800 b_3) T^2 + (-3695311872000 a_1^4 - 13276569600 a_1^3 a_2 + 442552320 a_1^2 a_2^2 + 593152874496000 a_1^2 b_1^2 \\
& - 1031146905600 a_1^2 b_1 b_2 + 442552320 a_1^2 b_2^2 - 13276569600 a_1 a_2 b_1^2 + 442552320 a_2^2 b_1^2 + 596848186368000 b_1^4 \\
& - 1031146905600 b_1^3 b_2 + 442552320 b_1^2 b_2^2 - 36292536000 a_1^2 - 673142400 a_1 a_2 - 1270080 a_2^2 - 25920 a_3^2 \\
& - 540960062328000 b_1^2 + 1678347216000 b_1 b_2 - 7482585600 b_1 b_3 - 1301832000 b_2^2 + 11612160 b_2 b_3 - 25920 b_3^2 \\
& - 120558375) T - 110377890000 b_1 + 194084100 b_2 - 1020600 b_3, \quad \mathbf{h}_1 = -4032 T^{15} a_1 + (117600 a_1 - 1176 a_2) T^{13} \\
& + (-584640 a_1 + 48384 a_2 - 1008 a_3) T^{11} + (-988404480 a_1 b_1 + 724416 a_1 b_2 + 103488 a_2 b_1) T^{10} + (276595200 a_1^3 \\
& - 829785600 a_1 b_1^2 + 127663200 a_1 + 3102120 a_2 - 25200 a_3) T^9 + (53826796800 a_1 b_1 - 80236800 a_1 b_2 + 302400 a_1 b_3
\end{aligned}$$

$$\begin{aligned}
& +6531840 a_2 b_1 + 24192 a_2 b_2 - 423360 a_3 b_1) T^8 + (-22285670400 a_1^3 + 158054400 a_1^2 a_2 - 53264332800 a_1 b_1^2 \\
& + 158054400 a_2 b_1^2 + 3680510400 a_1 + 181440 a_2 + 876960 a_3) T^7 + (-216850636800 a_1^3 b_1 - 216850636800 a_1 b_1^3 \\
& + 447897945600 a_1 b_1 - 105840000 a_1 b_2 - 2257920 a_1 b_3 + 3250558080 a_2 b_1 - 5588352 a_2 b_2 + 28224 a_2 b_3 + 31046400 a_3 b_1 \\
& - 28224 a_3 b_2) T^6 + (-53817523200 a_1^3 - 3279628800 a_1^2 a_2 - 23708160 a_1^2 a_3 + 18966528 a_1 a_2^2 + 19578119500800 a_1 b_1^2 \\
& - 27438243840 a_1 b_1 b_2 + 9483264 a_1 b_2^2 - 8558645760 a_2 b_1^2 + 9483264 a_2 b_1 b_2 - 23708160 a_3 b_1^2 - 13485074400 a_1 \\
& - 71918280 a_2 + 9707040 a_3) T^5 + (-11203949568000 a_1^3 b_1 + 10326220800 a_1^3 b_2 - 30978662400 a_1^2 a_2 b_1 \\
& + 36296666112000 a_1 b_1^3 - 30978662400 a_1 b_1^2 b_2 + 10326220800 a_2 b_1^3 - 14310838080000 a_1 b_1 + 21278073600 a_1 b_2 \\
& - 91324800 a_1 b_3 + 50972544000 a_2 b_1 - 77051520 a_2 b_2 + 302400 a_2 b_3 + 485654400 a_3 b_1 - 302400 a_3 b_2) T^4 \\
& - 545885125632000 a_1^3 b_1 + 817141248000 a_1^3 b_2 - 3477196800 a_1^3 b_3 - 10763188531200 a_1^2 a_2 b_1 + 15552552960 a_1^2 a_2 b_2 \\
& - 63221760 a_1^2 a_2 b_3 - 69227827200 a_1^2 a_3 b_1 + 63221760 a_1^2 a_3 b_2 - 15931883520 a_1 a_2^2 b_1 + 12644352 a_1 a_2^2 b_2 \\
& - 17936926866432000 a_1 b_1^3 + 47784586752000 a_1 b_1^2 b_2 - 3477196800 a_1 b_1^2 b_3 - 42232135680 a_1 b_1 b_2^2 + 12644352 a_1 b_2^3 \\
& - 12644352 a_2^3 b_1 - 24286322995200 a_2 b_1^3 + 41852805120 a_2 b_1^2 b_2 - 63221760 a_2 b_1^2 b_3 - 12644352 a_2 b_1 b_2^2 \\
& - 69227827200 a_3 b_1^3 + 63221760 a_3 b_1^2 b_2 + (105557760000 a_1^3 - 1433779200 a_1^2 a_2 - 553190400 a_1^2 a_3 - 36126720 a_1 a_2^2 \\
& - 238176804096000 a_1 b_1^2 + 348803481600 a_1 b_1 b_2 - 1174118400 a_1 b_1 b_3 - 22579200 a_1 b_2^2 + 451584 a_2^3 + 1229313254400 a_2 b_1^2 \\
& - 2063738880 a_2 b_1 b_2 + 4515840 a_2 b_1 b_3 + 451584 a_2 b_2^2 + 5814144000 a_3 b_1^2 - 4515840 a_3 b_1 b_2 - 8541288000 a_1 + 6350400 a_2 \\
& + 2494800 a_3) T^3 + (308392584192000 a_1^3 b_1 - 480169267200 a_1^3 b_2 + 2212761600 a_1^3 b_3 + 108425318400 a_1^2 a_2 b_1 \\
& - 6638284800 a_1^2 a_3 b_1 - 1254171147264000 a_1 b_1^3 + 1750294425600 a_1 b_1^2 b_2 - 6638284800 a_1 b_1^2 b_3 - 139403980800 a_2 b_1^3 \\
& + 2212761600 a_3 b_1^3 - 21779204832000 a_1 b_1 + 35251070400 a_1 b_2 - 166924800 a_1 b_3 - 284008939200 a_2 b_1 + 444528000 a_2 b_2 \\
& - 1995840 a_2 b_3 - 2115590400 a_3 b_1 + 1995840 a_3 b_2) T^2 + 4288933152000 a_1 b_1 - 7214054400 a_1 b_2 + 35380800 a_1 b_3 \\
& + 25477804800 a_2 b_1 - 40642560 a_2 b_2 + 181440 a_2 b_3 + 173275200 a_3 b_1 - 181440 a_3 b_2 + (5782683648000 a_1^5 \\
& + 289134182400 a_1^4 a_2 - 653443252224000 a_1^3 b_1^2 + 578268364800 a_1^3 b_1 b_2 - 659225935872000 a_1 b_1^4 + 578268364800 a_1 b_1^3 b_2 \\
& - 289134182400 a_2 b_1^4 - 1786663872000 a_1^3 - 40490150400 a_1^2 a_2 + 529804800 a_1^2 a_3 - 430133760 a_1 a_2^2 + 10644480 a_1 a_2 a_3 \\
& - 483840 a_1 a_3^2 - 12036334193856000 a_1 b_1^2 + 359094629376000 a_1 b_1 b_2 - 152448307200 a_1 b_1 b_3 - 26773286400 a_1 b_2^2 \\
& + 227404800 a_1 b_2 b_3 - 483840 a_1 b_3^2 - 2032128 a_2^3 + 96768 a_2^2 a_3 - 35851394880000 a_2 b_1^2 + 84312990720 a_2 b_1 b_2 \\
& - 232243200 a_2 b_1 b_3 - 45384192 a_2 b_2^2 + 193536 a_2 b_2 b_3 - 138041971200 a_3 b_1^2 + 232243200 a_3 b_1 b_2 - 96768 a_3 b_2^2 \\
& - 8811180000 a_1 - 140502600 a_2 - 4762800 a_3) T, \quad \mathbf{h}_0 = T^{19} + 93 T^{17} + 1428 T^{16} b_1 - 8604 T^{15} + (-238560 b_1 + 252 b_2) T^{14} \\
& + (493920 a_1^2 + 493920 b_1^2 - 701820) T^{13} + (12361440 b_1 - 31920 b_2 + 156 b_3) T^{12} + (-18863040 a_1^2 + 225792 a_1 a_2 \\
& - 129877440 b_1^2 + 112896 b_1 b_2 - 4810050) T^{11} + (-101418240 a_1^2 b_1 + 33806080 b_1^3 + 2397528000 b_1 - 2923452 b_2 \\
& + 7704 b_3) T^{10} + (-437992800 a_1^2 - 4179840 a_1 a_2 + 94080 a_1 a_3 + 15456 a_2^2 + 43934167200 b_1^2 - 63100800 b_1 b_2 + 107520 b_1 b_3 \\
& + 12768 b_2^2 - 54290250) T^9 + (41706604800 a_1^2 b_1 - 35562240 a_1^2 b_2 + 48160492800 b_1^3 - 35562240 b_1^2 b_2 - 50413217400 b_1 \\
& + 70784280 b_2 - 292140 b_3) T^8 + (-20652441600 a_1^4 + 20652441600 b_1^4 - 1138435200 a_1^2 - 57254400 a_1 a_2 - 806400 a_1 a_3 \\
& - 782208 a_2^2 + 16128 a_2 a_3 - 4050580867200 b_1^2 + 8723635200 b_1 b_2 - 18708480 b_1 b_3 - 4523904 b_2^2 + 16128 b_2 b_3 \\
& + 603911700) T^7 + (1430352806400 a_1^2 b_1 - 1552884480 a_1^2 b_2 + 3951360 a_1^2 b_3 + 3477196800 a_1 a_2 b_1 - 1580544 a_1 a_2 b_2 \\
& - 4741632 a_2^2 b_1 - 8994125145600 b_1^3 + 14647691520 b_1^2 b_2 + 3951360 b_1^2 b_3 - 6322176 b_1 b_2^2 + 159195002400 b_1 \\
& - 143757180 b_2 + 136080 b_3) T^6 + (82609766400 a_1^4 - 3097866240 a_1^3 a_2 + 10687638528000 a_1^2 b_1^2 - 9293598720 a_1^2 b_1 b_2 \\
& + 9293598720 a_1 a_2 b_1^2 - 3645155942400 b_1^4 + 3097866240 b_1^3 b_2 - 55351144800 a_1^2 - 1541030400 a_1 a_2 + 24192000 a_1 a_3 \\
& + 6223392 a_2^2 - 399168 a_2 a_3 + 4320 a_3^2 + 171956731312800 b_1^2 - 464351408640 b_1 b_2 + 1719325440 b_1 b_3 + 314598816 b_2^2 \\
& - 2334528 b_2 b_3 + 4320 b_3^2 + 1294290900) T^5 - 3172628078592000 a_1^4 b_1 + 4791366451200 a_1^4 b_2 - 20652441600 a_1^4 b_3 \\
& - 9830562201600 a_1^3 a_2 b_1 + 8260976640 a_1^3 a_2 b_2 + 41304883200 a_1^3 a_3 b_1 - 12391464960 a_1^2 a_2^2 b_1 + 16384408018944000 a_1^2 b_1^3 \\
& - 28500369408000 a_1^2 b_1^2 b_2 + 12391464960 a_1^2 b_1 b_2^2 + 28169930342400 a_1 a_2 b_1^3 - 24782929920 a_1 a_2 b_1^2 b_2 \\
& + 41304883200 a_1 a_3 b_1^3 + 4130488320 a_2^2 b_1^3 - 2293247115264000 b_1^5 + 4708756684800 b_1^4 b_2 + 20652441600 b_1^4 b_3 \\
& - 4130488320 b_1^3 b_2^2 + (-65324814912000 a_1^2 b_1 + 94886265600 a_1^2 b_2 - 386668800 a_1^2 b_3 + 454135449600 a_1 a_2 b_1 \\
& - 755274240 a_1 a_2 b_2 + 3386880 a_1 a_2 b_3 + 4651315200 a_1 a_3 b_1 - 3386880 a_1 a_3 b_2 + 254016000 a_2^2 b_1 - 112896 a_2^2 b_2 \\
& - 2257920 a_2 a_3 b_1 + 795450172608000 b_1^3 - 1754259897600 b_1^2 b_2 + 2966342400 b_1^2 b_3 + 1022837760 b_1 b_2^2 - 2257920 b_1 b_2 b_3 \\
& - 112896 b_2^3 - 588047040000 b_1 + 904932000 b_2 - 3685500 b_3) T^4 + (11063808000000 a_1^4 - 44255232000 a_1^3 a_2 \\
& - 1475174400 a_1^3 a_3 + 442552320 a_1^2 a_2^2 - 136704411648000 a_1^2 b_1^2 + 53106278400 a_1^2 b_1 b_2 - 4425523200 a_1^2 b_1 b_3 \\
& + 442552320 a_1^2 b_2^2 - 292084531200 a_1 a_2 b_1^2 + 4425523200 a_1 a_3 b_1^2 + 442552320 a_2^2 b_1^2 + 998899593216000 b_1^4 \\
& - 1516479283200 b_1^3 b_2 + 1475174400 b_1^3 b_3 + 442552320 b_1^2 b_2^2 + 130722984000 a_1^2 - 152409600 a_1 a_2 - 135475200 a_1 a_3 \\
& - 3810240 a_2^2 - 846720 a_2 a_3 + 8640 a_3^2 + 290942210664000 b_1^2 - 849708921600 b_1 b_2 + 3374784000 b_1 b_3 + 610908480 b_2^2 \\
& - 4717440 b_2 b_3 + 8640 b_3^2 + 93767625) T^3 - 114975812448000 a_1^2 b_1 + 186684825600 a_1^2 b_2 - 874540800 a_1^2 b_3 \\
& - 1774183219200 a_1 a_2 b_1 + 2760307200 a_1 a_2 b_2 - 12096000 a_1 a_2 b_3 - 12415334400 a_1 a_3 b_1 + 12096000 a_1 a_3 b_2 \\
& - 7642494720 a_2^2 b_1 + 11515392 a_2^2 b_2 - 48384 a_2^2 b_3 - 105960960 a_2 a_3 b_1 + 96768 a_2 a_3 b_2 - 241920 a_3^2 b_1 \\
& + 2574502074144000 b_1^3 - 10437009408000 b_1^2 b_2 - 11349676800 b_1^2 b_3 + 16414513920 b_1 b_2^2 + 2419200 b_1 b_2 b_3 - 241920 b_1 b_3^2 \\
& - 10160640 b_2^3 + 48384 b_2^2 b_3 + (-456832008192000 a_1^4 b_1 + 433701273600 a_1^4 b_2 - 867402547200 a_1^3 a_2 b_1 \\
& + 83848912896000 a_1^2 b_1^3 - 867402547200 a_1 a_2 b_1^3 + 540680921088000 b_1^5 - 433701273600 b_1^4 b_2 + 59604219360000 a_1^2 b_1 \\
& - 91437292800 a_1^2 b_2 + 417312000 a_1^2 b_3 - 628401715200 a_1 a_2 b_1 + 1032998400 a_1 a_2 b_2 - 4838400 a_1 a_2 b_3 - 6144768000 a_1 a_3 b_1
\end{aligned}$$

$$\begin{aligned}
& +4838400 a_1 a_3 b_2 + 5176846080 a_2^2 b_1 - 9144576 a_2^2 b_2 + 48384 a_2^2 b_3 + 99187200 a_2 a_3 b_1 - 96768 a_2 a_3 b_2 + 241920 a_3^2 b_1 \\
& -468230330328000 b_1^3 + 16452895737600 b_1^2 b_2 + 19596729600 b_1^2 b_3 - 22815717120 b_1 b_2^2 - 9192960 b_1 b_2 b_3 + 241920 b_1 b_3^2 \\
& +12531456 b_2^3 - 48384 b_2^2 b_3 + 904550976000 b_1 - 1546719300 b_2 + 7824600 b_3)T^2 + (94450499584000 a_1^6 \\
& +283351498752000 a_1^4 b_1^2 + 283351498752000 a_1^2 b_1^4 + 94450499584000 b_1^6 + 2225405952000 a_1^4 - 4741632000 a_1^3 a_2 \\
& +3161088000 a_1^3 a_3 - 126443520 a_1^2 a_2^2 - 63221760 a_1^2 a_2 a_3 + 12465412291584000 a_1^2 b_1^2 - 28741560422400 a_1^2 b_1 b_2 \\
& +75866112000 a_1^2 b_1 b_3 + 15299665920 a_1^2 b_2^2 - 63221760 a_1^2 b_2 b_3 + 12644352 a_1 a_2^3 + 19625614848000 a_1 a_2 b_1^2 \\
& -31610880000 a_1 a_2 b_1 b_2 + 12644352 a_1 a_2 b_2^2 + 3161088000 a_1 a_3 b_1^2 - 13403013120 a_2^2 b_1^2 + 12644352 a_2^2 b_1 b_2 \\
& -63221760 a_2 a_3 b_1^2 - 8439507514368000 b_1^4 + 24333107097600 b_1^3 b_2 + 75866112000 b_1^3 b_3 - 29587783680 b_1^2 b_2^2 \\
& -63221760 b_1^2 b_2 b_3 + 12644352 b_1 b_2^3 + 130167324000 a_1^2 + 2425852800 a_1 a_2 + 63504000 a_1 a_3 - 3810240 a_2^2 - 1270080 a_2 a_3 \\
& -64800 a_3^2 - 1137756096036000 b_1^2 + 3676716489600 b_1 b_2 - 17182368000 b_1 b_3 - 2970717120 b_2^2 + 27760320 b_2 b_3 - 64800 b_3^2 \\
& -66976875)T - 17104405500 b_1 + 30958200 b_2 - 170100 b_3, \\
\mathbf{q}_{20} &= 1, \quad \mathbf{q}_{19} = 0, \quad \mathbf{q}_{18} = 10 T^2 + 10, \quad \mathbf{q}_{17} = 1680 a_1, \quad \mathbf{q}_{16} = 45 T^4 - 270 T^2 - 5040 T b_1 + 405, \\
\mathbf{q}_{15} &= 6720 T^2 a_1 - 336 a_2, \quad \mathbf{q}_{14} = 120 T^6 - 1800 T^4 - 33600 T^3 b_1 + 1800 T^2 + 705600 a_1^2 + 705600 b_1^2 \\
& + (-1848000 b_1 + 1680 b_2)T + 16200, \quad \mathbf{q}_{13} = (33600 a_1 + 1680 a_2)T^2 + 240 a_3, \quad \mathbf{q}_{12} = 210 T^8 - 4200 T^6 \\
& -94080 T^5 b_1 + 6300 T^4 + (-5577600 b_1 + 5040 b_2)T^3 + (4939200 a_1^2 + 4939200 b_1^2 + 113400)T^2 + 1176000 a_1^2 \\
& -188160 a_1 a_2 + 430180800 b_1^2 - 376320 b_1 b_2 + (-227304000 b_1 + 362880 b_2 - 1680 b_3)T + 425250, \\
\mathbf{q}_{11} &= -47040 T^6 a_1 + (-470400 a_1 + 11760 a_2)T^4 + 61465600 a_1^3 - 184396800 a_1 b_1^2 + (-4334400 a_1 \\
& -40320 a_2 - 4320 a_3)T^2 + (146764800 a_1 b_1 - 188160 a_1 b_2 - 1317120 a_2 b_1)T + 604800 a_1 + 136080 a_2 - 4320 a_3, \\
\mathbf{q}_{10} &= 252 T^{10} - 3780 T^8 - 141120 T^7 b_1 + 63000 T^6 + (-168000 b_1 + 336 b_2)T^5 + (14817600 a_1^2 + 14817600 b_1^2 \\
& +718200)T^4 + (360292800 b_1 - 631680 b_2 + 3360 b_3)T^3 + (-160406400 a_1^2 - 3575040 a_1 a_2 - 1221628800 b_1^2 \\
& +1317120 b_1 b_2 + 3005100)T^2 + 41966400 a_1^2 - 4058880 a_1 a_2 + 215040 a_1 a_3 + 25536 a_2^2 + 66586094400 b_1^2 \\
& -112008960 b_1 b_2 + 188160 b_1 b_3 + 30912 b_2^2 + (-1659571200 a_1^2 b_1 + 553190400 b_1^3 + 4838097600 b_1 \\
& -7212240 b_2 + 30240 b_3)T + 1644300, \quad \mathbf{q}_9 = -117600 T^8 a_1 + (-2856000 a_1 + 21840 a_2)T^6 + (34104000 a_1 \\
& +470400 a_2 - 6000 a_3)T^4 + (-6039936000 a_1 b_1 + 6585600 a_1 b_2 + 940800 a_2 b_1)T^3 - 570752000 a_1^3 \\
& +79027200 a_1^2 a_2 + 8034432000 a_1 b_1^2 + 79027200 a_2 b_1^2 + (-2151296000 a_1^3 + 6453888000 a_1 b_1^2 \\
& -116424000 a_1 + 1587600 a_2 + 36000 a_3)T^2 + (-166475904000 a_1 b_1 + 238156800 a_1 b_2 - 940800 a_1 b_3 \\
& -45696000 a_2 b_1 + 53760 a_2 b_2 + 672000 a_3 b_1)T + 206388000 a_1 + 1058400 a_2 - 54000 a_3, \quad \mathbf{q}_8 = 210 T^{12} \\
& -567000)T^6 + (2873404800 b_1 - 4112640 b_2 + 15120 b_3)T^5 + (1852200000 a_1^2 + 2077992000 b_1^2 - 2822400 b_1 b_2 \\
& +23388750)T^4 + 51631104000 a_1^4 - 51631104000 b_1^4 + (13829760000 a_1^2 b_1 - 4609920000 b_1^3 + 31440528000 b_1 \\
& -424116000 b_2 + 151200 b_3)T^3 + (5089896000 a_1^2 + 18950400 a_1 a_2 - 1209600 a_1 a_3 + 262080 a_2^2 \\
& +137371752000 b_1^2 - 147168000 b_1 b_2 + 20160 b_2^2 - 31468500)T^2 - 2946888000 a_1^2 + 30240000 a_1 a_2 \\
& -2016000 a_1 a_3 + 544320 a_2^2 - 40320 a_2 a_3 + 6924150072000 b_1^2 - 16675142400 b_1 b_2 + 45561600 b_1 b_3 \\
& +9334080 b_2^2 - 40320 b_2 b_3 + (487992960000 a_1^2 b_1 - 395136000 a_1^2 b_2 + 410546304000 b_1^3 - 395136000 b_1^2 b_2 \\
& +56345436000 b_1 - 86940000 b_2 + 378000 b_3)T + 17435250, \quad \mathbf{q}_7 = -141120 T^{10} a_1 + (-6048000 a_1 \\
& +15120 a_2)T^8 + (-312681600 a_1 + 241920 a_2 + 8640 a_3)T^6 + (-15060326400 a_1 b_1 + 10160640 a_1 b_2 \\
& -1128960 a_2 b_1)T^5 + (5531904000 a_1^3 - 16595712000 a_1 b_1^2 - 1959552000 a_1 - 5896800 a_2 + 475200 a_3)T^4 \\
& + (-139184640000 a_1 b_1 + 88704000 a_1 b_2 + 559641600 a_2 b_1 - 645120 a_2 b_2 + 3225600 a_3 b_1)T^3 + 28336896000 a_1^3 \\
& -1433779200 a_1^2 a_2 + 11289600 a_1^2 a_3 - 18063360 a_1 a_2^2 - 17161208064000 a_1 b_1^2 + 30527078400 a_1 b_1 b_2 \\
& -13547520 a_1 b_2^2 + 4391654400 a_2 b_1^2 - 4515840 a_2 b_1 b_2 + 11289600 a_3 b_1^2 + (-45835776000 a_1^3 \\
& -632217600 a_1^2 a_2 - 311367168000 a_1 b_1^2 - 632217600 a_2 b_1^2 + 1014552000 a_1 + 7862400 a_2 + 1339200 a_3)T^2 \\
& + (-619573248000 a_1^3 b_1 - 619573248000 a_1 b_1^3 + 23627520000 a_1 b_1 - 155635200 a_1 b_2 + 1612800 a_1 b_3 \\
& +14288601600 a_2 b_1 - 20482560 a_2 b_2 + 80640 a_2 b_3 + 103219200 a_3 b_1 - 80640 a_3 b_2)T - 489888000 a_1 \\
& -10206000 a_2 - 648000 a_3, \quad \mathbf{q}_6 = 120 T^{14} + 5880 T^{12} - 47040 T^{11} b_1 + 476280 T^{10} + (27384000 b_1 \\
& -21840 b_2)T^9 + (24696000 a_1^2 + 24696000 b_1^2 + 16443000)T^8 + (3359664000 b_1 - 3628800 b_2 + 8640 b_3)T^7 \\
& + (19756800 a_1^2 + 7902720 a_1 a_2 + 10451347200 b_1^2 - 7902720 b_1 b_2 + 162729000)T^6 + (-7744665600 a_1^2 b_1 \\
& +2581555200 b_1^3 - 32128790400 b_1 + 73029600 b_2 - 423360 b_3)T^5 + (1220688000 a_1^2 + 107251200 a_1 a_2 \\
& -282240 a_2^2 + 416903760000 b_1^2 - 728179200 b_1 b_2 - 5644800 b_1 b_3 + 846720 b_2^2 - 154791000)T^4 \\
& -929359872000 a_1^4 - 10326220800 a_1^3 a_2 + 35625461760000 a_1^2 b_1^2 - 30978662400 a_1^2 b_1 b_2 \\
& +30978662400 a_1 a_2 b_1^2 - 10945794048000 b_1^4 + 10326220800 b_1^3 b_2 + (-77446656000 a_1^2 b_1 + 335602176000 b_1^3 \\
& -221681880000 b_1 + 393724800 b_2 - 2116800 b_3)T^3 + (-722835456000 a_1^4 + 722835456000 b_1^4 \\
& +31427424000 a_1^2 + 683020800 a_1 a_2 - 1612800 a_1 a_3 + 2257920 a_2^2 - 80640 a_2 a_3 + 21771584352000 b_1^2 \\
& -48855744000 b_1 b_2 + 40320000 b_1 b_3 + 30481920 b_2^2 - 80640 b_2 b_3 + 130977000)T^2 + 29942136000 a_1^2 \\
& +304819200 a_1 a_2 - 5644800 a_1 a_3 - 1270080 a_2^2 - 282240 a_2 a_3 + 14400 a_3^2 + 344794004856000 b_1^2 \\
& -1041872025600 b_1 b_2 + 4467456000 b_1 b_3 + 786179520 b_2^2 - 6733440 b_2 b_3 + 14400 b_3^2 + (-8850256128000 a_1^2 b_1 \\
& +15568358400 a_1^2 b_2 - 79027200 a_1^2 b_3 - 27817574400 a_1 a_2 b_1 + 31610880 a_1 a_2 b_2 + 31610880 a_2^2 b_1
\end{aligned}$$

$$\begin{aligned}
&+64945343232000 b_1^3 - 121306752000 b_1^2 b_2 - 79027200 b_1^2 b_3 + 63221760 b_1 b_2^2 + 559057464000 b_1 \\
&- 928746000 b_2 + 4536000 b_3)T + 130977000, \quad \mathbf{q}_5 = -94080 T^{12} a_1 + (-6216000 a_1 - 336 a_2)T^{10} \\
&+ (-257040000 a_1 - 1209600 a_2 + 15120 a_3)T^8 + (-3138508800 a_1 b_1 + 1128960 a_1 b_2 - 10160640 a_2 b_1)T^7 \\
&+ (2581555200 a_1^3 - 7744665600 a_1 b_1^2 - 1342051200 a_1 - 37467360 a_2 + 60480 a_3)T^6 + (-551722752000 a_1 b_1 \\
&+ 833172480 a_1 b_2 - 3386880 a_1 b_3 - 1957616640 a_2 b_1 + 1354752 a_2 b_2 - 3386880 a_3 b_1)T^5 + (171489024000 a_1^3 \\
&- 1106380800 a_1^2 a_2 + 326382336000 a_1 b_1^2 - 1106380800 a_2 b_1^2 - 15600816000 a_1 - 179928000 a_2 \\
&- 1360800 a_3)T^4 + (1445670912000 a_1^3 b_1 + 1445670912000 a_1 b_1^3 - 7175105280000 a_1 b_1 + 8992166400 a_1 b_2 \\
&- 29030400 a_1 b_3 + 4927910400 a_2 b_1 - 30481920 a_2 b_2 + 241920 a_2 b_3 + 188697600 a_3 b_1 - 241920 a_3 b_2)T^3 \\
&- 510854400000 a_1^3 - 7755955200 a_1^2 a_2 - 56448000 a_1^2 a_3 + 27095040 a_1 a_2^2 + 9031680 a_1 a_2 a_3 \\
&- 1530944835840000 a_1 b_1^2 + 3711568896000 a_1 b_1 b_2 - 9934848000 a_1 b_1 b_3 - 2108897280 a_1 b_2^2 \\
&+ 9031680 a_1 b_2 b_3 + 451584 a_2^3 + 2429894476800 a_2 b_1^2 - 3960391680 a_2 b_1 b_2 + 13547520 a_2 b_1 b_3 \\
&+ 451584 a_2 b_2^2 + 15071616000 a_3 b_1^2 - 13547520 a_3 b_1 b_2 + (694649088000 a_1^3 + 10826726400 a_1^2 a_2 \\
&- 237081600 a_1^2 a_3 + 63221760 a_1 a_2^2 - 72993473280000 a_1 b_1^2 + 99890380800 a_1 b_1 b_2 - 31610880 a_1 b_2^2 \\
&- 125416166400 a_2 b_1^2 + 94832640 a_2 b_1 b_2 - 237081600 a_3 b_1^2 + 7334712000 a_1 + 233377200 a_2 + 13608000 a_3)T^2 \\
&+ (53489823744000 a_1^3 b_1 - 41304883200 a_1^3 b_2 + 123914649600 a_1^2 a_2 b_1 - 136512638976000 a_1 b_1^3 \\
&+ 123914649600 a_1 b_1^2 b_2 - 41304883200 a_2 b_1^3 - 6186136320000 a_1 b_1 + 9551001600 a_1 b_2 - 41126400 a_1 b_3 \\
&+ 296792294400 a_2 b_1 - 470776320 a_2 b_2 + 2177280 a_2 b_3 + 2460326400 a_3 b_1 - 2177280 a_3 b_2)T \\
&- 2857680000 a_1 + 57153600 a_2 + 3402000 a_3, \quad \mathbf{q}_4 = 45 T^{16} + 5400 T^{14} + 459900 T^{12} + (16464000 b_1 \\
&- 11760 b_2)T^{11} + (14817600 a_1^2 + 14817600 b_1^2 + 19845000)T^{10} + (134904000 b_1 + 739200 b_2 - 6000 b_3)T^9 \\
&+ (-1054872000 a_1^2 + 2822400 a_1 a_2 + 2444904000 b_1^2 + 153798750)T^8 + (-16595712000 a_1^2 b_1 \\
&+ 5531904000 b_1^3 - 33203520000 b_1 + 53071200 b_2 - 216000 b_3)T^7 + (-23221296000 a_1^2 + 152409600 a_1 a_2 \\
&- 5644800 a_1 a_3 + 846720 a_2^2 - 591991344000 b_1^2 + 874944000 b_1 b_2 - 282240 b_2^2 + 702513000)T^6 \\
&+ (-1598720256000 a_1^2 b_1 + 1106380800 a_1^2 b_2 - 1134040320000 b_1^3 + 1106380800 b_1^2 b_2 + 426641040000 b_1 \\
&- 582120000 b_2 + 2268000 b_3)T^5 + (-97055280000 a_1^2 + 1185408000 a_1 a_2 - 84672000 a_1 a_3 + 29635200 a_2^2 \\
&+ 7687688400000 b_1^2 - 9991296000 b_1 b_2 + 84672000 b_1 b_3 - 4233600 b_2^2 - 89302500)T^4 + 885104640000 a_1^4 \\
&+ 88510464000 a_1^3 a_2 + 7375872000 a_1^3 a_3 + 2212761600 a_1^2 a_2^2 + 6020260485120000 a_1^2 b_1^2 \\
&- 9957427200000 a_1^2 b_1 b_2 + 22127616000 a_1^2 b_1 b_3 + 2212761600 a_1^2 b_2^2 + 88510464000 a_1 a_2 b_1^2 \\
&- 22127616000 a_1 a_3 b_1^2 + 2212761600 a_2^2 b_1^2 + 1760841922560000 b_1^4 - 3348645888000 b_1^3 b_2 \\
&- 7375872000 b_1^3 b_3 + 2212761600 b_1^2 b_2^2 + (29552221440000 a_1^2 b_1 - 66777984000 a_1^2 b_2 + 395136000 a_1^2 b_3 \\
&+ 151732224000 a_1 a_2 b_1 - 158054400 a_1 a_2 b_2 + 158054400 a_2^2 b_1 + 67184974080000 b_1^3 - 96808320000 b_1^2 b_2 \\
&+ 395136000 b_1^2 b_3 - 1025695440000 b_1 + 1902474000 b_2 - 10584000 b_3)T^3 + (1032622080000 a_1^4 \\
&+ 51631104000 a_1^3 a_2 - 178127308800000 a_1^2 b_1^2 + 154893312000 a_1^2 b_1 b_2 - 154893312000 a_1 a_2 b_1^2 \\
&+ 58343147520000 b_1^4 - 51631104000 b_1^3 b_2 + 613607400000 a_1^2 + 10499328000 a_1 a_2 - 435456000 a_1 a_3 \\
&+ 99489600 a_2^2 - 604800 a_2 a_3 + 43200 a_3^2 + 1033225903080000 b_1^2 - 3097809792000 b_1 b_2 + 13027392000 b_1 b_3 \\
&+ 2343297600 b_2^2 - 19958400 b_2 b_3 + 43200 b_3^2 + 1250235000)T^2 + 105787080000 a_1^2 - 3090528000 a_1 a_2 \\
&- 266112000 a_1 a_3 - 10584000 a_2^2 - 604800 a_2 a_3 + 43200 a_3^2 + 948764101320000 b_1^2 - 2947855680000 b_1 b_2 \\
&+ 12942720000 b_1 b_3 + 2284027200 b_2^2 - 19958400 b_2 b_3 + 43200 b_3^2 + (619396565760000 a_1^2 b_1 \\
&- 933028992000 a_1^2 b_2 + 4007808000 a_1^2 b_3 + 3968068608000 a_1 a_2 b_1 - 5622220800 a_1 a_2 b_2 + 22579200 a_1 a_2 b_3 \\
&+ 20772864000 a_1 a_3 b_1 - 22579200 a_1 a_3 b_2 + 3499776000 a_2^2 b_1 - 2257920 a_2^2 b_2 + 3682926616320000 b_1^3 \\
&- 9447193728000 b_1^2 b_2 - 1185408000 b_1^2 b_3 + 8128512000 b_1 b_2^2 - 2257920 b_2^3 - 2899116360000 b_1 \\
&+ 4858056000 b_2 - 23814000 b_3)T + 111628125, \quad \mathbf{q}_3 = -33600 T^{14} a_1 + (-3158400 a_1 - 5040 a_2)T^{12} \\
&+ (40152000 a_1 - 1061760 a_2 + 3360 a_3)T^{10} + (3894912000 a_1 b_1 - 940800 a_1 b_2 - 6585600 a_2 b_1)T^9 \\
&+ (-4609920000 a_1^3 + 13829760000 a_1 b_1^2 + 3190320000 a_1 - 28350000 a_2 + 151200 a_3)T^8 \\
&+ (660119040000 a_1 b_1 - 791884800 a_1 b_2 + 3225600 a_1 b_3 + 491904000 a_2 b_1 - 645120 a_2 b_2)T^7 \\
&+ (25815552000 a_1^3 + 851913216000 a_1 b_1^2 + 6932520000 a_1 - 190512000 a_2 + 7560000 a_3)T^6 \\
&+ (1445670912000 a_1^3 b_1 + 1445670912000 a_1 b_1^3 + 14030827776000 a_1 b_1 - 18881856000 a_1 b_2 + 72576000 a_1 b_3 \\
&+ 668739456000 a_2 b_1 - 84672000 a_2 b_2 + 241920 a_2 b_3 + 358041600 a_3 b_1 - 241920 a_3 b_2)T^5 + 57826836480000 a_1^5 \\
&+ 2891341824000 a_1^4 a_2 - 6534432522240000 a_1^3 b_1^2 + 5782683648000 a_1^3 b_1 b_2 - 6592259358720000 a_1 b_1^4 \\
&+ 5782683648000 a_1 b_1^3 b_2 - 2891341824000 a_2 b_1^4 + (1382976000000 a_1^3 - 2765952000 a_1^2 a_2 + 395136000 a_1^2 a_3 \\
&+ 211725722880000 a_1 b_1^2 - 360364032000 a_1 b_1 b_2 + 158054400 a_1 b_2^2 + 175835520000 a_2 b_1^2 - 158054400 a_2 b_1 b_2 \\
&+ 395136000 a_3 b_1^2 - 159500880000 a_1 - 1140426000 a_2 + 43848000 a_3)T^4 + (14456709120000 a_1^3 b_1 \\
&+ 14456709120000 a_1 b_1^3 - 51104632320000 a_1 b_1 + 71293824000 a_1 b_2 - 290304000 a_1 b_3 + 436738176000 a_2 b_1 \\
&- 643507200 a_2 b_2 + 2419200 a_2 b_3 + 3580416000 a_3 b_1 - 2419200 a_3 b_2)T^3 - 4967141760000 a_1^3 \\
&- 37086336000 a_1^2 a_2 + 3878784000 a_1^2 a_3 - 372556800 a_1 a_2^2 + 9676800 a_1 a_2 a_3 + 1612800 a_1 a_3^2 \\
&+ 32919971495040000 a_1 b_1^2 - 101916638208000 a_1 b_1 b_2 + 459389952000 a_1 b_1 b_3 + 78948172800 a_1 b_2^2 \\
&- 712857600 a_1 b_2 b_3 + 1612800 a_1 b_3^2 - 2257920 a_2^3 - 322560 a_2^2 a_3 + 102816362880000 a_2 b_1^2 \\
&- 251780659200 a_2 b_1 b_2 + 728985600 a_2 b_1 b_3 + 142248960 a_2 b_2^2 - 645120 a_2 b_2 b_3 + 410691456000 a_3 b_1^2
\end{aligned}$$

$$\begin{aligned}
& -728985600 a_3 b_1 b_2 + 322560 a_3 b_2^2 + (-3315755520000 a_1^3 - 1467648000 a_1^2 a_2 - 1467648000 a_1^2 a_3 \\
& -587059200 a_1 a_2^2 + 45158400 a_1 a_2 a_3 - 8329468008960000 a_1 b_1^2 + 19094777856000 a_1 b_1 b_2 \\
& -605122560000 a_1 b_1 b_3 - 9573580800 a_1 b_2^2 + 45158400 a_1 b_2 b_3 - 4515840 a_2^3 - 4943602944000 a_2 b_1^2 \\
& + 9257472000 a_2 b_1 b_2 - 4515840 a_2 b_2^2 + 7112448000 a_3 b_1^2 - 152885880000 a_1 - 1143072000 a_2 \\
& -20412000 a_3) T^2 + (1552030986240000 a_1^3 b_1 - 2788079616000 a_1^3 b_2 + 14751744000 a_1^3 b_3 \\
& + 309786624000 a_1^2 a_2 b_1 - 44255232000 a_1^2 a_3 b_1 - 8865060556800000 a_1 b_1^3 + 12081678336000 a_1 b_1^2 b_2 \\
& -44255232000 a_1 b_1^2 b_3 - 1342408704000 a_2 b_1^3 + 14751744000 a_3 b_1^3 + 156886208640000 a_1 b_1 \\
& -250163424000 a_1 b_2 + 1137024000 a_1 b_3 + 947013984000 a_2 b_1 - 1439424000 a_2 b_2 + 6048000 a_2 b_3 \\
& + 6241536000 a_3 b_1 - 6048000 a_3 b_2) T + 14288400000 a_1 + 214326000 a_2 + 6804000 a_3, \quad \mathbf{q}_2 = 10 T^{18} \\
& + 2250 T^{16} + 6720 T^{15} b_1 + 225000 T^{14} + (3662400 b_1 - 1680 b_2) T^{13} + (4939200 a_1^2 + 4939200 b_1^2 \\
& + 4422600) T^{12} + (-938952000 b_1 + 1249920 b_2 - 4320 b_3) T^{11} + (991132800 a_1^2 - 1317120 a_1 a_2 \\
& -4405296000 b_1^2 + 3575040 b_1 b_2 - 99508500) T^{10} + (6453888000 a_1^2 b_1 - 2151296000 b_1^3 - 80022600000 b_1 \\
& + 101833200 b_2 - 367200 b_3) T^9 + (25568424000 a_1^2 - 58464000 a_1 a_2 + 20160 a_2^2 - 250994520000 b_1^2 \\
& -16531200 b_1 b_2 - 1209600 b_1 b_3 + 262080 b_2^2 - 224248500) T^8 + (-482065920000 a_1^2 b_1 + 632217600 a_1^2 b_2 \\
& -1101639168000 b_1^3 + 632217600 b_1^2 b_2 - 1708876008000 b_1 + 2488752000 b_2 - 10411200 b_3) T^7 \\
& + (722835456000 a_1^4 - 722835456000 b_1^4 + 74215008000 a_1^2 - 3098995200 a_1 a_2 + 1612800 a_1 a_3 - 3386880 a_2^2 \\
& + 80640 a_2 a_3 - 11978110368000 b_1^2 + 23363827200 b_1 b_2 - 175795200 b_1 b_3 - 4515840 b_2^2 + 80640 b_2 b_3 \\
& + 9704205000) T^6 + 944504995840000 a_1^6 + 2833514987520000 a_1^4 b_1^2 + 2833514987520000 a_1^2 b_1^4 \\
& + 944504995840000 b_1^6 + (-70812322560000 a_1^2 b_1 + 86534784000 a_1^2 b_2 - 237081600 a_1^2 b_3 \\
& -141616742400 a_1 a_2 b_1 + 94832640 a_1 a_2 b_2 - 31610880 a_2^2 b_1 + 44045809920000 b_1^3 - 91434470400 b_1^2 b_2 \\
& -237081600 b_1^2 b_3 + 63221760 b_1 b_2^2 - 51559215120000 b_1 + 8707986000 b_2 - 40824000 b_3) T^5 \\
& + (11875153920000 a_1^4 + 51631104000 a_1^3 a_2 - 178127308800000 a_1^2 b_1^2 + 154893312000 a_1^2 b_1 b_2 \\
& -154893312000 a_1 a_2 b_1^2 + 47500615680000 b_1^4 - 51631104000 b_1^3 b_2 + 489139560000 a_1^2 - 15579648000 a_1 a_2 \\
& -84672000 a_1 a_3 + 40219200 a_2^2 - 4233600 a_2 a_3 + 43200 a_3^2 + 17061659370000000 b_1^2 - 4681853568000 b_1 b_2 \\
& + 16789248000 b_1 b_3 + 3249288000 b_2^2 - 23587200 b_2 b_3 + 43200 b_3^2 + 15181425000) T^4 + 40809646080000 a_1^4 \\
& + 312947712000 a_1^3 a_2 + 12644352000 a_1^3 a_3 - 7586611200 a_1^2 a_2^2 + 632217600 a_1^2 a_2 a_3 \\
& -106740047554560000 a_1^2 b_1^2 + 257410556928000 a_1^2 b_1 b_2 - 714405888000 a_1^2 b_1 b_3 - 144145612800 a_1^2 b_2^2 \\
& + 632217600 a_1^2 b_2 b_3 - 126443520 a_1 a_2^3 - 155275803648000 a_1 a_2 b_1^2 + 280704614400 a_1 a_2 b_1 b_2 \\
& -126443520 a_1 a_2 b_2^2 + 12644352000 a_1 a_3 b_1^2 + 142881177600 a_2^2 b_1^2 - 126443520 a_2^2 b_1 b_2 + 632217600 a_2 a_3 b_1^2 \\
& + 78835069071360000 b_1^4 - 232621304832000 b_1^3 b_2 - 714405888000 b_1^3 b_3 + 287026790400 b_1^2 b_2^2 \\
& + 632217600 b_1^2 b_2 b_3 - 126443520 b_1 b_2^3 + (-482297356800000 a_1^2 b_1 + 668005632000 a_1^2 b_2 - 2370816000 a_1^2 b_3 \\
& -1569705984000 a_1 a_2 b_1 + 1128960000 a_1 a_2 b_2 - 903168000 a_1 a_3 b_1 - 4154572800 a_2^2 b_1 + 4515840 a_2^2 b_2 \\
& -45158400 a_2 a_3 b_1 + 3193966702080000 b_1^3 - 4620946176000 b_1^2 b_2 + 48658176000 b_1^2 b_3 - 3296563200 b_1 b_2^2 \\
& -45158400 b_1 b_2 b_3 + 4515840 b_2^3 - 190707274800000 b_1 + 297198720000 b_2 - 129276000 b_3) T^3 \\
& + (49787136000000 a_1^4 - 132765696000 a_1^3 a_2 + 4425523200 a_1^2 a_2^2 + 4741948108800000 a_1^2 b_1^2 \\
& -9072322560000 a_1^2 b_1 b_2 + 4425523200 a_1^2 b_2^2 - 2611058688000 a_1 a_2 b_1^2 + 4425523200 a_2^2 b_1^2 \\
& + 7542197913600000 b_1^4 - 11550615552000 b_1^3 b_2 + 4425523200 b_1^2 b_2^2 + 1415186640000 a_1^2 \\
& + 7493472000 a_1 a_2 - 12700800 a_2^2 - 259200 a_3^2 - 5302830078000000 b_1^2 + 16496434080000 b_1 b_2 \\
& -76349952000 b_1 b_3 - 12713500800 b_2^2 + 116121600 b_2 b_3 - 259200 b_3^2 - 1920003750) T^2 - 264335400000 a_1^2 \\
& + 1905120000 a_1 a_2 + 254016000 a_1 a_3 + 114307200 a_2^2 + 12700800 a_2 a_3 + 388800 a_3^2 + 6177435266520000 b_1^2 \\
& -20354429088000 b_1 b_2 + 97886880000 b_1 b_3 + 16777756800 b_2^2 - 161481600 b_2 b_3 + 388800 b_3^2 \\
& + (-2775688151040000 a_1^4 b_1 + 2891341824000 a_1^4 b_2 - 5782683648000 a_1^3 a_2 b_1 + 1098709893120000 a_1^2 b_1^3 \\
& -5782683648000 a_1 a_2 b_1^3 + 3874398044160000 b_1^5 - 2891341824000 b_1^4 b_2 - 1072727349120000 a_1^2 b_1 \\
& + 1583535744000 a_1^2 b_2 - 6314112000 a_1^2 b_3 - 7736988672000 a_1 a_2 b_1 + 10770278400 a_1 a_2 b_2 \\
& -38707200 a_1 a_2 b_3 - 42384384000 a_1 a_3 b_1 + 38707200 a_1 a_3 b_2 - 132257664000 a_2^2 b_1 + 209986560 a_2^2 b_2 \\
& -967680 a_2^2 b_3 - 2119219200 a_2 a_3 b_1 + 1935360 a_2 a_3 b_2 - 4838400 a_3^2 b_1 + 77764203377280000 b_1^3 \\
& -281737782144000 b_1^2 b_2 - 243589248000 b_1^2 b_3 + 396705254400 b_1 b_2^2 + 48384000 b_1 b_2 b_3 - 4838400 b_1 b_3^2 \\
& -223534080 b_2^3 + 967680 b_2^2 b_3 - 5308140600000 b_1 + 9216018000 b_2 - 47628000 b_3) T + 223256250, \\
& \mathbf{q}_1 = -5040 T^{16} a_1 + (-638400 a_1 - 1680 a_2) T^{14} + (-53928000 a_1 - 120960 a_2 - 1680 a_3) T^{12} \\
& + (-1840204800 a_1 b_1 + 1317120 a_1 b_2 + 188160 a_2 b_1) T^{11} + (553190400 a_1^3 - 1659571200 a_1 b_1^2 \\
& -2187259200 a_1 + 1769040 a_2 - 211680 a_3) T^{10} + (-14292096000 a_1 b_1 - 81715200 a_1 b_2 \\
& + 672000 a_1 b_3 + 40857600 a_2 b_1 + 53760 a_2 b_2 - 940800 a_3 b_1) T^9 + (-33586560000 a_1^3 + 395136000 a_1^2 a_2 \\
& -265926528000 a_1 b_1^2 + 395136000 a_2 b_1^2 - 14978628000 a_1 + 112341600 a_2 - 6274800 a_3) T^8 \\
& + (-619573248000 a_1^3 b_1 - 619573248000 a_1 b_1^3 + 3908862720000 a_1 b_1 - 6198796800 a_1 b_2 + 25804800 a_1 b_3 \\
& + 9503424000 a_2 b_1 - 12741120 a_2 b_2 + 80640 a_2 b_3 + 40320000 a_3 b_1 - 80640 a_3 b_2) T^7 + (-975985920000 a_1^3 \\
& + 869299200 a_1^2 a_2 - 79027200 a_1^2 a_3 + 63221760 a_1 a_2^2 + 66086496000000 a_1 b_1^2 - 97361510400 a_1 b_1 b_2
\end{aligned}$$

$$\begin{aligned}
& +31610880 a_1 b_2^2 - 10826726400 a_2 b_1^2 + 31610880 a_2 b_1 b_2 - 79027200 a_3 b_1^2 - 80427816000 a_1 \\
& - 87318000 a_2 - 25704000 a_3) T^6 + (-62163849216000 a_1^3 b_1 + 41304883200 a_1^3 b_2 - 123914649600 a_1^2 a_2 b_1 \\
& + 127838613504000 a_1 b_1^3 - 123914649600 a_1 b_1^2 b_2 + 41304883200 a_2 b_1^3 - 36802176768000 a_1 b_1 \\
& + 47653401600 a_1 b_2 - 171763200 a_1 b_3 + 549080985600 a_2 b_1 - 877201920 a_2 b_2 + 4112640 a_2 b_3 \\
& + 4681152000 a_3 b_1 - 4112640 a_3 b_2) T^5 - 33043906560000 a_1^5 - 3304390656000 a_1^4 a_2 + 413048832000 a_1^4 a_3 \\
& - 82609766400 a_1^3 a_2^2 + 236024363581440000 a_1^3 b_1^2 - 381657120768000 a_1^3 b_1 b_2 + 826097664000 a_1^3 b_1 b_3 \\
& + 82609766400 a_1^3 b_2^2 + 570007388160000 a_1^2 a_2 b_1^2 - 495658598400 a_1^2 a_2 b_1 b_2 + 247829299200 a_1 a_2^2 b_1^2 \\
& - 200948256768000000 a_1 b_1^4 + 378352730112000 a_1 b_1^3 b_2 + 826097664000 a_1 b_1^3 b_3 - 247829299200 a_1 b_1^2 b_2^2 \\
& - 186698072064000 a_2 b_1^4 + 165219532800 a_2 b_1^3 b_2 - 413048832000 a_3 b_1^4 + (-3170684160000 a_1^3 \\
& + 65536128000 a_1^2 a_2 - 5927040000 a_1^2 a_3 + 1083801600 a_1 a_2^2 - 749722579200000 a_1 b_1^2 + 925295616000 a_1 b_1 b_2 \\
& - 9031680000 a_1 b_1 b_3 + 519321600 a_1 b_2^2 + 2257920 a_2^3 + 5564926080000 a_2 b_1^2 - 9686476800 a_2 b_1 b_2 \\
& + 22579200 a_2 b_1 b_3 + 2257920 a_2 b_2^2 + 29070720000 a_3 b_1^2 - 22579200 a_3 b_1 b_2 - 41436360000 a_1 - 2571912000 a_2 \\
& - 241542000 a_3) T^4 + (973762621440000 a_1^3 b_1 - 2375030784000 a_1^3 b_2 + 14751744000 a_1^3 b_3 \\
& - 929359872000 a_1^2 a_2 b_1 - 44255232000 a_1^2 a_3 b_1 - 7543304294400000 a_1 b_1^3 + 10842531840000 a_1 b_1^2 b_2 \\
& - 44255232000 a_1 b_1^2 b_3 - 929359872000 a_2 b_1^3 + 14751744000 a_3 b_1^3 + 91024940160000 a_1 b_1 \\
& - 184965984000 a_1 b_2 + 1112832000 a_1 b_3 - 1192647456000 a_2 b_1 + 1405555200 a_2 b_2 - 3628800 a_2 b_3 \\
& - 8249472000 a_3 b_1 + 3628800 a_3 b_2) T^3 + 13224919680000 a_1^3 + 419295744000 a_1^2 a_2 + 12410496000 a_1^2 a_3 \\
& + 3420748800 a_1 a_2^2 + 106444800 a_1 a_2 a_3 - 4838400 a_1 a_3^2 - 111656974020480000 a_1 b_1^2 \\
& + 344031141888000 a_1 b_1 b_2 - 1489259520000 a_1 b_1 b_3 - 264210508800 a_1 b_2^2 + 2274048000 a_1 b_2 b_3 - 4838400 a_1 b_3^2 \\
& + 13547520 a_2^3 + 967680 a_2^2 a_3 - 269769733632000 a_2 b_1^2 + 693565286400 a_2 b_1 b_2 - 1983744000 a_2 b_1 b_3 \\
& - 419973120 a_3 b_2^2 + 1935360 a_2 b_2 b_3 - 1019039616000 a_3 b_1^2 + 1983744000 a_3 b_1 b_2 - 967680 a_3 b_2^2 \\
& + (57826836480000 a_1^5 + 2891341824000 a_1^4 a_2 - 6534432522240000 a_1^3 b_1^2 + 5782683648000 a_1^3 b_1 b_2 \\
& - 6592259358720000 a_1 b_1^4 + 5782683648000 a_1 b_1^3 b_2 - 2891341824000 a_2 b_1^4 + 26325371520000 a_1^3 \\
& + 562391424000 a_1^2 a_2 - 13668480000 a_1^2 a_3 + 3285273600 a_1 a_2^2 + 106444800 a_1 a_2 a_3 \\
& - 4838400 a_1 a_3^2 - 123143455612800000 a_1 b_1^2 + 364044893184000 a_1 b_1 b_2 - 1505516544000 a_1 b_1 b_3 \\
& - 271526169600 a_1 b_2^2 + 2274048000 a_1 b_2 b_3 - 4838400 a_1 b_3^2 - 20321280 a_2^3 + 967680 a_2^2 a_3 \\
& - 371107723392000 a_2 b_1^2 + 854509824000 a_2 b_1 b_2 - 2322432000 a_2 b_1 b_3 - 453841920 a_2 b_2^2 + 1935360 a_2 b_2 b_3 \\
& - 1418352768000 a_3 b_1^2 + 2322432000 a_3 b_1 b_2 - 967680 a_3 b_2^2 - 221470200000 a_1 - 4072194000 a_2 \\
& - 156492000 a_3) T^2 + (17687109242880000 a_1^3 b_1 - 25965176832000 a_1^3 b_2 + 107476992000 a_1^3 b_3 \\
& - 131178829824000 a_1^2 a_2 b_1 + 240242688000 a_1^2 a_2 b_2 - 1264435200 a_1^2 a_2 b_3 - 1561577472000 a_1^2 a_3 b_1 \\
& + 1264435200 a_1^2 a_3 b_2 - 247829299200 a_1 a_2^2 b_1 + 252887040 a_1 a_2^2 b_2 - 423777796485120000 a_1 b_1^3 \\
& + 1076242987008000 a_1 b_1^2 b_2 + 107476992000 a_1 b_1^2 b_3 - 915451084800 a_1 b_1 b_2^2 + 252887040 a_1 b_2^3 \\
& - 252887040 a_2^3 b_1 - 564500772864000 a_2 b_1^3 + 907864473600 a_2 b_1^2 b_2 - 1264435200 a_2 b_1^2 b_3 \\
& - 252887040 a_2 b_1 b_2^2 - 1561577472000 a_3 b_1^3 + 1264435200 a_3 b_1^2 b_2 + 376012264320000 a_1 b_1 \\
& - 625387392000 a_1 b_2 + 3030048000 a_1 b_3 + 2565561600000 a_2 b_1 - 4064256000 a_2 b_2 + 18144000 a_2 b_3 \\
& + 17835552000 a_3 b_1 - 18144000 a_3 b_2) T - 15360030000 a_1 - 142884000 a_2 - 3402000 a_3, \quad \mathbf{q}_0 = T^{20} + 370 T^{18} \\
& + 1680 T^{17} b_1 + 44325 T^{16} + 336 T^{15} b_2 + (705600 a_1^2 + 705600 b_1^2 + 2208600) T^{14} + 240 T^{13} b_3 + (51979200 a_1^2 \\
& + 376320 a_1 a_2 - 128654400 b_1^2 + 188160 b_1 b_2 + 62795250) T^{12} + (-184396800 a_1^2 b_1 + 61465600 b_1^3 \\
& + 5745801600 b_1 - 8159760 b_2 + 41760 b_3) T^{11} + (1920072000 a_1^2 + 24756480 a_1 a_2 + 188160 a_1 a_3 + 30912 a_2^2 \\
& + 77399112000 b_1^2 - 111901440 b_1 b_2 + 215040 b_1 b_3 + 25536 b_2^2 + 693384300) T^{10} + (79158912000 a_1^2 b_1 \\
& - 79027200 a_1^2 b_2 + 114808960000 b_1^3 - 79027200 b_1^2 b_2 + 177128532000 b_1 - 280476000 b_2 + 1414800 b_3) T^9 \\
& + (-51631104000 a_1^4 + 51631104000 b_1^4 + 55460664000 a_1^2 + 858412800 a_1 a_2 + 9273600 a_1 a_3 - 342720 a_2^2 \\
& + 40320 a_2 a_3 - 6252079176000 b_1^2 + 15883257600 b_1 b_2 - 33465600 b_1 b_3 - 10100160 b_2^2 + 40320 b_2 b_3 \\
& + 6641129250) T^8 + (8847320832000 a_1^2 b_1 - 8862336000 a_1^2 b_2 + 11289600 a_1^2 b_3 + 11199283200 a_1 a_2 b_1 \\
& - 4515840 a_1 a_2 b_2 - 13547520 a_2^2 b_1 - 21328876800000 b_1^3 + 38689459200 b_1^2 b_2 + 11289600 b_1^2 b_3 \\
& - 18063360 b_1 b_2^2 + 4896031392000 b_1 - 6993378000 b_2 + 29592000 b_3) T^7 + (-1652195328000 a_1^4 \\
& - 10326220800 a_1^3 a_2 + 35625461760000 a_1^2 b_1^2 - 30978662400 a_1^2 b_1 b_2 + 30978662400 a_1 a_2 b_1^2 \\
& - 10222958592000 b_1^4 + 10326220800 b_1^3 b_2 + 559142136000 a_1^2 + 237081600 a_1 a_2 + 422553600 a_1 a_3 \\
& + 20744640 a_2^2 - 40320 a_2 a_3 + 14400 a_3^2 + 413028968184000 b_1^2 - 1097213644800 b_1 b_2 + 4668249600 b_1 b_3 \\
& + 743843520 b_2^2 - 6491520 b_2 b_3 + 14400 b_3^2 + 4346055000) T^6 + 944504995840000 a_1^6 \\
& + 2833514987520000 a_1^4 b_1^2 + 2833514987520000 a_1^2 b_1^4 + 944504995840000 b_1^6 + (-176294442240000 a_1^2 b_1 \\
& + 343824768000 a_1^2 b_2 - 1862784000 a_1^2 b_3 + 1940456448000 a_1 a_2 b_1 - 3147540480 a_1 a_2 b_2 + 13547520 a_1 a_2 b_3 \\
& + 19869696000 a_1 a_3 b_1 - 13547520 a_1 a_3 b_2 + 763176960 a_2^2 b_1 - 451584 a_2^2 b_2 - 9031680 a_2 a_3 b_1 \\
& + 2987238885120000 b_1^3 - 6485028480000 b_1^2 b_2 + 12813696000 b_1^2 b_3 + 3712020480 b_1 b_2^2 - 9031680 b_1 b_2 b_3 \\
& - 451584 b_2^3 + 254736770400000 b_1 - 40318689600 b_2 + 184842000 b_3) T^5 + (-2729072640000 a_1^4 \\
& - 634324992000 a_1^3 a_2 - 7375872000 a_1^3 a_3 + 2212761600 a_1^2 a_2^2 + 146706094080000 a_1^2 b_1^2 \\
& - 354041856000 a_1^2 b_1 b_2 - 22127616000 a_1^2 b_1 b_3 + 2212761600 a_1^2 b_2^2 - 1460422656000 a_1 a_2 b_1^2
\end{aligned}$$

$$\begin{aligned}
&+22127616000 a_1 a_3 b_1^2 + 2212761600 a_2^2 b_1^2 + 5357980938240000 b_1^4 - 7788920832000 b_1^3 b_2 \\
&+ 7375872000 b_1^3 b_3 + 2212761600 b_1^2 b_2^2 + 1046916360000 a_1^2 + 38864448000 a_1 a_2 + 2274048000 a_1 a_3 \\
&+ 201096000 a_2^2 - 9072000 a_2 a_3 + 388800 a_3^2 + 9992405573640000 b_1^2 - 29340837792000 b_1 b_2 \\
&+ 126766080000 b_1 b_3 + 21377563200 b_2^2 - 183254400 b_2 b_3 + 388800 b_3^2 + 14042818125)T^4 \\
&+ 131426749440000 a_1^4 + 3282112512000 a_1^3 a_2 - 98896896000 a_1^3 a_3 + 46061568000 a_1^2 a_2^2 \\
&- 1986969600 a_1^2 a_2 a_3 + 45158400 a_1^2 a_3^2 + 1261464395950080000 a_1^2 b_1^2 - 3708301234176000 a_1^2 b_1 b_2 \\
&+ 15222445056000 a_1^2 b_1 b_3 + 2717451878400 a_1^2 b_2^2 - 22217932800 a_1^2 b_2 b_3 + 45158400 a_1^2 b_3^2 \\
&+ 397393920 a_1 a_2^3 - 18063360 a_1 a_2^2 a_3 + 6434560806912000 a_1 a_2 b_1^2 - 15355662336000 a_1 a_2 b_1 b_2 \\
&+ 41545728000 a_1 a_2 b_1 b_3 + 8489779200 a_1 a_2 b_2^2 - 36126720 a_1 a_2 b_2 b_3 + 23789896704000 a_1 a_3 b_1^2 \\
&- 41545728000 a_1 a_3 b_1 b_2 + 18063360 a_1 a_3 b_2^2 + 1806336 a_2^4 + 6934523904000 a_2^2 b_1^2 - 11957944320 a_2^2 b_1 b_2 \\
&+ 18063360 a_2^2 b_1 b_3 + 3612672 a_2^2 b_2^2 + 39558758400 a_2 a_3 b_1^2 - 36126720 a_2 a_3 b_1 b_2 + 45158400 a_3^2 b_1^2 \\
&+ 420447434480640000 b_1^4 - 1863670910976000 b_1^3 b_2 - 8666348544000 b_1^3 b_3 + 3805769318400 b_1^2 b_2^2 \\
&+ 19327795200 b_1^2 b_2 b_3 + 45158400 b_1^2 b_3^2 - 3865559040 b_1 b_2^3 - 18063360 b_1 b_2^2 b_3 + 1806336 b_2^4 \\
&+ (-2775688151040000 a_1^4 b_1 + 2891341824000 a_1^4 b_2 - 5782683648000 a_1^3 a_2 b_1 + 1098709893120000 a_1^2 b_1^3 \\
&- 5782683648000 a_1 a_2 b_1^3 + 3874398044160000 b_1^5 - 2891341824000 b_1^4 b_2 + 119027664000000 a_1^2 b_1 \\
&+ 180690048000 a_1^2 b_2 - 3088512000 a_1^2 b_3 + 29223355392000 a_1 a_2 b_1 - 39897446400 a_1 a_2 b_2 + 148377600 a_1 a_2 b_3 \\
&+ 210115584000 a_1 a_3 b_1 - 148377600 a_1 a_3 b_2 + 38305612800 a_2^2 b_1 - 60963840 a_2^2 b_2 + 322560 a_2^2 b_3 \\
&+ 480614400 a_2 a_3 b_1 - 645120 a_2 a_3 b_2 + 1612800 a_3^2 b_1 + 1322893031040000 b_1^3 + 34047457920000 b_1^2 b_2 \\
&+ 375854976000 b_1^2 b_3 - 114171724800 b_1 b_2^2 - 241920000 b_1 b_2 b_3 + 1612800 b_1 b_3^2 + 83543040 b_2^3 \\
&- 322560 b_2^2 b_3 - 7764316560000 b_1 + 11787930000 b_2 - 47628000 b_3)T^3 + (944504995840000 a_1^6 \\
&+ 2833514987520000 a_1^4 b_1^2 + 2833514987520000 a_1^2 b_1^4 + 944504995840000 b_1^6 - 114052055040000 a_1^4 \\
&- 2525709312000 a_1^3 a_2 + 31610880000 a_1^3 a_3 - 1264435200 a_1^2 a_2^2 - 632217600 a_1^2 a_2 a_3 \\
&+ 14850238233600000 a_1^2 b_1^2 - 331670836224000 a_1^2 b_1 b_2 + 581640192000 a_1^2 b_1 b_3 + 188400844800 a_1^2 b_2^2 \\
&- 632217600 a_1^2 b_2 b_3 + 126443520 a_1 a_2^3 + 275207482368000 a_1 a_2 b_1^2 - 386917171200 a_1 a_2 b_1 b_2 \\
&+ 126443520 a_1 a_2 b_2^2 + 385652736000 a_1 a_3 b_1^2 - 98625945600 a_2^2 b_1^2 + 126443520 a_2^2 b_1 b_2 - 632217600 a_2 a_3 b_1^2 \\
&- 56130143569920000 b_1^4 + 201200090112000 b_1^3 b_2 + 935682048000 b_1^3 b_3 - 295877836800 b_1^2 b_2^2 \\
&- 632217600 b_1^2 b_2 b_3 + 126443520 b_1 b_2^3 + 1395659160000 a_1^2 - 127008000 a_1 a_2 - 816480000 a_1 a_3 \\
&+ 63504000 a_2^2 + 16329600 a_2 a_3 + 1425600 a_3^2 + 26993024052120000 b_1^2 - 85679723808000 b_1 b_2 \\
&+ 391438656000 b_1 b_3 + 68038185600 b_2^2 - 622339200 b_2 b_3 + 1425600 b_3^2 + 2902331250)T^2 + 755856360000 a_1^2 \\
&+ 17908128000 a_1 a_2 + 526176000 a_1 a_3 + 114307200 a_2^2 + 7257600 a_2 a_3 + 129600 a_3^2 + 1702675757160000 b_1^2 \\
&- 5826237984000 b_1 b_2 + 29592864000 b_1 b_3 + 4991414400 b_2^2 - 50803200 b_2 b_3 + 129600 b_3^2 \\
&+ (-76984041308160000 a_1^4 b_1 + 107392696320000 a_1^4 b_2 - 413048832000 a_1^4 b_3 - 219741978624000 a_1^3 a_2 b_1 \\
&+ 165219532800 a_1^3 a_2 b_2 + 826097664000 a_1^3 a_3 b_1 - 247829299200 a_1^2 a_2^2 b_1 + 327225545687040000 a_1^2 b_1^3 \\
&- 570007388160000 a_1^2 b_1^2 b_2 + 247829299200 a_1^2 b_1 b_2^2 + 540267872256000 a_1 a_2 b_1^3 - 495658598400 a_1 a_2 b_1^2 b_2 \\
&+ 826097664000 a_1 a_3 b_1^3 + 82609766400 a_2^2 b_1^3 - 327960772608000000 b_1^5 + 82609766400000 b_1^4 b_2 \\
&+ 413048832000 b_1^4 b_3 - 82609766400 b_1^3 b_2^2 + 2319398928000000 a_1^2 b_1 - 3826497024000 a_1^2 b_2 \\
&+ 18603648000 a_1^2 b_3 + 63310270464000 a_1 a_2 b_1 - 100048435200 a_1 a_2 b_2 + 454809600 a_1 a_2 b_3 \\
&+ 476679168000 a_1 a_3 b_1 - 454809600 a_1 a_3 b_2 + 431725593600 a_2^2 b_1 - 663828480 a_2^2 b_2 + 2903040 a_2^2 b_3 \\
&+ 6357657600 a_2 a_3 b_1 - 5806080 a_2 a_3 b_2 + 14515200 a_3^2 b_1 - 183630192600960000 b_1^3 + 711772475904000 b_1^2 b_2 \\
&+ 690463872000 b_1^2 b_3 - 1069136409600 b_1 b_2^2 - 145152000 b_1 b_2 b_3 + 14515200 b_1 b_3^2 + 636733440 b_2^3 \\
&- 2903040 b_2^2 b_3 + 2482252290000 b_1 - 4429404000 b_2 + 23814000 b_3)T + 22325625
\end{aligned}$$