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# PRECISION CONCERNING THE COMPLEMENTS OF THE "RIEMANN HYPOTHESIS PROOF"

MUSTAPHA BEKKHOUCHA

ABSTRACT. It is about the infinite partition of the real interval  $(-\infty, 1/2)$  associated with each complex  $\zeta(s)$ -zero, or more precisely, with the two  $\zeta^{*'}(s)$ -zeros going with the latter. One shows that this partition is regular, with the points of division  $k \cdot \frac{1}{2}$ ,  $k \in \mathbb{Z}$  and  $k \leq 1$ .

## 1. Introduction

One establishes here the connection with a result given at the end of the complements. It is about the property that each zero  $\rho$  of  $\zeta(s)$  is accompanied on the axis  $Re(s) = 1/2$  by some points where  $\zeta^*(s)$  takes the value  $\zeta^*(\rho)$  (the last of these points in number  $\geq 2$ ) satisfying moreover  $\zeta^{*'}(s) = 0$ .

In order to go further, and in particular to specify the partition of the interval  $(-\infty, 1/2)$  in partial intervals where  $\omega$  must depend on whether it is one or the other of the  $\zeta^{*'}(s)$ -zeros which is used, one starts by making the simplest assumption, namely that the number of these zeros is 2, that their order of multiplicity is 1.

Now, it is clear that the functioning of this set of points associated to  $\rho$  (let us call "motif") depending on the position of  $\omega$ , as it was described at the end of the complements, is incomplete or insufficient. Indeed, in order to avoid that the angle  $\Phi$  remains  $> -\pi$ , once reached the lower limit  $\omega_1$  of the first partial interval, because it would result from it an indetermination on  $\omega$ , one has to assume that  $\Phi$  becomes  $< -\pi$ , by putting  $\Phi = -\pi + \beta$  with  $\beta < 0$ . This implies that the situation relating of the respective tangents to the  $\zeta^*$ -curves images of the half-straight lines  $\omega S'$ ,  $\omega S_1$ , changes direction completely, so that, in  $\omega_1$ , the roles of the upper and lower half-plane were exchanged.

That is the indication of intervention of a second "motif", which would have function, in synchronising with the first "motif", and which, while the first sends a wave in the upper half-plane, sends a wave in the lower half-plane, waves which will cross in  $\omega_1$ .

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In a more precise way, one will consider a new entity (let us call it "bimotif") by associating the two motifs : the first with its two remarkable points

$$s_{11} = \frac{1}{2} + i\tau_1 \quad , \quad s_{12} = \frac{1}{2} + i\tau_2 \quad , \quad \tau_1 > \tau_2 > 0$$

the second, with the two corresponding points

$$s_{21} = \frac{\gamma}{2} - i\tau_1 \quad , \quad s_{22} = \frac{\gamma}{2} - i\tau_2 \quad , \quad \gamma > 1.$$

## 2. The intrication of the two motifs in the bimotif

It is supposed from now that the orthogonal projection  $\omega_0$  of the double zero  $z_0$  of  $\zeta^*(s)$  on the real axis, is not other than  $1/2$ . Then one assign the origin 0 as lower limit  $\omega_1$  of the first interval partial of the partition of  $(-\infty, 1/2)$  in the initial functioning of the motif. Here why :

Let us call  $\omega$  the point describing the segment of origin  $1/2$ , and, end 0,  $\omega'$  the point describing the segment of origin  $\gamma/2$ , and end 0, by dividing it in the ratio that  $\omega$  divides the segment  $[1/2, 0]$ .

By putting

$$\frac{1}{2} - \omega = \tau_1 \tan \alpha \quad , \quad \frac{\gamma}{2} - \omega' = \tau_2 \tan \alpha'$$

that result in

$$\tau_1 \tan \alpha = \frac{1}{\gamma} \tau_2 \tan \alpha'.$$

One knows (cf [1] lemma1.p8) that the upper vertical half-straight of abscissa  $\omega_0 (= 1/2)$  is the first, while coming by the right, to have its  $\zeta^*$ -image which has points in the lower half-plane, and that the point crossing of the real axis is the origin 0.

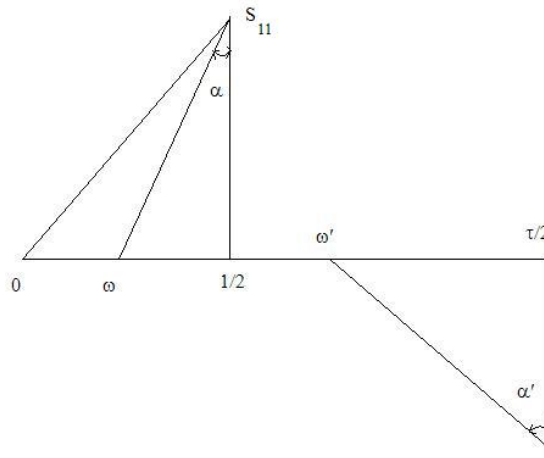


FIGURE 1

The transposition of this result to the function in  $(x', y')$ :  $\theta(x', y')$  reciprocal image of  $\zeta^*(x, y)(= \zeta^*(s))$  by the dilatation of the real half-axis, of ratio  $\gamma$

$$x' = x/\gamma \quad , \quad y' = y$$

show that the lower half-vertical of abscissa  $\gamma/2$  is the first, while coming by the right, to have its  $\theta$ -image which has points in the upper half-plane, and which the point of crossing the real axis is the origin.

Thus, the two waves:  $\zeta^*(s)$  emitted by the first motif in the upper half-plane,  $c.\zeta^*(s - \gamma)$  emitted by the second motif in the lower half-plane, cross at the point O,  $c$  being already given according to  $\gamma$ , such that

$$\zeta^*\left(\frac{1}{2}\right) = c.\zeta^*\left(\frac{1}{2} - \gamma\right)$$

(the two waves coincide in  $1/2$ ). One also has, according to what precedes

$$\zeta^*(0) = c.\zeta^*(0 - \gamma)$$

(by making expressly the point  $z_0$  of the first motif to intervene, and also its homologous  $z'_0$  in the second motif).

These two relations are one of the elements of the intrication of the two motifs constituting the bimotif. Another element of this intrication is that the values  $\alpha_0, \alpha'_0$  of  $\alpha, \alpha'$ , for  $\omega = 0$ , are complementary. It is easy to see that is equivalent to  $OS_{11} \perp OS_{22}$ , or  $\widehat{S_{11}OS_{22}} =$  a right angle, or  $2\widehat{S_{11}OS_{22}} =$  a flat angle. (As this last relation suggest it, the condition posed is a manner of establishing a connection between the bimotif when  $\omega$  comes on 0, and the motif isolated initially, when  $\Phi$  reaches its limit value  $\Phi_1 = -\pi$ , what, by taking again the initial notations, corresponds to  $2\widehat{s_1\omega_1s_2} =$  flat angle (cf last line of p...))

The intrication is confirmed by posing that  $\left\| \overrightarrow{OS_{11}} \right\| = \left\| \overrightarrow{OS_{22}} \right\|$ . Then the two relations between  $\overrightarrow{OS_{11}}$  and  $\overrightarrow{OS_{22}}$  are expressed by

$$\begin{cases} 4\tau_1\tau_2 = \gamma \\ \frac{1}{4} + \tau_1^2 = \frac{\gamma^2}{4} + \tau_2^2 \end{cases}$$

or

$$\tau_1 = \frac{\gamma}{2} \quad , \quad \tau_2 = \frac{1}{2}$$

### 3. The antibimotif

Returning once again to the functioning of the motif isolated, one saw how the first phase starts at  $\frac{1}{2}$  and finishes in 0, by integrating this functioning in that of the bimotif. The two motifs act in the same direction; from the right hand towards the left.

To be able to pass on the left from 0, as planned for the second phase, it will be necessary to associate to the bimotif, the bimotif which acts on contrary direction (like the manner of the alternating current). It will be called *antibimotif*.

**Definition 1.** *The antibimotif associated to the bimotif, with its motifs of respective abscissas  $\frac{1}{2}, \frac{\gamma}{2}$ , carrying its couple of remarkable points  $(s_{11}, s_{22}), (s_{12}, s_{21})$ , is the bimotif symmetrical relatively to the point  $\frac{1}{2}$ . The waves which are associated to it are thus  $\zeta^*(1-s), c.\zeta^*(1-s-\gamma)$ , rising from the points symmetrical of  $s_{11}, s_{22}, s_{12}, s_{21}$ .*

Functioning of the system:

It is necessary to have  $\gamma = 1$  to make functioning the system. It is indeed under that condition, that the two characteristic triangles (rectangles-isosceles), have in common their two apex  $\frac{1}{2}(1+i), \frac{1}{2}(1-i)$  (thus forming a square of center  $\frac{1}{2}$ , and whose two other apex are 0 and 1). So, one can consider that wave emitted by bimotif (functioning between 0 and  $\frac{1}{2}$ ), and the wave emitted by the antibimotif (between  $\frac{1}{2}$  and 1), are the same waves functioning between 0 and 1.

Because the system, at the end of the cycle, returns to its initial state, it is possible to repeat this cycle indefinitely throughout the real axis. Let us express the system bimotif-antibimotif (with its two right-isosceles triangles) corresponding to  $\gamma$  in a same context : the one of complex plane network.

Because the symmetrical role of the upper and lower half-planes in the bimotif and in the antibimotif, because also the shape of the intrication of the two motifs in these latter, one is leading to use the orthogonal network generated by  $\frac{1+i}{2}$  and  $\frac{1-i}{2}$ .

Now, in this representation, the intrication in the system, corresponding to  $\gamma = 1$ , corresponds to the central symmetry relatively to the origin. Then, one adopts this same correspondance to define the intrication of the two components of the system for the general case with  $\gamma =$  an appropriate integer. Moreover, the fact of generating the same network shows that they emit the same wave.  $\gamma$  must be such that

$$2\sqrt{1+\gamma^2}/2 \times \frac{\sqrt{2}}{2} = q \text{ (integer)}$$

or

$$1 + \gamma^2 = 2q^2 \quad q \in \mathbb{N}$$

Clearly, The points  $k.\frac{1}{2}, k \in \mathbb{Z}$  are center of symmetry for the network, and therefore the points of division for the real axis, what answers the question asked initially about the partition of the interval  $(-\infty, 1/2)$ .

## REFERENCES

- [1] Bekkhoucha, M., *Riemann hypothesis proof*, submitted.

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