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Phoebe's orbit from ground-based and space-based observations^{★,★★}

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ABSTRACT

Context. The ephemeris of Phoebe, the ninth satellite of Saturn, is not very accurate. Previous dynamical models were usually too simplified, the astrometry is heterogeneous and, the Saturn's ephemeris itself is an additional source of error.

Aims. The aim is to improve Phoebe's ephemeris by using a large set of observations, correcting some systematic errors and updating the dynamical model.

Methods. The dynamical model makes use of the most recent ephemeris of planets and Saturnian satellites. The astrometry of Phoebe is improved by using a compilation of ground-based and space-based observations and by correcting the bias in stellar catalogues used for the reduction.

Results. We present an accurate ephemeris of Phoebe with residuals of 0.45 arcsec and with an estimated accuracy of Phoebe's position of less than 100 km on 1990–2020 period.

Key words. ephemerides – astrometry – planets and satellites: individual: Phoebe

1. Introduction

Phoebe is the biggest irregular satellite of Saturn (220 km in diameter). It has a retrograde orbit with an inclination of about 176 degrees on the ecliptic plane orbiting at about 13 million km (0.086 AU) from Saturn. The ephemeris of Phoebe is not very accurate for three reasons. The main reason is the astrometry of Phoebe. Because of its faintness (about magnitude 16), Phoebe is not easy to observe and was especially difficult to observe in the past. Most of the available observations have been realized in the last twenty years. There are only a few observations in the past and they are not very accurate. Another reason is that the dynamical models of Phoebe's motion are usually not complete and take into account only a few perturbations. However, while the astrometry is not good, a simple dynamical model could be enough to compute the position of Phoebe. The last reason is the dependence on Saturn's position. When comparing the observed and computed positions of Phoebe, the position of Saturn, or more exactly the barycentre of the Saturnian system is required because the dynamical model provides the position of Phoebe in relation to the Saturnian barycentre. Consequently, the ephemeris of Saturn can be the source of systematic error in Phoebe's position.

In this paper, we develop a new ephemeris of Phoebe by improving the three previous limiting factors. In particular, we first develop a new flexible dynamical model that can take into account the perturbations of the planets, the main Saturnian satellites, and the flatness of Saturn, using the most recent planetary and satellite ephemerides (Sect. 2).

An important step is realized for astrometry. We compile a large part of ground-based observations and include also observations from Voyager and *Cassini* spacecrafts (Sect. 3). Moreover, we study old observations and try to reduce some old observations with modern techniques (Sect. 4.1). In addition, most of the observations (78%) are also corrected of bias in the stellar catalogue used for their reduction (Sect. 4.2).

After presenting the fitting process (Sect. 5), we compare four different models of Phoebe's motion (Sect. 6) and highlight the one with the best residuals (Sect. 7). Finally, by using statistical methods, we also compute the estimated accuracy of Phoebe's position during the 1875–2020 period (Sect. 8).

2. A flexible dynamical model

According to Jacobson (1998), the previous dynamical models usually took into account the perturbation of some planets. Rose (1979) includes only the Sun's perturbation, Bykova & Shikhalev (1982) added Jupiter's perturbation and Bec-Borsenberger & Rocher (1982) also added the perturbation of Titan. For more recent models, Jacobson (1998) took into account the perturbations of the Sun, Jupiter, and Uranus, as well as the perturbations of the main Saturnian satellites for

* The catalogue of ground-based observations of Phoebe is available at the CDS via anonymous ftp to cdsarc.u-strasbg.fr (130.79.128.5) or via

<http://cdsarc.u-strasbg.fr/viz-bin/qcat?J/A+A/553/A36>

** Table 5 is available in electronic form at <http://www.aanda.org>

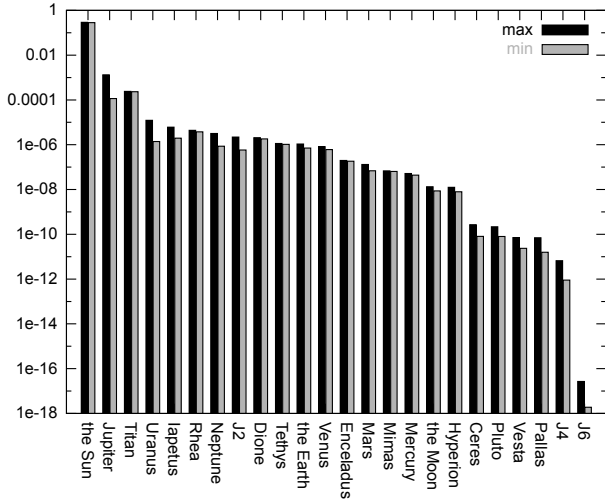


Fig. 1. Comparison of the maximum and minimum magnitude of gravitational perturbations compared to Saturn's acceleration.

post-1966 observations and the perturbation of an orbiting Titan for pre-1966 observations. Shen et al. (2005, 2011) included the perturbations of the Sun, Jupiter and Uranus and the flatness (J_2) of Saturn, whereas Emelyanov (2007) took into account the perturbations of the Sun, Jupiter, Uranus, Neptune and Saturn's flatness (J_2, J_4). For the two last models, the Saturnian satellites are taken into account by adding their masses to the mass of Saturn and by correcting the J_2 and J_4 values reported by Emelyanov (2007). Finally, Jacobson et al. (2006) proposed a complete model of the Saturnian system by determining the motion of all Saturnian satellites including Phoebe.

Figure 1 represents the ratio of maximum and minimum magnitude of gravitational accelerations compared to Saturn's acceleration. The main perturbers of Phoebe's motion are the Sun, Jupiter, Titan, Uranus, and Iapetus. Taking into account the flatness of Saturn does not seem justified since other perturbations like those due to the main Saturnian satellites are not included. In particular, the perturbation due to J_4 is less important than the gravitational perturbations of the main asteroids.

To measure the effect of perturbers on Phoebe's motion, we have developed a *flexible* model. This dynamical model is adjustable, in the sense that it is possible to take into account or not the perturbations of the Sun, the eight planets and the Moon, the eight major Saturnian satellites and the flatness of Saturn (J_2). The masses of the planets are from Folkner et al. (2009) and the other parameters of the Saturnian system are from Jacobson et al. (2006) and are provided in Table 1.

If a planet is taken into account, it is considered as a point mass and its position is from JPL Planetary Ephemeris DE421 (Folkner et al. 2009). In the opposite case, the mass of the planet is added to the Sun's mass. Likewise, if a Saturnian satellite is taken into account, it is considered as a point mass and its position is given by Lainey et al. (2012) and in the opposite case, its mass is added to Saturn's mass.

For our model, we use a numerical integration of the equations of motion using a Gauss-Radau integrator (Everhart 1985). These equations are written in the Saturnian system barycentre. In order to fit the dynamical model, the equations of variation are also integrated with the equations of motion (as in Lainey et al. 2004).

As a large part of astrometric observations is given in absolute coordinates and not in relative coordinates compared to another satellite as for the other main Saturnian satellites, the

Table 1. Dynamical constants of Saturn's system (Jacobson et al. 2006).

Name	Value	Units
Phoebe GM	0.5534	$\text{km}^3 \text{s}^{-2}$
Mimas GM	2.5023	$\text{km}^3 \text{s}^{-2}$
Enceladus GM	7.2096	$\text{km}^3 \text{s}^{-2}$
Tethys GM	41.2097	$\text{km}^3 \text{s}^{-2}$
Dione GM	73.1127	$\text{km}^3 \text{s}^{-2}$
Rhea GM	153.9416	$\text{km}^3 \text{s}^{-2}$
Titan GM	8978.1356	$\text{km}^3 \text{s}^{-2}$
Hyperion GM	0.3727	$\text{km}^3 \text{s}^{-2}$
Iapetus GM	120.5117	$\text{km}^3 \text{s}^{-2}$
Saturn equatorial radius	60 330	km
Saturn $J_2 (\times 10^6)$	16 290.71	
Saturn pole α_P	40.5955	degrees
Saturn pole δ_P	83.5381	degrees
Saturn polar rate $\dot{\alpha}_P$	-0.04229	deg century^{-1}
Saturn polar rate $\dot{\delta}_P$	-0.00444	deg century^{-1}

computed position of Phoebe depends on the computed position of Saturn. Consequently, the position of Saturn can become a source of systematic error in the determination of Phoebe's orbit.

In particular, some differences in the position of Saturn between several planetary ephemeris exist. In previous models of Phoebe, JPL ephemeris DE406 was usually used. Compared to the most recent ephemeris, DE421, the difference can reach 0.15 arcsec during the 1900–2020 period. In comparison, the difference between INPOP10a (Fienga et al. 2011) and DE421 is less than 2 mas during the same period. In these recent ephemerides, the orbit of Saturn is determined using *Cassini* tracking and ground-based astrometry. For DE421, the accuracy of Saturn's position is tens of km (Folkner et al. 2009).

3. Observations of Phoebe

3.1. Ground-based observations

The ground-based observations of Phoebe come from four different sources:

- NSDC: the Natural Satellite Data Center¹ provides a large number of observations of natural satellites (Arlot & Emelyanov 2009). For Phoebe, 3404 observations are available from 1904 to 2011 concerning almost 120 different references;
- MPC: the Minor Planet Center provides observations of natural outer irregular satellites of the giant planets². As of 19 September 2012, 2040 observations of Phoebe from 1898 to 2012 are available in the database. The old observations before 1981, usually provided in another frame (such as B1950.0) have been rotated to the J2000. All the observations are given in the J2000 reference frame. Most of the observations come from CCD (about 90%) and 5 observations made on 13 and 14 June 2010 come from the WISE spacecraft (see Sect. 3.2);

¹ NSDC observations of Phoebe are available on this website: http://www.imcce.fr/hosted_sites/saimirror/bsapooe.htm

² <http://www.minorplanetcenter.net/iau/ECS/MPCAT-OBS/MPCAT-OBS.html>

- NOFS: 205 NOFS observations (U.S.N.O 2012) from the Flagstaff Observatory were obtained from 2000 to 2011. All the data are available on the web site of the FASTT Planetary Satellite Observations <http://www.nofs.navy.mil/data/plansat.html>;
- Pickering: Pickering (1908) published 42 observations of Phoebe made in Arequipa, the Harvard observatory station from 1898 to 1904. The observations are provided in B1875 reference frame. They represent the oldest observations of Phoebe. The time of observation is given without indication of seconds, i.e. to the nearest minute. A specific study of these observations has been done in order to improve their astrometry (Sect. 4.1).

Many observations appear in several databases. In particular, about 1950 appear in both MPC and NSDC. In that context and as a general rule, we favour observations in NSDC because they are given in their original format (similar to the publication) whereas MPC turns the observations into J2000 reference system with their own routine. Consequently, only 83 observations from MPC (and only in that database) have been selected.

In the same way, some observations appear in both the NSDC and the NOFS databases. In particular, observations from Stone (2000, 2001) and observations from NOFS compiled in NDSC (so0023 and so0029) are replaced by NOFS observations (U.S.N.O 2012) that use a more recent reduction process and are provided with more digits for time and coordinates. This is also the case of MPC observations provided by the NSDC database (MPC63362, MPC63363, MPC67130, MPC75144, MPC75145) replaced in our compilation by U.S.N.O (2012).

Finally, there are also several doubles in the NSDC database. Some observations appear in several references but with different digits for time, or coordinates. In this context, we keep the observations with more digits. For example, observations in MPC52887 are replaced by those of NSDC (so0022 file). Observations in Mulholland & Shelus (1980) appear twice in the NDSC database (so0001 and so0019) and we keep those in Strugnell & Taylor (1990) (so0001).

3.2. Space-based observations

During its encounter with Saturn in 1981, Voyager 2 performed eight imaging observations of Phoebe. These observations from 17 June to 15 August 1981 were published in Jacobson (1998). They are provided in specific coordinates which are pixel and line locations in the Voyager camera frame. The Saturn barycentric position and the velocity of Voyager 2 during the eight observation times are also provided in the paper. The process of reduction that takes into account the camera pointing, the gnomonic projection, the electromagnetic and optical distortion, is fully described in Jacobson (1998). In this paper, we also use pixel and line locations in the camera frame as coordinates and the positions of Voyager given in Jacobson (1998) are also used.

The *Cassini* spacecraft entered Saturn's orbit at the end of June 2004. Ever since, its ISS Narrow Angle (NAC) and Wide Angle (WAC) cameras have sent tens of thousands of images of Saturn, its rings, and its satellites. Right before entering Saturn's orbit, *Cassini* performed a Phoebe flyby on 11 June 2004, where it reached a minimum distance of about 2068 km. *Cassini*'s ISS NAC kept observing that satellite from 6 to 12 June 2004, producing hundreds of high-resolution images of Phoebe that can be downloaded from Planetary Data System³.

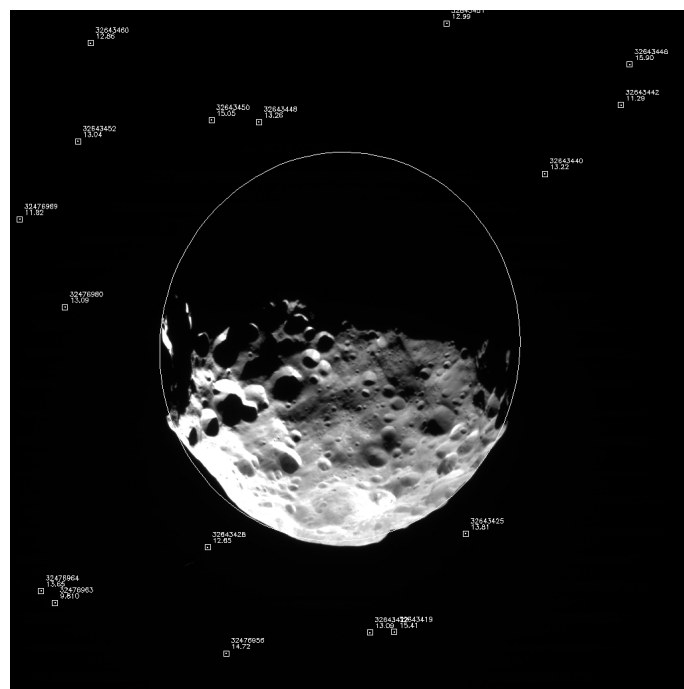


Fig. 2. Example of a *Cassini* astrometric reduction of Phoebe. UCAC2 catalogue stars are superimposed on the image.

A total number of 223 images from Phoebe's flyby were used for astrometric reduction. The spacecraft's position and camera's pointing vector were computed using SPICE library⁴ (Acton 1996). Since there is an error in the camera's pointing vector, its correction was done using the UCAC2 star catalogue (Zacharias et al. 2004). The satellite's position measurement was done using the ellipse projection of Phoebe's triaxial shape (Thomas 2010). Nonetheless, a difficulty was encountered for the centre-of-figure measurement since Phoebe has an irregular shape and fitting an ellipse is not the best way to determine its position. The astrometric model that was used for pointing correction and satellite centre-of-figure measurement is well described in Tajeddine et al. (2013). The number of detected stars varied from 2 to 43 stars per image, depending on the satellite's size on the image and camera exposure length.

Figure 2 shows an example of Phoebe's astrometric reduction from the *Cassini* spacecraft and the resolution reached. With an obtained accuracy of a few kilometers, the *Cassini* observations have proven to be critical in Phoebe orbit modelling.

Finally, the WISE spacecraft (Wide-Field Infrared Survey Explorer) also provided five more space-based observations in 2009. But unlike to *Cassini* or Voyager 2, this spacecraft was in Earth-orbit and not in the vicinity of Saturn system. These observations can be treated as ground-based observations.

3.3. Catalogue of observations of Phoebe

A total of 3598 observations (3367 ground-based and 231 space-based) from 1898 to 2012 have been compiled in a catalogue. For comparison, in previous studies Emelyanov (2007) used 1606 observations from 1904 to 2007 and Shen et al. (2011) used 2994 observations from 1904 to 2009.

For the catalogue of observations, we adopted the format presented in Desmars et al. (2009b) and we added some

³ <http://pds.nasa.gov/>

⁴ <http://naif.jpl.nasa.gov/naif/>

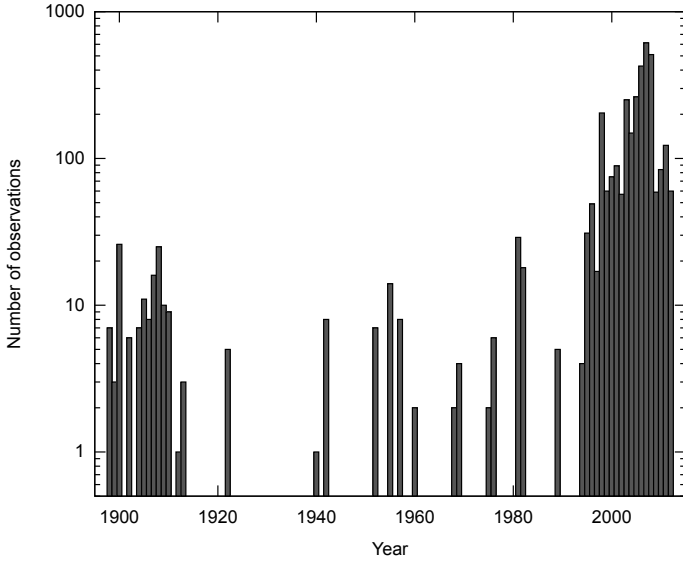


Fig. 3. Logarithmic distribution of the Phoebe observations.

information about the catalogue used for astrometric reduction (using the MPC flag), the correction applied on observations, and the weight used for the fit. For specific observations, we also add the year of epoch or the angular parameters of the Voyager camera. The full catalogue can be requested from the authors.

Figure 3 represents the distribution of Phoebe observations and reveals its strong sparsity. Most of the observations have been realized since the mid-1990s. Other observations were from the early 20th century and there are a few data between these two periods. This strong disparity is a problem for determining the accurate orbit of Phoebe.

4. Improvement of astrometry

In this section, we try to improve the astrometry of Phoebe in two ways. The first is a new reduction of old data, and the second is the correction of bias in stellar catalogues.

4.1. New reduction of old observations

In some older publications such as [Pickering \(1908\)](#) and [Perrine \(1904\)](#), authors gave the spherical positions of the natural satellite and its relative positions to reference stars on the photographic plates. In their papers, the position of Phoebe is deduced from the positions of reference stars of the Cape Photographic Durchmusterung (CPD) catalogue. Unfortunately, the number of reference stars of this catalogue on photographic plates is small and their positions are inaccurate. As an illustration, Fig. 4 represents the statistics of the difference in angular separation⁵ of the stars in CPD and modern catalogues used in current reduction. For many stars, the difference in position between the older and recent catalogues is more than 5 arcsec. The positions of the reference stars in the CPD catalogue represent a source of systematic errors on Phoebe's positions.

Nowadays, although these plates are not available, it is possible to reduce the positions of Phoebe from data provided by these publications with modern and precise astrometric catalogues such as TYCHO and UCAC4 ([Zacharias et al. 2013](#)).

⁵ In this context, angular separation is given by $s = \sqrt{\Delta\alpha^2 \cos^2 \delta + \Delta\delta^2}$ where $\Delta\alpha$ and $\Delta\delta$ are the difference between right ascension and declination given by older and modern catalogues.

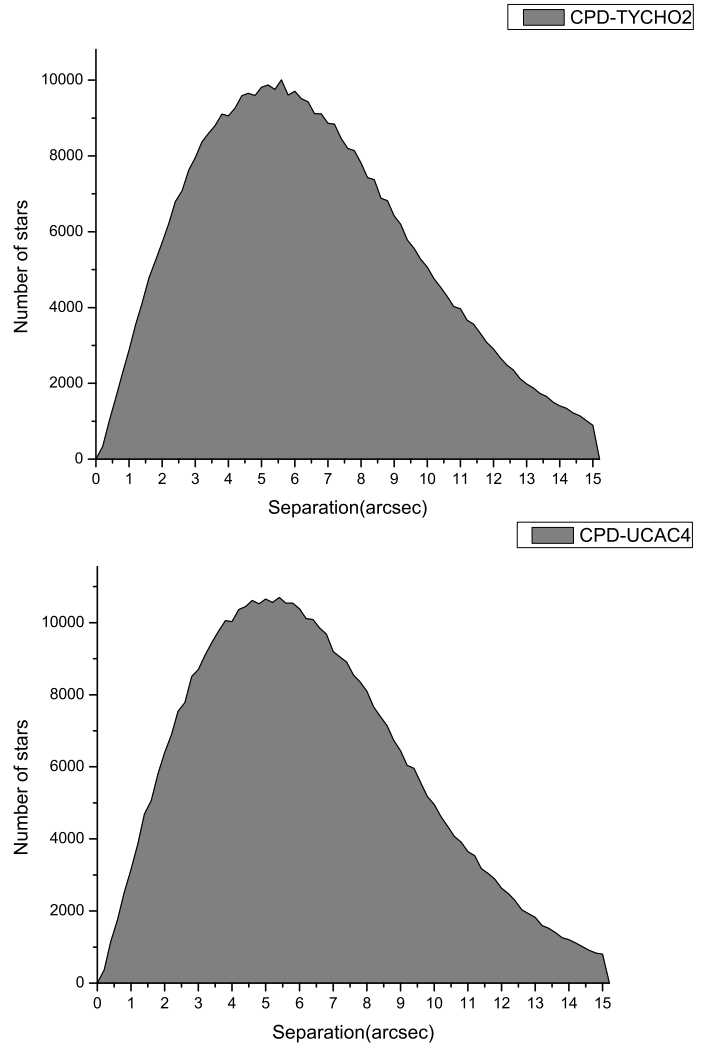


Fig. 4. Distribution of the separation of pairs of stars in the Tycho2-CPD catalogues (*top*) and UCAC4-CPD catalogues (*bottom*).

The authors gave the position of Phoebe relative to the position of the reference stars in CPD catalogue. With accurate positions of reference stars given in the modern catalogues, it is possible to deduce the spherical position of Phoebe on these plates. The method of reduction consists in identifying the reference stars on the plates and applying corrections of proper motion for each star at the date of observation. In the identification process, two stars in two different catalogues are considered a same star if the distance between their position is smaller than 15 arcsec and if the difference of their magnitude is less than one.

We tried to reduce observations from [Pickering \(1908\)](#) with this method and by using several stellar catalogues (TYCHO, USNOB, and UCAC4). The residuals obtained are at the same level of accuracy as the positions provided by Pickering (2–3 arcsec). In fact, the main difficulty of this method is the poor knowledge of the proper motion of reference stars. The observations were realized more than 100 years ago and consequently inaccurate values of proper motion lead to inaccurate positions of stars at the date of observation. For example, Fig. 5 represents the position of a reference star given by four different catalogues at J2000.0 and B1900.0 epochs using the star proper motions from these catalogues. For the J2000 epoch, the positions of the star are quite similar for the different catalogues. For B1900.0, the difference in the positions of the star

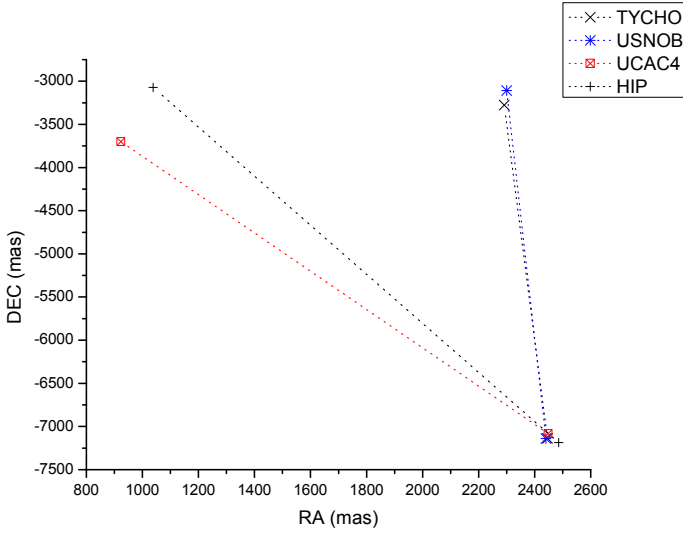


Fig. 5. The positions of one reference star in epoch J2000.0 (lower) and B1900.0 (upper) calculated with the positions and the proper motions in four different catalogues: TYCHO, USNOB1.0, UCAC4, and HIPPARCOS.

can reach more than 1 arcsec, according to the catalogue used. The HIPPARCOS catalogue is expected to give accurate values of proper motions but only a small number of stars from this catalogue is available on the plate (less than four).

The inaccurate values of proper motion of stars are currently a difficulty in the new reduction of old data. Finally, for the Pickering and Perrine observations, we used the positions provided in the papers. A possible solution would be given by the Gaia space mission and its astrometric stellar catalogue. Gaia will provide accurate positions and accurate proper motions of stars (Mignard et al. 2007). In that context, the method described in the section could be successfully applied.

4.2. Stellar catalogue correction

Stellar catalogues used in astrometric reduction are usually a source of systematic error. This error depends on the catalogue used and on the zone on the celestial sphere.

Chesley et al. (2010) propose a treatment of star catalogue biases in asteroid astrometric observations. Considering the 2MASS stellar catalogue as the reference, they have detected a bias in star positions of five stellar catalogues (Tycho-2, UCAC2, USNO B1.0, USNO A2.0, USNO A1.0), deduced from the residuals of numbered asteroid observations. They also provided a method to remove the biases for observations.

This method can be applied for the first time to a natural satellite such as Phoebe because most of its observations are in absolute coordinates, which is not the case for inner or major satellites (where inter-satellite positions are also used). Table 2 provides statistics on the catalogues used to reduce the astrometric positions of Phoebe. The stellar catalogue can be identified for 2915 observations (86.6%). Among these observations, 2625 (78.0%) use one of the five catalogues studied in Chesley et al. (2010) and are pertained to the treatment. Figure 6 represents the bias correction applied to Phoebe's observations. As many observations of Phoebe have been realized in the same zone on the celestial sphere, these observations have the same bias correction. Consequently, the number of observations for a specific bias correction is also indicated in Fig. 6. The bias removal can reach about 0.5 arcsec both in right ascension and in declination.

Table 2. Statistics on catalogues used for the reduction of the Phoebe observations.

Code	Catalogue	Number	Percentage	Time-span
a	USNO A1.0	8	0.2%	2000–2000
b	USNO SA1.0	3	0.1%	2000–2000
c	USNO A2.0	384	11.4%	1998–2012
d	USNO SA2.0	12	0.4%	2001–2003
g	Tycho-2	236	7.0%	2000–2011
l	ACT	5	0.1%	2000–2000
o	USNO B1.0	272	8.1%	2005–2012
r	UCAC2	1725	51.2%	1996–2012
t	UCAC3-beta	6	0.2%	2011–2012
u	UCAC3	76	2.3%	2010–2012
v	NOMAD	95	2.8%	2008–2009
w	CMC	2	0.1%	2010–2010
z	GSC (generic)	27	0.8%	2000–2000
	Unknown	516	15.3%	1898–2010

Notes. Code is similar to MPC flag.

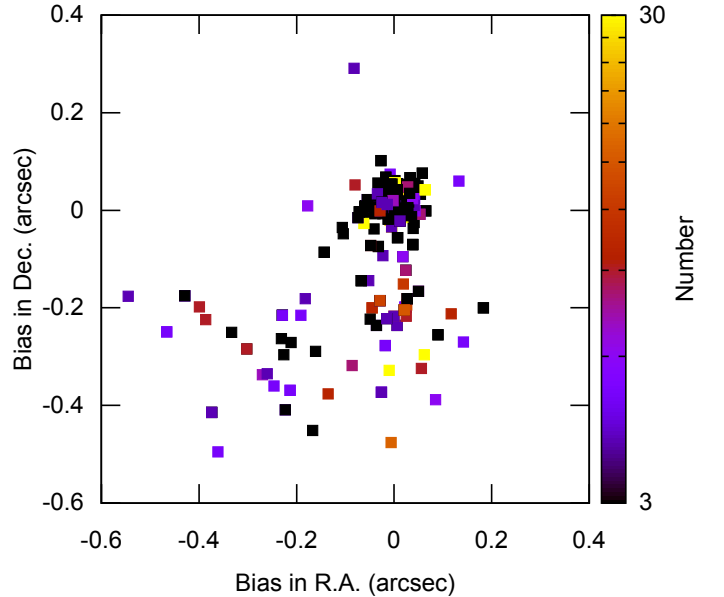


Fig. 6. Bias in right ascension and declination, and number of observations concerned by bias correction (see text).

5. Fitting process

5.1. Weighting process

The fitting process consists in the determination of the epoch state vector of Phoebe (components of position and velocity at a given epoch) that minimizes the residuals (observed position minus computed position). The classic least-squares method (LSM) is used for this determination (for more information, see for example Desmars et al. 2009a). In the LSM, a weighting matrix V_{obs} is required and usually considered as a diagonal matrix where the diagonal components are $\epsilon_i^2 = 1/\sigma_i^2$ where σ_i^2 is the estimated variance of the observation i .

For our process, we first fit the model to all observations by considering their weight as identical and $\sigma = 1$ arcsec for each observation. Then, the root mean square (rms) of the O-C in both coordinates (right ascension and declination) is computed for each reference and each observatory. Finally, we fit again the

model by considering these rms depending on reference and observatory as their weight.

5.2. Correction applied to observations

In the fitting process, observations can be corrected for different bias. In particular, as described in Sect. 4.2, correction of stellar catalogue bias can be corrected. Moreover, for prior 1940 observations, a correction in right ascension of 0.75 arcsec should be applied as explained in Jacobson (2000).

5.3. Change of reference frames

Most of observations of Phoebe are provided in absolute coordinates (right ascension and declination) but in a different reference system (ICRF⁶, J2000, B1950, true equator and equinox of the date, mean equator and equinox of the date, mean equator and equinox at 1st January of the year of observation, mean equator and equinox at 1st January of a specific year, mean equator and equinox of specific epoch).

Because the dynamical model allows the computation of the positions of Phoebe to ICRF, we have to transform observations in ICRF in order to compare computed and observed positions. The classical transformations have been done with the routine of Standards of Fundamental Astronomy (SOFA, IAU SOFA Board 2010). For transformation from B1950, the technique from Murray (1989) is applied.

The case of observations from Pickering (1908) published in B1875 is particular. The observations are first corrected from elliptical aberration, then transformed to mean equator and equinox of the date using the precession value of Newcomb (Kinoshita 1975). Finally, the coordinates are transformed to ICRF by using current values of precession (IAU Resolutions 2006).

6. Comparison of dynamical models

The flexible model developed in Sect. 2 allows the definition of four different models of the motion of Phoebe:

- Model 1 is the most complete version of the flexible model with all the perturbations: gravitational perturbations of the Sun, of all the planets and the Moon, then the perturbations of the eight main satellites of Saturn, and the flatness parameter (J_2);
- Model 2 includes the perturbations of the Sun, Jupiter, Uranus and Neptune, then the three main Saturnian satellites (Titan, Iapetus, Rhea), and the J_2 parameter;
- Model 3 includes only perturbations of the Sun, Jupiter, and Titan;
- Model 4 includes the perturbations of the Sun and Jupiter.

The aim of the four models is to determine if a full model is required to compute the position of Phoebe or if a simple model is enough. In that context, we compute the residuals of the Phoebe observations after fitting to the four models. Table 3 provides the mean and the root mean square of the O–C for the different models. If O–C were greater than 5 arcsec, they were rejected. For each case, 3319 observations are accepted among a total number of 3367 ground-based observations and all *Cassini* data are accepted, representing 223 observations.

The residuals for the four models are quite similar in particular for Models 1 and 2. As expected, the perturbations included

Table 3. Mean μ and root mean square (rms) of O–C in arcsec for the different models.

		<i>Cassini</i> obs.			
		μ	rms	μ	rms
Model 1	α	0.0089	0.4467	0.6304	2.5353
	δ	0.0564	0.4617	−0.1291	1.5782
Model 2	α	0.0088	0.4468	0.6368	2.5392
	δ	0.0564	0.4617	−0.1277	1.5782
Model 3	α	0.0084	0.4514	0.6506	2.5614
	δ	0.0532	0.4612	−0.1232	1.5793
Model 4	α	0.0079	0.4675	0.6897	2.6214
	δ	0.0458	0.4649	−0.1162	1.5813

Table 4. Post-fit residuals (mean μ and root mean square) after correction or not of bias in stellar catalogue.

		Uncorrected		Corrected		Number
		μ	rms	μ	rms	
α		0.0241	0.4590	0.0089	0.4467	3319/3367
δ		0.0857	0.4856	0.0564	0.4617	3319/3367

Notes. Residuals greater than 5 arcsec were rejected.

in the first model but not in the second⁷ have no main influence on the motion of Phoebe.

The residuals for ground-based observations represent about 4000 km in distance whereas the residuals for *Cassini* data represents only 4.7 km and provide an important constraint on the motion of Phoebe (see also Sect. 8).

With regard to Table 3, the Model 1 has the best residuals, but a simple model with only the perturbations of the Sun and Jupiter can be enough to compute the motion of Phoebe. Even if the dynamical model is enhanced, there is no clear improvement in the residuals. Although the astrometry is better (large number of observations, correction of bias, addition of space-based observations), it remains unreliable. Consequently, regular observations of Phoebe are still necessary. The method of reduction of old observations presented in Sect. 4.1 and the future Gaia stellar catalogue also appear to be a good opportunity for the improvement of the astrometry. Even if the difference in the residuals of the four models are not important, the full Model 1 provides the best residuals and we deal with this model hereafter.

Table 4 presents the residuals computed with Model 1 with and without correction of the bias in the stellar catalogue (see Sect. 4.2). The correction of the bias can help to reduce the residuals, in particular in declination.

7. Ephemeris of Phoebe

In Sect. 6, we have shown that Model 1 including the perturbations of planets and satellites, and the flatness parameter provides the best residuals. In this context, Model 1 fitted to the whole set of observations is now considered our Phoebe ephemeris and is called PH12. The statistics of the residuals for each reference is presented in Table 5. Figure 7 represents the residuals in right ascension and declination for all ground-based observations.

⁷ Gravitational perturbations of Dione, Tethys, Earth, Venus, Enceladus, Mars, Mimas, Mercury, the Moon, and Hyperion.

⁶ International Celestial Reference Frame (ICRF).

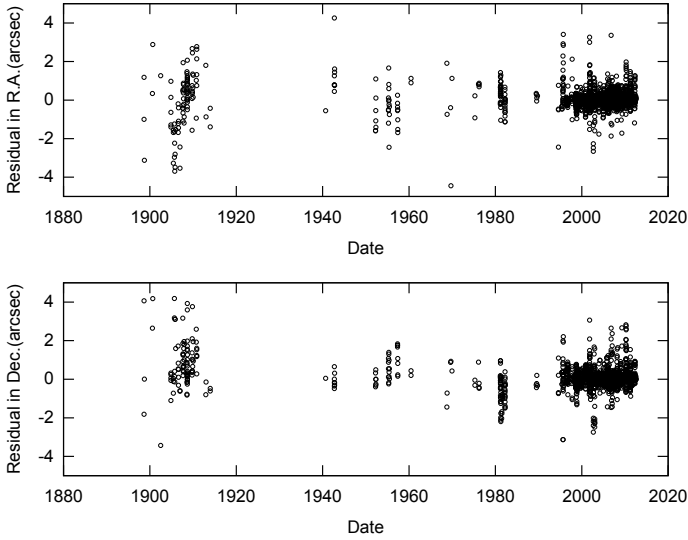


Fig. 7. Residuals in right ascension (RA) and declination (Dec.) of observations.

We have derived from the numerical integration a binary file for ephemeris computation using the Chebychev polynomials available at <ftp://ftp.imcce.fr/pub/ephem/satel/ph12/>. The PH12 ephemeris is available from 1876 to 2022.

8. Accuracy of the extrapolated position of Phoebe

The accuracy of the position of Phoebe can be estimated with statistical methods. Desmars et al. (2009a) presented and developed some statistical methods for estimating the accuracy of satellite ephemerides. Emelyanov (2010) also estimated the ephemeris precision by three different methods for all outer satellites including Phoebe. These methods are Monte Carlo processes and consist in modifying the nominal orbit with specific assumptions. The modified orbit is then compared to the nominal orbit and the process is repeated many times giving a statistical estimation of the accuracy.

In order to estimate the position uncertainty of Phoebe we use two of these methods given in Desmars et al. (2009a). The first, Monte Carlo using Covariance Matrix (MCCM) consists in adding a random noise to observations, whereas the second, bootstrap resampling (BR) consists in randomly sampling the observations. For both methods we generate $K = 200$ clones of the nominal orbit.

A measure of the accuracy in distance σ_d can be computed with this relation

$$\sigma_d(t) = \sqrt{\frac{1}{K} \sum_{k=1}^K d_k(t)^2}, \quad (1)$$

where $d_k(t)$ is the distance between the position on the nominal orbit and the position on orbit k at time t .

The position uncertainty obtained by the two methods is represented in Fig. 8. Both methods give similar results. The accuracy of the Phoebe ephemeris is quite good during the 2000 s thanks to a lot of ground-based observations and, in particular, thanks to *Cassini* data. During this period, the accuracy is about 50 km in distance (which represents about 0.01 arcsec in angular separation) and less than 10 km during the *Cassini* flyby (June 2004). Going back to the past, the accuracy deteriorates owing to the lack of observations and to inaccurate observations. The accuracy reaches about 1000 km (about 0.2 arcsec)

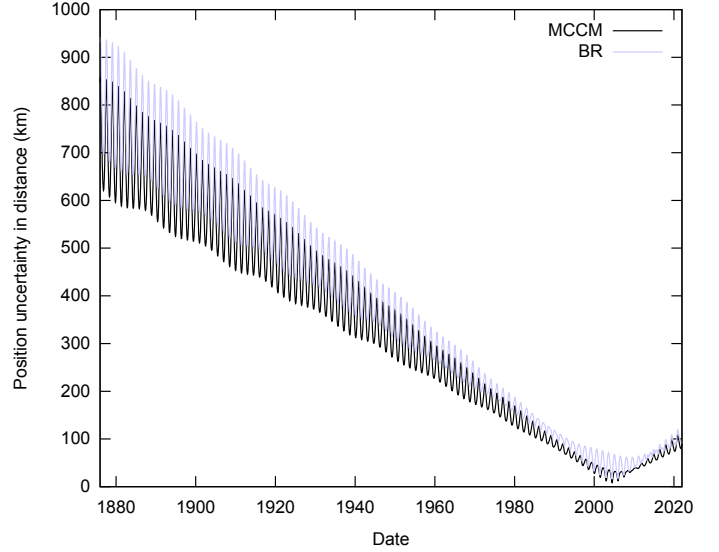


Fig. 8. Position uncertainty of the Phoebe ephemeris in distance calculated with Monte Carlo using Covariance Matrix (MCCM) and Bootstrap Resampling (BR) as a function of time in years.

in the early 20th century. In the near future, the accuracy will remain less than 100 km (about 0.02 arcsec). As a comparison, Emelyanov (2010) provides an accuracy of 0.05–0.06 arcsec during 2010–2020, but he used only 1606 observations until 2007 and the *Cassini* data were not fitted.

9. Conclusion

In this paper we have developed a flexible dynamical model of Phoebe that includes the perturbations of the Sun, all the planets, the Moon, the main Saturnian satellites, and the flatness parameter. We used the most recent theories of planets and satellites. An important step in the improvement of astrometry has been realized. We extended the catalogue of available observations of Phoebe from 1898 to 2012, then we dealt with the treatment of old observations, and the correction of bias in the stellar catalogue. Finally, we have produced a new ephemeris of Phoebe. The residuals are 0.45 arcsec and the estimated accuracy of Phoebe's position is less than 100 km for the 1990–2020 period. The comparison of the different models also shows that a simple model is enough to determine the position of Phoebe, even if the more complete model provides the best residuals.

The main constraint in the development of an accurate ephemeris of Phoebe is the astrometry. Consequently, this is the point to improve in the future. One way is to regularly observe the satellite. Another way is to improve the process of reduction. Recently, Peng et al. (2012) dealt with the geometric distortion in astrometric images and they successfully applied their method to the observations of Phoebe. Finally, the main progress for the Phoebe ephemeris will be achieved with the Gaia space mission. Indeed, Gaia will provide an accurate stellar catalogue (with accurate positions of stars and accurate proper motions). Thus, the method described in Sect. 4.1 could be effective and provide accurate old data. Moreover, CCD frames and old photographic plates could be reduced again with this stellar catalogue.

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Table 5. Statistics of astrometric residuals.

Code bib.	Reference	n/N	μ_α	μ_δ	rms_α	rms_δ	Time-span
Observations in Strugnelli & Taylor (1990)							
13	Chernykh & Chernykh (1971)	2/2	0.5854	-1.0816	1.4508	1.1405	1968–1968
34	Mulholland & Shelus (1980)	6/8	0.4136	-0.0970	0.7573	0.4710	1975–1976
Observations in Bec-Borsenberger & Rocher (1982)							
51	Pickering (1904b)	0/1	–	–	–	–	1904–1904*
52	Pickering (1904a)	0/1	–	–	–	–	1904–1904*
53	Perrine (1904)	5/5	-0.4413	-0.1065	0.9992	0.5282	1904–1904*
54	Albrecht & Smitil (1909)	11/11	-2.3742	1.3441	2.5230	2.1368	1905–1905*
55	Perrine et al. (1909)	10/10	-1.0285	0.0567	1.6793	0.7945	1906–1908*
56	MNRAS68-1908 (1908)	15/15	-0.1002	1.2292	0.6667	1.4391	1907–1907*
57	MNRAS69-1909 (1909)	23/23	0.6177	0.8746	0.8672	1.3982	1908–1908*
58	MNRAS70-1910 (1910)	12/12	0.7487	1.1878	1.3078	1.5321	1909–1910*
59	MNRAS71-1911 (1911)	7/7	1.3857	1.4758	1.8302	1.6171	1910–1910*
60	Barnard (1913)	2/2	0.4634	-0.4752	1.4150	0.5778	1912–1913*
61	Barnard (1914)	2/2	-0.9066	-0.5416	1.0271	0.5457	1913–1913*
62	van Biesbroeck (1922)	0/5	–	–	–	–	1922–1922*
63	Richmond & Nicholson (1943)	1/1	-0.5528	0.0497	0.5528	0.0497	1940–1940
64	van Biesbroeck (1944)	8/8	1.4202	-0.0666	1.8178	0.3589	1942–1942
65	Bobone (1953)	7/7	-0.7151	-0.0210	1.1774	0.3068	1952–1952
66	van Biesbroeck (1957)	13/14	-0.2220	0.5337	0.9805	0.7550	1955–1955
67	van Biesbroeck (1958)	8/8	-0.6959	1.1637	0.9347	1.3354	1957–1957
68	Roemer & Lloyd (1966)	2/2	1.0106	0.3198	1.0171	0.3413	1960–1960
69	van Biesbroeck et al. (1976)	3/4	-1.2365	0.7450	2.6553	0.7780	1969–1969
71	Veillet & Debehogne (1981)	21/21	0.5240	-0.1129	0.6780	0.5557	1981–1981
72	Debehogne (1981)						
72	Smith et al. (1981)	8/8	0.3358	-1.5206	0.8243	1.6113	1981–1981
73	Debehogne H. priv. comm. 1981	18/18	-0.2414	-0.6121	0.5502	0.8142	1982–1982
74	Dourneau et al. (1991)	5/5	0.1846	-0.2148	0.2326	0.3068	1989–1989
Observations in NSDC							
102	Ledovskaya et al. (1999)	9/9	0.0327	-0.3741	0.0795	0.3846	1998–1998*
103	Stone & Harris (2000)	38/38	-0.0901	-0.0090	0.2336	0.3486	1998–1999
104	Stone (2000)	19/19	0.0158	0.0829	0.2770	0.2154	1999–1999
105	Derek Jones comm to NSDC	7/7	-0.0385	0.0701	0.1007	0.1803	1995–1997
106	Veiga et al. (2000)	60/60	-0.0745	0.2488	0.1828	0.3986	1995–1997
107	MPC comm. (2001–2002a)	22/22	-0.2601	0.2854	0.4536	0.4563	2000–2000*
108	MPC comm. (2001–2002b)	9/9	-0.3174	0.5602	0.4106	0.5755	2000–2000*
109	MPC comm. (2001–2002c)	6/6	-0.0137	-0.0085	0.0318	0.0306	2001–2001*
110	MPC comm. (2001–2002d)	18/18	0.0451	0.1962	0.4876	0.2463	2001–2001*
111	MPC comm. (2001–2002e)	3/3	-0.4311	-0.1218	0.4765	0.3222	2001–2001
113	Comm. to NSDC (2002)	3/3	0.5031	-0.7376	0.7787	0.7420	2002–2002*
114	MPC comm. (2002)	3/3	-0.0223	0.1123	0.0453	0.1131	2002–2002*
115	Comm. to NSDC (2003a)	22/22	0.0162	-0.4346	0.6929	1.2151	2002–2003*
116	Comm. to NSDC (2003b)	38/38	0.0143	0.1562	1.1432	1.0772	1998–2003*
117	Comm. to NSDC (2003c)	16/16	-0.0630	0.0440	0.0857	0.0564	2003–2003*
118	Fienga et al. (2002)	162/162	0.0720	-0.1076	0.1855	0.2209	1998–1999*
120	Comm. to NSDC (2003d)	2/2	0.0209	-0.0003	0.0423	0.0036	2003–2003*
121	Comm. to NSDC (2003–2004)	34/34	-0.1025	0.0402	0.3632	0.1568	2003–2003*
122	Comm. to NSDC (2004)	5/5	-0.0698	-0.0109	0.0772	0.0693	2004–2004*
125	Peng & Zhang (2006)	210/210	0.0269	-0.0163	0.0592	0.0575	2003–2005*
127	Qiao et al. (2006)	115/115	-0.0074	0.0046	0.1191	0.1508	2003–2004*
137	MPC 70653 WISE Observations	5/5	-0.0593	0.4536	0.1572	0.5128	2010–2010
139	Qiao et al. (2011)	1173/1173	-0.0725	0.0251	0.1486	0.1099	2005–2008*
200	MPC24160	4/4	-0.6614	-0.0325	1.3239	0.7528	1994–1994
201	MPC38967	8/8	-0.1696	-0.0770	0.1981	0.0933	1999–1999
202	MPC40910	38/39	0.6800	-0.1226	1.2349	1.1008	1995–1998
203	MPC40911	17/17	-0.1276	0.0833	0.3540	0.3465	1998–2000
204	MPC41261	3/3	-0.1428	0.3942	0.1443	0.3948	2000–2000
205	MPC41449	10/10	-0.1008	0.1553	0.3738	0.3201	2000–2000
206	MPC41830	3/3	-0.2477	0.3817	0.2626	0.4060	2000–2000
207	MPC43757	8/8	0.3832	0.8383	1.3833	1.3944	2001–2001*

Notes. Code bib. is the bibliographic code used in the catalogue, n is the number of accepted observations for each reference, N is the total number of observations of the reference, μ_α and μ_δ are the mean of the residuals for each coordinate in arcsec, rms_α and rms_δ are the root mean squares for each coordinate in arcsec, and the time-span is also indicated. The symbol * indicates that the observations have been corrected for bias.

Table 5. continued.

Code bib.	Reference	n/N	μ_α	μ_δ	rms_α	rms_δ	Time-span
208	MPC44182	11/11	0.4054	-0.1473	0.6700	0.3502	2001–2001*
209	MPC46761	6/6	0.0458	0.0652	0.0739	0.0988	2002–2002*
210	MPC50594	7/7	0.0202	0.0621	0.0927	0.1233	2004–2004*
211	MPC51181	3/3	0.0779	0.0451	0.0801	0.0453	2004–2004
212	MPC51367	6/6	0.0582	0.0325	0.0659	0.0418	2004–2004
213	MPC51497	4/4	0.0246	0.0406	0.0403	0.0412	2004–2004*
215	MPC53171	3/3	-0.1010	-0.0407	0.1257	0.0479	1996–1996*
216	MPC53464	19/19	-0.0555	-0.0195	0.1914	0.1828	2004–2005
217	MPC53629	8/8	-0.0015	-0.0514	0.1990	0.1518	2005–2005
218	MPC53948	36/36	0.0253	-0.0210	0.2774	0.2015	2004–2005*
219	MPC54163	22/22	0.0228	0.0053	0.2475	0.1784	2004–2005*
220	MPC55508	5/5	-0.0147	0.8286	0.2316	0.8815	2005–2005*
221	MPC55975	6/6	0.3247	-0.0217	0.7387	0.2284	2005–2005*
222	MPC55976	72/72	0.0081	0.0544	0.2003	0.2815	2005–2006*
223	MPC56148	34/34	0.0053	0.0140	0.2814	0.2085	2006–2006*
224	MPC56608	33/33	0.1470	-0.0966	0.2252	0.1823	2006–2006*
225	MPC56609	1/1	0.2920	-0.2526	0.2920	0.2526	2006–2006*
226	MPC57946	7/7	-0.5558	0.6623	0.8614	1.5035	2006–2006*
227	MPC57947	3/3	1.5135	1.2146	1.9990	1.2276	2006–2006*
228	MPC58097	10/10	0.0283	-0.1981	0.2801	0.4356	2006–2006*
229	MPC58523	12/12	0.0619	-0.0699	0.5181	0.2336	2006–2006*
230	MPC58764	34/34	0.2153	0.0840	0.4006	0.6965	2007–2007*
231	MPC59029	9/9	-0.0607	0.1404	0.2501	0.5894	2007–2007*
232	MPC59305	23/23	0.1357	0.0140	0.2595	0.2105	2007–2007*
233	MPC59581	18/18	0.1645	-0.0935	0.2798	0.1918	2007–2007*
234	MPC59860	17/17	0.2167	-0.1131	0.3165	0.2162	2007–2007*
235	MPC60086	14/14	0.0379	0.0344	0.3345	0.2506	2007–2007*
236	MPC61685	1/1	1.2399	-0.2102	1.2399	0.2102	2008–2008*
237	MPC61686	3/3	0.3235	-0.1693	0.4012	0.2109	2008–2008*
238	MPC61978	30/30	-0.0142	0.0457	0.2321	0.2438	2008–2008*
239	MPC62254	20/20	0.0054	0.1463	0.3018	0.4611	2008–2008*
240	MPC62566	23/23	0.0406	-0.0538	0.2190	0.2306	2008–2008*
241	MPC62864	8/8	0.2757	0.1044	0.4027	0.1738	2008–2008
242	MPC62865	2/2	0.4133	-0.0853	0.4201	0.0967	2008–2008
243	MPC63123	14/14	0.3811	-0.2029	0.4459	0.2786	2008–2008
246	MPC63584	3/3	0.2071	0.2070	0.3299	0.2858	2008–2008
247	MPC64482	4/4	-0.0818	0.3491	0.4814	0.5066	2008–2008*
248	MPC64749	6/6	0.0161	0.5311	0.2223	0.6249	2008–2008*
249	MPC65036	9/9	0.0818	0.0208	0.1599	0.0503	2009–2009*
250	MPC65325	4/4	-0.5536	0.2077	0.6222	0.2978	2009–2009
251	MPC65628	15/15	0.1077	-0.0344	0.2934	0.2110	2009–2009
252	MPC65920	16/16	-0.1531	0.3044	0.5297	0.8047	2009–2009*
253	MPC66188	2/2	-0.5985	0.0678	0.6111	0.3130	2009–2009
254	MPC66450	6/6	0.0406	0.0375	0.2085	0.1453	2009–2009*
255	MPC66686	4/4	0.1597	-0.1154	0.2672	0.2136	2009–2009
257	MPC68212	2/2	-0.0147	-0.1053	0.0530	0.1070	2010–2010*
258	MPC68669	6/6	-0.0511	0.0803	0.2327	0.1850	2010–2010*
259	MPC69202	30/30	0.0601	0.3838	0.3508	0.8498	2010–2010*
260	MPC69724	21/21	0.6952	0.9287	0.9956	1.3787	2010–2010*
261	MPC70193	4/4	0.0733	0.1649	0.1064	0.1968	2010–2010*
262	MPC71062	1/1	0.0851	0.1336	0.0851	0.1336	2010–2010*
263	MPC73667	2/2	-0.0898	0.0278	0.1128	0.0305	2011–2011*
264	MPC74025	21/21	0.0002	-0.0429	0.1241	0.1678	2011–2011*
267	MPC74380	39/39	0.2880	0.1457	0.5760	0.5211	2011–2011*
268	MPC74811	29/29	0.1555	0.0254	0.3181	0.1771	2011–2011*
270	MPC75400	3/3	0.2502	0.0030	0.2608	0.0727	2011–2011*
271	MPC75593	9/9	0.3081	-0.0401	0.3166	0.1026	2011–2011*
272	MPC75849	6/6	0.0941	-0.2012	0.1434	0.2334	2011–2011*
Other observations							
300	Pickering (1908)	6/42	0.2576	0.9413	1.9223	3.0575	1898–1902*
301	U.S.N.O (2012)	205/205	0.0201	0.0420	0.2786	0.3851	2000–2011*
350	Voyager data – Jacobson (1998)	8/8	-6.6018	-1.1079	6.6194	1.2836	1981–1981
Jacobson (1998)							
351	Cassini data	223/223	0.6304	-0.1291	2.5353	1.5782	2004–2004
Tajeddine R., privat comm. 2012							
500	Minor Planet Center (2011)	83/83	0.1518	0.1092	0.4838	0.3341	1907–2012*