



HAL
open science

Six-parameters deformations of fourth order Peregrine breather solutions of the NLS equation.

Pierre Gaillard

► **To cite this version:**

Pierre Gaillard. Six-parameters deformations of fourth order Peregrine breather solutions of the NLS equation.. 2013. hal-00798669v1

HAL Id: hal-00798669

<https://hal.science/hal-00798669v1>

Preprint submitted on 9 Mar 2013 (v1), last revised 23 Apr 2013 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Six-parameters deformations of fourth order Peregrine breather solutions of the NLS equation.

+Pierre Gaillard, + Université de Bourgogne, Dijon, France :
e-mail: Pierre.Gaillard@u-bourgogne.fr,

March 9, 2013

Abstract

We construct solutions of the focusing NLS equation as a quotient of two determinants. This formulation gives in the case of the order 4, new deformations of the Peregrine breather with 6 real parameters.

We construct families of quasi-rational solutions of the NLS equation and describe the apparition of multi rogue waves. With this method, we construct the analytical expressions of deformations of the Peregrine breather of order 4 with 6 real parameters and plot different types of rogue waves.

1 Introduction

The nonlinear Schrödinger equation was first solved in 1972 by Zakharov and Shabat, using the inverse scattering method. The first quasi-rational solutions of NLS equation were constructed in 1983 by Peregrine. In 1986 Eleonski, Akhmediev and Kulagin obtained the two-phase almost periodic solution to the NLS equation and obtained the first higher order analogue of the Peregrine breather[3]. In a series of articles [1, 2] , other families of higher order were constructed by Akhmediev et al., using Darboux transformations.

In 2010, it has been proved that rational solutions of the NLS equation can be written as a quotient of two Wronskians in [8] . Recently, a new representation of the solutions of the NLS equation has been constructed in [11], in terms of a ratio of two Wronskians determinants of even order $2N$ composed of elementary functions; these solutions of NLS are called of order N . By the passage to the limit when some parameter tended to 0, it has been possible to obtain quasi-rational solutions of the NLS equation.

It can be compared with the results obtained recently by Akhmediev et al. in [5] with Darboux dressing method and numerical approach.

In 2012, an other representation of the solutions of the focusing NLS equation, as a ratio of two determinants has been given in [14] using generalized Darboux transform.

Ohta and Yang [18] have given a new approach where solutions of the focusing NLS equation by means of a determinant representation, obtained from Hirota bilinear method.

At the beginning of the year 2012, one obtained a representation in terms of determinants which does not involve limits [13].

But, these first two formulations given in [11, 13] did depend in fact only on two parameters; this remark was first pointed out by V.B. Matveev. Then we found for the order N (for determinants of order $2N$), solutions depending on $2N - 2$ real parameters.

With this new method, we construct new deformations at order 4 with 6 real parameters, which are similar to those first found by V.B. Matveev and P. Dubard. The following orders will be able to be the object of other publications.

2 Wronskian representation of solutions of NLS equation and quasi-rational limit

2.1 Solutions of NLS equation in terms of Wronskian determinant

We consider the focusing NLS equation

$$iv_t + v_{xx} + 2|v|^2v = 0. \quad (1)$$

We recall briefly the results obtained in [11]. The solution of the NLS equation can be written in the form

$$v(x, t) = \frac{\det(I + A_3(x, t))}{\det(I + A_1(x, t))} \exp(2it - i\varphi). \quad (2)$$

In (2), the matrix $A_r = (a_{\nu\mu})_{1 \leq \nu, \mu \leq 2N}$ ($r = 3, 1$) is defined by

$$a_{\nu\mu} = (-1)^{\epsilon_\nu} \prod_{\lambda \neq \mu} \left| \frac{\gamma_\lambda + \gamma_\nu}{\gamma_\lambda - \gamma_\mu} \right| \exp(i\kappa_\nu x - 2\delta_\nu t + x_{r,\nu} + e_\nu). \quad (3)$$

κ_ν , δ_ν , γ_ν are functions of the parameters λ_ν , $\nu = 1, \dots, 2N$ satisfying the relations

$$0 < \lambda_j < 1, \quad \lambda_{N+j} = -\lambda_j, \quad 1 \leq j \leq N. \quad (4)$$

They are given by the following equations,

$$\kappa_\nu = 2\sqrt{1 - \lambda_\nu^2}, \quad \delta_\nu = \kappa_\nu \lambda_\nu, \quad \gamma_\nu = \sqrt{\frac{1 - \lambda_\nu}{1 + \lambda_\nu}}, \quad (5)$$

and

$$\kappa_{N+j} = \kappa_j, \quad \delta_{N+j} = -\delta_j, \quad \gamma_{N+j} = 1/\gamma_j, \quad j = 1 \dots N. \quad (6)$$

The terms $x_{r,\nu}$ ($r = 3, 1$) are defined by

$$x_{r,\nu} = (r-1) \ln \frac{\gamma_\nu - i}{\gamma_\nu + i}, \quad 1 \leq j \leq 2N. \quad (7)$$

The parameters e_ν are defined by

$$e_j = ia_j - b_j, \quad e_{N+j} = ia_j + b_j, \quad 1 \leq j \leq N, \quad (8)$$

where a_j and b_j , for $1 \leq j \leq N$ are arbitrary real numbers.

The terms ϵ_ν are defined by :

$$\begin{aligned} \epsilon_\nu &= 0, & 1 \leq \nu \leq N \\ \epsilon_\nu &= 1, & N+1 \leq \nu \leq 2N. \end{aligned}$$

We use the following notations :

$$\Theta_{r,\nu} = \kappa_\nu x/2 + i\delta_\nu t - ix_{r,\nu}/2 + \gamma_\nu y - ie_\nu, \quad 1 \leq \nu \leq 2N.$$

We consider the functions

$$\begin{aligned} \phi_{r,\nu}(y) &= \sin \Theta_{r,\nu}, & 1 \leq \nu \leq N, \\ \phi_{r,\nu}(y) &= \cos \Theta_{r,\nu}, & N+1 \leq \nu \leq 2N. \end{aligned} \quad (9)$$

$W_r(y) = W(\phi_{r,1}, \dots, \phi_{r,2N})$ is the Wronskian

$$W_r(y) = \det[(\partial_y^{\mu-1} \phi_{r,\nu})_{\nu, \mu \in [1, \dots, 2N]}]. \quad (10)$$

Then we get the following link between Fredholm and Wronskian determinants [12]

Theorem 2.1

$$\det(I + A_r) = k_r(0) \times W_r(\phi_{r,1}, \dots, \phi_{r,2N})(0), \quad (11)$$

where

$$k_r(y) = \frac{2^{2N} \exp(i \sum_{\nu=1}^{2N} \Theta_{r,\nu})}{\prod_{\nu=2}^{2N} \prod_{\mu=1}^{\nu-1} (\gamma_\nu - \gamma_\mu)}.$$

In (11), the matrix A_r is defined by (3).

It can be deduced the following result :

Theorem 2.2 *The function v defined by*

$$v(x, t) = \frac{W_3(0)}{W_1(0)} \exp(2it - i\varphi). \quad (12)$$

is solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2 v = 0.$$

2.2 Quasi-rational solutions of NLS equation

Here, we take the limit when the parameters $\lambda_j \rightarrow 1$ for $1 \leq j \leq N$ and $\lambda_j \rightarrow -1$ for $N+1 \leq j \leq 2N$.

We consider the parameter λ_j written in the form

$$\lambda_j = 1 - 2j^2\epsilon^2, \quad 1 \leq j \leq N. \quad (13)$$

When ϵ goes to 0, we realize limited expansions at order p , for $1 \leq j \leq N$, of the terms

$$\begin{aligned} \kappa_j &= 4j\epsilon(1 - \epsilon^2j^2)^{1/2}, \quad \delta_j = 4j\epsilon(1 - 2\epsilon^2j^2)(1 - \epsilon^2j^2)^{1/2}, \\ \gamma_j &= j\epsilon(1 - \epsilon^2j^2)^{-1/2}, \quad x_{r,j} = (r-1) \ln \frac{1+i\epsilon j(1-\epsilon^2j^2)^{-1/2}}{1-i\epsilon j(1-\epsilon^2j^2)^{-1/2}}, \\ \kappa_{N+j} &= 4j\epsilon(1 - \epsilon^2j^2)^{1/2}, \quad \delta_{N+j} = -4j\epsilon(1 - 2\epsilon^2j^2)(1 - \epsilon^2j^2)^{1/2}, \\ \gamma_{N+j} &= 1/(j\epsilon)(1 - \epsilon^2j^2)^{1/2}, \quad x_{r,N+j} = (r-1) \ln \frac{1-i\epsilon j(1-\epsilon^2j^2)^{-1/2}}{1+i\epsilon j(1-\epsilon^2j^2)^{-1/2}}. \end{aligned}$$

Then we get quasi-rational solutions of the NLS equation given by :

Theorem 2.3 *With the parameters λ_j defined by (13), a_j and b_j chosen as in (8), for $1 \leq j \leq N$, the function v defined by*

$$v(x, t) = \exp(2it - i\varphi) \lim_{\epsilon \rightarrow 0} \frac{W_3(0)}{W_1(0)}, \quad (14)$$

is a quasi-rational solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2v = 0.$$

3 Expression of solutions of NLS equation in terms of a ratio of two determinants

We give the construction of the solutions of the NLS equation expressed as a quotient of two determinants which does not involve a passage to the limit.

For this we need the following notations :

$$A_\nu = \kappa_\nu x/2 + i\delta_\nu t - ix_{3,\nu}/2 - ie_\nu/2,$$

$$B_\nu = \kappa_\nu x/2 + i\delta_\nu t - ix_{1,\nu}/2 - ie_\nu/2,$$

for $1 \leq \nu \leq 2N$, with κ_ν , δ_ν , $x_{r,\nu}$ defined in (5), (6) and (7).

The parameters e_ν are defined by (8).

With particular special choices of the parameters a_j and b_j , for $1 \leq N$, we get new deformations depending on six parameters.

Below we use the following functions :

$$\begin{aligned} f_{4j+1,k} &= \gamma_k^{4j-1} \sin A_k, & f_{4j+2,k} &= \gamma_k^{4j} \cos A_k, \\ f_{4j+3,k} &= -\gamma_k^{4j+1} \sin A_k, & f_{4j+4,k} &= -\gamma_k^{4j+2} \cos A_k, \end{aligned} \quad (15)$$

for $1 \leq k \leq N$, and

$$\begin{aligned} f_{4j+1,k} &= \gamma_k^{2N-4j-2} \cos A_k, & f_{4j+2,k} &= -\gamma_k^{2N-4j-3} \sin A_k, \\ f_{4j+3,k} &= -\gamma_k^{2N-4j-4} \cos A_k, & f_{4j+4,k} &= \gamma_k^{2N-4j-5} \sin A_k, \end{aligned} \quad (16)$$

for $N+1 \leq k \leq 2N$.

We define the functions $g_{j,k}$ for $1 \leq j \leq 2N$, $1 \leq k \leq 2N$ in the same way, we replace only the term A_k by B_k .

$$\begin{aligned} g_{4j+1,k} &= \gamma_k^{4j-1} \sin B_k, & g_{4j+2,k} &= \gamma_k^{4j} \cos B_k, \\ g_{4j+3,k} &= -\gamma_k^{4j+1} \sin B_k, & g_{4j+4,k} &= -\gamma_k^{4j+2} \cos B_k, \end{aligned} \quad (17)$$

for $1 \leq k \leq N$, and

$$\begin{aligned} g_{4j+1,k} &= \gamma_k^{2N-4j-2} \cos B_k, & g_{4j+2,k} &= -\gamma_k^{2N-4j-3} \sin B_k, \\ g_{4j+3,k} &= -\gamma_k^{2N-4j-4} \cos B_k, & g_{4j+4,k} &= \gamma_k^{2N-4j-5} \sin B_k, \end{aligned} \quad (18)$$

for $N+1 \leq k \leq 2N$.

Then we get the following result :

Theorem 3.1 *The function v defined by*

$$v(x, t) = \exp(2it - i\varphi) \times \frac{\det((n_{jk})_{j,k \in [1, 2N]})}{\det((d_{jk})_{j,k \in [1, 2N]})} \quad (19)$$

is a quasi-rational solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2 v = 0,$$

where

$$n_{j1} = f_{j,1}(x, t, 0), \quad 1 \leq j \leq 2N \quad n_{jk} = \frac{\partial^{2k-2} f_{j,1}}{\partial \epsilon^{2k-2}}(x, t, 0), \quad 2 \leq k \leq N, \quad 1 \leq j \leq 2N$$

$$n_{jN+1} = f_{j,N+1}(x, t, 0), \quad 1 \leq j \leq 2N \quad n_{jN+k} = \frac{\partial^{2k-2} f_{j,N+1}}{\partial \epsilon^{2k-2}}(x, t, 0), \quad 2 \leq k \leq N, \quad 1 \leq j \leq 2N$$

$$d_{j1} = g_{j,1}(x, t, 0), \quad 1 \leq j \leq 2N \quad d_{jk} = \frac{\partial^{2k-2} g_{j,1}}{\partial \epsilon^{2k-2}}(x, t, 0), \quad 2 \leq k \leq N, \quad 1 \leq j \leq 2N$$

$$d_{jN+1} = g_{j,N+1}(x, t, 0), \quad 1 \leq j \leq 2N \quad d_{jN+k} = \frac{\partial^{2k-2} g_{j,N+1}}{\partial \epsilon^{2k-2}}(x, t, 0), \quad 2 \leq k \leq N, \quad 1 \leq j \leq 2N$$

The functions f and g are defined in (15), (16), (17), (18).

We don't give here the proof of this result. The ideas are the same as exposed in [13]. We will give the proof in a forthcoming paper.

The solutions of the NLS equation can also be written in the form :

$$v(x, t) = \exp(2it - i\varphi) \times Q(x, t)$$

where $Q(x, t)$ is defined by :

$$Q(x, t) := \frac{\begin{vmatrix} f_{1,1}[0] & \dots & f_{1,1}[N-1] & f_{1,N+1}[0] & \dots & f_{1,N+1}[N-1] \\ f_{2,1}[0] & \dots & f_{2,1}[N-1] & f_{2,N+1}[0] & \dots & f_{2,N+1}[N-1] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_{2N,1}[0] & \dots & f_{2N,1}[N-1] & f_{2N,N+1}[0] & \dots & f_{2N,N+1}[N-1] \end{vmatrix}}{\begin{vmatrix} g_{1,1}[0] & \dots & g_{1,1}[N-1] & g_{1,N+1}[0] & \dots & g_{1,N+1}[N-1] \\ g_{2,1}[0] & \dots & g_{2,1}[N-1] & g_{2,N+1}[0] & \dots & g_{2,N+1}[N-1] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ g_{2N,1}[0] & \dots & g_{2N,1}[N-1] & g_{2N,N+1}[0] & \dots & g_{2N,N+1}[N-1] \end{vmatrix}} \quad (20)$$

4 Quasi-rational solutions of order 4 with six parameters

We have already constructed in [11] solutions for the cases $N = 1, 2, 3, 4$, and in [13] with two parameters.

4.1 Analytical expressions of the solutions of NLS equation with six parameters

Because of the length of the expression v of the solution of NLS equation with six, we only give in the appendix.

4.2 Plots of the solutions of NLS equation with six parameters

Conversely to the study with two parameters given in preceding works [10, 11, 13], we get other type of symmetries in the plots in the (x, t) plane. We give some examples of this fact in the following.

4.2.1 Peregrine breather of order 4

If we choose $\tilde{a}_1 = \tilde{b}_1 = \tilde{a}_2 = \tilde{b}_2 = \tilde{a}_3 = \tilde{b}_3 = 0$, we obtain the classical Peregrine breather in figure 1.

With other choices of parameters, we obtain all types of configurations : triangles, overlapping triangular-circular and circular configurations with a maximum of 10 peaks.

4.2.2 Variation of one parameter

We obtain triangles in the cases $\tilde{a}_1 \neq 0$ or $\tilde{b}_1 \neq 0$, overlapping triangular-circular configurations in the cases $\tilde{a}_2 \neq 0$ or $\tilde{b}_2 \neq 0$, circular configurations in the cases $\tilde{a}_3 \neq 0$ or $\tilde{b}_3 \neq 0$. These plots are given in the figures 2 and 3.

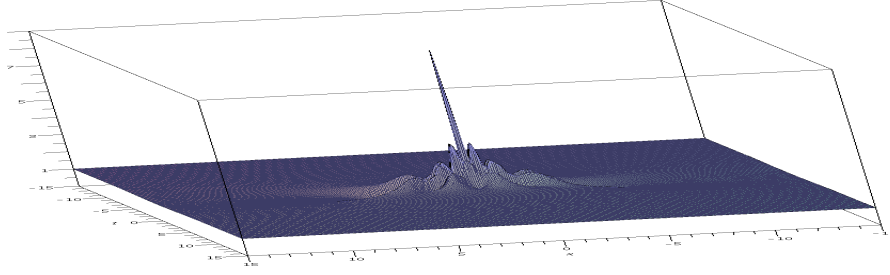


Figure 1: Solution of NLS, $N=4$, $\tilde{a}_1 = \tilde{b}_1 = \tilde{a}_2 = \tilde{b}_2 = \tilde{a}_3 = \tilde{b}_3 = 0$.

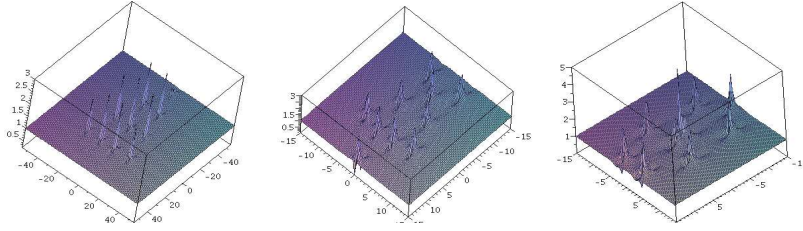


Figure 2: Solution of NLS, $N=4$, $\tilde{a}_1 = 10^3$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$ on the left, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 10^5$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$ in the center, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 10^8$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$ on the right.

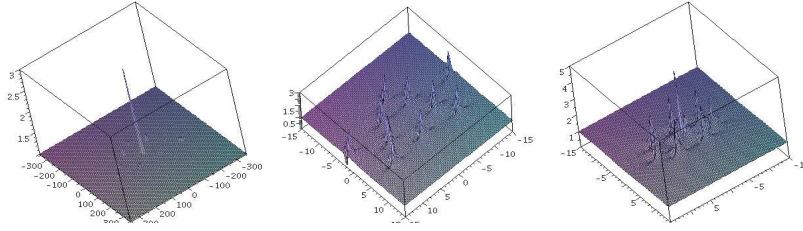


Figure 3: Solution of NLS, $N=4$, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 10^5$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$ on the left, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 10^5$, $\tilde{b}_3 = 0$ in the center, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 10^6$ on the right.

4.2.3 Circular, triangular and overlapping triangular-circular configurations

In general we obtain regular forms as triangular, overlapping triangular-circular and circular configurations. We present here some examples in the figure 4.

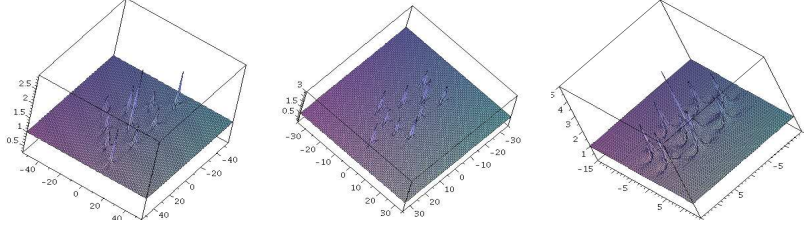


Figure 4: Solution of NLS, $N=4$, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 10^3$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$ on the left, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 10^6$, $\tilde{a}_3 = 0$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$ in the center, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, $\tilde{a}_3 = 10^7$, $\tilde{b}_1 = 0$, $\tilde{b}_2 = 0$, $\tilde{b}_3 = 0$ on the right.

5 Conclusion

The method described in the present paper provides a new tool to get explicitly solutions of the NLS equation of order N with $2N - 2$ real parameters.

By different choices of these parameters, we obtained new patterns in the $(x; t)$ plane; we recognized circular shape as already observed in the case of deformations depending on two parameters [11, 13]. We get new triangular shapes and overlapping circular-triangular shape.

We chose to present here the solutions of the NLS equation in the case $N = 4$ with six real parameters, only in order not to weigh down the text with . We postpone the presentation of the higher orders in another publication.

It would be very interesting to carry out experiments to observe these phenomena.

Acknowledgments

I would like to thank deeply V.B. Matveev for the long and fruitful discussions which we had. I am infinitely grateful to him.

References

- [1] N. Akhmediev, A. Ankiewicz, J.M. Soto-Crespo, Rogue waves and rational solutions of nonlinear Schrödinger equation, *Physical Review E*, V. **80**, N. 026601, (2009).
- [2] N. Akhmediev, V. Eleonskii, N. Kulagin, Exact first order solutions of the nonlinear Schrödinger equation, *Th. Math. Phys.*, V. **72**, N. 2, 183-196, (1987).
- [3] N. Akhmediev, V. Eleonsky, N. Kulagin, Generation of periodic trains of picosecond pulses in an optical fiber : exact solutions, *Sov. Phys. J.E.T.P.*, V. **62**, 894-899, (1985).

- [4] N. Akhmediev, A. Ankiewicz, P.A. Clarkson, Rogue waves, rational solutions, the patterns of their zeros and integral relations, *J. Phys. A : Math. Theor.*, V. **43**, 122002, 1-9, (2010).
- [5] N. Akhmediev, A. Ankiewicz, D. J. Kedziora, Circular rogue wave clusters, *Phys. Review E*, V. **84**, 1-7, 2011
- [6] E.D. Belokolos, A.i. Bobenko, A.R. Its, V.Z. Enolskij and V.B. Matveev, *Algebro-geometric approach to nonlinear integrable equations*, Springer series in nonlinear dynamics, Springer Verlag, 1-360, (1994).
- [7] A. Chabchoub, H. Hoffmann, M. Onorato, N. Akhmediev, Super rogue waves : observation of a higher-order breather in water waves, *Phys. Review X*, V. **2**, 1-6, (2012).
- [8] P. Dubard, P. Gaillard, C. Klein, V.B. Matveev, On multi-rogue waves solutions of the NLS equation and positon solutions of the KdV equation, *Eur. Phys. J. Special Topics*, V. **185**, 247-258, (2010).
- [9] V. Eleonskii, I. Krichever, N. Kulagin, Rational multisoliton solutions to the NLS equation, *Soviet Doklady 1986 sect. Math. Phys.*, V. **287**, 606-610, (1986).
- [10] P. Gaillard, Higher order Peregrine breathers and multi-rogue waves solutions of the NLS equation, [arXiv:1105.0058](#), 2011
- [11] P. Gaillard, Families of quasi-rational solutions of the NLS equation and multi-rogue waves, *J. Phys. A : Meth. Theor.*, V. **44**, 1-15, 2011
- [12] P. Gaillard, Wronskian representation of solutions of the NLS equation and higher Peregrine breathers, *Scientific Advances*, V. **13**, N. 2, 71-153, 2012
- [13] P. Gaillard, Degenerate determinant representation of solution of the NLS equation, higher Peregrine breathers and multi-rogue waves, *Jour. Of Math. Phys.*, V. **54**, 013504-1-32, 2013
- [14] B. Guo, L. Ling, Q.P. Liu, Nonlinear Schrodinger equation: Generalized Darboux transformation and rogue wave solutions, *Phys. Rev. E*, V. **85**, 026607, (2012).
- [15] A.R. Its, A.V. Rybin, M.A. Salle, Exact integration of nonlinear Schrödinger equation, *Teore. i Mat. Fiz.*, V. **74**, N. 1, 29-45, (1988).
- [16] A.R. Its, V.P. Kotlyarov, Explicit expressions for the solutions of nonlinear Schrödinger equation, *Dokl. Akad. Nauk. SSSR, S. A*, V. **965**, N. 11, (1976).
- [17] V.B. Matveev, M.A. Salle, *Darboux transformations and solitons*, Series in Nonlinear Dynamics, Springer Verlag, Berlin, (1991).

- [18] Y Ohta, J. Yang, General high-order rogue waves and their dynamics in the nonlinear Schrödinger equation, arXiv : 1110.5873 [nlin.SI] 26 Oct. 2011.
- [19] D. Peregrine, Water waves, nonlinear Schrödinger equations and their solutions, J. Austral. Math. Soc. Ser. B, V. **25**, 16-43, (1983).
- [20] V. E. Zakharov, Stability of periodic waves of finite amplitude on a surface of a deep fluid, J. Appl. Tech. Phys, V. **9**, 86-94, (1968)
- [21] V. E. Zakharov, A.B. Shabat Exact theory of two dimensional self focusing and one dimensional self modulation of waves in nonlinear media, Sov. Phys. JETP, V. **34**, 62-69, (1972)

Appendix

The solution of NLS equation takes the form

$$v_N(x, t) = \frac{n(x, t)}{d(x, t)} \exp(2it) = \left(1 - \alpha_N \frac{G_N(2x, 4t) + iH_N(2x, 4t)}{Q_N(2x, 4t)}\right) e^{2it}$$

with

$$G_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{g}_k(T) X^k, \quad H_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{h}_k(T) X^k, \quad Q_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{q}_k(T) X^k.$$

Here, we present the expressions of the polynomials G_N , H_N and Q_N , for $N = 4$.

$$\begin{aligned}
& \alpha_4 = 40, \quad \mathbf{g}_{20} = 0, \quad \mathbf{g}_{19} = 0, \quad \mathbf{g}_{18} = 1, \quad \mathbf{g}_{17} = 0, \quad \mathbf{g}_{16} = 27T^2 + 27, \quad \mathbf{g}_{15} = 672a_1, \quad \mathbf{g}_{14} = 180T^4 \\
& -360T^2 - 10080Tb_1 + 900, \quad \mathbf{g}_{13} = 3360a_1 + 168a_2, \quad \mathbf{g}_{12} = 23520T^6 - 218400T^4 - 3763200T^3b_1 \\
& -50400T^2 + 19756800a_1^2 + 19756800b_1^2 + (-70694400b_1 + 60480b_2)T + 7560000, \quad \mathbf{g}_{11} = -2822400T^4a_1 \\
& + 282240T^2a_2 + 5241600a_1 + 403200a_2 - 17280a_3, \quad \mathbf{g}_{10} = 45360T^8 - 705600T^6 - 11854080T^5b_1 \\
& + 756000T^4 + (4569600b_1 + 13440b_2)T^3 + (355622400a_1^2 + 355622400b_1^2 - 1512000)T^2 + 69619200a_1^2 \\
& - 14300160a_1a_2 - 6071923200b_1^2 + 5268480b_1b_2 + (5834707200b_1 - 9112320b_2 + 40320b_3)T - 4309200, \\
& \mathbf{g}_9 = -13171200T^6a_1 + (-39648000a_1 + 1310400a_2)T^4 - 8605184000a_1^3 + 25815552000a_1b_1^2 \\
& + (-108864000a_1 + 4435200a_2 - 144000a_3)T^2 + (-87230976000a_1b_1 + 79027200a_1b_2 + 11289600a_2b_1)T \\
& + 622944000a_1 + 1512000a_2 - 144000a_3, \quad \mathbf{g}_8 = 55440T^{10} - 831600T^8 - 16934400T^7b_1 + 9424800T^6 \\
& + (1591833600b_1 - 1270080b_2)T^5 + (1481760000a_1^2 + 1481760000b_1^2 + 6804000)T^4 + (102791808000b_1 \\
& - 149587200b_2 + 604800b_3)T^3 + (14817600000a_1^2 + 67652928000b_1^2 - 67737600b_1b_2 - 227934000)T^2 \\
& - 9275616000a_1^2 + 75801600a_1a_2 - 4838400a_1a_3 + 1048320a_2^2 + 2788128000b_1^2 - 91929600b_1b_2 \\
& + 80640b_2^2 + (165957120000a_1^2b_1 - 55319040000b_1^3 + 75926592000b_1 - 126705600b_2 + 604800b_3)T \\
& - 19278000, \quad \mathbf{g}_7 = -25401600T^8a_1 + (-203212800a_1 + 1693440a_2)T^6 + (-3794112000a_1 + 806400a_2 \\
& + 518400a_3)T^4 + (-526546944000a_1b_1 + 406425600a_1b_2 - 45158400a_2b_1)T^3 + 82188288000a_1^3 \\
& - 2528870400a_1^2a_2 + 82188288000a_1b_1^2 - 2528870400a_2b_1^2 + (132765696000a_1^3 - 398297088000a_1b_1^2 \\
& + 7499520000a_1 - 6048000a_2 + 3801600a_3)T^2 + (-208889856000a_1b_1 + 251596800a_1b_2 + 8251084800a_2b_1 \\
& - 7741440a_2b_2 + 38707200a_3b_1)T - 393120000a_1 - 38707200a_2 - 2246400a_3, \quad \mathbf{g}_6 = 43680T^{12} - 73920T^{10} \\
& - 10348800T^9b_1 + 27871200T^8 + (3965875200b_1 - 3144960b_2)T^7 + (2765952000a_1^2 + 2765952000b_1^2 \\
& + 224380800)T^6 + (190382169600b_1 - 233694720b_2 + 725760b_3)T^5 + (-4346496000a_1^2 + 474163200a_1a_2 \\
& + 577293696000b_1^2 - 474163200b_1b_2 - 1370628000)T^4 - 2891341824000a_1^4 + 2891341824000b_1^4 \\
& + (-309786624000a_1^2b_1 + 103262208000b_1^3 - 1231751808000b_1 + 1927699200b_2 - 8870400b_3)T^3 \\
& + (24554880000a_1^2 + 677376000a_1a_2 - 6773760a_2^2 + 6346730880000b_1^2 - 15579648000b_1b_2 \\
& - 135475200b_1b_3 + 20321280b_2^2 + 793800000)T^2 + 32570496000a_1^2 + 293529600a_1a_2 - 6451200a_1a_3 \\
& - 4515840a_2^2 - 322560a_2a_3 + 47868863616000b_1^2 - 121724467200b_1b_2 + 341913600b_1b_3 + 72253440b_2^2 \\
& - 322560b_2b_3 + (-309786624000a_1^2b_1 + 516311040000b_1^3 + 829319904000b_1 - 1524096000b_2 \\
& + 8467200b_3)T - 47628000, \quad \mathbf{g}_5 = -24837120T^{10}a_1 + (-373766400a_1 - 60480a_2)T^8 + (-6457651200a_1 \\
& - 93703680a_2 + 1693440a_3)T^6 + (-210528460800a_1b_1 + 94832640a_1b_2 - 853493760a_2b_1)T^5 \\
& + (154893312000a_1^3 - 464679936000a_1b_1^2 - 1270080000a_1 - 419126400a_2 - 3628800a_3)T^4 \\
& + (-22157420544000a_1b_1 + 32694681600a_1b_2 - 135475200a_1b_3 - 67556966400a_2b_1 + 54190080a_2b_2 \\
& - 135475200a_3b_1)T^3 - 1381395456000a_1^3 - 9799372800a_1^2a_2 - 948326400a_1^2a_3 + 252887040a_1a_2^2 \\
& - 130796338176000a_1b_1^2 + 257944780800a_1b_1b_2 - 126443520a_1b_2^2 - 413154201600a_2b_1^2 \\
& + 379330560a_2b_1b_2 - 948326400a_3b_1^2 + (1017870336000a_1^3 - 26553139200a_1^2a_2 + 2257016832000a_1b_1^2 \\
& - 26553139200a_2b_1^2 + 69600384000a_1 + 812851200a_2 + 10886400a_3)T^2 + (17348050944000a_1^3b_1 \\
& + 17348050944000a_1b_1^3 - 9584870400000a_1b_1 + 16324761600a_1b_2 - 77414400a_1b_3 + 349593753600a_2b_1 \\
& - 582543360a_2b_2 + 2903040a_2b_3 + 3077222400a_3b_1 - 2903040a_3b_2)T - 127008000a_1 - 184161600a_2 \\
& - 10886400a_3, \quad \mathbf{g}_4 = (3452736000b_1 - 2587200b_2)T^9 - 734317920000a_1^2 + 3874167995040000b_1^2 \\
& + 36136800T^{10} + (2667168000a_1^2 + 2667168000b_1^2 + 431676000)T^8 + (-80373888000b_1 + 168537600b_2 \\
& - 864000b_3)T^7 + (-101549952000a_1^2 + 316108800a_1a_2 + 157659264000b_1^2 - 246078000)T^6 \\
& + (-1394039808000a_1^2b_1 + 464679936000b_1^3 - 1416054528000b_1 + 1816214400b_2 - 6048000b_3)T^5 \\
& + (-211821120000a_1^2 + 5983488000a_1a_2 - 338688000a_1a_3 + 50803200a_2^2 - 30449885760000b_1^2 \\
& + 46174464000b_1b_2 - 16934400b_2^2 + 714420000)T^4 + 206524416000a_1^3a_2 - 712509235200000a_1^2b_1^2 \\
& - 206524416000b_1^3b_2 + (-54655211520000a_1^2b_1 + 44255232000a_1^2b_2 - 48459479040000b_1^3 \\
& + 44255232000b_1^2b_2 + 2810263680000b_1 - 4233600000b_2 + 18144000b_3)T^3 + 619573248000a_1^2b_1b_2 \\
& - 619573248000a_1a_2b_1^2 - 2419200a_2a_3 - 14902272000a_1a_2 - 11931978240000b_1b_2 - 79833600b_2b_3 \\
& + 21600T^{14} - 387072000a_1a_3 + 52109568000b_1b_3 - 76204800a_2^2 + 9169977600b_2^2 \\
& + (689638763520000a_1^2b_1 - 1066867200000a_1^2b_2 + 4741632000a_1^2b_3 + 2174828544000a_1a_2b_1 \\
& - 1896652800a_1a_2b_2 + 1896652800a_2^2b_1 + 696909265920000b_1^3 - 1073189376000b_1^2b_2 + 4741632000b_1^2b_3 \\
& - 25363497600000b_1 + 4032504000b_2 - 18144000b_3)T + 655200T^{12} + (906837120000a_1^2 + 18966528000a_1a_2 \\
& - 677376000a_1a_3 + 237081600a_2^2 + 59675978880000b_1^2 - 79930368000b_1b_2 + 677376000b_1b_3 - 33868800b_2^2 \\
& + 2619540000)T^2 + 4130488320000a_1^4 + 233372590080000b_1^4 + 714420000 + 172800a_3^2 + 172800b_3^2, \\
& \mathbf{g}_3 = -21702675456000a_2b_1^2 - 138636288000a_1^2a_2 - 47628000000a_1 - 381024000a_2 - 9072000a_3 \\
& + (45650626560000a_1b_1 - 64621670400a_1b_2 + 270950400a_1b_3 + 54596505600a_2b_1 - 54190080a_2b_2)T^5 \\
& + (2581555200000a_1^3 + 23233996800000a_1b_1^2 - 34503840000a_1 + 1016064000a_2 + 90720000a_3)T^4 \\
& + (57826836480000a_1^3b_1 + 57826836480000a_1b_1^3 + 193896622080000a_1b_1 - 279304704000a_1b_2 \\
& + 1096704000a_1b_3 + 1953326592000a_2b_1 - 2664345600a_2b_2 + 9676800a_2b_3 + 12515328000a_3b_1 \\
& - 9676800a_3b_2)T^3 + (-28765900800000a_1^3 - 66382848000a_1^2a_2 + 94832640000a_1^2a_3 \\
& + 4926524037120000a_1b_1^2 - 8648736768000a_1b_1b_2 + 3793305600a_1b_2^2 + 4220052480000a_2b_1^2 \\
& - 3793305600a_2b_1b_2 + 9483264000a_3b_1^2 + 76204800000a_1 - 1968624000a_2 - 181440000a_3)T^2 \\
& - 28075713361920000a_1b_1^2 - 6624737280000a_1^3 - 12230400T^{12}a_1 + (-295680000a_1 - 1330560a_2)T^{10} \\
& + (121464000000a_1 - 117331200a_2 + 604800a_3)T^8 + (434423808000a_1b_1 - 135475200a_1b_2 \\
& - 948326400a_2b_1)T^7 + (-516311040000a_1^3 + 1548933120000a_1b_1^2 + 75188736000a_1 - 330220800a_2 \\
& + 5644800a_3)T^6 - 40823193600a_1b_1^2 - 3161088000a_3b_1^2 - 191471616000a_1b_1b_3 + (57826836480000a_1^3b_1 \\
& + 57826836480000a_1b_1^3 + 184286914560000a_1b_1 - 310915584000a_1b_2 + 1548288000a_1b_3 \\
& + 1229437440000a_2b_1 - 2032128000a_2b_2 + 9676800a_2b_3 + 8902656000a_3b_1 - 9676800a_3b_2)T
\end{aligned}$$

$$\begin{aligned}
& -18063360 a_2^3 + 180633600 a_1 b_2 b_3 + 180633600 a_1 a_2 a_3 + 69753470976000 a_1 b_1 b_2 + 39558758400 a_2 b_1 b_2 \\
& -18063360 a_2 b_2^2 + 13095936000 a_1^2 a_3 - 2348236800 a_1 a_2^2, \quad \mathbf{g}_2 = (241540992000 a_1^2 - 2122444800 a_1 a_2 \\
& + 2257920 a_2^2 - 4142436480000 b_1^2 - 18921369600 b_1 b_2 - 135475200 b_1 b_3 + 29352960 b_2^2 + 10128888000) T^6 \\
& + 17702092800 b_1^2 b_2^2 + (-213754464000 b_1 + 273772800 b_2 - 950400 b_3) T^9 + (928166400 b_1 - 524160 b_2) T^{11} \\
& + (1303948800 a_1^2 + 1303948800 b_1^2 - 467208000) T^{10} + 608368320000 a_1^2 + 17186518077120000 b_1^2 \\
& + 1947230208000 a_1^3 a_2 + 21322170777600000 a_1^2 b_1^2 - 38767583232000 b_1^3 b_2 - 38767583232000 a_1^2 b_1 b_2 \\
& + 1947230208000 a_1 a_2 b_1^2 + 17702092800 a_1^2 a_2^2 + 29030400 a_2 a_3 + 13716864000 a_1 a_2 - 56076064128000 b_1 b_2 \\
& + (-1358694328320000 a_1^2 b_1 + 2155410432000 a_1^2 b_2 - 9483264000 a_1^2 b_3 - 5787500544000 a_1 a_2 b_1 \\
& + 5960908800 a_1 a_2 b_2 - 10838016000 a_1 a_3 b_1 - 57441484800 a_2^2 b_1 + 54190080 a_2^2 b_2 - 541900800 a_2 a_3 b_1 \\
& + 17716864112640000 b_1^3 - 22089231360000 b_1^2 b_2 + 602864640000 b_1^2 b_3 - 54731980800 b_1 b_2^2 \\
& - 541900800 b_1 b_2 b_3 + 54190080 b_2^3 - 4158876960000 b_1 + 6477408000 b_2 - 27216000 b_3) T^7 \\
& + (929359872000 a_1^2 b_1 - 309786624000 b_1^3 - 4748962176000 b_1 + 6380640000 b_2 - 24537600 b_3) T^7 \\
& + (111523184640000 a_1^4 + 1239146496000 a_1^3 a_2 - 4275055411200000 a_1^2 b_1^2 + 3717439488000 a_1^2 b_1 b_2 \\
& - 3717439488000 a_1 a_2 b_1^2 + 1313495285760000 b_1^4 - 1239146496000 b_1^3 b_2 + 4657383360000 a_1^2 \\
& + 77220864000 a_1 a_2 - 870912000 a_1 a_3 + 152409600 a_2^2 - 43545600 a_2 a_3 + 1036800 a_3^2 \\
& + 28730656221120000 b_1^2 - 84414597120000 b_1 b_2 + 349526016000 b_1 b_3 + 61725888000 b_2^2 - 508032000 b_2 b_3 \\
& + 1036800 b_3^2 + 18574920000) T^2 - 435456000 b_2 b_3 + 532800 T^{14} + 580608000 a_1 a_3 + 266499072000 b_1 b_3 \\
& + 254016000 a_2^2 + 45773683200 b_2^2 + (-2098741155840000 a_1^2 b_1 + 2753307648000 a_1^2 b_2 - 9483264000 a_1^2 b_3 \\
& - 4956585984000 a_1 a_2 b_1 + 3793305600 a_1 a_2 b_2 - 1264435200 a_2^2 b_1 + 1929117173760000 b_1^3 \\
& - 3657378816000 b_1^2 b_2 - 9483264000 b_1^2 b_3 + 2528870400 b_1 b_2^2 + 42731841600000 b_1 - 69282864000 b_2 \\
& + 326592000 b_3) T^3 + (-53504575488000 a_1^2 b_1 + 53106278400 a_1^2 b_2 - 78287505408000 b_1^3 \\
& + 53106278400 b_1^2 b_2 - 18308584224000 b_1 + 29364249600 b_2 - 134265600 b_3) T^5 + 17702092800 a_2^2 b_1^2 \\
& + 22024800 T^{12} + 2822400 T^{13} b_1 + (43370127360000 a_1^4 - 43370127360000 b_1^4 - 2383516800000 a_1^2 \\
& - 40981248000 a_1 a_2 + 96768000 a_1 a_3 - 135475200 a_2^2 + 4838400 a_2 a_3 - 694350195840000 b_1^2 \\
& + 1686327552000 b_1 b_2 - 7838208000 b_1 b_3 - 745113600 b_2^2 + 4838400 b_2 b_3 - 3572100000) T^4 \\
& + 752338944000000 a_1^4 + 21246936883200000 b_1^4 + (106094016000 a_1^2 - 237081600 a_1 a_2 - 798880320000 b_1^2 \\
& + 643507200 b_1 b_2 - 6925338000) T^8 + 893025000 + 6120 T^{16} + 17702092800 a_1^2 b_2^2 + 1036800 a_3^2 \\
& + 1036800 b_3^2, \quad \mathbf{g}_1 = -1148200413696000 a_2 b_1^2 + 312270336000 a_1^2 a_2 - 100971360000 a_1 - 1047816000 a_2 \\
& - 27216000 a_3 - 2419200 T^{14} a_1 + (99574272000 a_1^3 - 298722816000 a_1 b_1^2 - 37431072000 a_1 + 545832000 a_2 \\
& - 19958400 a_3) T^8 + (-85612800 a_1 - 611520 a_2) T^{12} + (-4171507200 a_1 - 6209280 a_2 - 443520 a_3) T^{10} \\
& + (-385878528000 a_1 b_1 + 289766400 a_1 b_2 + 41395200 a_2 b_1) T^9 - 19353600 a_1 b_3^2 + 3870720 a_2^2 a_3 \\
& - 456870823718400000 a_1 b_1^2 - 12751603200000 a_1^3 + 7741440 a_2 b_2 b_3 - 1061177241600 a_1 b_2^2 \\
& - 4353882624000 a_3 b_1^2 + (-2139592949760000 a_1^3 b_1 + 1652195328000 a_1^3 b_2 - 4956585984000 a_1^2 a_2 b_1 \\
& + 5460505559040000 a_1 b_1^3 - 4956585984000 a_1 b_1^2 b_2 + 1652195328000 a_2 b_1^3 - 116711884800000 a_1 b_1 \\
& + 167989248000 a_1 b_2 - 677376000 a_1 b_3 + 9901882368000 a_2 b_1 - 15308697600 a_2 b_2 + 67737600 a_2 b_3 \\
& + 79252992000 a_3 b_1 - 67737600 a_3 b_2) T^3 + (90274222080000 a_1 b_1 - 18721382400 a_1 b_2 + 96768000 a_1 b_3 \\
& + 1277337600 a_2 b_1 + 7741440 a_2 b_2 - 135475200 a_3 b_1) T^7 + (-3142121472000 a_1^3 + 44255232000 a_1^2 a_2 \\
& - 16772732928000 a_1 b_1^2 + 44255232000 a_2 b_1^2 + 328442688000 a_1 + 2641766400 a_2 - 88300800 a_3) T^6 \\
& + (-52044152832000 a_1^3 b_1 - 52044152832000 a_1 b_1^3 + 174379613184000 a_1 b_1 - 229156300800 a_1 b_2 \\
& + 812851200 a_1 b_3 + 789075302400 a_2 b_1 - 1287014400 a_2 b_2 + 6773760 a_2 b_3 + 5825433600 a_3 b_1 \\
& - 6773760 a_3 b_2) T^5 + 11565367296000 a_1^4 a_2 - 26137730088960000 a_1^3 b_1^2 + 231307345920000 a_1^5 \\
& + 8205926400 a_3 b_1 b_2 - 8205926400 a_2 b_1 b_3 - 19353600 a_1 a_3^2 - 26369037434880000 a_1 b_1^4 \\
& - 11565367296000 a_2 b_1^4 + (-13861370880000 a_1^3 - 36352512000 a_1^2 a_2 - 4741632000 a_1^2 a_3 \\
& + 3793305600 a_1 a_2^2 + 3381463249920000 a_1 b_1^2 - 5133606912000 a_1 b_1 b_2 + 1896652800 a_1 b_2^2 \\
& - 1446197760000 a_2 b_1^2 + 1896652800 a_2 b_1 b_2 - 4741632000 a_3 b_1^2 - 52708320000 a_1 + 1365336000 a_2 \\
& + 308448000 a_3) T^4 - 6000390144000 a_1 b_1 b_3 + 27095040 a_2^3 + 9096192000 a_1 b_2 b_3 + 425779200 a_1 a_2 a_3 \\
& + 23130734592000 a_1^3 b_1 b_2 + 23130734592000 a_1 b_1^3 b_2 - 3870720 a_3 b_2^2 + (35995760640000 a_1^3 \\
& + 833172480000 a_1^2 a_2 - 47416320000 a_1^2 a_3 + 10838016000 a_1 a_2^2 - 3822500505600000 a_1 b_1^2 \\
& + 4150960128000 a_1 b_1 b_2 - 65028096000 a_1 b_1 b_3 + 4877107200 a_1 b_2^2 + 54190080 a_2^3 + 143060456448000 a_2 b_1 b_2 \\
& - 240062054400 a_2 b_1 b_3 + 541900800 a_2 b_1 b_3 + 541900800 a_2 b_2^2 + 640797696000 a_3 b_1^2 - 541900800 a_3 b_1 b_2 \\
& - 285768000000 a_1 + 381024000 a_2 + 81648000 a_3) T^2 + 1394229473280000 a_1 b_1 b_2 + 2892395520000 a_2 b_1 b_2 \\
& - 1706987520 a_2 b_2^2 + (24126182277120000 a_1^3 b_1 - 384135413760000 a_1^3 b_2 + 1770209280000 a_1^3 b_3 \\
& - 1239146496000 a_1^2 a_2 b_1 - 531062784000 a_1^2 a_3 b_1 - 78078620712960000 a_1 b_1^3 + 120197210112000 a_1 b_1^2 b_2 \\
& - 531062784000 a_1 b_1^2 b_3 - 1239146496000 a_2 b_1^3 + 177020928000 a_3 b_1^3 + 329641643520000 a_1 b_1 \\
& - 520732800000 a_1 b_2 + 2322432000 a_1 b_3 + 2278523520000 a_2 b_1 - 3454617600 a_2 b_2 + 14515200 a_2 b_3 \\
& + 15386112000 a_3 b_1 - 14515200 a_3 b_2) T + 40158720000 a_1^2 a_3 + 4470681600 a_1 a_2^2, \quad \mathbf{g}_0 = -178605000 \\
& - 1077298790400 b_1^2 b_2^2 - 2528870400 b_1^2 b_2 b_3 - 221628960000 a_1^2 - 7634662348320000 b_1^2 \\
& + 9015422976000 a_1^3 a_2 + 390736397721600000 a_1^2 b_1^2 + 817545867264000 b_1^3 b_2 - 979722326016000 a_1^2 b_1 b_2 \\
& + 468511174656000 a_1 a_2 b_1^2 + 101154816000 a_1^2 a_2^2 - 21772800 a_2 a_3 - 20829312000 a_1 a_2 \\
& + 25545373056000 b_1 b_2 + (53106278400 b_1^2 b_2^2 - 765858240000 a_1^2 + 65375128848960000 b_1^2 \\
& - 2832334848000 a_1^3 a_2 - 9291828510720000 a_1^2 b_1^2 - 169586049024000 b_1^3 b_2 - 1062125568000 a_1^2 b_1 b_2 \\
& - 12745506816000 a_1 a_2 b_1^2 + 53106278400 a_1^2 a_2^2 - 14515200 a_2 a_3 - 164602368000 a_1 a_2 \\
& - 204085598976000 b_1 b_2 - 177020928000 a_1^3 a_3 + 177020928000 b_1^3 b_3 - 1407974400 b_2 b_3 - 10450944000 a_1 a_3 \\
& + 901103616000 b_1 b_3 - 558835200 a_2^2 + 159369638400 b_2^2 + 53106278400 a_2^2 b_1^2 + 531062784000 a_1 a_3 b_1^2
\end{aligned}$$

$$\begin{aligned}
& -164629463040000 a_1^4 + 104477751705600000 b_1^4 - 531062784000 a_1^2 b_1 b_3 + 8394435000 \\
& + 53106278400 a_1^2 b_2^2 + 3110400 a_3^2 + 3110400 b_3^2) T^2 - 227598336000 a_1^3 a_3 + 2680602624000 b_1^3 b_3 \\
& - 2528870400 a_1^2 b_2 b_3 + 210470400 b_2 b_3 + 5176800 T^{14} - 943488000 a_1 a_3 - 125556480000 b_1 b_3 - 254016000 a_2^2 \\
& - 21388147200 b_2^2 + 505774080 a_1 a_2^3 + 11334059950080000 a_1^4 b_1^2 + 11334059950080000 a_1^2 b_1^4 \\
& + (4494087198720000 a_1^2 b_1 - 478904832000 a_1^2 b_2 + 948326400 a_1^2 b_3 + 728314675200 a_1 a_2 b_1 \\
& - 379330560 a_1 a_2 b_2 - 1137991680 a_2^2 b_1 - 1893147153408000 b_1^3 + 3303020851200 b_1^2 b_2 + 948326400 b_1^2 b_3 \\
& - 1517322240 b_1 b_2^2 + 29073147264000 b_1 - 41309352000 b_2 + 156038400 b_3) T^5 - 571524710400 a_2^2 b_1^2 \\
& + 505774080 a_1 a_2 b_2^2 + 760 T^{18} + 3778019983360000 a_1^6 + 3778019983360000 b_1^6 + 913920 T^{15} b_1 \\
& + (-75264000 b_1 + 141120 b_2) T^{13} + (256838400 a_1^2 + 256838400 b_1^2 + 13154400) T^{12} + (2880460800 b_1 \\
& - 8628480 b_2 + 74880 b_3) T^{11} + (3580684800 a_1^2 + 99348480 a_1 a_2 - 46424716800 b_1^2 + 49674240 b_1 b_2 \\
& + 194518800) T^{10} + (-40567296000 a_1^2 b_1 + 13522432000 b_1^3 + 900736704000 b_1 - 1277438400 b_2 \\
& + 5212800 b_3) T^9 + (35092512000 a_1^2 + 1475712000 a_1 a_2 + 33868800 a_1 a_3 + 5564160 a_2^2 + 13852079136000 b_1^2 \\
& - 20819635200 b_1 b_2 + 38707200 b_1 b_3 + 4596480 b_2^2 - 8061606000) T^8 + (12372498432000 a_1^2 b_1 \\
& - 11379916800 a_1^2 b_2 + 15263840256000 b_1^3 - 11379916800 b_1^2 b_2 + 2339668800000 b_1 - 4311014400 b_2 \\
& + 21945600 b_3) T^7 + (-5782683648000 a_1^4 + 5782683648000 b_1^4 + 594228096000 a_1^2 + 12599193600 a_1 a_2 \\
& + 406425600 a_1 a_3 - 88058880 a_2^2 + 4515840 a_2 a_3 - 841622576256000 b_1^2 + 1985930956800 b_1 b_2 \\
& - 4560998400 b_1 b_3 - 1144765440 b_2^2 + 4515840 b_2 b_3 - 7382340000) T^6 - 2528870400 a_2 a_3 b_1^2 \\
& + 505774080 a_2^2 b_1 b_2 - 981201715200 a_1 a_2 b_1 b_2 - 227598336000 a_1 a_3 b_1^2 + (-4858056000 b_2 + 27216000 b_3 \\
& + 657248256000 a_1 a_3 b_1 + 3597967296000000 a_1^2 b_1 - 222072217920000000 b_1^3 - 839946240 a_2^2 b_2 \\
& + 40016170844160000 b_1^5 - 6580707840000 a_1^2 b_2 + 905023784448000 b_1^2 b_2 + 36094464000 a_1^2 b_3 \\
& + 521444044800 a_2^2 b_1 + 1472905728000 b_1^2 b_3 - 39784863498240000 a_1^4 b_1 + 34696101888000 a_1^4 b_2 \\
& + 231307345920000 a_1^2 b_1^3 - 34696101888000 b_1^4 b_2 + 3870720 a_2^2 b_3 + 19353600 a_3^2 b_1 + 19353600 b_1 b_3^2 \\
& - 3870720 b_2^2 b_3 - 7741440 a_2 a_3 b_2 - 69392203776000 a_1^3 a_2 b_1 - 69392203776000 a_1 a_2 b_1^3 \\
& - 141707059200 a_1 a_2 b_2 + 83674902528000 a_1 a_2 b_1 - 1450262016000 b_1 b_2^2 + 894136320 b_2^3 - 735436800 b_1 b_2 b_3 \\
& + 7934976000 a_2 a_3 b_1 - 696729600 a_1 a_3 b_2 + 696729600 a_1 a_2 b_3 + 2655737280000 b_1) T + 404113489920000 a_1^4 \\
& - 258487349944320000 b_1^4 + 2680602624000 a_1^2 b_1 b_3 + 505774080 b_1 b_2^3 + (-41304883200000 a_1^4 \\
& - 619573248000 a_1^3 a_2 + 2137527705600000 a_1^2 b_1^2 - 1858719744000 a_1^2 b_1 b_2 + 1858719744000 a_1 a_2 b_1^2 \\
& - 671204352000000 b_1^4 + 619573248000 b_1^3 b_2 - 6123690720000 a_1^2 - 49448448000 a_1 a_2 + 4644864000 a_1 a_3 \\
& + 364089600 a_2^2 - 21772800 a_2 a_3 + 864000 a_3^2 + 22894956352800000 b_1^2 - 66642960384000 b_1 b_2 \\
& + 278933760000 b_1 b_3 + 48627129600 b_2^2 - 408844800 b_2 b_3 + 864000 b_3^2 + 58344300000) T^4 \\
& - 2528870400 a_1^2 a_2 a_3 + (-4334696110080000 a_1^2 b_1 + 6678475776000 a_1^2 b_2 - 30256128000 a_1^2 b_3 \\
& + 79439044608000 a_1 a_2 b_1 - 123372748800 a_1 a_2 b_2 + 541900800 a_1 a_2 b_3 + 693633024000 a_1 a_3 b_1 \\
& - 541900800 a_1 a_3 b_2 + 27998208000 a_2^2 b_1 - 18063360 a_2^2 b_2 - 361267200 a_2 a_3 b_1 + 101424127150080000 b_1^3 \\
& - 239087987712000 b_1^2 b_2 + 455648256000 b_1^2 b_3 + 148480819200 b_1 b_2^2 - 361267200 b_1 b_2 b_3 - 18063360 b_2^3 \\
& - 88659204480000 b_1 + 143392032000 b_2 - 662256000 b_3) T^3 + 132600 T^{16} + 576582451200 a_1^2 b_2^2 - 518400 a_3^2 \\
& - 518400 b_3^2, \\
\mathbf{h}_{20} = 0, \quad \mathbf{h}_{19} = 0, \quad \mathbf{h}_{18} = T, \quad \mathbf{h}_{17} = 0, \quad \mathbf{h}_{16} = 9 T^3 - 27 T - 252 b_1, \quad \mathbf{h}_{15} = 672 T a_1, \quad \mathbf{h}_{14} = 36 T^5 \\
& - 600 T^3 - 5040 T^2 b_1 - 540 T - 95760 b_1 + 84 b_2, \quad \mathbf{h}_{13} = (3360 a_1 + 168 a_2) T, \quad \mathbf{h}_{12} = 3360 T^7 - 117600 T^5 \\
& - 94080 T^4 b_1 + 50400 T^3 + (-28761600 b_1 + 30240 b_2) T^2 + (19756800 a_1^2 + 19756800 b_1^2 - 554400) T \\
& - 530544000 b_1 + 792960 b_2 - 3360 b_3, \quad \mathbf{h}_{11} = -564480 T^5 a_1 + (3763200 a_1 + 94080 a_2) T^3 + 398899200 a_1 b_1 \\
& - 376320 a_1 b_2 - 2634240 a_2 b_1 + (5241600 a_1 - 161280 a_2 - 17280 a_3) T, \quad \mathbf{h}_{10} = 5040 T^9 - 262080 T^7 \\
& - 1975680 T^6 b_1 + 1038240 T^5 + (29366400 b_1 + 3360 b_2) T^4 + (118540800 a_1^2 + 118540800 b_1^2 - 2116800) T^3 \\
& - 3319142400 a_1^2 b_1 + 1106380800 b_1^3 + (2572214400 b_1 - 4193280 b_2 + 20160 b_3) T^2 + (-641625600 a_1^2 \\
& - 14300160 a_1 a_2 - 5834841600 b_1^2 + 5268480 b_1 b_2 - 7938000) T - 3345148800 b_1 + 4929120 b_2 - 20160 b_3, \\
\mathbf{h}_9 = -1881600 T^7 a_1 + (18412800 a_1 + 262080 a_2) T^5 + (197568000 a_1 - 48000 a_3) T^3 + (-39927552000 a_1 b_1 \\
& + 39513600 a_1 b_2 + 5644800 a_2 b_1) T^2 - 274471680000 a_1 b_1 + 423628800 a_1 b_2 - 1881600 a_1 b_3 - 144076800 a_2 b_1 \\
& + 107520 a_2 b_2 + 1344000 a_3 b_1 + (-8605184000 a_1^3 + 25815552000 a_1 b_1^2 + 98784000 a_1 - 4939200 a_2 \\
& + 144000 a_3) T, \quad \mathbf{h}_8 = 5040 T^{11} - 310800 T^9 - 2116800 T^8 b_1 + 2872800 T^7 + (318931200 b_1 - 211680 b_2) T^6 \\
& + (296352000 a_1^2 + 296352000 b_1^2 - 21319200) T^5 + (23716224000 b_1 - 36086400 b_2 + 151200 b_3) T^4 \\
& + (6914880000 a_1^2 + 16623936000 b_1^2 - 22579200 b_1 b_2 - 24570000) T^3 + 865347840000 a_1^2 b_1 - 790272000 a_1^2 b_2 \\
& + 916978944000 b_1^3 - 790272000 b_1^2 b_2 + (82978560000 a_1^2 b_1 - 27659520000 b_1^3 - 50416128000 b_1 \\
& + 74541600 b_2 - 302400 b_3) T^2 + (-3348576000 a_1^2 + 75801600 a_1 a_2 - 4838400 a_1 a_3 + 1048320 a_2^2 \\
& + 211250592000 b_1^2 - 272563200 b_1 b_2 + 80640 b_2^2 + 119070000) T - 67908456000 b_1 + 104932800 b_2 \\
& - 453600 b_3, \quad \mathbf{h}_7 = -2822400 T^9 a_1 + (38707200 a_1 + 241920 a_2) T^7 + (-1036224000 a_1 - 3548160 a_2 \\
& + 103680 a_3) T^5 + (-141120000000 a_1 b_1 + 101606400 a_1 b_2 - 11289600 a_2 b_1) T^4 - 1239146496000 a_1^3 b_1 \\
& - 1239146496000 a_1 b_1^3 + (44255232000 a_1^3 - 132765696000 a_1 b_1^2 + 2935296000 a_1 - 31046400 a_2 \\
& + 1497600 a_3) T^3 + (320044032000 a_1 b_1 - 280627200 a_1 b_2 + 3809433600 a_2 b_1 - 3870720 a_2 b_2 \\
& + 19353600 a_3 b_1) T^2 + 739146240000 a_1 b_1 - 995097600 a_1 b_2 + 3225600 a_1 b_3 + 21513139200 a_2 b_1 \\
& - 34513920 a_2 b_2 + 161280 a_2 b_3 + 174182400 a_3 b_1 - 161280 a_3 b_2 + (-183343104000 a_1^3 - 2528870400 a_1^2 a_2 \\
& - 183343104000 a_1 b_1^2 - 2528870400 a_2 b_1^2 - 3005856000 a_1 - 16934400 a_2 - 1555200 a_3) T, \\
\mathbf{h}_6 = 1630994400 b_2 - 7862400 b_3 - 28056236544000 a_1^2 b_1 + 2872800 T^9 + 139892368896000 b_1^3 - 194880 T^{11} \\
& + 39987763200 a_1^2 b_2 - 251464550400 b_1^2 b_2 - 1034880 T^{10} b_1 + (540892800 b_1 - 393120 b_2) T^8 + (395136000 a_1^2 \\
& + 395136000 b_1^2 + 11894400) T^7 + (26002771200 b_1 - 33868800 b_2 + 120960 b_3) T^6 + (-8613964800 a_1^2 \\
& + 94832640 a_1 a_2 + 112139596800 b_1^2 - 94832640 b_1 b_2 - 291060000) T^5 + (-77446656000 a_1^2 b_1
\end{aligned}$$

$$\begin{aligned}
& +25815552000 b_1^3 - 811849248000 b_1 + 1204459200 b_2 - 5040000 b_3)T^4 + 3360 T^{13} + 63221760 a_1 a_2 b_2 \\
& - 73337241600 a_1 a_2 b_1 + (-15523200000 a_1^2 - 406425600 a_1 a_2 - 2257920 a_2^2 + 1508121216000 b_1^2 \\
& - 4560998400 b_1 b_2 - 45158400 b_1 b_3 + 6773760 b_2^2 + 1746360000)T^3 - 158054400 a_1^2 b_3 + 63221760 a_2^2 b_1 \\
& - 158054400 b_1^2 b_3 + 126443520 b_1 b_2^2 + (154893312000 a_1^2 b_1 + 980990976000 b_1^3 + 199931760000 b_1 \\
& - 330220800 b_2 + 1814400 b_3)T^2 + (-2891341824000 a_1^4 + 2891341824000 b_1^4 - 82922112000 a_1^2 \\
& - 1061222400 a_1 a_2 - 6451200 a_1 a_3 + 9031680 a_2^2 - 322560 a_2 a_3 + 61878466944000 b_1^2 - 143626291200 b_1 b_2 \\
& + 387072000 b_1 b_3 + 76769280 b_2^2 - 322560 b_2 b_3 + 587412000)T - 986195952000 b_1, \quad \mathbf{h}_5 = -2257920 T^{11} a_1 \\
& + (41260800 a_1 - 6720 a_2)T^9 + (16128000 a_1 - 16128000 a_2 + 241920 a_3)T^7 + (-39513600000 a_1 b_1 \\
& + 15805440 a_1 b_2 - 142248960 a_2 b_1)T^6 + (30978662400 a_1^3 - 92935987200 a_1 b_1^2 + 15325632000 a_1 \\
& - 16087680 a_2 - 5080320 a_3)T^5 + (-5090480640000 a_1 b_1 + 8015616000 a_1 b_2 - 33868800 a_1 b_3 \\
& - 16098969600 a_2 b_1 + 13547520 a_2 b_2 - 33868800 a_3 b_1)T^4 + 95414280192000 a_1^3 b_1 - 82609766400 a_1^3 b_2 \\
& + 247829299200 a_1^2 a_2 b_1 - 284590645248000 a_1 b_1^3 + 247829299200 a_1 b_1^2 b_2 - 82609766400 a_2 b_1^3 \\
& + (545814528000 a_1^3 - 8851046400 a_1^2 a_2 + 2611058688000 a_1 b_1^2 - 8851046400 a_2 b_1^2 + 16087680000 a_1 \\
& + 3894912000 a_2 - 10886400 a_3)T^3 + (8674025472000 a_1^3 b_1 + 8674025472000 a_1 b_1^3 + 13051003392000 a_1 b_1 \\
& - 20829312000 a_1 b_2 + 96768000 a_1 b_3 + 206362598400 a_2 b_1 - 318366720 a_2 b_2 + 1451520 a_2 b_3 + 1538611200 a_3 b_1 \\
& - 1451520 a_3 b_2)T^2 + 11627159040000 a_1 b_1 - 17611776000 a_1 b_2 + 72576000 a_1 b_3 - 188920166400 a_2 b_1 \\
& + 304819200 a_2 b_2 - 1451520 a_2 b_3 - 1640217600 a_3 b_1 + 1451520 a_3 b_2 + (300303360000 a_1^3 \\
& + 43306905600 a_1^2 a_2 - 948326400 a_1^2 a_3 + 252887040 a_1 a_2^2 - 210544266240000 a_1 b_1^2 + 328753152000 a_1 b_1 b_2 \\
& - 126443520 a_1 b_2^2 - 430856294400 a_2 b_1^2 + 379330560 a_2 b_1 b_2 - 948326400 a_3 b_1^2 - 45849888000 a_1 \\
& - 692193600 a_2 - 32659200 a_3)T, \quad \mathbf{h}_4 = 2286144000 b_2 - 11340000 b_3 - 236391805440000 a_1^2 b_1 \\
& + 6232571573760000 b_1^3 + 1440 T^{15} - 4515840 b_2^3 + (353740800 b_1 - 258720 b_2)T^{10} + (296352000 a_1^2 \\
& + 296352000 b_1^2 - 23436000)T^9 + (-19177200000 b_1 + 28828800 b_2 - 108000 b_3)T^8 + (-26361216000 a_1^2 \\
& + 45158400 a_1 a_2 + 20151936000 b_1^2 - 1313172000)T^7 + (-232339968000 a_1^2 b_1 + 77446656000 b_1^3 \\
& - 136745280000 b_1 - 2116800 b_2 + 1008000 b_3)T^6 + (27969984000 a_1^2 + 564480000 a_1 a_2 - 67737600 a_1 a_3 \\
& + 10160640 a_2^2 - 7053318720000 b_1^2 + 9867110400 b_1 b_2 - 3386880 b_2^2 + 4302396000)T^5 + 554400 T^{11} \\
& + 349864704000 a_1^2 b_2 - 16766975232000 b_1^2 b_2 + 45158400 a_1 a_2 b_3 + 54190080000 a_1 a_3 b_1 \\
& + (-11340403200000 a_1^2 b_1 + 11063808000 a_1^2 b_2 - 12889336320000 b_1^3 + 11063808000 b_1^2 b_2 \\
& + 5475103200000 b_1 - 8149680000 b_2 + 34776000 b_3)T^4 + (20603520000 a_1^2 - 4064256000 a_1 a_2 \\
& + 225792000 a_1 a_3 + 56448000 a_2^2 + 93843953280000 b_1^2 - 139313664000 b_1 b_2 + 451584000 b_1 b_3 \\
& + 11289600 b_2^2 + 3095820000)T^3 - 45158400 a_1 a_3 b_2 - 4515840 a_2^2 b_2 + (347695971840000 a_1^2 b_1 \\
& - 533433600000 a_1^2 b_2 + 2370816000 a_1^2 b_3 + 998903808000 a_1 a_2 b_1 - 948326400 a_1 a_2 b_2 + 948326400 a_2^2 b_1 \\
& + 446922524160000 b_1^3 - 625105152000 b_1^2 b_2 + 2370816000 b_1^2 b_3 + 5749652160000 b_1 - 9414468000 b_2 \\
& + 45360000 b_3)T^2 + (4130488320000 a_1^4 + 206524416000 a_1^3 a_2 - 712509235200000 a_1^2 b_1^2 \\
& + 619573248000 a_1^2 b_1 b_2 - 619573248000 a_1 a_2 b_1^2 + 233372590080000 b_1^4 - 206524416000 b_1^3 b_2 \\
& - 1508220000000 a_1^2 - 176117760000 a_1 a_2 + 967680000 a_1 a_3 - 143942400 a_2^2 - 2419200 a_2 a_3 + 172800 a_3^2 \\
& + 4042609391520000 b_1^2 - 12222572544000 b_1 b_2 + 52786944000 b_1 b_3 + 9237715200 b_2^2 - 79833600 b_2 b_3 \\
& + 172800 b_3^2 + 2143260000)T - 50400 T^{13} - 9980006400 a_1 a_2 b_2 + 6317660160000 a_1 a_2 b_1 - 1467648000 a_1^2 b_3 \\
& + 4470681600 a_2^2 b_1 + 790272000 b_1^2 b_3 + 14992588800 b_1 b_2^2 - 134977520000 b_1, \quad \mathbf{h}_3 = -88510464000 a_1 b_1^2 b_3 \\
& - 100568321280000 a_1 b_1 + 167565888000 a_1 b_2 - 1000399680000 a_2 b_1 + 29503488000 a_3 b_1^3 \\
& + 619573248000 a_1^2 a_2 b_1 + 19206770688000 a_1 b_1^2 b_2 + (115199078400000 a_1^3 - 5870592000 a_1^2 a_2 \\
& - 5870592000 a_1^2 a_3 - 2348236800 a_1 a_2^2 + 180633600 a_1 a_2 a_3 - 34790433269760000 a_1 b_1^2 \\
& + 81006944256000 a_1 b_1 b_2 - 216760320000 a_1 b_1 b_3 - 43352064000 a_1 b_2^2 + 180633600 a_1 b_2 b_3 - 18063360 a_2^3 \\
& - 24250512384000 a_2 b_1^2 + 42087628800 a_2 b_1 b_2 - 18063360 a_2 b_2^2 + 3161088000 a_3 b_1^2 - 118752480000 a_1 \\
& + 508032000 a_2 + 63504000 a_3)T - 822528000 a_1 b_3 + 1591833600 a_2 b_2 - 7354368000 a_3 b_1 \\
& + (-28913418240000 a_1^3 b_1 - 28913418240000 a_1 b_1^3 - 336066554880000 a_1 b_1 + 507693312000 a_1 b_2 \\
& - 2128896000 a_1 b_3 - 1189133568000 a_2 b_1 + 1557964800 a_2 b_2 - 4838400 a_2 b_3 - 5806080000 a_3 b_1 \\
& + 4838400 a_3 b_2)T^2 + (1106380800000 a_1^3 - 22127616000 a_1^2 a_2 + 3161088000 a_1^2 a_3 + 1619456993280000 a_1 b_1^2 \\
& - 2882912256000 a_1 b_1 b_2 + 1264435200 a_1 b_2^2 + 1406684160000 a_2 b_1^2 - 1264435200 a_2 b_1 b_2 + 3161088000 a_3 b_1^2 \\
& + 236234880000 a_1 + 698544000 a_2 - 84672000 a_3)T^3 + (7022582784000 a_1 b_1 - 10454169600 a_1 b_2 \\
& + 45158400 a_1 b_3 + 11312179200 a_2 b_1 - 9031680 a_2 b_2)T^6 + (1961981952000 a_1^3 + 2788079616000 a_1 b_1^2 \\
& - 8255520000 a_1 + 1439424000 a_2 + 22982400 a_3)T^5 + 4971042693120000 a_1^3 b_1 - 12063091138560000 a_1 b_1^3 \\
& - 7257600 a_2 b_3 + 7257600 a_3 b_2 + 29503488000 a_1^3 b_3 - 7228354560000 a_1^3 b_2 + 619573248000 a_2 b_1^3 \\
& - 940800 T^{13} a_1 + (22041600 a_1 - 120960 a_2)T^{11} + (3388896000 a_1 - 10483200 a_2 + 67200 a_3)T^9 \\
& + (62205696000 a_1 b_1 - 16934400 a_1 b_2 - 118540800 a_2 b_1)T^8 + (-73758720000 a_1^3 + 221276160000 a_1 b_1^2 \\
& + 12950784000 a_1 + 227001600 a_2 - 806400 a_3)T^7 - 88510464000 a_1^2 a_3 b_1 + (14456709120000 a_1^3 b_1 \\
& + 14456709120000 a_1 b_1^3 - 20968174080000 a_1 b_1 + 33812352000 a_1 b_2 - 177408000 a_1 b_3 + 453108096000 a_2 b_1 \\
& - 620928000 a_2 b_2 + 2419200 a_2 b_3 + 3354624000 a_3 b_1 - 2419200 a_3 b_2)T^4, \quad \mathbf{h}_2 = 360 T^{17} + 7763364000 b_2 \\
& - 40824000 b_3 + 6129644666880000 b_1^5 + 96768000 b_1 b_2 b_3 - 4238438400 a_2 a_3 b_1 + 982694764800000 a_1^2 b_1 \\
& + 149713838265600000 b_1^3 + 4800 T^{15} - 447068160 b_2^3 - 1498694400000 a_1^2 b_2 - 551640450816000 b_1^2 b_2 \\
& - 77414400 a_1 a_2 b_3 - 84768768000 a_1 a_3 b_1 + 201600 T^{14} b_1 + (67939200 b_1 - 43680 b_2)T^{12} + (118540800 a_1^2 \\
& + 118540800 b_1^2 - 160574400)T^{11} + (-26239651200 b_1 + 30320640 b_2 - 95040 b_3)T^{10} + (9285696000 a_1^2 \\
& - 26342400 a_1 a_2 - 94954944000 b_1^2 + 71500800 b_1 b_2 - 1231146000)T^9 + (116169984000 a_1^2 b_1 \\
& - 38723328000 b_1^3 - 138617136000 b_1 + 139557600 b_2 - 561600 b_3)T^8 + (-2104784640000 a_1^2 + 329011200 a_1 a_2 \\
& + 322560 a_2^2 + 820552320000 b_1^2 - 4057804800 b_1 b_2 - 19353600 b_1 b_3 + 4193280 b_2^2 + 8341704000)T^7 \\
& + (-11085935616000 a_1^2 b_1 + 8851046400 a_1^2 b_2 - 13151179776000 b_1^3 + 8851046400 b_1^2 b_2 + 5947848144000 b_1
\end{aligned}$$

$$\begin{aligned}
& -7341062400 b_2 + 23587200 b_3)T^6 + (8674025472000 a_1^4 - 8674025472000 b_1^4 - 1764733824000 a_1^2 \\
& -10634803200 a_1 a_2 + 19353600 a_1 a_3 - 40642560 a_2^2 + 967680 a_2 a_3 - 124736927616000 b_1^2 \\
& + 359889868800 b_1 b_2 - 1432166400 b_1 b_3 - 189665280 b_2^2 + 967680 b_2 b_3 - 17860500000)T^5 \\
& -7170527723520000 a_1^4 b_1 + 77414400 a_1 a_3 b_2 + 419973120 a_2^2 b_2 - 11565367296000 a_1^3 a_2 b_1 \\
& -11565367296000 a_1 a_2 b_1^3 - 1040883056640000 a_1^2 b_1^3 - 5782683648000 b_1^4 b_2 + 5782683648000 a_1^4 b_2 \\
& + (-493485350400000 a_1^2 b_1 + 644071680000 a_1^2 b_2 - 2370816000 a_1^2 b_3 - 1327656960000 a_1 a_2 b_1 \\
& + 948326400 a_1 a_2 b_2 - 316108800 a_2^2 b_1 + 652440660480000 b_1^3 - 1047110400000 b_1^2 b_2 - 2370816000 b_1^2 b_3 \\
& + 632217600 b_1 b_2^2 + 52368256080000 b_1 - 81078732000 b_2 + 353808000 b_3)T^4 + (-20652441600000 a_1^4 \\
& + 413048832000 a_1^3 a_2 - 1425018470400000 a_1^2 b_1^2 + 1239146496000 a_1^2 b_1 b_2 - 1239146496000 a_1 a_2 b_1^2 \\
& + 495658598400000 b_1^4 - 413048832000 b_1^3 b_2 + 688806720000 a_1^2 + 46061568000 a_1 a_2 - 677376000 a_1 a_3 \\
& + 321753600 a_2^2 - 33868800 a_2 a_3 + 345600 a_3^2 + 12876021506880000 b_1^2 - 35919894528000 b_1 b_2 \\
& + 141087744000 b_1 b_3 + 24639552000 b_2^2 - 188697600 b_2 b_3 + 345600 b_3^2 + 17622360000)T^3 - 1935360 a_2^2 b_3 \\
& + 3870720 a_2 a_3 b_2 - 9676800 a_3^2 b_1 - 9676800 b_1 b_3^2 + 1935360 b_2^2 b_3 + (1240729297920000 a_1^2 b_1 \\
& -1587091968000 a_1^2 b_2 + 4741632000 a_1^2 b_3 + 1278885888000 a_1 a_2 b_1 - 812851200 a_1 a_2 b_2 - 5419008000 a_1 a_3 b_1 \\
& -24927436800 a_2^2 b_1 + 27095040 a_2^2 b_2 - 270950400 a_2 a_3 b_1 + 10996623590400000 b_1^3 - 13899078144000 b_1^2 b_2 \\
& + 310915584000 b_1^2 b_3 - 27365990400 b_1 b_2^2 - 270950400 b_1 b_2 b_3 + 27095040 b_2^3 - 15384796560000 b_1^4 \\
& + 25528608000 b_2 - 122472000 b_3)T^2 + (-147812474880000 a_1^4 - 531062784000 a_1^3 a_2 + 17702092800 a_1^2 a_2^2 \\
& + 23726114979840000 a_1^2 b_1^2 - 41245876224000 a_1^2 b_1 b_2 + 17702092800 a_1^2 b_2^2 - 531062784000 a_1 a_2 b_1^2 \\
& + 17702092800 a_2^2 b_1^2 + 238739274547200000 b_1^4 - 41245876224000 b_1^3 b_2 + 17702092800 b_1^2 b_2^2 \\
& -1451701440000 a_1^2 - 26925696000 a_1 a_2 - 50803200 a_2^2 - 1036800 a_3^2 - 21638402493120000 b_1^2 \\
& + 67133888640000 b_1 b_2 - 299303424000 b_1 b_3 - 52073280000 b_2^2 + 464486400 b_2 b_3 - 1036800 b_3^2 \\
& -4822335000)T - 756000 T^{13} + 13953945600 a_1 a_2 b_2 - 7697700864000 a_1 a_2 b_1 + 6338304000 a_1^2 b_3 \\
& -264515328000 a_2^2 b_1 - 468211968000 b_1^2 b_3 + 785823897600 b_1 b_2^2 - 4415115600000 b_1, \\
\mathbf{h}_1 = & (4222310400000 a_1^3 - 57351168000 a_1^2 a_2 - 22127616000 a_1^2 a_3 - 1445068800 a_1 a_2^2 \\
& -9527072163840000 a_1 b_1^2 + 13952139264000 a_1 b_1 b_2 - 46964736000 a_1 b_1 b_3 - 903168000 a_1 b_2^2 + 18063360 a_2^3 \\
& + 49172530176000 a_2^2 b_1 - 82549555200 a_2 b_1 b_2 + 180633600 a_2 b_1 b_3 + 18063360 a_2 b_2^2 + 232565760000 a_3 b_1^2 \\
& -180633600 a_3 b_1 b_2 - 341651520000 a_1 + 254016000 a_2 + 99792000 a_3)T^3 - 139087872000 a_1 b_1^2 b_3 \\
& + 171557326080000 a_1 b_1 - 288562176000 a_1 b_2 + 1019112192000 a_2 b_1 - 2769113088000 a_3 b_1^3 \\
& + (12335703367680000 a_1^3 b_1 - 19206770688000 a_1^3 b_2 + 88510464000 a_1^3 b_3 + 4337012736000 a_1^2 a_2 b_1 \\
& -265531392000 a_1^2 a_3 b_1 - 50166845890560000 a_1 b_1^3 + 70011777024000 a_1 b_1^2 b_2 - 265531392000 a_1 b_1^2 b_3 \\
& -5576159232000 a_2 b_1^3 + 88510464000 a_3 b_1^3 - 871168193280000 a_1 b_1 + 1410042816000 a_1 b_2 - 6676992000 a_1 b_3 \\
& -11360357568000 a_2 b_1 + 17781120000 a_2 b_2 - 79833600 a_2 b_3 - 84623616000 a_3 b_1 + 79833600 a_3 b_2)T^2 \\
& -430527541248000 a_1^2 a_2 b_1 + 1911383470080000 a_1 b_1^2 b_2 + 1415232000 a_1 b_3 + 1625702400 a_2 b_2 \\
& + 6931008000 a_3 b_1 + (9289728000 a_3 b_1 b_2 - 9289728000 a_2 b_1 b_3 + 23130734592000 a_1^3 b_1 b_2 \\
& + 23130734592000 a_1 b_1^3 b_2 + 7741440 a_2 b_2 b_3 - 3524472000000 a_1 - 5620104000 a_2 + 11565367296000 a_1^4 a_2 \\
& -26137730088960000 a_1^3 b_1^2 - 26369037434880000 a_1 b_1^4 - 11565367296000 a_2 b_1^4 - 19353600 a_1 a_3^2 \\
& -19353600 a_1 b_3^2 + 3870720 a_2^2 a_3 - 3870720 a_3 b_2^2 - 190512000 a_3 - 81285120 a_2^3 + 21192192000 a_1^2 a_3 \\
& -17205350400 a_1 a_2^2 - 1070931456000 a_1 b_2^2 - 5521678848000 a_3 b_1^2 - 481453367754240000 a_1 b_1^2 \\
& -71466554880000 a_1^3 + 425779200 a_1 a_2 a_3 - 6097932288000 a_1 b_1 b_3 + 9096192000 a_1 b_2 b_3 - 1815367680 a_2 b_2^2 \\
& -1619606016000 a_1^2 a_2 - 1434055795200000 a_2 b_1^2 + 1436378517504000 a_1 b_1 b_2 + 3372519628800 a_2 b_1 b_2 \\
& + 231307345920000 a_1^5)T + 622102118400 a_1^2 a_2 b_2 - 2528870400 a_1^2 a_2 b_3 - 161280 T^{15} a_1 + (4704000 a_1 \\
& -47040 a_2)T^{13} + (-23385600 a_1 + 1935360 a_2 - 40320 a_3)T^{11} + (-39536179200 a_1 b_1 + 28976640 a_1 b_2 \\
& + 4139520 a_2 b_1)T^{10} + (11063808000 a_1^3 - 33191424000 a_1 b_1^2 + 5106528000 a_1 + 124084800 a_2 \\
& -1008000 a_3)T^9 + (2153071872000 a_1 b_1 - 3209472000 a_1 b_2 + 12096000 a_1 b_3 + 261273600 a_2 b_1 + 967680 a_2 b_2 \\
& -16934400 a_3 b_1)T^8 + (-891426816000 a_1^3 + 6322176000 a_1^2 a_2 - 2130573312000 a_1 b_1^2 + 6322176000 a_2 b_1^2 \\
& + 147220416000 a_1 + 7257600 a_2 + 35078400 a_3)T^7 + (-8674025472000 a_1^3 b_1 - 8674025472000 a_1 b_1^3 \\
& + 17915917824000 a_1 b_1 - 4233600000 a_1 b_2 - 90316800 a_1 b_3 + 130022323200 a_2 b_1 - 223534080 a_2 b_2 \\
& + 1128960 a_2 b_3 + 1241856000 a_3 b_1 - 1128960 a_3 b_2)T^6 - 21835405025280000 a_1^3 b_1 - 717477074657280000 a_1 b_1^3 \\
& + 7257600 a_2 b_3 - 7257600 a_3 b_2 - 139087872000 a_1^3 b_3 - 1689285427200 a_1 b_1 b_2^2 + (-2152700928000 a_1^3 \\
& -131185152000 a_1^2 a_2 - 948326400 a_1^2 a_3 + 758661120 a_1 a_2^2 + 783124780032000 a_1 b_1^2 - 1097529753600 a_1 b_1 b_2 \\
& + 379330560 a_1 b_2^2 - 342345830400 a_2 b_1^2 + 379330560 a_2 b_1 b_2 - 948326400 a_3 b_1^2 - 539402976000 a_1 \\
& -2876731200 a_2 + 388281600 a_3)T^5 + (-448157982720000 a_1^3 b_1 + 413048832000 a_1^3 b_2 \\
& -1239146496000 a_1^2 a_2 b_1 + 1451866644480000 a_1 b_1^3 - 1239146496000 a_1 b_1^2 b_2 + 413048832000 a_2 b_1^3 \\
& -572433523200000 a_1 b_1 + 851122944000 a_1 b_2 - 3652992000 a_1 b_3 + 2038901760000 a_2 b_1 - 3082060800 a_2 b_2 \\
& + 12096000 a_2 b_3 + 19426176000 a_3 b_1 - 12096000 a_3 b_2)T^4 + 32685649920000 a_1^3 b_2 - 971452919808000 a_2 b_1^3 \\
& + 505774080 a_1 b_2^3 - 505774080 a_2^3 b_1 + 1674112204800 a_2 b_1^2 b_2 - 2769113088000 a_1^2 a_3 b_1 - 2528870400 a_2 b_1^2 b_3 \\
& -505774080 a_2 b_1 b_2^2 + 2528870400 a_3 b_1^2 b_2 + 2528870400 a_1^2 a_3 b_2 - 637275340800 a_1 a_2^2 b_1 + 505774080 a_1 a_2^2 b_2, \\
\mathbf{h}_0 = & 1652195328000 a_1^3 a_3 b_1 + 330439065600 a_1^3 a_2 b_2 - 991317196800 a_1 a_2 b_1^2 b_2 + 1652195328000 a_1 a_3 b_1^3 \\
& + 826097664000 b_1^4 b_3 + 3720 T^{17} + 1238328000 b_2 - 6804000 b_3 - 91729884610560000 b_1^5 \\
& -1140014776320000 a_1^2 b_1^2 b_2 + 96768000 b_1 b_2 b_3 - 4238438400 a_2 a_3 b_1 - 4599032497920000 a_1^2 b_1 \\
& + 102980082965760000 b_1^3 - 495658598400 a_1^2 a_2^2 b_1 - 344160 T^{15} - 406425600 b_2^3 + (-2679075000 \\
& + 505774080 b_1 b_2^3 + 126443520000 a_1^3 a_3 + 5206692960000 a_1^2 - 45510243841440000 b_1^2 - 5057740800 a_1^2 a_2^2 \\
& + 611986636800 a_1^2 b_2^2 - 536120524800 a_2^2 b_1^2 - 1183511347200 b_1^2 b_2^2 + 97034112000 a_1 a_2 \\
& + 147068659584000 b_1 b_2 + 505774080 a_1 a_2 b_2^2 - 2528870400 a_1^2 a_2 a_3 - 2528870400 a_1^2 b_2 b_3
\end{aligned}$$

$$\begin{aligned}
& -1149662416896000 a_1^2 b_1 b_2 + 785024593920000 a_1 a_2 b_1^2 - 2592000 a_3^2 - 2592000 b_3^2 - 189665280000 a_1^3 a_2 \\
& + 498616491663360000 a_1^2 b_1^2 + 973324283904000 b_1^3 b_2 + 3034644480000 b_1^3 b_3 - 2528870400 b_1^2 b_2 b_3 \\
& - 1264435200000 a_1 a_2 b_1 b_2 + 2540160000 a_1 a_3 + 505774080 a_1 a_2^3 - 152409600 a_2^2 - 118828684800 b_2^2 \\
& + 11334059950080000 a_1^4 b_1^2 + 11334059950080000 a_1^2 b_1^4 + 3034644480000 a_1^2 b_1 b_3 + 126443520000 a_1 a_3 b_1^2 \\
& + 505774080 a_2^2 b_1 b_2 - 2528870400 a_2 a_3 b_1^2 - 687294720000 b_1 b_3 - 50803200 a_2 a_3 + 1110412800 b_2 b_3 \\
& + 89016238080000 a_1^4 - 337580300574720000 b_1^4 + 3778019983360000 b_1^6 + 3778019983360000 a_1^6 T \\
& + 495658598400 a_1^2 b_1 b_2^2 + 7467393024000 a_1^2 b_2 - 417480376320000 b_1^2 b_2 - 483840000 a_1 a_2 b_3 \\
& - 496613376000 a_1 a_3 b_1 - 126905123143680000 a_1^4 b_1 + 483840000 a_1 a_3 b_2 + 460615680 a_2^2 b_2 \\
& - 393222488064000 a_1^3 a_2 b_1 + 1126797213696000 a_1 a_2 b_1^3 + 655376320757760000 a_1^2 b_1^3 \\
& + 188350267392000 b_1^4 b_2 + 191654658048000 a_1^4 b_2 + 40 T^{19} - 1935360 a_2^2 b_3 + 3870720 a_2 a_3 b_2 \\
& - 9676800 a_3^2 b_1 - 9676800 b_1 b_3^2 + 1935360 b_2^2 b_3 + (-61868772000 b_2 + 312984000 b_3 \\
& + 21627236843520000 b_1^5 - 367718400 b_1 b_2 b_3 + 3967488000 a_2 a_3 b_1 + 2384168774400000 a_1^2 b_1 \\
& - 187292132133120000 b_1^3 + 501258240 b_2^3 - 3657491712000 a_1^2 b_2 + 658115829504000 b_1^2 b_2 \\
& - 193536000 a_1 a_2 b_3 - 245790720000 a_1 a_3 b_1 - 18273280327680000 a_1^4 b_1 + 193536000 a_1 a_3 b_2 \\
& - 365783040 a_2^2 b_2 - 34696101888000 a_1^3 a_2 b_1 - 34696101888000 a_1 a_2 b_1^3 + 3353956515840000 a_1^2 b_1^3 \\
& - 17348050944000 b_1^4 b_2 + 17348050944000 a_1^4 b_2 + 1935360 a_2^2 b_3 - 3870720 a_2 a_3 b_2 + 9676800 a_3^2 b_1 \\
& + 9676800 b_1 b_3^2 - 1935360 b_2^2 b_3 + 41319936000 a_1 a_2 b_2 - 25136068608000 a_1 a_2 b_1 + 16692480000 a_1^2 b_3 \\
& + 207073843200 a_2^2 b_1 + 783869184000 b_1^2 b_3 - 912628684800 b_1 b_2^2 + 36182039040000 b_1 T^2 \\
& + 165219532800 a_2^2 b_1^3 + 110412288000 a_1 a_2 b_2 - 70967328768000 a_1 a_2 b_1 - 34981632000 a_1^2 b_3 \\
& - 305699788800 a_2^2 b_1 - 453987072000 b_1^2 b_3 + 656580556800 b_1 b_2^2 - 165219532800 b_1^3 b_2^2 \\
& - 826097664000 a_1^4 b_3 + (-59006976000 a_1^3 a_3 + 5228919360000 a_1^2 + 11637688426560000 b_1^2 \\
& + 17702092800 a_1^2 a_2 + 17702092800 a_1^2 b_2^2 + 17702092800 a_2^2 b_1^2 + 17702092800 b_1^2 b_2^2 - 6096384000 a_1 a_2 \\
& - 33988356864000 b_1 b_2 + 2124251136000 a_1^2 b_1 b_2 - 11683381248000 a_1 a_2 b_1^2 + 345600 a_3^2 + 345600 b_3^2 \\
& - 1770209280000 a_1^3 a_2 - 5468176465920000 a_1^2 b_1^2 - 60659171328000 b_1^3 b_2 + 59006976000 b_1^3 b_3 \\
& - 5419008000 a_1 a_3 - 152409600 a_2^2 + 24436339200 b_2^2 - 177020928000 a_1^2 b_1 b_3 + 177020928000 a_1 a_3 b_1^2 \\
& + 134991360000 b_1 b_3 - 33868800 a_2 a_3 - 188697600 b_2 b_3 + 44255232000000 a_1^4 + 39955983728640000 b_1^4 \\
& + 3750705000 T^3 - 684176220000 b_1 + 57120 T^{16} b_1 + (-9542400 b_1 + 10080 b_2) T^{14} + (19756800 a_1^2 \\
& + 19756800 b_1^2 - 28072800) T^{13} + (494457600 b_1 - 1276800 b_2 + 6240 b_3) T^{12} + (-754521600 a_1^2 \\
& + 9031680 a_1 a_2 - 5195097600 b_1^2 + 4515840 b_1 b_2 - 192402000) T^{11} + (-4056729600 a_1^2 b_1 + 1352243200 b_1^3 \\
& + 95901120000 b_1 - 116938080 b_2 + 308160 b_3) T^{10} + (-17519712000 a_1^2 - 167193600 a_1 a_2 + 3763200 a_1 a_3 \\
& + 618240 a_2^2 + 1757366688000 b_1^2 - 2524032000 b_1 b_2 + 4300800 b_1 b_3 + 510720 b_2^2 - 2171610000) T^9 \\
& + (1668264192000 a_1^2 b_1 - 1422489600 a_1^2 b_2 + 1926419712000 b_1^3 - 1422489600 b_1^2 b_2 - 2016528696000 b_1 \\
& + 2831371200 b_2 - 11685600 b_3) T^8 + (-826097664000 a_1^4 + 826097664000 b_1^4 - 45537408000 a_1^2 \\
& - 2290176000 a_1 a_2 - 32256000 a_1 a_3 - 31288320 a_2^2 + 645120 a_2 a_3 - 162023234688000 b_1^2 + 348945408000 b_1 b_2 \\
& - 748339200 b_1 b_3 - 180956160 b_2^2 + 645120 b_2 b_3 + 24156468000) T^7 + (57214112256000 a_1^2 b_1 \\
& - 62115379200 a_1^2 b_2 + 158054400 a_1^2 b_3 + 139087872000 a_1 a_2 b_1 - 63221760 a_1 a_2 b_2 - 189665280 a_2^2 b_1 \\
& - 359765005824000 b_1^3 + 585907660800 b_1^2 b_2 + 158054400 b_1^2 b_3 - 252887040 b_1 b_2^2 + 6367800096000 b_1 \\
& - 5750287200 b_2 + 5443200 b_3) T^6 + (3304390656000 a_1^4 - 123914649600 a_1^3 a_2 + 427505541120000 a_1^2 b_1^2 \\
& - 371743948800 a_1^2 b_1 b_2 + 371743948800 a_1 a_2 b_1^2 - 145806237696000 b_1^4 + 123914649600 b_1^3 b_2 \\
& - 2214045792000 a_1^2 - 616412160000 a_1 a_2 + 967680000 a_1 a_3 + 248935680 a_2^2 - 15966720 a_2 a_3 + 172800 a_3^2 \\
& + 6878269252512000 b_1^2 - 18574056345600 b_1 b_2 + 68773017600 b_1 b_3 + 12583952640 b_2^2 - 93381120 b_2 b_3 \\
& + 172800 b_3^2 + 51771636000) T^5 + (-2612992596480000 a_1^2 b_1 + 3795450624000 a_1^2 b_2 - 15466752000 a_1^2 b_3 \\
& + 18165417984000 a_1 a_2 b_1 - 30210969600 a_1 a_2 b_2 + 135475200 a_1 a_2 b_3 + 186052608000 a_1 a_3 b_1 \\
& - 135475200 a_1 a_3 b_2 + 10160640000 a_2^2 b_1 - 4515840 a_2^2 b_2 - 90316800 a_2 a_3 b_1 + 31818006904320000 b_1^3 \\
& - 70170395904000 b_1^2 b_2 + 118653696000 b_1^2 b_3 + 40913510400 b_1 b_2^2 - 90316800 b_1 b_2 b_3 - 4515840 b_2^3 \\
& - 23521881600000 b_1 + 36197280000 b_2 - 147420000 b_3) T^4, \\
\mathbf{q}_{20} = 1, \quad \mathbf{q}_{19} = 0, \quad \mathbf{q}_{18} = 10 T^2 + 10, \quad \mathbf{q}_{17} = 1680 a_1, \quad \mathbf{q}_{16} = 45 T^4 - 270 T^2 - 5040 T b_1 + 405, \\
\mathbf{q}_{15} = 6720 T^2 a_1 - 336 a_2, \quad \mathbf{q}_{14} = 120 T^6 - 1800 T^4 - 33600 T^3 b_1 + 1800 T^2 + 705600 a_1^2 + 705600 b_1^2 \\
+ (-1848000 b_1 + 1680 b_2) T + 16200, \quad \mathbf{q}_{13} = (33600 a_1 + 1680 a_2) T^2 + 240 a_3, \quad \mathbf{q}_{12} = 210 T^8 - 4200 T^6 \\
- 94080 T^5 b_1 + 6300 T^4 + (-5577600 b_1 + 5040 b_2) T^3 + (4939200 a_1^2 + 4939200 b_1^2 + 113400) T^2 + 1176000 a_1^2 \\
- 188160 a_1 a_2 + 430180800 b_1^2 - 376320 b_1 b_2 + (-227304000 b_1 + 362880 b_2 - 1680 b_3) T + 425250, \\
\mathbf{q}_{11} = -47040 T^6 a_1 + (-470400 a_1 + 11760 a_2) T^4 + 61465600 a_1^3 - 184396800 a_1 b_1^2 + (-4334400 a_1 \\
- 40320 a_2 - 4320 a_3) T^2 + (146764800 a_1 b_1 - 188160 a_1 b_2 - 1317120 a_2 b_1) T + 604800 a_1 + 136080 a_2 - 4320 a_3, \\
\mathbf{q}_{10} = 252 T^{10} - 3780 T^8 - 141120 T^7 b_1 + 63000 T^6 + (-168000 b_1 + 336 b_2) T^5 + (14817600 a_1^2 + 14817600 b_1^2 \\
+ 718200) T^4 + (360292800 b_1 - 631680 b_2 + 3360 b_3) T^3 + (-160406400 a_1^2 - 3575040 a_1 a_2 - 1221628800 b_1^2 \\
+ 1317120 b_1 b_2 + 3005100) T^2 + 41966400 a_1^2 - 4058880 a_1 a_2 + 215040 a_1 a_3 + 25536 a_2^2 + 66586094400 b_1^2 \\
- 112008960 b_1 b_2 + 188160 b_1 b_3 + 30912 b_2^2 + (-1659571200 a_1^2 b_1 + 553190400 b_1^3 + 4838097600 b_1 \\
- 7212240 b_2 + 30240 b_3) T + 1644300, \quad \mathbf{q}_9 = -117600 T^8 a_1 + (-2856000 a_1 + 21840 a_2) T^6 + (34104000 a_1 \\
+ 470400 a_2 - 6000 a_3) T^4 + (-6039936000 a_1 b_1 + 6585600 a_1 b_2 + 940800 a_2 b_1) T^3 - 570752000 a_1^3 \\
+ 79027200 a_1^2 a_2 + 8034432000 a_1 b_1^2 + 79027200 a_2 b_1^2 + (-2151296000 a_1^3 + 6453888000 a_1 b_1^2 \\
- 116424000 a_1 + 1587600 a_2 + 36000 a_3) T^2 + (-166475904000 a_1 b_1 + 238156800 a_1 b_2 - 940800 a_1 b_3 \\
- 45696000 a_2 b_1 + 53760 a_2 b_2 + 672000 a_3 b_1) T + 206388000 a_1 + 1058400 a_2 - 54000 a_3, \quad \mathbf{q}_8 = 210 T^{12} \\
+ 1260 T^{10} - 117600 T^9 b_1 + 255150 T^8 + (18144000 b_1 - 15120 b_2) T^7 + (24696000 a_1^2 + 24696000 b_1^2 \\
- 567000) T^6 + (2873404800 b_1 - 4112640 b_2 + 15120 b_3) T^5 + (1852200000 a_1^2 + 2077992000 b_1^2 - 2822400 b_1 b_2
\end{aligned}$$

$$\begin{aligned}
&+23388750)T^4 + 51631104000 a_1^4 - 51631104000 b_1^4 + (13829760000 a_1^2 b_1 - 4609920000 b_1^3 + 31440528000 b_1 \\
&- 42411600 b_2 + 151200 b_3)T^3 + (5089896000 a_1^2 + 18950400 a_1 a_2 - 1209600 a_1 a_3 + 262080 a_2^2 \\
&+ 137371752000 b_1^2 - 147168000 b_1 b_2 + 20160 b_2^2 - 31468500)T^2 - 2946888000 a_1^2 + 30240000 a_1 a_2 \\
&- 2016000 a_1 a_3 + 544320 a_2^2 - 40320 a_2 a_3 + 6924150072000 b_1^2 - 16675142400 b_1 b_2 + 45561600 b_1 b_3 \\
&+ 9334080 b_2^2 - 40320 b_2 b_3 + (487992960000 a_1^2 b_1 - 395136000 a_1^2 b_2 + 410546304000 b_1^3 - 395136000 b_1^2 b_2 \\
&+ 56345436000 b_1 - 86940000 b_2 + 378000 b_3)T + 17435250, \quad \mathbf{q}_7 = -141120 T^{10} a_1 + (-6048000 a_1 \\
&+ 15120)T^8 + (-312681600 a_1 + 241920 a_2 + 8640 a_3)T^6 + (-15060326400 a_1 b_1 + 10160640 a_1 b_2 \\
&- 1128960 a_2 b_1)T^5 + (5531904000 a_1^3 - 16595712000 a_1 b_1^2 - 1959552000 a_1 - 5896800 a_2 + 475200 a_3)T^4 \\
&+ (-139184640000 a_1 b_1 + 88704000 a_1 b_2 + 559641600 a_2 b_1 - 645120 a_2 b_2 + 3225600 a_3 b_1)T^3 + 28336896000 a_1^3 \\
&- 1433779200 a_1^2 a_2 + 11289600 a_1^2 a_3 - 18063360 a_1 a_2^2 - 17161208064000 a_1 b_1^2 + 30527078400 a_1 b_1 b_2 \\
&- 13547520 a_1 b_1^2 + 4391654400 a_2 b_1^2 - 4515840 a_2 b_1 b_2 + 11289600 a_3 b_1^2 + (-45835776000 a_1^3 \\
&- 632217600 a_1^2 a_2 - 311367168000 a_1 b_1^2 - 632217600 a_2 b_1^2 + 1014552000 a_1 + 7862400 a_2 + 1339200 a_3)T^2 \\
&+ (-619573248000 a_1^3 b_1 - 619573248000 a_1 b_1^3 + 23627520000 a_1 b_1 - 155635200 a_1 b_2 + 1612800 a_1 b_3 \\
&+ 14288601600 a_2 b_1 - 20482560 a_2 b_2 + 80640 a_2 b_3 + 103219200 a_3 b_1 - 80640 a_3 b_2)T - 489888000 a_1 \\
&- 10206000 a_2 - 648000 a_3, \quad \mathbf{q}_6 = 120 T^{14} + 5880 T^{12} + 47040 T^{11} b_1 + 476280 T^{10} + (27384000 b_1 \\
&- 21840 b_2)T^9 + (24696000 a_1^2 + 24696000 b_1^2 + 16443000)T^8 + (3359664000 b_1 - 3628800 b_2 + 8640 b_3)T^7 \\
&+ (19756800 a_1^2 + 7902720 a_1 a_2 + 10451347200 b_1^2 - 7902720 b_1 b_2 + 162729000)T^6 + (-7744665600 a_1^2 b_1 \\
&+ 2581555200 b_1^3 - 32128790400 b_1 + 73029600 b_2 - 423360 b_3)T^5 + (1220688000 a_1^2 + 107251200 a_1 a_2 \\
&- 282240 a_2^2 + 416903760000 b_1^2 - 728179200 b_1 b_2 - 5644800 b_1 b_3 + 846720 b_2^2 - 154791000)T^4 \\
&- 929359872000 a_1^4 - 10326220800 a_1^3 a_2 + 35625461760000 a_1^2 b_1^2 - 30978662400 a_1^2 b_1 b_2 \\
&+ 30978662400 a_1 a_2 b_1^2 - 10945794048000 b_1^4 + 10326220800 b_1^3 b_2 + (-77446656000 a_1^2 b_1 + 35602176000 b_1^3 \\
&- 221681880000 b_1 + 393724800 b_2 - 2116800 b_3)T^3 + (-7228354560000 a_1^4 + 7228354560000 a_1^4 \\
&+ 31427424000 a_1^2 + 683020800 a_1 a_2 - 1612800 a_1 a_3 + 2257920 a_2^2 - 80640 a_2 a_3 + 21771584352000 b_1^2 \\
&- 48855744000 b_1 b_2 + 40320000 b_1 b_3 + 30481920 b_2^2 - 80640 b_2 b_3 + 130977000)T^2 + 29942136000 a_1^2 \\
&+ 304819200 a_1 a_2 - 5644800 a_1 a_3 - 1270080 a_2^2 - 282240 a_2 a_3 + 14400 a_3^2 + 344794004856000 b_1^2 \\
&- 1041872025600 b_1 b_2 + 4467456000 b_1 b_3 + 786179520 b_2^2 - 6733440 b_2 b_3 + 14400 b_3^2 + (-8850256128000 a_1^2 b_1 \\
&+ 15568358400 a_1^2 b_2 - 79027200 a_1^2 b_3 - 27817574400 a_1 a_2 b_1 + 31610880 a_1 a_2 b_2 + 31610880 a_2^2 b_1 \\
&+ 64945343232000 b_1^3 - 121306752000 b_1^2 b_2 - 79027200 b_1^2 b_3 + 63221760 b_1 b_2^2 + 559057464000 b_1 \\
&- 928746000 b_2 + 4536000 b_3)T + 130977000, \quad \mathbf{q}_5 = -94080 T^{12} a_1 + (-6216000 a_1 - 336 a_2)T^{10} \\
&+ (-257040000 a_1 - 1209600 a_2 + 15120 a_3)T^8 + (-3138508800 a_1 b_1 + 1128960 a_1 b_2 - 10160640 a_2 b_1)T^7 \\
&+ (2581555200 a_1^3 - 7744665600 a_1 b_1^2 - 1342051200 a_1 - 37467360 a_2 + 60480 a_3)T^6 + (-551722752000 a_1 b_1 \\
&+ 833172480 a_1 b_2 - 3386880 a_1 b_3 - 1957616640 a_2 b_1 + 1354752 a_2 b_2 - 3386880 a_3 b_1)T^5 + (171489024000 a_1^3 \\
&- 1106380800 a_1^2 a_2 + 326382336000 a_1 b_1^2 - 1106380800 a_2 b_1^2 - 15600816000 a_1 - 179928000 a_2 \\
&- 1360800 a_3)T^4 + (1445670912000 a_1^3 b_1 + 1445670912000 a_1 b_1^3 - 7175105280000 a_1 b_1 + 8992166400 a_1 b_2 \\
&- 29030400 a_1 b_3 + 4927910400 a_2 b_1 - 30481920 a_2 b_2 + 241920 a_2 b_3 + 188697600 a_3 b_1 - 241920 a_3 b_2)T^3 \\
&- 510854400000 a_1^3 - 7755955200 a_1^2 a_2 - 56448000 a_1^2 a_3 + 27095040 a_1 a_2^2 + 9031680 a_1 a_2 a_3 \\
&- 1530944835840000 a_1 b_1^2 + 3711568896000 a_1 b_1 b_2 - 9934848000 a_1 b_1 b_3 - 2108897280 a_1 b_2^2 \\
&+ 9031680 a_1 b_2 b_3 + 451584 a_2^3 + 2429894476800 a_2 b_1^2 - 3960391680 a_2 b_1 b_2 + 13547520 a_2 b_1 b_3 \\
&+ 451584 a_2 b_2^2 + 15071616000 a_3 b_1^2 - 13547520 a_3 b_1 b_2 + (694649088000 a_1^3 + 10826726400 a_1^2 a_2 \\
&- 237081600 a_1^2 a_3 + 63221760 a_1 a_2^2 - 72993473280000 a_1 b_1^2 + 99890380800 a_1 b_1 b_2 + 31610880 a_1 b_2^2 \\
&- 125416166400 a_2 b_1^2 + 94832640 a_2 b_1 b_2 - 237081600 a_3 b_1^2 + 7334712000 a_1 + 233377200 a_2 + 13608000 a_3)T^2 \\
&+ (53489823744000 a_1^3 b_1 - 41304883200 a_1^3 b_2 + 123914649600 a_1^2 a_2 b_1 - 136512638976000 a_1 b_1^3 \\
&+ 123914649600 a_1 b_1^2 b_2 - 41304883200 a_2 b_1^3 - 6186136320000 a_1 b_1 + 9551001600 a_1 b_2 - 41126400 a_1 b_3 \\
&+ 296792294400 a_2 b_1 - 470776320 a_2 b_2 + 2177280 a_2 b_3 + 2460326400 a_3 b_1 - 2177280 a_3 b_2)T \\
&- 2857680000 a_1 + 57153600 a_2 + 3402000 a_3, \quad \mathbf{q}_4 = 45 T^{16} + 5400 T^{14} + 459900 T^{12} + (16464000 b_1 \\
&- 11760 b_2)T^{11} + (14817600 a_1^2 + 14817600 b_1^2 + 19845000)T^{10} + (134904000 b_1 + 739200 b_2 - 6000 b_3)T^9 \\
&+ (-1054872000 a_1^2 + 2822400 a_1 a_2 + 2444904000 b_1^2 + 153798750)T^8 + (-16595712000 a_1^2 b_1 \\
&+ 5531904000 b_1^3 - 33203520000 b_1 + 53071200 b_2 - 216000 b_3)T^7 + (-23221296000 a_1^2 + 152409600 a_1 a_2 \\
&- 5644800 a_1 a_3 + 846720 a_2^2 - 591991344000 b_1^2 + 874944000 b_1 b_2 - 282240 b_2^2 + 702513000)T^6 \\
&+ (-1598720256000 a_1^2 b_1 + 1106380800 a_1^2 b_2 - 1134040320000 b_1^3 + 1106380800 b_1^2 b_2 + 426641040000 b_1 \\
&- 582120000 b_2 + 2268000 b_3)T^5 + (-97055280000 a_1^2 + 1185408000 a_1 a_2 - 84672000 a_1 a_3 + 29635200 a_2^2 \\
&+ 7687688400000 b_1^2 - 9991296000 b_1 b_2 + 84672000 b_1 b_3 - 4233600 b_2^2 - 89302500)T^4 + 885104640000 a_1^4 \\
&+ 88510464000 a_1^3 a_2 + 7375872000 a_1^3 a_3 + 2212761600 a_1^2 a_2^2 + 6020260485120000 a_1^2 b_1^2 \\
&- 9957427200000 a_1^2 b_1 b_2 + 22127616000 a_1^2 b_1 b_3 + 2212761600 a_1^2 b_2^2 + 88510464000 a_1 a_2 b_1^2 \\
&- 22127616000 a_1 a_3 b_1^2 + 2212761600 a_2^2 b_1^2 + 1760841922560000 b_1^4 - 3348645888000 b_1^3 b_2 \\
&- 7375872000 b_1^3 b_3 + 2212761600 b_1^2 b_2^2 + (29552221440000 a_1^2 b_1 - 66777984000 a_1^2 b_2 + 395136000 a_1^2 b_3 \\
&+ 151732224000 a_1 a_2 b_1 - 158054400 a_1 a_2 b_2 + 158054400 a_2^2 b_1 + 67184974080000 b_1^3 - 96808320000 b_1^2 b_2 \\
&+ 395136000 b_1^2 b_3 - 1025695440000 b_1 + 1902474000 b_2 - 10584000 b_3)T^3 + (1032622080000 a_1^4 \\
&+ 51631104000 a_1^3 a_2 - 178127308800000 a_1^2 b_1^2 + 154893312000 a_1^2 b_1 b_2 - 154893312000 a_1 a_2 b_1^2 \\
&+ 58343147520000 b_1^4 - 51631104000 b_1^3 b_2 + 613607400000 a_1^2 + 10499328000 a_1 a_2 - 435456000 a_1 a_3 \\
&+ 99489600 a_2^2 - 604800 a_2 a_3 + 43200 a_3^2 + 1033225903080000 b_1^2 - 3097809792000 b_1 b_2 + 13027392000 b_1 b_3 \\
&+ 2343297600 b_2^2 - 19958400 b_2 b_3 + 43200 b_3^2 + 1250235000)T^2 + 105787080000 a_1^2 - 3090528000 a_1 a_2 \\
&- 266112000 a_1 a_3 - 10584000 a_2^2 - 604800 a_2 a_3 + 43200 a_3^2 + 948764101320000 b_1^2 - 2947855680000 b_1 b_2 \\
&+ 12942720000 b_1 b_3 + 2284027200 b_2^2 - 19958400 b_2 b_3 + 43200 b_3^2 + (619396565760000 a_1^2 b_1
\end{aligned}$$

$$\begin{aligned}
& -933028992000 a_1^2 b_2 + 4007808000 a_1^2 b_3 + 3968068608000 a_1 a_2 b_1 - 5622220800 a_1 a_2 b_2 + 22579200 a_1 a_2 b_3 \\
& + 20772864000 a_1 a_3 b_1 - 22579200 a_1 a_3 b_2 + 3499776000 a_2^2 b_1 - 2257920 a_2^2 b_2 + 3682926616320000 b_1^3 \\
& - 9447193728000 b_1^2 b_2 - 1185408000 b_1^2 b_3 + 8128512000 b_1 b_2^2 - 2257920 b_2^3 - 2899116360000 b_1 \\
& + 4858056000 b_2 - 23814000 b_3 T + 111628125, \quad \mathbf{q}_3 = -33600 T^{14} a_1 + (-3158400 a_1 - 5040 a_2) T^{12} \\
& + (40152000 a_1 - 1061760 a_2 + 3360 a_3) T^{10} + (3894912000 a_1 b_1 - 940800 a_1 b_2 - 6585600 a_2 b_1) T^9 \\
& + (-4609920000 a_1^3 + 13829760000 a_1 b_1^2 + 3190320000 a_1 - 28350000 a_2 + 151200 a_3) T^8 \\
& + (660119040000 a_1 b_1 - 791884800 a_1 b_2 + 3225600 a_1 b_3 + 491904000 a_2 b_1 - 645120 a_2 b_2) T^7 \\
& + (25815552000 a_1^3 + 851913216000 a_1 b_1^2 + 6932520000 a_1 - 190512000 a_2 + 7560000 a_3) T^6 \\
& + (1445670912000 a_1^3 b_1 + 1445670912000 a_1 b_1^3 + 14030827776000 a_1 b_1 - 18881856000 a_1 b_2 + 72576000 a_1 b_3 \\
& + 66873945600 a_2 b_1 - 84672000 a_2 b_2 + 241920 a_2 b_3 + 358041600 a_3 b_1 - 241920 a_3 b_2) T^5 + 57826836480000 a_1^5 \\
& + 2891341824000 a_1^4 a_2 - 653443252240000 a_1^3 b_1^2 + 5782683648000 a_1^3 b_1 b_2 - 6592259358720000 a_1 b_1^4 \\
& + 5782683648000 a_1 b_1^3 b_2 - 2891341824000 a_2 b_1^4 + (138297600000 a_1^3 - 2765952000 a_1^2 a_2 + 395136000 a_1^2 a_3 \\
& + 211725722880000 a_1 b_1^2 - 360364032000 a_1 b_1 b_2 + 158054400 a_1 b_2^2 + 175835520000 a_2 b_1^2 - 158054400 a_2 b_1 b_2 \\
& + 395136000 a_3 b_1^2 - 159500880000 a_1 - 1140426000 a_2 + 43848000 a_3) T^4 + (14456709120000 a_1^3 b_1 \\
& + 14456709120000 a_1 b_1^3 - 51104632320000 a_1 b_1 + 71293824000 a_1 b_2 - 290304000 a_1 b_3 + 436738176000 a_2 b_1 \\
& - 643507200 a_2 b_2 + 2419200 a_2 b_3 + 3580416000 a_3 b_1 - 2419200 a_3 b_2) T^3 - 4967141760000 a_1^3 \\
& - 37086336000 a_1^2 a_2 + 3878784000 a_1^2 a_3 - 372556800 a_1 a_2^2 + 9676800 a_1 a_2 a_3 + 1612800 a_1 a_3^2 \\
& + 32919971495040000 a_1 b_1^2 - 101916638208000 a_1 b_1 b_2 + 459389952000 a_1 b_1 b_3 + 78948172800 a_1 b_2^2 \\
& - 712837200 a_1 b_2 b_3 + 1612800 a_1 b_3^2 - 2257920 a_2^3 - 322560 a_2^2 a_3 + 102816362880000 a_2 b_1^2 \\
& - 251780659200 a_2 b_1 b_2 + 728985600 a_2 b_1 b_3 + 142248960 a_2 b_2^2 - 645120 a_2 b_2 b_3 + 410691456000 a_3 b_1^2 \\
& - 728985600 a_3 b_1 b_2 + 322560 a_3 b_2^2 + (-3315755520000 a_1^3 - 1467648000 a_1^2 a_2 - 1467648000 a_1^2 a_3 \\
& - 587059200 a_1 a_2^2 + 45158400 a_1 a_2 a_3 - 8329468008960000 a_1 b_1^2 + 19094777856000 a_1 b_1 b_2 \\
& - 60512256000 a_1 b_1 b_3 - 9573580800 a_1 b_2^2 + 45158400 a_1 b_2 b_3 - 4515840 a_2^3 - 4943602944000 a_2 b_1^2 \\
& + 9257472000 a_2 b_1 b_2 - 4515840 a_2 b_2^2 + 7112448000 a_3 b_1^2 - 152885880000 a_1 - 1143072000 a_2 \\
& - 20412000 a_3) T^2 + (1552030986240000 a_1^3 b_1 - 2788079616000 a_1^3 b_2 + 14751744000 a_1^3 b_3 \\
& + 309786624000 a_1^2 a_2 b_1 - 44255232000 a_1^2 a_3 b_1 - 8865060556800000 a_1 b_1^3 + 12081678336000 a_1 b_1^2 b_2 \\
& - 44255232000 a_1 b_1^2 b_3 - 1342408704000 a_2 b_1^3 + 14751744000 a_3 b_1^3 + 156886208640000 a_1 b_1 \\
& - 250163424000 a_1 b_2 + 1137024000 a_1 b_3 + 947013984000 a_2 b_1 - 1439424000 a_2 b_2 + 6048000 a_2 b_3 \\
& + 6241536000 a_3 b_1 - 6048000 a_3 b_2) T + (3662400 b_1 - 1680 b_2) T^{13} + (4939200 a_1^2 + 4939200 b_1^2 \\
& + 2250 T^{16} + 6720 T^{15} b_1 + 225000 T^{14} + (3662400 b_1 - 1680 b_2) T^{13} + (4939200 a_1^2 + 4939200 b_1^2 \\
& + 4422600) T^{12} + (-938952000 b_1 + 1249920 b_2 - 4320 b_3) T^{11} + (991132800 a_1^2 - 1317120 a_1 a_2 \\
& - 4405296000 b_1^2 + 3575040 b_1 b_2 - 99508500) T^{10} + (6453888000 a_1^2 b_1 - 2151296000 b_1^3 - 80022600000 b_1 \\
& + 101833200 b_2 - 367200 b_3) T^9 + (25568424000 a_1^2 - 58464000 a_1 a_2 + 20160 a_2^2 - 250994520000 b_1^2 \\
& - 16531200 b_1 b_2 - 1209600 b_1 b_3 + 262080 b_2^2 - 224248500) T^8 + (-482065920000 a_1^2 b_1 + 632217600 a_1^2 b_2 \\
& - 1101639168000 b_1^3 + 632217600 b_1^2 b_2 - 1708876008000 b_1 + 2488752000 b_2 - 10411200 b_3) T^7 \\
& + (722835456000 a_1^4 - 722835456000 b_1^4 + 74215008000 a_1^2 - 3098995200 a_1 a_2 + 1612800 a_1 a_3 - 3386880 a_2^2 \\
& + 80640 a_2 a_3 - 11978110368000 b_1^2 + 23363827200 b_1 b_2 - 175795200 b_1 b_3 - 4515840 b_2^2 + 80640 b_2 b_3 \\
& + 9704205000) T^6 + 944504995840000 a_1^6 + 2833514987520000 a_1^4 b_1^2 + 2833514987520000 a_1^2 b_1^4 \\
& + 944504995840000 b_1^6 + (-70812322560000 a_1^2 b_1 + 86534784000 a_1^2 b_2 - 237081600 a_1^2 b_3 \\
& - 141616742400 a_1 a_2 b_1 + 94832640 a_1 a_2 b_2 - 31610880 a_2^2 b_1 + 44045809920000 b_1^3 - 91434470400 b_1^2 b_2 \\
& - 237081600 b_1^2 b_3 + 63221760 b_1 b_2^2 - 5155921512000 b_1 + 8707986000 b_2 - 40824000 b_3) T^5 \\
& + (11875153920000 a_1^4 + 51631104000 a_1^3 a_2 - 178127308800000 a_1^2 b_1^2 + 154893312000 a_1^2 b_1 b_2 \\
& - 154893312000 a_1 a_2 b_1^2 + 47500615680000 b_1^4 - 51631104000 b_1^3 b_2 + 489139560000 a_1^2 - 15579648000 a_1 a_2 \\
& - 84672000 a_1 a_3 + 40219200 a_2^2 - 4233600 a_2 a_3 + 43200 a_3^2 + 1706165937000000 b_1^2 - 4681853568000 b_1 b_2 \\
& + 16789248000 b_1 b_3 + 3249288000 b_2^2 - 23587200 b_2 b_3 + 43200 b_3^2 + 15181425000) T^4 + 40809646080000 a_1^4 \\
& + 312947712000 a_1^3 a_2 + 12644352000 a_1^3 a_3 - 7586611200 a_1^2 a_2^2 + 632217600 a_1^2 a_2 a_3 \\
& - 106740047554560000 a_1^2 b_1^2 + 257410556928000 a_1^2 b_1 b_2 - 714405888000 a_1^2 b_1 b_3 - 144145612800 a_1^2 b_2^2 \\
& + 632217600 a_1^2 b_2 b_3 - 126443520 a_1 a_2^3 - 155275803648000 a_1 a_2 b_1^2 + 280704614400 a_1 a_2 b_1 b_2 \\
& - 126443520 a_1 a_2 b_2^2 + 12644352000 a_1 a_3 b_1^2 + 142881177600 a_2^2 b_1^2 - 126443520 a_2^2 b_1 b_2 + 632217600 a_2 a_3 b_1^2 \\
& + 78835069071360000 b_1^4 - 232621304832000 b_1^3 b_2 - 714405888000 b_1^3 b_3 + 287026790400 b_1^2 b_2^2 \\
& + 632217600 b_1^2 b_2 b_3 - 126443520 b_1 b_2^3 + (-482297356800000 a_1^2 b_1 + 668005632000 a_1^2 a_2 - 2370816000 a_1^2 b_3 \\
& - 1569705984000 a_1 a_2 b_1 + 1128960000 a_1 a_2 b_2 - 903168000 a_1 a_3 b_1 - 4154572800 a_2^2 b_1 + 4515840 a_2^2 b_2 \\
& - 45158400 a_2 a_3 b_1 + 3193966702080000 b_1^3 - 4620946176000 b_1^2 b_2 + 48658176000 b_1^2 b_3 - 3296563200 b_1 b_2^2 \\
& - 45158400 b_1 b_2 b_3 + 4515840 b_2^3 - 19070727480000 b_1 + 29719872000 b_2 - 129276000 b_3) T^3 \\
& + (49787136000000 a_1^4 - 132765696000 a_1^3 a_2 + 4425523200 a_1^2 a_2^2 + 4741948108800000 a_1^2 b_1^2 \\
& - 9072322560000 a_1^2 b_1 b_2 + 4425523200 a_1^2 b_2^2 - 2611058688000 a_1 a_2 b_1^2 + 4425523200 a_2^2 b_1^2 \\
& + 7542197913600000 b_1^4 - 11550615552000 b_1^3 b_2 + 4425523200 b_1^2 b_2^2 + 1415186640000 a_1^2 \\
& + 7493472000 a_1 a_2 - 12700800 a_2^2 - 259200 a_3^2 - 5302830078000000 b_1^2 + 16496434080000 b_1 b_2 \\
& - 76349952000 b_1 b_3 - 12713500800 b_2^2 + 116121600 b_2 b_3 - 259200 b_3^2 - 1920003750) T^2 - 264335400000 a_1^2 \\
& + 1905120000 a_1 a_2 + 254016000 a_1 a_3 + 114307200 a_2^2 + 12700800 a_2 a_3 + 388800 a_3^2 + 6177435266520000 b_1^2 \\
& - 20354429088000 b_1 b_2 + 97886880000 b_1 b_3 + 16777756800 b_2^2 - 161481600 b_2 b_3 + 388800 b_3^2 \\
& + (-2775688151040000 a_1^4 b_1 + 2891341824000 a_1^4 b_2 - 5782683648000 a_1^3 a_2 b_1 + 1098709893120000 a_1^2 b_1^3 \\
& - 5782683648000 a_1 a_2 b_1^3 + 3874398044160000 b_1^5 - 2891341824000 b_1^4 b_2 - 1072727349120000 a_1^2 b_1 \\
& + 15833535744000 a_1^2 b_2 - 6314112000 a_1^2 b_3 - 7736988672000 a_1 a_2 b_1 + 10770278400 a_1 a_2 b_2
\end{aligned}$$

$$\begin{aligned}
& -38707200 a_1 a_2 b_3 - 42384384000 a_1 a_3 b_1 + 38707200 a_1 a_3 b_2 - 132257664000 a_2^2 b_1 + 209986560 a_2^2 b_2 \\
& -967680 a_2^2 b_3 - 2119219200 a_2 a_3 b_1 + 1935360 a_2 a_3 b_2 - 4838400 a_3^2 b_1 + 77764203377280000 b_1^3 \\
& -281737782144000 b_1^2 b_2 - 243589248000 b_1^2 b_3 + 396705254400 b_1 b_2^2 + 48384000 b_1 b_2 b_3 - 4838400 b_1 b_3^2 \\
& -223534080 b_2^3 + 967680 b_2^2 b_3 - 5308140600000 b_1 + 9216018000 b_2 - 47628000 b_3)T + 2232562650, \\
\mathbf{q}_1 = & -5040 T^{16} a_1 + (-638400 a_1 - 1680 a_2)T^{14} + (-53928000 a_1 - 120960 a_2 - 1680 a_3)T^{12} \\
& + (-1840204800 a_1 b_1 + 1317120 a_1 b_2 + 188160 a_2 b_1)T^{11} + (553190400 a_1^3 - 1659571200 a_1 b_1^2 \\
& -2187259200 a_1 + 1769040 a_2 - 211680 a_3)T^{10} + (-14292096000 a_1 b_1 - 81715200 a_1 b_2 \\
& +672000 a_1 b_3 + 40857600 a_2 b_1 + 53760 a_2 b_2 - 940800 a_3 b_1)T^9 + (-33586560000 a_1^3 + 395136000 a_1^2 a_2 \\
& -265926528000 a_1 b_1^2 + 395136000 a_2 b_1^2 - 14978628000 a_1 + 112341600 a_2 - 6274800 a_3)T^8 \\
& + (-619573248000 a_1^3 b_1 - 619573248000 a_1 b_1^3 + 3908862720000 a_1 b_1 - 6198796800 a_1 b_2 + 25804800 a_1 b_3 \\
& +9503424000 a_2 b_1 - 12741120 a_2 b_2 + 80640 a_2 b_3 + 40320000 a_3 b_1 - 80640 a_3 b_2)T^7 + (-975985920000 a_1^3 \\
& +869299200 a_1^2 a_2 - 79027200 a_1^2 a_3 + 63221760 a_1 a_2^2 + 66086496000000 a_1 b_1^2 - 97361510400 a_1 b_1 b_2 \\
& +31610880 a_1 b_2^2 - 10826726400 a_2 b_1^2 + 31610880 a_2 b_1 b_2 - 79027200 a_3 b_1^2 - 80427816000 a_1 \\
& -87318000 a_2 - 25704000 a_3)T^6 + (-62163849216000 a_1^3 b_1 + 41304883200 a_1^3 b_2 - 123914649600 a_1^2 a_2 b_1 \\
& +127838613504000 a_1 b_1^3 - 123914649600 a_1 b_1^2 b_2 + 41304883200 a_3 b_1^3 - 36802176768000 a_1 b_1 \\
& +47653401600 a_1 b_2 - 171763200 a_1 b_3 + 549080985600 a_2 b_1 - 877201920 a_2 b_2 + 4112640 a_2 b_3 \\
& +4681152000 a_3 b_1 - 4112640 a_3 b_2)T^5 - 33043906560000 a_1^5 - 3304390656000 a_1^4 a_2 + 413048832000 a_1^4 a_3 \\
& -82609766400 a_1^3 a_2^2 + 236024363581440000 a_1^3 b_1^2 - 381657120768000 a_1^3 b_1 b_2 + 826097664000 a_1^3 b_1 b_3 \\
& +82609766400 a_1^3 b_2^2 + 570007388160000 a_1^2 a_2 b_1^2 - 495658598400 a_1^2 a_2 b_1 b_2 + 247829299200 a_1^2 a_2 b_1^2 \\
& -200948256768000000 a_1 b_1^4 + 378352730112000 a_1 b_1^3 b_2 + 826097664000 a_1 b_1^3 b_3 - 247829299200 a_1 b_1^2 b_2^2 \\
& -186698072064000 a_2 b_1^4 + 165219532800 a_2 b_1^3 b_2 - 413048832000 a_3 b_1^4 + (-3170684160000 a_1^3 \\
& +65536128000 a_1^2 a_2 - 5927040000 a_1^2 a_3 + 1083801600 a_1 a_2^2 - 749722579200000 a_1 b_1^2 + 925295616000 a_1 b_1 b_2 \\
& -9031680000 a_1 b_1 b_3 + 519321600 a_1 b_2^2 + 2257920 a_2^3 + 5564926080000 a_2 b_1^2 - 9686476800 a_2 b_1 b_2 \\
& +22579200 a_2 b_1 b_3 + 2257920 a_2 b_2^2 + 29070720000 a_3 b_1^2 - 22579200 a_3 b_1 b_2 - 41436360000 a_1 - 2571912000 a_2 \\
& -241542000 a_3)T^4 + (973762621440000 a_1^3 b_1 - 2375030784000 a_1^3 b_2 + 14751744000 a_1^3 b_3 \\
& -929359872000 a_1^2 a_2 b_1 - 44255232000 a_1^2 a_3 b_1 - 75433042944000000 a_1 b_1^3 + 10842531840000 a_1 b_1^2 b_2 \\
& -44255232000 a_1 b_1^2 b_3 - 929359872000 a_2 b_1^3 + 14751744000 a_3 b_1^3 + 91024940160000 a_1 b_1 \\
& -184965984000 a_1 b_2 + 1112832000 a_1 b_3 - 1192647456000 a_2 b_1 + 1405555200 a_2 b_2 - 3628800 a_2 b_3 \\
& -8249472000 a_3 b_1 + 3628800 a_3 b_2)T^3 + 132249196800000 a_1^3 + 419295744000 a_1^2 a_2 + 12410496000 a_1^2 a_3 \\
& +3420748800 a_1 a_2^2 + 106444800 a_1 a_2 a_3 - 4838400 a_1 a_3^2 - 111656974020480000 a_1 b_1^2 \\
& +344031141888000 a_1 b_1 b_2 - 1489259520000 a_1 b_1 b_3 - 264210508800 a_1 b_2^2 + 2274048000 a_1 b_2 b_3 - 4838400 a_1 b_3^2 \\
& +13547520 a_2^3 + 967680 a_2^2 a_3 - 269769733632000 a_2 b_1^2 + 693565286400 a_2 b_1 b_2 - 1983744000 a_2 b_1 b_3 \\
& -419973120 a_2 b_2^2 + 1935360 a_2 b_2 b_3 - 1019039616000 a_3 b_1^2 + 1983744000 a_3 b_1 b_2 - 967680 a_3 b_2^2 \\
& + (57826836480000 a_1^5 + 2891341824000 a_1^4 a_2 - 6534432522240000 a_1^3 b_1^2 + 5782683648000 a_1^3 b_1 b_2 \\
& -6592259358720000 a_1 b_1^4 + 5782683648000 a_1 b_1^3 b_2 - 2891341824000 a_2 b_1^4 + 26325371520000 a_1^3 \\
& +562391424000 a_1^2 a_2 - 136684800000 a_1^2 a_3 + 3285273600 a_1 a_2^2 + 106444800 a_1 a_2 a_3 \\
& -4838400 a_1 a_3^2 - 123143455612800000 a_1 b_1^2 + 364044893184000 a_1 b_1 b_2 - 1505516544000 a_1 b_1 b_3 \\
& -271526169600 a_1 b_2^2 + 2274048000 a_1 b_2 b_3 - 4838400 a_1 b_3^2 - 20321280 a_2^3 + 967680 a_2^2 a_3 \\
& -371107723392000 a_2 b_1^2 + 854509824000 a_2 b_1 b_2 - 2322432000 a_2 b_1 b_3 - 453841920 a_2 b_2^2 + 1935360 a_2 b_2 b_3 \\
& -1418352768000 a_3 b_1^2 + 2322432000 a_3 b_1 b_2 - 967680 a_3 b_2^2 - 2214702000000 a_1 - 4072194000 a_2 \\
& -156492000 a_3)T^2 + (17687109242880000 a_1^3 b_1 - 25965176832000 a_1^3 b_2 + 107476992000 a_1^3 b_3 \\
& -131178829824000 a_1^2 a_2 b_1 + 240242688000 a_1^2 a_2 b_2 - 1264435200 a_1^2 a_2 b_3 - 1561577472000 a_1^2 a_3 b_1 \\
& +1264435200 a_1^2 a_3 b_2 - 247829299200 a_1 a_2^2 b_1 + 252887040 a_1 a_2^2 b_2 - 423777964851200000 a_1 b_1^3 \\
& +1076242987008000 a_1 b_1^2 b_2 + 107476992000 a_1 b_1^2 b_3 - 915451084800 a_1 b_1 b_2^2 + 252887040 a_1 b_2^3 \\
& -252887040 a_2^3 b_1 - 564500772864000 a_2 b_1^3 + 907864473600 a_2 b_1^2 b_2 - 1264435200 a_2 b_1^2 b_3 \\
& -252887040 a_2 b_1 b_2^2 - 1561577472000 a_3 b_1^3 + 1264435200 a_3 b_1^2 b_2 + 376012264320000 a_1 b_1 \\
& -625387392000 a_1 b_2 + 3030048000 a_1 b_3 + 2565561600000 a_2 b_1 - 4064256000 a_2 b_2 + 18144000 a_2 b_3 \\
& +17835552000 a_3 b_1 - 18144000 a_3 b_2)T - 15360030000 a_1 - 142884000 a_2 - 3402000 a_3, \quad \mathbf{q}_0 = T^{20} + 370 T^{18} \\
& +1680 T^{17} b_1 + 44325 T^{16} + 336 T^{15} b_2 + (705600 a_1^2 + 705600 b_1^2 + 2208600)T^{14} + 240 T^{13} b_3 + (51979200 a_1^2 \\
& +376320 a_1 a_2 - 128654400 b_1^2 + 188160 b_1 b_2 + 62795250)T^{12} + (-184396800 a_1^2 b_1 + 61465600 b_1^3 \\
& +5745801600 b_1 - 8159760 b_2 + 41760 b_3)T^{11} + (1920072000 a_1^2 + 24756480 a_1 a_2 + 188160 a_1 a_3 + 30912 a_2^2 \\
& +77399112000 b_1^2 - 111901440 b_1 b_2 + 215040 b_1 b_3 + 25536 b_2^2 + 693384300)T^{10} + (79158912000 a_1^2 b_1 \\
& -79027200 a_1^2 b_2 + 114808960000 b_1^3 - 79027200 b_1^2 b_2 + 177128532000 b_1 - 280476000 b_2 + 1414800 b_3)T^9 \\
& + (-51631104000 a_1^4 + 51631104000 b_1^4 + 55460664000 a_1^2 + 858412800 a_1 a_2 + 9273600 a_1 a_3 - 342720 a_2^2 \\
& +40320 a_2 a_3 - 6252079176000 b_1^2 + 15883257600 b_1 b_2 - 33465600 b_1 b_3 - 10100160 b_2^2 + 40320 b_2 b_3 \\
& +6641129250)T^8 + (8847320832000 a_1^2 b_1 - 8862336000 a_1^2 b_2 + 11289600 a_1^2 b_3 + 11199283200 a_1 a_2 b_1 \\
& -4515840 a_1 a_2 b_2 - 13547520 a_2^2 b_1 - 213288768000000 b_1^3 + 38689459200 b_1^2 b_2 + 11289600 b_1^2 b_3 \\
& -18063360 b_1 b_2^2 + 4896031392000 b_1 - 6993378000 b_2 + 29592000 b_3)T^7 + (-1652195328000 a_1^4 \\
& -10326220800 a_1^3 a_2 + 35625461760000 a_1^2 b_1^2 - 30978662400 a_1^2 b_1 b_2 + 30978662400 a_1 a_2 b_1^2 \\
& -10222958592000 b_1^4 + 10326220800 b_1^3 b_2 + 559142136000 a_1^2 + 237081600 a_1 a_2 + 422553600 a_1 a_3 \\
& +20744640 a_2^2 - 40320 a_2 a_3 + 14400 a_3^2 + 413028968184000 b_1^2 - 1097213644800 b_1 b_2 + 4668249600 b_1 b_3 \\
& +743843520 b_2^2 - 6491520 b_2 b_3 + 14400 b_3^2 + 4346055000)T^6 + 944504995840000 a_1^6 \\
& +2833514987520000 a_1^4 b_1^2 + 2833514987520000 a_1^2 b_1^4 + 944504995840000 b_1^6 + (-176294442240000 a_1^2 b_1 \\
& +343824768000 a_1^2 b_2 - 1862784000 a_1^2 b_3 + 1940456448000 a_1 a_2 b_1 - 3147540480 a_1 a_2 b_2 + 13547520 a_1 a_2 b_3
\end{aligned}$$

$$\begin{aligned}
&+19869696000 a_1 a_3 b_1 - 13547520 a_1 a_3 b_2 + 763176960 a_2^2 b_1 - 451584 a_2^2 b_2 - 9031680 a_2 a_3 b_1 \\
&-298723885120000 b_1^3 - 6485028480000 b_1^2 b_2 + 12813696000 b_1^2 b_3 + 3712020480 b_1 b_2^2 - 9031680 b_1 b_2 b_3 \\
&-451584 b_2^3 + 25473677040000 b_1 - 40318689600 b_2 + 184842000 b_3)T^5 + (-2729072640000 a_1^4 \\
&-634324992000 a_1^3 a_2 - 7375872000 a_1^3 a_3 + 2212761600 a_1^2 a_2^2 + 146706094080000 a_1^2 b_1^2 \\
&-354041856000 a_1^2 b_1 b_2 - 22127616000 a_1^2 b_1 b_3 + 2212761600 a_1^2 b_2^2 - 1460422656000 a_1 a_2 b_1^2 \\
&+22127616000 a_1 a_3 b_1^2 + 2212761600 a_2^2 b_1^2 + 5357980938240000 b_1^4 - 7788920832000 b_1^3 b_2 \\
&+7375872000 b_1^3 b_3 + 2212761600 b_1^2 b_2^2 + 1046916360000 a_1^2 + 38864448000 a_1 a_2 + 2274048000 a_1 a_3 \\
&+201096000 a_2^2 - 9072000 a_2 a_3 + 388800 a_3^2 + 9992405573640000 b_1^2 - 29340837792000 b_1 b_2 \\
&+126766080000 b_1 b_3 + 21377563200 b_2^2 - 183254400 b_2 b_3 + 388800 b_3^2 + 14042818125)T^4 \\
&+131426749440000 a_1^4 + 3282112512000 a_1^3 a_2 - 98896896000 a_1^3 a_3 + 46061568000 a_1^2 a_2^2 \\
&-1986969600 a_1^2 a_2 a_3 + 45158400 a_1^2 a_3^2 + 1261464395950080000 a_1^2 b_1^2 - 3708301234176000 a_1^2 b_1 b_2 \\
&+15222445056000 a_1^2 b_1 b_3 + 2717451878400 a_1^2 b_2^2 - 22217932800 a_1^2 b_2 b_3 + 45158400 a_1^2 b_3^2 \\
&+397393920 a_1 a_2^3 - 18063360 a_1 a_2^2 a_3 + 6434560806912000 a_1 a_2 b_1^2 - 15355662336000 a_1 a_2 b_1 b_2 \\
&+41545728000 a_1 a_2 b_1 b_3 + 8489779200 a_1 a_2 b_2^2 - 36126720 a_1 a_2 b_2 b_3 + 23789896704000 a_1 a_3 b_1^2 \\
&-41545728000 a_1 a_3 b_1 b_2 + 18063360 a_1 a_3 b_2^2 + 1806336 a_2^4 + 6934523904000 a_2^2 b_1^2 - 11957944320 a_2^2 b_1 b_2 \\
&+18063360 a_2^2 b_1 b_3 + 3612672 a_2^2 b_2^2 + 39558758400 a_2 a_3 b_1^2 - 36126720 a_2 a_3 b_1 b_2 + 45158400 a_3^2 b_1^2 \\
&+420447434480640000 b_1^4 - 1863670910976000 b_1^3 b_2 - 8666348544000 b_1^3 b_3 + 3805769318400 b_1^2 b_2^2 \\
&+19327795200 b_1^2 b_2 b_3 + 45158400 b_1^2 b_3^2 - 3865559040 b_1 b_2^3 - 18063360 b_1 b_2^2 b_3 + 1806336 b_2^4 \\
&+(-2775688151040000 a_1^4 b_1 + 2891341824000 a_1^4 b_2 - 5782683648000 a_1^3 a_2 b_1 + 1098709893120000 a_1^2 b_1^3 \\
&-5782683648000 a_1 a_2 b_1^3 + 3874398044160000 b_1^5 - 2891341824000 b_1^4 b_2 + 119027664000000 a_1^2 b_1 \\
&+180690048000 a_1^2 b_2 - 3088512000 a_1^2 b_3 + 29223355392000 a_1 a_2 b_1 - 39897446400 a_1 a_2 b_2 + 148377600 a_1 a_2 b_3 \\
&+210115584000 a_1 a_3 b_1 - 148377600 a_1 a_3 b_2 + 38305612800 a_2^2 b_1 - 60963840 a_2^2 b_2 + 322560 a_2^2 b_3 \\
&+480614400 a_2 a_3 b_1 - 645120 a_2 a_3 b_2 + 1612800 a_3^2 b_1 + 1322893031040000 b_1^3 + 34047457920000 b_1^2 b_2 \\
&+375854976000 b_1^2 b_3 - 114171724800 b_1 b_2^2 - 241920000 b_1 b_2 b_3 + 1612800 b_1 b_3^2 + 83543040 b_2^3 \\
&-322560 b_2^2 b_3 - 7764316560000 b_1 + 11787930000 b_2 - 47628000 b_3)T^3 + (944504995840000 a_1^6 \\
&+2833514987520000 a_1^4 b_1^2 + 2833514987520000 a_1^2 b_1^4 + 944504995840000 b_1^6 - 114052055040000 a_1^4 \\
&-2525709312000 a_1^3 a_2 + 31610880000 a_1^3 a_3 - 1264435200 a_1^2 a_2^2 - 632217600 a_1^2 a_2 a_3 \\
&+148502382336000000 a_1^2 b_1^2 - 331670836224000 a_1^2 b_1 b_2 + 581640192000 a_1^2 b_1 b_3 + 188400844800 a_1^2 b_2^2 \\
&-632217600 a_1^2 b_2 b_3 + 126443520 a_1 a_2^3 + 275207482368000 a_1 a_2 b_1^2 - 386917171200 a_1 a_2 b_1 b_2 \\
&+126443520 a_1 a_2 b_2^2 + 385652736000 a_1 a_3 b_1^2 - 98625945600 a_2^2 b_1^2 + 126443520 a_2^2 b_1 b_2 - 632217600 a_2 a_3 b_1^2 \\
&-56130143569920000 b_1^4 + 201200090112000 b_1^3 b_2 + 935682048000 b_1^3 b_3 - 295877836800 b_1^2 b_2^2 \\
&-632217600 b_1^2 b_2 b_3 + 126443520 b_1 b_2^3 + 1395659160000 a_1^2 - 127008000 a_1 a_2 - 81648000 a_1 a_3 \\
&+63504000 a_2^2 + 16329600 a_2 a_3 + 1425600 a_3^2 + 26993024052120000 b_1^2 - 85679723808000 b_1 b_2 \\
&+391438656000 b_1 b_3 + 68038185600 b_2^2 - 622339200 b_2 b_3 + 1425600 b_3^2 + 2902331250)T^2 + 755856360000 a_1^2 \\
&+17908128000 a_1 a_2 + 5261760000 a_1 a_3 + 114307200 a_2^2 + 7257600 a_2 a_3 + 129600 a_3^2 + 1702675757160000 b_1^2 \\
&-5826237984000 b_1 b_2 + 29592864000 b_1 b_3 + 4991414400 b_2^2 - 50803200 b_2 b_3 + 129600 b_3^2 \\
&+(-76984041308160000 a_1^4 b_1 + 107392696320000 a_1^4 b_2 - 413048832000 a_1^4 b_3 - 219741978624000 a_1^3 a_2 b_1 \\
&+165219532800 a_1^3 a_2 b_2 + 826097664000 a_1^3 a_3 b_1 - 247829299200 a_1^2 a_2^2 b_1 + 327225545687040000 a_1^2 b_1^3 \\
&-570007388160000 a_1^2 b_1^2 b_2 + 247829299200 a_1^2 b_1 b_2^2 + 540267872256000 a_1 a_2 b_1^3 - 495658598400 a_1 a_2 b_1^2 b_2 \\
&+826097664000 a_1 a_3 b_1^3 + 82609766400 a_2^2 b_1^3 - 32796077260800000 b_1^5 + 82609766400000 b_1^4 b_2 \\
&+413048832000 b_1^4 b_3 - 82609766400 b_1^3 b_2^2 + 2319398928000000 a_1^2 b_1 - 3826497024000 a_1^2 b_2 \\
&+18603648000 a_1^2 b_3 + 63310270464000 a_1 a_2 b_1 - 100048435200 a_1 a_2 b_2 + 454809600 a_1 a_2 b_3 \\
&+476679168000 a_1 a_3 b_1 - 454809600 a_1 a_3 b_2 + 431725593600 a_2^2 b_1 - 663828480 a_2^2 b_2 + 2903040 a_2^2 b_3 \\
&+6357657600 a_2 a_3 b_1 - 5806080 a_2 a_3 b_2 + 14515200 a_3^2 b_1 - 183630192600960000 b_1^3 + 711772475904000 b_1^2 b_2 \\
&+690463872000 b_1^2 b_3 - 1069136409600 b_1 b_2^2 - 145152000 b_1 b_2 b_3 + 14515200 b_1 b_3^2 + 636733440 b_2^3 \\
&-2903040 b_2^2 b_3 + 2482252290000 b_1 - 4429404000 b_2 + 23814000 b_3)T + 22325625
\end{aligned}$$