Modelling effective permeability of fracture networks in permeable rock formation by singular integral equations method

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Abstract

In this paper, theoretical and numerical formulations of plane steady-state fluid flow in a fractured porous rock are used to investigate its effective permeability. If the far field inflow is uniform, the theoretical solution shows that the pressure field in the matrix is a function of the discharge in the fracture network. A numerical resolution based on singular integral equations is employed to derive the general problem of many intersected fractures in order to obtain the pressure field in anisotropic matrix. This solution allows computing the flux in the fractures which is the key issue for upscalling the equivalent permeability. This paper presents in detail the method for deriving the equivalent permeability from this solution. This method is applied to two real cases: an Excavation Damage Zone (EDZ) around a deep underground gallery and a geological rock formation presenting several families of fractures. The results of the both cases show that the developed method provides an easy and efficient way to determine the equivalent permeability of the fractured porous rock medium. This equivalent permeability can be implemented in analytical and numerical tools for continuous media towards estimating the flow characteristics in the rock formation.

Keywords: facture network, porous rock, steady-state flow, effective permeability

1 Introduction

In the context of important industrial applications such as underground nuclear waste disposal in claystone, geological CO_2 storage or hydrocabure reservoir, the numerical modelling of fluid flow in fractured porous materials is intensively investigated. More specifically, in damaged zones, cracks and geological faults generally have a great effect on the effective permeability of porous media or rock masses. The interest of this question also extends to other fields, such as hydrogeology for water resources management in aquifers, etc.

Different approaches are used to study the problem of flow in porous media containing fractures with mass exchange between matrix and fracture, a review of which can be found in Sanchez-Vila et al.[16], Goméz-Hernandez and Wen [7] and Renard and de Marsily [15]. However, these works are essentially based on some numerical-empirical modelling. Pouya and Fouché [11] presented some rigorous theoretical-numerical results based on homogenisation theories for the equivalent permeability of heterogeneous or fractured media. The self-consistent scheme is investigated by Dormieux and Kondo [5] and Barthélémy [1] for upscalling the effective permeability of cracked porous media. In their method, cracks are assimilated to ellipsoidal inclusion obeying to a Darcy's law of flow. Following a different way, based on theory of Cauchy integral, Liolios and Exadaktylos [9] studied mathematically the problem of plane steady-state flow in an infinite isotropic porous media with multiple non-intersecting fractures. In this approach, the fracture is considered as zero thickness discontinuity line and complex numbers are used. Recently, Pouya and Ghabezloo [14] used a direct approach to obtain a general potential pressure solution function of flux in the fracture network for anisotropic matrix containing intersecting curvilinear fractures. By continuing this work, Pouya and Vu [13] employed the singular integral equation method to establish a fast numerical calculation that can be used to derive the general solution and to show the fluid flow around many intersecting fractures.

Using the recent theoretical and numerical advances, this paper presents a method to evaluate the effective permeability of the permeable rock formation containing a dense fracture network. The numerical solution of pressure and discharge in the fracture, and then, the equivalent permeability using the formulas of average velocity and of average pressure gradient. The numerical calculation is applied to two cases. The first one is an EDZ around a deep underground gallery where the fractures are curved surfaces with a well defined shape and are almost regularly distributed in this zone. The second is a fractured geological formation with several families of fractures in which the fractures are plane surfaces and their characteristic parameters (orientation, length and position) are defined by a stochastic law. This approach provides an easy and efficient tool to determine the effective permeability of fractured porous rock formation which includes large applied areas.

2 Mathematical formulation

An infinite homogeneous body Ω containing a fracture network is considered (fig.1). Fracture number m is denoted Γ_m. It is generally presented by a smooth curve $\underline{z}^m$ of the curvilinear abscise s. The intersection points of two or more fractures and the extremities of fractures ending in the matrix constitute a set of singular point S.
Fluid velocity $\underline{v}(\underline{x})$ in the matrix is given by Darcy's law:

$$\forall \underline{x} \in \Omega - \Gamma \qquad \underline{v}(\underline{x}) = -\mathbf{k}(\underline{x}).\underline{\nabla} p(\underline{x}) \tag{1}$$

where **k** is the matrix permeability and $p(\underline{x})$ the pressure field. The flow through the discontinuity is usually expressed in terms of the Poiseuille's law:

$$\forall s \in \Gamma \qquad q(s) = -c(s).\partial_s p \tag{2}$$

where, s is the abscise along the fracture and c is the fracture's conductivity.

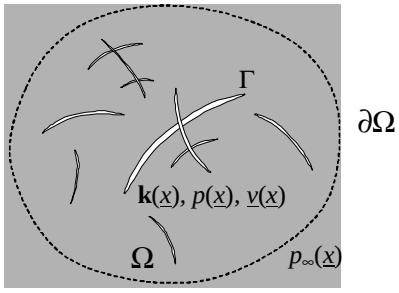


Figure 1: Rock mass containing fractures

Mass conservation in the matrix reads:

$$\forall \underline{x} \in \Omega - \Gamma \qquad \underline{\nabla}.\underline{v}(\underline{x}) = 0 \tag{3}$$

The fracture-matrix mass exchange law on the fracture excluding singular points is obtained by considering masse balance in a portion ds of the fractures (fig.2a)

$$\forall s \in \Gamma \qquad [\![\underline{v}(\underline{z})]\!]\underline{n}(s) + \partial_s p(s) = 0 \tag{4}$$

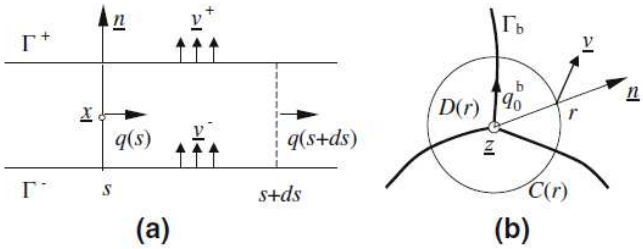


Figure 2: a. Mass exchange between matrix and fracture. b Mass balance in a disc around an intersection point (Pouya and Ghabezloo [14])

At the intersection points of fractures (fig.2b), Pouya and Ghabezloo [14] analyzed the masse balance in a small disc centred on the singular point and deduced the following relationship:

$$\forall \underline{z} \in S \qquad \underline{\nabla}.\underline{v}(\underline{x}) + \left(\sum_b q_0^b \right) \delta(\underline{x} - \underline{z}) = 0 \tag{5}$$

with q_0^b is the outgoing flow on the branch of fracture b and δ is the Dirac distribution. Nevertheless, a deeper mathematical analysis allows us to go further than eqn. (5) (Pouya and Vu [13]) and shows that:

$$\sum_b q_0^b = 0 \tag{6}$$

In order to calculate the effective permeability, it is sufficient to impose a constant pressure gradient at infinity: $p_\infty(\underline{x}) = \underline{A}.\underline{x}$. The previous problem was derived rigorously by Pouya and Ghabezloo [14] and a general solution of potential was given:

$$p(\underline{x}) = p_\infty(\underline{x}) + \frac{1}{2\pi\kappa} \sum_{\mathrm{m}} \int_{\Gamma_{\mathrm{m}}} q^{\mathrm{m}}(s) \frac{\underline{x} - \underline{z}^{\mathrm{m}}(s)}{\left\| \sqrt{\mathbf{k}^{-1}}.\left[\underline{x} - \underline{z}^{\mathrm{m}}(s) \right] \right\|^2} . \mathbf{k}^{-1} . \underline{t}^{\mathrm{m}}(s)\, ds \tag{7}$$

where, $\underline{x}$ is a current point in the material, $\underline{z}^{\mathrm{m}}(s)$ is the point the Γ_m at the curvilinear abscise s, $q^m(s)$ and $\underline{t}^m(s)$ are respectively the discharge along the fracture and the unit vector tangent to the fracture at this point, and κ is the square root of determinant of $\boldsymbol{k}$: $\kappa = \sqrt{|\mathbf{k}|}$.

3 Numerical calculation

When field point $\underline{x}$ is located in the fractures; we obtain a singular integral equation of unknown pressure field in the fracture networks by replacing eqn. (2) into eqn. (7). This equation was resolved numerically by collocation method (Bonnet [3]) which consists of enforcing exactly eqn.(7) at finite number of points called collocation point.

Curvilinear fractures are approximated by a series of small linear segments and are discretized by E elements numbered n and denoted E_n ($1 \le n \le E$). Thus, eqn.(7) is written as follows:

$$p(\underline{x}) = p_\infty(\underline{x}) + \sum_{n=1}^{N} I_n \tag{8}$$

with:

$$I_n(\underline{x}) = \int_{E_{\mathrm{n}}} q^{\mathrm{n}}(s) \frac{\underline{x} - \underline{z}^{\mathrm{n}}(s)}{\left\| \sqrt{\mathbf{k}^{-1}}.\left[\underline{x} - \underline{z}^{\mathrm{n}}(s) \right] \right\|^2} . \mathbf{k}^{-1} . \underline{t}^{\mathrm{n}}\, ds \tag{9}$$

Two types of elements are distinguished: extremity elements and current elements. Linear interpolation of pressure is used for current elements. General theoretical results of velocity field singularity around a fracture tip lead us to choose an interpolation function for the flux q(s) corresponding to a variation as $s^{1/2}$ where s is the distance on the fracture line to the extremity point. Therefore, interpolation function for pressure on the left extremity elements (tip at $s=0$), current element, and right extremity elements (tip at $s=L$) are respectively expressed as following:

$$p(s) = p_1 + \frac{p_2 - p_1}{\sqrt{L}^3}\sqrt{s}^3$$

$$p(s) = p_1 + \frac{p_2 - p_1}{L}s \qquad (10)$$

$$p(s) = p_2 - \frac{p_2 - p_1}{\sqrt{L}^3}\sqrt{L-s}^3$$

At first, a collocation point $\underline{x}$ is selected per element which verifies $I_n(\underline{x}) = 0$. And then, the set of extremity points of fracture network is added as supplementary collocation points. It should be noted that no matter how the fractures intersect and how the discretization is made, the number of nodes N is always smaller than number of collocation points M. Otherwise, this method of selection of collocation points always leads to the number of equations M larger than the number of nodal pressure unknown. It is interesting that all elementary integrals $I_n(\underline{x})$ in eqn. (9) can be calculated analytically by using the variable interpolations in eqn. (10), which provides a very fast calculation method.

Enforcing eqn. (7) in M collocation points leads to following matrix equation:

$$\mathbf{H}.P=Y \qquad (11)$$

where, P(Nx1) is the column of nodal pressure unknown, Y(Mx1) is the column of infinite pressure field at collocation points and $\mathbf{H}$(MxN) is computed from the assembly operation after computing all elementary integrals. The approximated solution can be found by the least squares fitting method. Pouya and Vu [13] have well shown the validity of this method by comparing the numerical solution of a single straight fracture in an infinite homogeneous media with the known closed-form solution of the same problem.

Once the eqn. (11) is solved, the nodal pressure P$\{p_n\}$ ($1 \le n \le N$) are known, using eqn. (2), we can deduce the flux in each element. In addition, in the post-processing stage, eqn. (11) shall be employed again to compute directly the field pressure in porous matrix.

For a homogeneous rock mass Ω, containing a fractures network Γ_m that is considered as the limit case of thin permeable layers, the average velocity and pressure gradient in Ω is given, respectively, by:

$$\underline{V} \equiv \frac{1}{\Omega}\left[\int_{\Omega}\underline{v}d\Omega + \sum_m \int_{\Gamma_m} q\underline{t}ds\right];\ \underline{G} \equiv \frac{1}{\Omega}\int_{\Omega}\underline{\nabla}p d\Omega \qquad (12)$$

Let us to suppose that a linear pressure $p(\underline{x}) = \underline{A}.\underline{x}$ is applied at the infinity. The pressure field in matrix is known, especially on the boundary $\partial\Omega$, hence, the vector $\underline{G}$ can be deduced such as:

$$\underline{G} \equiv \frac{1}{\Omega}\int_{\partial\Omega} p.\underline{n}ds \tag{13}$$

where, $\underline{n}$ is the unit outward normal on $\partial\Omega$. Besides, Pouya and Fouché [11] demonstrated that $\frac{1}{\Omega}\int_{\Omega}\underline{v}d\Omega = -\mathbf{k}.\underline{G}$. Hence, the linearity of all velocities and flux with respect to $\underline{G}$ implies that there exists a tensor $\mathbf{k}^f$ that satisfies the following relationship:

$$\frac{1}{\Omega}\sum_m \int_{\Gamma_m} q\underline{t}ds = -\mathbf{k}^f.\underline{G} \tag{14}$$

Thus, under boundary condition of linear pressure, the global permeability tensor of Ω domain is given by $\mathbf{k}^p=\mathbf{k}+\mathbf{k}^f$. In the case of weak fracture density, the Mori-Tanaka estimation of $\mathbf{k}^p$ is determined by neglecting the interaction between fractures i.e. assuming that the flux q in the left-hand side of eqn. (14) is deduced from the results obtained for a single fracture in an infinite body which is given in Pouya and Ghabezloo [14]. However, in the case of high fracture density (or dense fracture networks), the integral of flux on the fracture network, i.e. the term described in left-hand side of eqn. (13) could be computed numerically for a fracture network discretized by E element.
In the next section, we shall take up some real examples for calculating the effective permeability in using the recent advance of theoretical development and numerical tools which are described above.

4 Application

4.1 Effective permeability of EDZ

Excavation of underground galleries generates cracks and fractures in a zone around the gallery called Excavation Damaged Zone (EDZ). The effect of fractures on the hydraulic properties of EDZ can be estimated through a numerical method taking into account the complex geometry and density of discontinuities. Three zones around the gallery (fig. 3) can be distinguished: the zone crossed by fractures; damaged or micro-fractured zone beyond fractured zone and the zone of intact rock beyond EDZ. The first zone is considered here to calculate its equivalent permeability. The fractures appearing in this zone has more or less regular geometry with the same shape, spacing, extensions that are repeated along the axis of gallery.

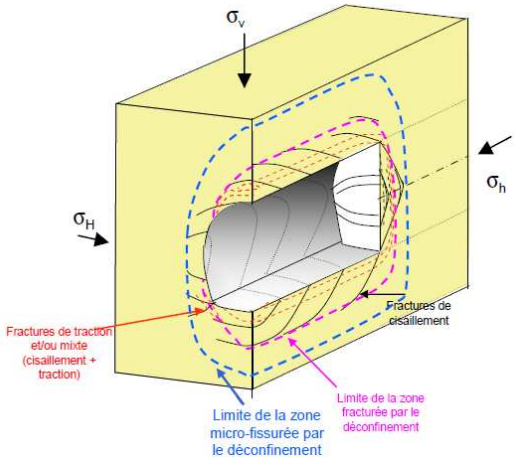


Figure 3: Representation of the EDZ around the gallery and the shape of the fracture (ANDRA 2008 [18])

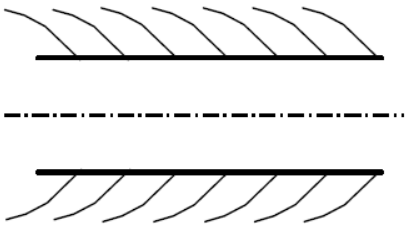


Figure 4: Representation of the shape of the "chevron" fractures in a vertical plane

Field observations show that the section of the fracture surfaces in the plane orthogonal to gallery axis (Z=const) is elliptical; the trace of fracture in vertical plane has a curvature form as a hyperbolic tangent function; the extension fracture in the horizontal fracture is more or less constant. This observation leads to take the following expression (eqn. 15) to represent the equation of the fracture surface Pouya et al. [17].

$$\frac{X^2}{a^2}+\frac{Y^2}{b^2\tanh(\lambda Z)}=1 \qquad (15)$$

The parameters a, b, λ depend on the orientation of the gallery. And then, the values: $a=b=4$, $\lambda=0.57$ are determined by comparing the trace of theoretical surface to observed fracture in field. In addition, the last parameter d=60 cm of spacing between fractures is fixed by the field measurement.

The above geometrical model is used to evaluate the effective permeability of EDZ (fig. 5) with the isotropic matrix permeability k = 10^{-12}m/s and the hydraulic conductivity of fracture c=10^{-11}m^2/s. The chevron fracture form is discretized by series of small linear segments.

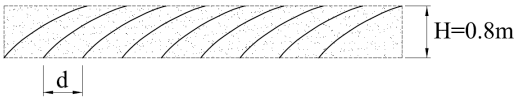


Figure 5: Representative segment of EDZ in calculating the effective permeability

The linear pressure $p(\underline{x}) = \underline{A}.\underline{x}$ is imposed at infinity. By solving the problem for two vectors of pressure gradient A_1=(1,0) and A_2=(0,1), two average velocity vectors $\underline{V}^1$, $\underline{V}^2$ and two average pressure gradient vectors $\underline{G}^1$, $\underline{G}^2$ are obtained. The equivalent permeability $\mathbf{k}^p$ is then deduced from the equation $\underline{V} = -\mathbf{k}^p.\underline{G}$. Launching the numerical program for the EDZ zone, the following effective permeability matrix is found:

$$\mathbf{k}^p = \begin{bmatrix} 9.09 & -5,39 \\ -5,26 & 8.19 \end{bmatrix} \text{ x } 10^{-12}\text{(m/s)}$$

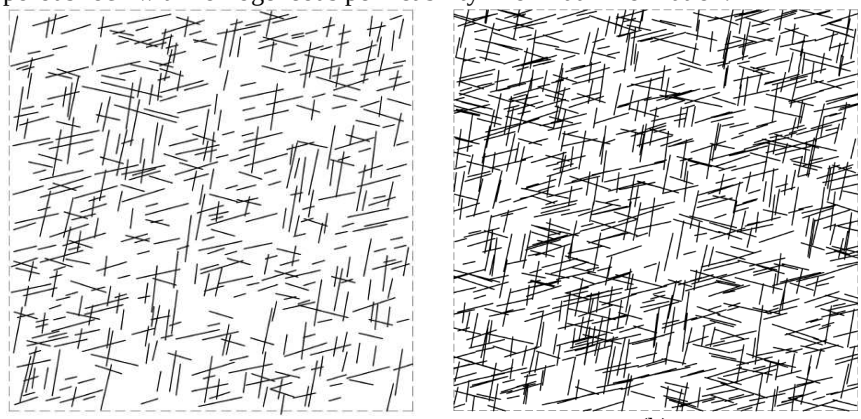
This result shows that the effective permeability matrix is almost symmetric according to the results announced by Pouya and Fouché [11]. The principal directions of this permeability tensor can be calculated easily. They are found to be approximately parallel and orthogonal to the fractures surfaces. Otherwise, permeability predicted in this way is smaller than that given by the Mori-Tanaka estimation based on theoretical results of Pouya and Ghabezloo [14]. The difference can be explained by the fact that the Mori-Tanaka estimation does not take into account the fractures interaction.

4.2 Fracture network in a permeable rock

The study of fluid flow in fractured permeable rocks needs, first, a geometrical model of fractures that may be described the deterministic models (Kolditz and Clauser [8]) or by stochastic approach (Cacas et al. [4]; Billaux [2]; Gervais [6] and Maleki and Pouya [10]). The stochastic simulation of joint network is based on a hierarchical probabilistic model that realistically reproduces fracture connectivity using minimal data such as the number of fracture sets, the fracture length, spacing and density. In practice, the stochastic distribution is often used for modelling fluid flow in hydrogeology or for natural geological reservoir since their large flexibility. In this work, the study introduced by Maleki and Pouya [10] is adopted. A square domain is characterized by two corners (X_{min},Y_{min}) and (X_{max},Y_{max}). Three fracture families are defined each one by a density ρ (number of fracture per area unit), the half of fracture length r = L/2 and the orientation θ (angle between fracture line and x-axis, $0 \leq \theta < \pi$). The fracture line is completely determined by these parameters and the coordinates (x_0,y_0) of its center. A uniform distribution of the fracture center points and of the angle θ; and an exponential law of length distribution are supposed.

In order to study the effect of fracture density on the effective permeability of fractured porous media, three families are generated in a domain 10x10m^2 of

rock formation. Several configurations are considered for which all the fractures parameters are constant except for the density that increases (fig. 6). Herein, we introduce the dimensionless parameter $\lambda = c / 2\pi k\xi$, with $\xi = 1m$; and then we use the numerical procedure to compute the equivalent permeability of fractured porous rock with homogeneous permeability *k* for matrix formation.



(a) (b)

Figure 6: Fracture network in porous rock formation: (a) non-percolated network, (b) percolated network.

Figure 7 depicts the evolution of the determinant of the effective permeability tensor $K^{eff} = \left|\mathbf{k}^{p}\right|$ with the fracture density for different value of λ.

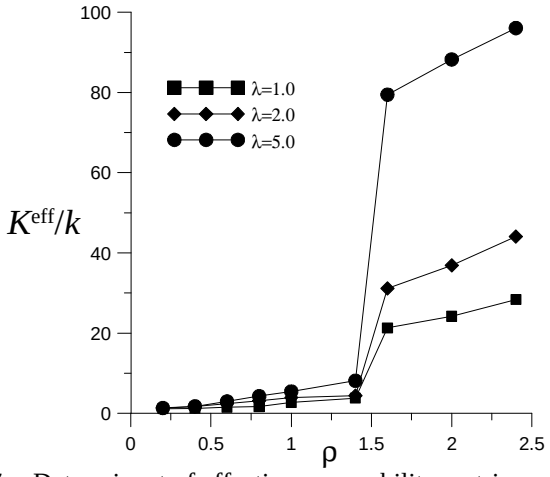


Figure 7: Determinant of effective permeability matrix varies with fracture density for different values of λ.

The equivalent permeability increases first linearly with the fracture density ρ, for small values of the density. A sharp increase is observed for a critical density ρ≈1.5 that represents the percolation threshold. The dimensionless density

$\bar{\rho} = \pi \bar{R}^2 \rho$, where $\bar{R}$ is the mean half-length of the fractures, is found to be around 1.2. After percolation threshold, the effective permeability function of density increases with a different pace as observed also by Maleki and Pouya [10].

5 Conclusion

The effective permeability of fractured porous rocks is studied numerically using analytical and numerical developments based on the singular integral equations. This method that is clearly presented in this paper allows taking into account the real geometry of the fractures and of their interactions effects. The geometrical shape of individual fractures may be introduced precisely in the model as it has been done for the case of the EDZ studied here above. In the case of great number of fractures, the geometry of the fractures network can be described and generated numerically in the model by stochastic laws. The parameters of these stochastic laws are deduced from observation data. The numerical program that has been developed using this method provides an efficient tool for quick evaluation of the effective permeability of fractured porous rocks. The results presented in this paper to illustrate this method show also the important effect of percolation which modifies significantly the magnitude of effective permeability.

Acknowledgement

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