Precision concerning the complements of the "Riemann hypothesis proof"

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PRECISION CONCERNING THE COMPLEMENTS OF THE "RIEMANN HYPOTHESIS PROOF"

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Abstract. It is about the infinite partition of th real interval $(-\infty, 1/2)$ associated with each complex $\zeta(s)$-zero, or more precisely, with the two $\zeta^{*'}(s)$-zeros going with the latter. One shows that this partition is regular, with the points of division $k.\frac{1}{2}$, $k \in \mathbb{Z}$ and $k \leq 1$. In addition, by looking further this question, it appears that certain parabolic forms could play a role in quantum mechanics, at the level of elementary particles and beyond.

1. Introduction

One establishes here the connection with a result given at the end of the complements. It is about the property that each zero ρ of $\zeta(s)$ is accompanied on the axis $\Re(s) = 1/2$ by some points where $\zeta^*(s)$ takes the value $\zeta^*(\rho)$(the last of these points in number ≥ 2) satisfying moreover $\zeta^{*'}(s) = 0$.

In order to go further, and in particular to specify the partition of the interval $(-\infty, 1/2)$ in partial intervals where ω must depends on whether it is one or the other of the $\zeta^{*'}(s)$-zeros which is used, one start by making the simplest assumption, namely that the number of these zeros is 2, that their order of multiplicity is 1.

Now, it is clear that the functioning of this set of points associated to ρ (let us call "*motif*") depending on the position of ω, as it was described at the end of the complements, is incomplete or insufficient. Indeed, in order to avoid that the angle Φ remains $> -\pi$, once reached the lower limit ω_1 of the first partial interval, because it would result from it an indetermination on ω, one has to assume that Φ becomes $< -\pi$, by putting $\Phi = -\pi + \beta$ with $\beta < 0$. This implies that the situation relating of the respective tangents to the ζ^*-curves images of the half-straights $\omega S'$, ωS_1, changes direction completely, so that, in ω_1, the roles of the upper and lower half-plane were exchanged.

That is the indication of intervention of a second "motif", which would have function, in synchronizing with the first "motif", and which, wile the first sends a wave in the upper half-plane, sends a wave in the lower half-plane, waves which will cross in ω_1.

In a more precise way, one will consider a new entity (let us call it "*bimotif*") by associating

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the two motifs : the first with its two remarkable points

$$s_{11} = \frac{1}{2} + i\tau_1 \quad , \quad s_{12} = \frac{1}{2} + i\tau_2 \quad , \quad \tau_1 > \tau_2 > 0$$

the second, with the two corresponding points

$$s_{21} = \frac{\gamma}{2} - i\tau_1 \quad , \quad s_{22} = \frac{\gamma}{2} - i\tau_2 \quad , \quad \gamma > 1.$$

2. **The intrication of the two motifs in the bimotif**

It is supposed from now that the orthogonal projection ω_0 of the double zero z_0 of $\zeta^*(s)$ on the real axis, is not other that $1/2$. Then one assign the origin 0 as lower limit ω_1 of the first interval partial of the partition of $(-\infty, 1/2)$ in the initial functioning of the motif. Here why :

Let us call ω the point describing the segment of origin $1/2$, and, end 0, ω' the point describing the segment of origin $\gamma/2$, and end 0, by dividing it in the ratio that ω divides the segment $[1/2, 0]$.
By putting

$$\frac{1}{2} - \omega = \tau_1 \tan\alpha \quad , \quad \frac{\gamma}{2} - \omega' = \tau_2 \tan\alpha'$$

that result in

$$\tau_1 \tan\alpha = \frac{1}{\gamma}\tau_2 \tan\alpha'.$$

One knows (cf [1] lemma1.p8) that the upper vertical half-straight of abscissa $\omega_0(= 1/2)$ is the first, while coming by the right, to have its ζ^*-image which has points in the lower half-plane, and that the point crossing of the real axis is the origin 0. The transposition of

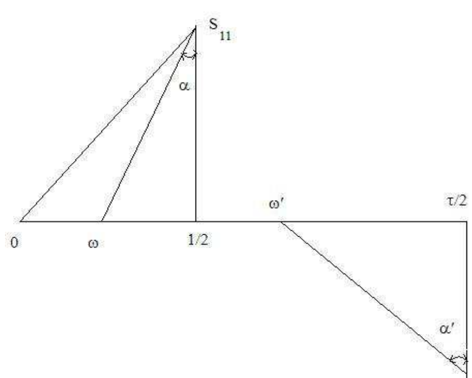

FIGURE 1

this result to the function in (x', y'): $\theta(x', y')$ reciprocal image of $\zeta^*(x, y)(= \zeta^*(s))$ by the dilatation of the real half-axis, of ratio γ

$$x' = x/\gamma \quad , \quad y' = y$$

show that the lower half-vertical of abscissa $\gamma/2$ is the first, while coming by the right, to have its θ-image which has points in the upper half-plane, and which the point of crossing the real axis is the origin.

Thus, the two waves: $\zeta^*(s)$ emitted by the first motif in the upper half-plane, $c.\zeta^*(s-\gamma)$ emitted by the second motif in the lower half-plane, cross at the point O, c being already given according to γ, such that

$$\zeta^*(\frac{1}{2}) = c.\zeta^*(\frac{1}{2}-\gamma)$$

(the two waves coincide in 1/2). One also has, according to what precedes

$$\zeta^*(0) = c.\zeta^*(0-\gamma)$$

(by making expressly the point z_0 of the first motif to intervene, and also its homologous z'_0 in the second motif).

These two relations are one of the elements of the intrication of the two motifs constituting the bimotif. Another element of this intrication is that the values α_0, α'_0 of α, α', for $\omega = 0$, are complementary. It is easy to see that is equivalent to $OS_{11} \perp OS_{22}$, or $\widehat{S_{11}OS_{22}}$= a right angle, or $2\widehat{S_{11}OS_{22}}$=a flat angle. (As this last relation suggest it, the condition posed is a manner of establishing a connection between the bimotif when ω comes on 0, and the motif isolated initially, when Φ reaches its limit value $\Phi_1 = -\pi$, what, by taking again the initial notations, corresponds to $2\widehat{s_1\omega_1 s_2}$=flat angle (cf [1] last line of p 21))

The intrication is confirmed by putting $\left\|\overrightarrow{OS_{11}}\right\| = \left\|\overrightarrow{OS_{22}}\right\|$. Then the two relations between $\overrightarrow{OS_{11}}$ and $\overrightarrow{OS_{22}}$ are expressed by

$$\begin{cases} 4\tau_1\tau_2 = \gamma \\ \frac{1}{4} + \tau_1^2 = \frac{\gamma^2}{4} + \tau_2^2 \end{cases}$$

or

$$\tau_1 = \frac{\gamma}{2} \quad , \quad \tau_2 = \frac{1}{2}$$

3. **The antibimotif**

Returning once again to the functioning of the motif isolated, one saw how the first phase starts at $\frac{1}{2}$ and finishes in 0, by integrating this functioning in that of the bimotif. The two motifs act in the same direction; from the right hand towards the left.

To be able to pass on the left from 0, as planned for the second phase, it will be necessary to associate to the bimotif, the bimotif which acts on contrary direction (like the manner of the alternating current). It will be called *antibimotif.*

Definition 1. *The antibimotif associated to the bimotif, with its motifs of respective abscissas* $\frac{1}{2}, \frac{\gamma}{2}$, *carrying its couple of remarkable points* (s_{11}, s_{22}), (s_{12}, s_{21}), *is the bimotif symmetrical* *relatively to the point* $\frac{1}{2}$. *The waves which are associated to it are thus* $\zeta^*(1-s),\ c.\zeta^*(1-s-\gamma)$, *rising from the points symmetrical of* $s_{11}, s_{22}, s_{12}, s_{21}$.

Functioning of the system:
It is necessary to have $\gamma = 1$ to make functioning the system. It is indeed under that condition, that the two characteristic triangles (rectangles-isosceles), have in common their two apex $\frac{1}{2}(1+i),\ \frac{1}{2}(1-i)$ (thus forming a square of center $\frac{1}{2}$, and whose two other apex are 0 and 1). So, one can consider that wave emitted by bimotif (functioning between 0 and $\frac{1}{2}$), and the wave emitted by the antibimotif (between $\frac{1}{2}$ and 1), are the same waves functioning between 0 and 1.

Because the system, at the end of the cycle, returns to its initial state, it is possible to repeat this cycle indefinitely throughout the real axis. Let us express the system bimotif-antibimotif (with its two right-isosceles triangles) corresponding to γ in a same context : the one of complex plane network.

Because the symmetrical role of the upper and lower half-planes in the bimotif and in the antibimotif, because also the shape of the intrication of the two motifs in these latter, one is leading to use the orthogonal network generated by $\frac{1+i}{2}$ and $\frac{1-i}{2}$.

Now, in this representation, the intrication in the system, corresponding to $\gamma = 1$, corresponds to the central symmetry relatively to the origin. Then, one adopts this same correspondence to define the intrication of the two components of the system for the general case with $\gamma =$ an appropriate integer. Moreover, the fact of generating the same network shows that they emit the same wave. γ must be such that

$$2\sqrt{1+\gamma^2}/2 \ \times \ \frac{\sqrt{2}}{2} = q \text{ (integer)}$$

or

$$1+\gamma^2 = 2q^2 \qquad\qquad q \in \mathbb{N}$$

Clearly, The points $k.\frac{1}{2},\ k \in \mathbb{Z}$ are center of symmetry for the network, and therefore the points of division for the real axis, what answers the question asked initially about the partition of the interval $(-\infty, 1/2)$.

4. **Note about the introduction of the lattice generated by the "bimotif"**

Whereas initially one considered the introduction of the case $\gamma = 1$ only like borderline case, it seems that this case is the normal case at certain point of view. Indeed, one sees well, according to the calculations of page 3, that the general case $\gamma > 1$, yields a distortion of the geometry of the space (represented by one of its directions, the axis $\Re(z) = 1/2$) with $\tau_2 = 1/2$ too lower relatively to $\tau_1 = \gamma/2$. The rotation carried out to bring the axis of symmetry of the characteristic triangle to the horizontal position like for the case $\gamma = 1$, would be thus a manner of correcting the preceding distortion, and, in the same time, of preserving the symmetry of the role played by the upper and lower half-plane. One will thus start with :

4.1. **The study of the case $\gamma = 1$.** The two waves discussed before, are then $\zeta^*(s)$ and $c.\zeta^*(s-1)$ (with the value of c corresponding to $\gamma = 1$). It was shown that these two waves emitted respectively in the upper half-plane and the lower half-plane, cross the real axis while meeting in the origin. It is specified now that they cross with vertically directed and opposite speeds. Indeed, the half-line Oz_0, having polar angle $\pi/4$, one saw in section 1 (cf [1] p12) that the "retrogression" tangent in $\zeta^*(z_0)(=0)$ by the ζ^*-image of the half-line, has a polar angle $= \frac{\pi}{4} \times 2 = \frac{\pi}{2}$.
The same goes symmetrically relatively to the real axis, except that $\pi/2$ must be replaced by $-\pi/2$. By symmetry relatively to $1/2$, there is a similar result for the fundamental antibimotif. The corresponding waves cross in 1, with opposite speeds and directed vertically. Thus, for the states of the waves in $0, 1/2, 1$, it seems the one which cut across orthogonally the real axis in these points with this order (or the conjugated wave for the opposite order), by passing through $1/2$, with the angle $\pm\frac{\pi}{4}$. Moreover, the state of this wave is the same in 0 and 1, which suggest, using the translation of $\mathbb{Z}$, to extend it to all points $k.\frac{1}{2}, k \in \mathbb{Z}$. In fact, this extension can be obtained by the rational function $\Delta(z) = (-z^2(1-z^2))^{-2}$ (with exponent -2 like for the parabolic form of weight 12: in fact the parabolic form Δ is given, with $q = \exp 2i\pi z$, by

$$\Delta(q) = (2\pi)^{12} q \prod_{n=1}^{\infty} (1-q^n)^{24}$$

but the two expressions differ only by a multiplicative constant, since they concern modular forms of the same weight 12).
Let us finish the study of the fundamental case, while returning to the property sought initially, in connection with the partition of the interval $(-\infty, 1/2)$. It is clear that the symmetries mentioned above, or only the subgroup of translations ($k \in \mathbb{Z}$) are sufficient for leading that the fundamental square of apex $0, 1, \frac{-(1+i)}{2}, \frac{(1-i)}{2}$ yields by its images, copies throughout the real axis, and in particular, one thus obtains the partition of $(-\infty, +\infty)$ by the points $k.\frac{1}{2}, k \in \mathbb{Z}$.
Also, let us note that one provide not only the symmetry of the role played in the system fundamental bimotif-antibimotif, by the two half-planes, but also the symmetry between the two directions of the real axis.

4.2. **Study of the general case.** It is sufficient to suppose that the numbers γ and $q = \frac{\sqrt{2}}{2}\sqrt{1+\gamma^2}$ are integers, in order to make the lattice generated by the characteristic triangle of the bimotif (once its axis brought back to horizontal) a sub-lattice of the fundamental lattice.
One saw before that this condition is equivalent to (q, γ) is an integer solution of the "diophantienne" equation $2q^2 - \gamma^2 = 1$, or $q\sqrt{2} - \gamma$ is invertible in the euclidean ring $\mathbb{Z}\sqrt{2}$, and consequently is of the form $(-1+\sqrt{2})^n, n \in \mathbb{N}$. It results from this that the sought solutions are an increasing sequence $\frac{\gamma_n}{q_n}$ of rational numbers tending to $\sqrt{2}$.

5. A special intrication between the fundamental level (corresponding to Δ) and each lattice corresponding to the modular forms G_2, G_3, G_4, G_5, G_7

All start with the wave Δ, operating in a space-time of dimension 2, with the space of dimension 1. The wave ΔG_2 operates in a space-time of dimension 3, with the space of dimension 2, but G_2 operates in a space-time of dimension 2 (thus with the space of dimension 1) and can consequently be identified with Δ under some condition, more exactly, when the corresponding lattice is a sub-lattice of the lattice corresponding to Δ. This intrication is repeated twice, like the number of spatial dimensions. Generally speaking, the wave $\Delta G_k, k = 2, 3, 4, , 5, 7$ operates in a space-time of dimension > 2, whereas G_k operates in a space-time of dimension 2, and the intrication appears when the lattice corresponding to this last is a sub-lattice of the fundamental lattice. This intrication is repeated as many as the number of spatial dimensions.

In terms of the physics of elementary particles, and even in terms of entities of higher scales, let us agree to associate Δ on the scale of the motif (space of dimension 1), ΔG_2 on the scale of the proton-neutron (space of dimension 2 : $\mathbb{R}^2$), ΔG_3 on the scale of the nucleus (space of dimension 3 : $\mathbb{R}^3$), ΔG_4 on the scale of the atom (space of dimension 4: $\mathbb{R}^4 = \mathbb{R}^3 \times \mathbb{R}$, $\mathbb{R}^3$ for the nucleus, $\mathbb{R}$ for the orbits of the electrons), ΔG_5 on the scale of the star (space of the atoms with a new dimension for the gravity), ΔG_7 on the scale of the galaxy (space of dimension 7 : 3 for the galaxy, and 3 probably for the supermassif black hole which is associated to it, and 1 for the gravity).

6. Consequences of the special intrications

(1) Intrication appearing thanks to the property that the lattice associated to G_k is a sub-lattice of that associated to Δ, since the symmetries of a sub-lattice are those of the lattice, the answer to the question asked initially will be the same: the time is marked by the sequence of points of the real axis $\left\{k.\frac{1}{2},\ k \in \mathbb{Z}\right\}$. Thus, for all the scales, it is the same time.
(2) On the scale of the star, or the galaxy, since a cell representing the sub-lattice is made by cells representing the lattice of Δ, the electrical current is annihilated inside,

because each of the interior side of the elementary cells is covered in one direction and the opposite direction. There remains only the current of one of edge. The interior matter having lost its electric and luminous properties, and became this black matter of which is question in the cosmos.

(3) On the scale of the galaxy, while returning to the antibimotif-bimotif complex which is associated to it (after switching the axis of the bimotif on the real axis, and the replacement of symmetry relatively to 1/2, by symmetry relatively to 0), one saw higher that the position in space, and the time q_n made to reach this distance are connected by the relation $2q_n^2 - \gamma_n = 1$ (what implies the double inequality $q_n < \gamma_n < q_n\sqrt{2}$) and that $q_n\sqrt{2} - \gamma_n = (\sqrt{2}-1)^n$.
It results from this last relation that $\sqrt{2} - \dfrac{\gamma_n}{q_n} = \dfrac{(\sqrt{2}-1)^n}{q_n} \to 0$ while decreasing. Thus $\dfrac{\gamma_n}{q_n}$ is an increasing sequence, and consequently whereas q_n is measured with a uniformly passing time, it is not similar for γ_n : the measuring unit of the space does nothing but increase, thus giving the impression of an accelerated expansion of the space, unless one suppose that the space is marked previously with a speed exceeding the one of the light.

(4) Solar neutrinos: the deficit of solar neutrinos in the detectors could be due to the special intrication of the fundamental lattice (corresponding to Δ) and the sub-lattice (G_5). Indeed, the last which appears at long distances, sees its cells (assemblies of elementary cells) to lose the electromagnetic properties of interior cells, as explained precedently (black matter), as that only the fraction of elementary cells not belonging to these last, can react with the detector.

7. **Conclusion**

The development which one gave to the initial question, shows that the macroscopic world is already annouced in the microscopic world. Like this, the black matter, which is revealed only at the level of the star and the galaxy, is already present at the level of the motif, through the existence of a sector (of apex ω), between two successive motifs, sector which, contrary to the preceeding sector, is not corresponding to any property of the matter, except for this form of the gravity that goes with the appearance of each sector and point out by the term $2k\pi i$ in $(\ln_{\omega,k}((1-s)\zeta^*(s) + 2k\pi i))^{-1}$ in order to bring the system down the fundamental state (cf.[1] p16).

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