



HAL
open science

On the construction of mean residual life for maintenance decision-making

Khac Tuan Huynh, Inma T. Castro, Anne Barros, Christophe Bérenguer

► **To cite this version:**

Khac Tuan Huynh, Inma T. Castro, Anne Barros, Christophe Bérenguer. On the construction of mean residual life for maintenance decision-making. SAFEPROCESS 2012 - 8th IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes, Aug 2012, Mexico City, Mexico. pp.654-659, 10.3182/20120829-3-MX-2028.00144 . hal-00793253

HAL Id: hal-00793253

<https://hal.science/hal-00793253>

Submitted on 21 Feb 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

On the Construction of Mean Residual Life for Maintenance Decision-Making

K.T. Huynh* I.T. Castro** A. Barros* C. Bérenguer*.*.*.*

* *LM2S, ICD - STMR UMR CNRS 6279, Université de technologie de Troyes, 10010 Troyes cedex, France (e-mail: tuan.huynh@utt.fr, anne.barros@utt.fr, christophe.berenguer@utt.fr)*

** *Departamento de Matemáticas, Escuela Politécnica, Universidad de Extremadura, 10071 Cáceres, Spain (e-mail: inmatorres@unex.es)*

*** *Gipsa-lab, Département Automatique, CNRS, Grenoble INP, 38402 Saint Martin d'Hères cedex, France (e-mail: christophe.berenguer@grenoble-inp.fr)*

Abstract: This paper provides a methodology to assess the efficiency of *mean residual life* (MRL) in condition-based maintenance decision-making for a system subjected to the competing and dependent risks of degradation and shock. Based on this system, the cost models of two quite simple policies implementing MRL-based or degradation-based preventive replacement are developed and compared with each other. Analyzing the equivalence and the performance of both policies allow us to give some conclusions on the interest of the MRL indicator.

Keywords: Competing risks model; Condition-based maintenance; Condition monitoring; Conditional indicator; Mean residual life; Reliability; Stochastic process.

1. INTRODUCTION

With the dissemination of *condition monitoring* (CM) techniques, *condition-based maintenance* (CBM) is nowadays a promising approach to improve the durability, reliability and maintainability of industrial systems. The majority of CBM models are of the control-limit type where a maintenance decision is made whenever a quantity related to the system state reaches a predetermined threshold (see Wang (2002)). Such a quantity is called conditional indicator. The construction of this kind of indicator is an important task that decides the performance of a CBM strategy, and its quality depend closely on the capacity to characterize precisely the system health state.

For a deteriorating system, the degradation level is a traditional conditional indicator for CBM decision-making. Numerous degradation-based maintenance models have been developed and their performance has been proved through practical applications (see e.g., Jardine et al. (2006), Van Noortwijk (2009)). However, since the system failure in the reality is not solely due to degradation but also is due to the other causes, the degradation level does not suffice to describe completely the system health state. In this context, it is necessary to construct a more robust indicator for CBM decision. The MRL, that corresponds to the expected time remaining to system failure (see Sikorskaa et al. (2011)), is an other way to reflect the system health state. The advantage is that the definition of MRL is not strictly limited to a failure mechanism (e.g., degradation, shock, etc.), and it may be then a good alternative to characterizing a system with multiple failure modes. However, if we agree that the MRL is very promising for maintenance planning (see e.g., Peng et al. (2010), Ahmad and Kamaruddin (2012)), it is surprising

that very few CBM strategies based on the MRL are proposed. Hence the following question arises naturally: how to develop such a strategy, and can the introducing of MRL in maintenance decision-making effectively lead to more saving in maintenance cost when degradation level is already available? The present paper aims to answer this question by proposing a MRL-based maintenance policy and investigating its performance by comparing with a classical degradation-based maintenance policy. Both policies are applied for a system which is subjected to the dependent and competing failures due to degradation and shock. Comparisons of the optimal maintenance cost rate for these policies allow us to show the efficiency of MRL indicator and to give the appropriate conditions for its application.

The remainder of this paper is organized as follows. Section 2 deals with the system modeling. In section 3, a condition approach is used to compute the MRL of the system. The structures and the cost models of two CBM policies (i.e. the policy implementing MRL-based preventive decision and the policy implementing degradation-based preventive decision) are introduced in section 4. The sections 5 and 6 are devoted to analyze and discuss the equivalence and the performance of both proposed policies. Finally, we conclude the paper in section 7.

2. SYSTEM MODELING

The considered system is subjected to the competing failures of degradation and shock. Such a system can be described by a *Degradation-Threshold-Shock* (DTS) model where the system fails when the degradation process reaches a critical threshold or when a shock occurs (see Lehmann (2009)). As advocated by Singpurwalla (1995),

this kind of model can be seen as a versatile and realistic extension of many classical failure models based either only on degradation or only on parametric lifetime distributions. The ‘‘classical’’ DTS models assume that the degradation process and the shock process are independent (see e.g., Li and Pham (2005), Van Noortwijk et al. (2007)). But, in many practical situations, the dependence between them is of importance and should not be neglected in the modeling. Therefore, the present paper aims to add this dimension to the DTS model by considering that the shock failure rate depends on the degradation level of the system. And we call the proposed model *Degradation-Threshold-Dependent Shock* (DTDS).

2.1 Degradation modeling

We consider a stochastic deteriorating system, for which the degradation level at time t can be summarized by a random aging variable X_t . Without any repair or replacement action, $\{X_t\}_{t \geq 0}$ is a continuous-time monotonically increasing stochastic process, with $X_0 = 0$. In this paper, we assume that $\{X_t\}_{t \geq 0}$ is a homogeneous Gamma process (see Van Noortwijk (2009) for a thorough review on the use of Gamma processes in maintenance modelling). So, for all $0 \leq s \leq t$, the increment of $\{X_t\}_{t \geq 0}$ between s and t , $X_t - X_s$, follows a Gamma density function with shape parameter $\alpha \cdot (t - s)$ and scale parameter β :

$$f_{\alpha \cdot (t-s), \beta}(u) = \frac{\beta^{\alpha \cdot (t-s)} u^{\alpha \cdot (t-s)-1} e^{-\beta u} \cdot 1_{\{u \geq 0\}}}{\Gamma(\alpha \cdot (t-s))}, \quad (1)$$

where $1_{\{\cdot\}}$ and $\Gamma(\cdot)$ denote the indicator function and Gamma function respectively

$$\Gamma(y) = \int_0^{\infty} u^{y-1} e^{-u} du, \quad \text{for } y > 0. \quad (2)$$

Such a process depends on both parameters α and β which allows to model various degradation behaviors from almost deterministic to highly variable. The average degradation rate is $m = \frac{\alpha}{\beta}$ and its variance is $\sigma^2 = \frac{\alpha}{\beta^2}$. The system is considered to be in the failed state due to degradation when X_t is greater than a fixed failure threshold L .

To characterize the degradation-based failures, we study two types of hitting time of the deteriorating Gamma process. The first hitting time, denoted by τ_A , is the time at which the degradation reaches a level A . As shown in Van Noortwijk (2009), its survival function is given by:

$$\bar{F}_{\tau_A}(t) = 1 - \frac{\Gamma(\alpha t, A\beta)}{\Gamma(\alpha t)}, \quad (3)$$

and its density function is computed as:

$$f_{\tau_A}(t) = \frac{\alpha}{\Gamma(\alpha t)} \int_{A\beta}^{\infty} \{\log(z) - \psi(\alpha t)\} z^{\alpha t-1} e^{-z} dz, \quad (4)$$

where $\Gamma(\alpha, x) = \int_x^{\infty} z^{\alpha-1} e^{-z} dz$ and $\psi(\alpha) = \frac{\Gamma'(\alpha)}{\Gamma(\alpha)}$ are respectively the Gamma function and Digamma function. Considering now another degradation level $B > A$, $\tau_B - \tau_A$ is the second type of hitting time considered here. Its exact survival function has been derived by Bérenguer et al. (2003), but in a complex form. In order to simplify and reduce the numerical computation, we use the approximation proposed by Huynh et al. (2011) as follows:

$$\bar{F}_{\tau_B - \tau_A}(t) \simeq \bar{F}_{\tau_{B-A-\frac{1}{2\beta}}}(t), \quad (5)$$

where the condition of application is $1 < \beta A < \beta B - \frac{1}{2}$.

2.2 Shock failures modeling

In most practical situations the system failure is not solely due to degradation but also to shocks. We assume in this paper that the system fails when a shock occurs. Moreover, the degradation process and the shock process can depend on each other: the higher the degradation the more the system is vulnerable to shocks (see Huynh et al. (2010); Ye et al. (2011)). Hence if N_t is the number of shocks in $(0, t]$, the shock arrival times can be modeled by a non-homogeneous Poisson process $\{N_t\}_{t \geq 0}$ with stochastic increasing intensity which depends on both degradation level X_t and working time t :

$$r(t, X_t) = r_1(t) 1_{\{X_t < M_s\}} + r_2(t) 1_{\{X_t \geq M_s\}}, \quad (6)$$

where $r_1(t)$ and $r_2(t)$ denote two continuous and non-decreasing failure rates at time t with $r_1(t) \leq r_2(t)$, $\forall t \geq 0$. The quantity M_s represents a fixed degradation level. The expression 6 means that the system is more prone to shock failures when the degradation increases and exceeds a given level.

From (6), the survival function of failure times due to shocks can be computed by:

$$\bar{F}_s(t) = \bar{F}_1(t) \bar{F}_{\tau_{M_s}}(t) + \bar{F}_2(t) \int_0^t \frac{\bar{F}_1(u)}{\bar{F}_2(u)} f_{\tau_{M_s}}(u) du, \quad (7)$$

where $\bar{F}_{\tau_{M_s}}$ and $f_{\tau_{M_s}}$ are given from (3) and (4), \bar{F}_i , $i = 1, 2$, is the survival function associated with the failure rate r_i :

$$\bar{F}_i(t) = e^{-\int_0^t r_i(u) du}. \quad (8)$$

Fig. 1 shows an illustration of the proposed DTDS model.

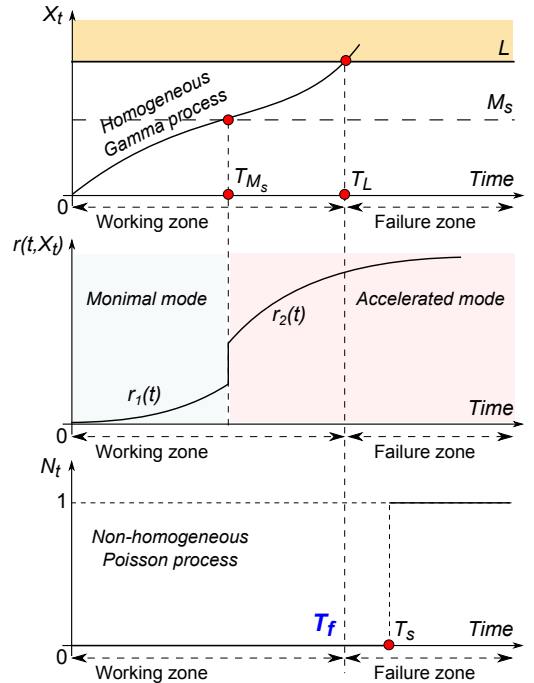


Fig. 1. Illustration of DTDS model

3. CONDITIONAL MEAN RESIDUAL LIFE

Mean residual life is an important reliability indicator for maintenance decision-making since it can provide an

idea about how long a system at any particular age and deteriorating is expected to survive. Generally, as shown in Jardine et al. (2006), the MRL function at time t can be computed by:

$$E[\tau_f - t \mid \tau_f > t, \mathbf{Z}_{0:t}], \quad (9)$$

where τ_f is the system lifetime, and $\mathbf{Z}_{0:t}$ is the CM information related to system state obtained at time t . In the literature, the calculation of MRL function is usually free from the effect of CM data (see e.g. Bebbington et al. (2008), Navarro and Rychlik (2010)). However, since this approach can lead to a large variability of MRL, the MRL is then not very useful for maintenance planning (see Banjevic (2009)). In order to reduce this inaccuracy, as proposed by Wang and Zhang (2005), the system CM data must be taken into account when estimating the MRL. In this paper, the system monitoring is assumed perfect and the system evolution is memoryless, the CM data of the system are then limited to its degradation level (i.e., $\mathbf{Z}_{0:t} \equiv X_t$). Hence from (9), the MRL at time t , given the degradation level $X_t = z$, is calculated by:

$$m_z(t) = E[\tau_f - t \mid \tau_f > t, X_t = z] = \int_t^\infty R(u \mid z) du, \quad (10)$$

where $R(u \mid z)$ is the conditional reliability of system at time $u > t$, given $X_t = z$ and that the system is still running at time t .

In the considered DTDS model, the system fails whenever degradation level exceeds a fixed threshold L or a shock occurs, $R(u \mid z)$ is hence computed by:

$$\begin{aligned} R(u \mid z) &= P(X_u < L, N_u = 0 \mid X_t = z) \\ &= R_1 \cdot 1_{\{z < M_s\}} + R_2 \cdot 1_{\{z \geq M_s\}}, \end{aligned} \quad (11)$$

where R_1 and R_2 are given respectively by:

$$\begin{aligned} R_1 &= \bar{F}_{\tau_{M_s-z}}(u-t) \frac{\bar{F}_1(u)}{\bar{F}_1(t)} + \int_t^u \bar{F}_{\tau_L-\tau_{M_s}}(u-v) \\ &\quad \times \frac{\bar{F}_1(v)}{\bar{F}_1(t)} \frac{\bar{F}_2(u)}{\bar{F}_2(v)} f_{\tau_{M_s-z}}(v-t) dv, \end{aligned} \quad (12)$$

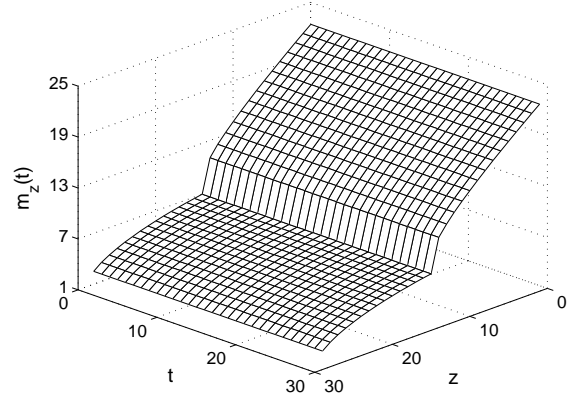
$$R_2 = \bar{F}_{\tau_L-z}(u-t) \frac{\bar{F}_2(u)}{\bar{F}_2(t)}, \quad (13)$$

where $\bar{F}_{\tau_{M_s-z}}$, \bar{F}_{τ_L-z} , $f_{\tau_{M_s-z}}$ and $\bar{F}_{\tau_L-\tau_{M_s}}$ are calculated from (3), (4) and (5) respectively, \bar{F}_i , $i = 1, 2$, is given by (8). As such, the conditional MRL of DTDS model is computed by (10) and (11).

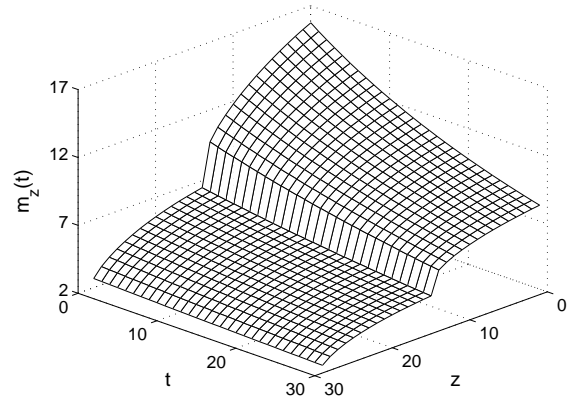
Fig. 2 shows the MRL surfaces as a function of t and z when the shock rate functions $r_1(t)$ and $r_2(t)$ are constant (see Fig. 2a) or linear (see Fig. 2b) of time t .

Remark 1. Since the conditional MRL of DTDS model, $m_z(t)$, depend on both variables t and z , it reflect not only the system degradation level z but also the shock failure rate $r(t, X_t)$.

- When $r(t, X_t)$ is constant of time t , $m_z(t)$ does not depend on t (see Fig. 2a), hence $m_z(t)$ and X_t return the same information on the system. The MRL prediction may be useless, and a solitary degradation level is sufficient for maintenance decision-making.
- When $r(t, X_t)$ depend on time t , $m_z(t)$ depends on both z and t (see Fig. 2b), so it contains more



(a) $r_1(t) = 0.01$ and $r_2(t) = 0.1$



(b) $r_1(t) = 0.0025t + 0.01$ and $r_2(t) = 0.0025t + 0.1$

Fig. 2. MRL - $\alpha = 0.1$, $\beta = 0.1$, $L = 30$, $M_s = 15$.

information on the system compared to degradation level. In this cas, it is interesting to predict and integrate the MRL in maintenance decision-making.

Lemma 2. For a fixed degradation level z , $0 \leq z \leq L$, the conditional MRL, $m_z(t)$, is a non-decreasing function of time. For a fixed time t , $0 \leq t$, the conditional MRL, $m_z(t)$, is a non-decreasing function of degradation level.

Proof. Detailed proof is represented in Huynh (2011).

Corollary 3. For all times $0 \leq u \leq v$ and all degradation levels $0 \leq x \leq y \leq L$, the conditional MRL of DTDS model has the following properties:

$$m_z(u) \geq m_z(v) \geq m_y(v) \text{ and } m_z(u) \geq m_y(u) \geq m_y(v).$$

4. CONDITION-BASED MAINTENANCE MODELS

In order to quantify the value of the MRL indicator in maintenance decision-making, we develop in this section the cost model of two quite simple CBM policies. For the first policy, the preventive replacement decision bases on the condition MRL, while for the second one, it relies directly on the degradation level.

4.1 Assumptions and objective cost function

Consider a system described by the DTDS model as in section 2. We assume that the system degradation is hid-

den and that the system failure is non-self-announcing. This means that the systems reveals only its degradation state and its failures through an instantaneous, perfect and non-destructive inspection (cost C_i). Two maintenance operations are available on the system: a preventive replacement (cost $C_p > C_i$) and a corrective replacement (cost $C_c > C_p$). A replacement can be carried out only at inspections, restores the system to an as-good-as-new (AGAN) condition, and takes negligible time. Therefore, there exists an inactivity time interval after a system failure, and an additional cost is incurred from the failure time until the next inspection time at a cost rate C_d .

To evaluate the performance of the maintenance policies, we use in this work a cost criterion which is the long-run expected maintenance cost rate including the unavailability cost C_∞ . Since the system is AGAN after a replacement, we can use the classical renewal theorem to evaluate analytically C_∞ (see Tijms (2003)):

$$C_\infty = \lim_{t \rightarrow \infty} \frac{E[C(t)]}{t} = \frac{E[C(S)]}{E[S]}, \quad (14)$$

where $C(\cdot)$ is the cumulative maintenance cost, and S is the length of a renewal cycle.

4.2 MRL-based maintenance model - $(\Delta T, m)$

Under the $(\Delta T, m)$ policy, the system is regularly inspected with period ΔT . At each inspection, the conditional MRL is estimated from the detected degradation level of system. If the value of MRL reaches threshold m and if no failure occurred, the system is preventively replaced. But if the system has already failed, it is correctively replaced. In this case, because of the system inactivity after failure, an addition cost is incurred from the failure time until the replacement time. The inspection period ΔT and the preventive replacement threshold m are the two decision variables to be optimized.

Applying (14), the long-run expected maintenance cost rate of $(\Delta T, m)$ policy can be given by:

$$C_\infty^{\Delta T, m}(\Delta T, m) = \frac{C_i E[N_i^{\Delta T, m}] + C_c (1 - P_p^{\Delta T, m})}{E[S^{\Delta T, m}]} + \frac{C_p P_p^{\Delta T, m} + C_d E[W_d^{\Delta T, m}]}{E[S^{\Delta T, m}]}, \quad (15)$$

where $N_i^{\Delta T, m}$, $P_p^{\Delta T, m}$ and $W_d^{\Delta T, m}$ are respectively the inspection number, the probability of preventive replacement, and the inactivity time interval in a renewal cycle $S^{\Delta T, m}$ under $(\Delta T, m)$ policy. The optimal values of decision variables ΔT_{opt} and m_{opt} are obtained by minimizing $C_\infty^{\Delta T, m}(\Delta T, m)$ where $0 \leq \Delta T$ and $0 \leq m \leq MTTF$.

4.3 Degradation-based maintenance model - $(\Delta T, M)$

The $(\Delta T, M)$ policy is used as a benchmark here. Under this policy, the system is regularly inspected with period ΔT . At each inspection, if the system degradation level exceeds a degradation threshold M and if no failure occurred, the system is preventively replaced. But if the system has already failed, it is correctively replaced. In this case, because of the system inactivity after failure,

an addition cost is incurred from the failure time until the replacement time. The inspection period ΔT and the preventive replacement threshold M are the two decision variables to be optimized.

Applying (14), the long-run expected maintenance cost rate of $(\Delta T, M)$ policy can be given by:

$$C_\infty^{\Delta T, M}(\Delta T, M) = \frac{C_i E[N_i^{\Delta T, M}] + C_c (1 - P_p^{\Delta T, M})}{E[S^{\Delta T, M}]} + \frac{C_p P_p^{\Delta T, M} + C_d E[W_d^{\Delta T, M}]}{E[S^{\Delta T, M}]}, \quad (16)$$

where $N_i^{\Delta T, M}$, $P_p^{\Delta T, M}$ and $W_d^{\Delta T, M}$ are respectively the inspection number, the probability of preventive replacement, and the inactivity time interval in a renewal cycle $S^{\Delta T, M}$ under the $(\Delta T, M)$ policy. The optimal values of decision variables ΔT_{opt} and M_{opt} are obtained by minimizing $C_\infty^{\Delta T, M}(\Delta T, M)$ where $0 \leq \Delta T$ and $0 \leq M \leq L$.

5. ANALYZING THE EQUIVALENCE OF $(\Delta T, M)$ POLICY AND $(\Delta T, m)$ POLICY

The structures of the considered policies show that the difference between them resides in the way to make the preventive replacement decision. So, to analyze the equivalence of them, it is sufficient to study the equivalence of their preventive replacement probabilities. We consider two cases according to $r(t, X_t)$ independent or dependent of time t .

5.1 Case of $r(t, X_t)$ independent of time

When the shock rate $r(t, X_t)$ is independent of time, the value of the conditional MRL depend only on the degradation level $X_t = z$, and we denote it simply by m_z . Let z_m be the minimal degradation level z at which the value of the conditional MRL is less than m :

$$z_m = \inf \{0 \leq z \leq L, m_z \leq m\}. \quad (17)$$

The structure of $(\Delta T, m)$ policy and the independence in time of MRL lead to represent the event of preventive replacement E_p at time $k\Delta T$ as follows:

$$\begin{aligned} E_p &\triangleq \{m_{z_{(k-1)\Delta T}} > m \geq m_{z_{k\Delta T}}, k\Delta T < \tau_L, \} \\ &\Leftrightarrow \{X_{(k-1)\Delta T} < z_m \leq X_{k\Delta T}, k\Delta T < \tau_L, N_{k\Delta T} = 0\} \\ &\Leftrightarrow \{(k-1)\Delta T < \tau_{z_m} \leq k\Delta T < \tau_L, N_{k\Delta T} = 0\}, \end{aligned} \quad (18)$$

where τ_{z_m} is the hitting time of z_m . This preventive replacement event is similar to the one of $(\Delta T, M)$ policy. Thus, the preventive replacement probabilities of both policies are equivalent:

$$P_p^{\Delta T, m}(\Delta T, m) \equiv P_p^{\Delta T, M}(\Delta T, z_m). \quad (19)$$

Remark 4. For the DTDS model, when the shock rate function is independent of time, both $(\Delta T, m)$ policy and $(\Delta T, M)$ policy are equivalent.

5.2 Case of $r(t, X_t)$ dependent on time

When the shock rate $r(t, X_t)$ is dependent of time, the value of the conditional MRL depend on both the degradation level $X_t = z$ and on the time t . For the fixed time

t , let $z_{t,m}$ be the minimal degradation level z at which the value of the conditional MRL is less than m :

$$z_{t,m} = \inf \{0 \leq z \leq L, m_z(t) \leq m\}. \quad (20)$$

According to $(\Delta T, m)$ policy, a preventive replacement is performed at $k\Delta T$ when:

$$\begin{aligned} E_p &\triangleq \{m_{z_{(k-1)\Delta T}}((k-1)\Delta T) > m \geq m_{z_{k\Delta T}}(k\Delta T), \\ &\quad k\Delta T < \tau_L, N_{k\Delta T} = 0\} \\ &\Leftrightarrow \{X_{(k-1)\Delta T} < z_{(k-1)\Delta T, m}, z_{k\Delta T, m} \leq X_{k\Delta T}, \\ &\quad k\Delta T < \tau_L, N_{k\Delta T} = 0\} \\ &\Leftrightarrow \{(k-1)\Delta T < \tau_{z_{(k-1)\Delta T, m}}, \tau_{z_{k\Delta T, m}} \leq k\Delta T < \tau_L, \\ &\quad N_{k\Delta T} = 0\}, \end{aligned} \quad (21)$$

where $\tau_{z_{t,m}}$ is the hitting time of $z_{t,m}$. This event is completely different to the preventive replacement event in $(\Delta T, M)$ policy. So, the preventive replacement probabilities of both policies are not equivalent in this case:

$$P_p^{\Delta T, m}(\Delta T, m) \neq P_p^{\Delta T, M}(\Delta T, z_{t,m}). \quad (22)$$

Remark 5. For the DTDS model, when the shock rate depend on time, the $(\Delta T, m)$ policy is not equivalent to the $(\Delta T, M)$ policy.

6. ANALYZING THE PERFORMANCE OF $(\Delta T, M)$ POLICY AND $(\Delta T, m)$ POLICY

We have shown in the previous section that the $(\Delta T, M)$ policy and the $(\Delta T, m)$ policy are equivalent (resp. different) when the shock rate of DTDS model, $r(t, X_t)$, is independent (resp. dependent) of time. In this section, we study in detail the performance of $(\Delta T, m)$ policy when $r(t, X_t)$ depends on time. For this end, we compare the optimal maintenance cost rates of the two considered policies in different configurations of maintenance costs and system characteristics. The comparison results allow weighing the benefit of integrating the MRL in maintenance decision-making instead of the degradation level.

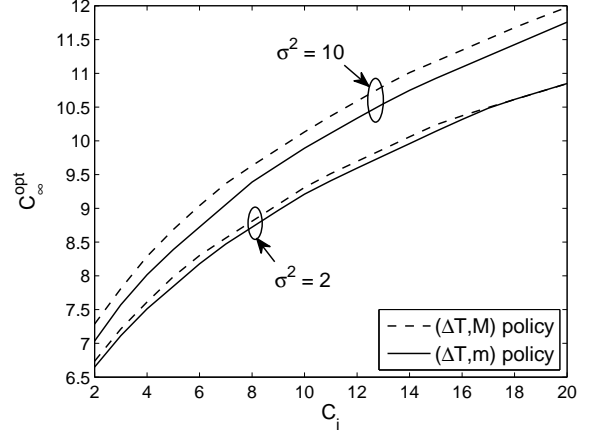
6.1 Numerical results

We consider that case that the shock rates are linear in time: $r_1(t) = 0.0025t + 0.01$ and $r_2(t) = 0.0025t + 0.1$. Three situations of maintenance costs are investigated:

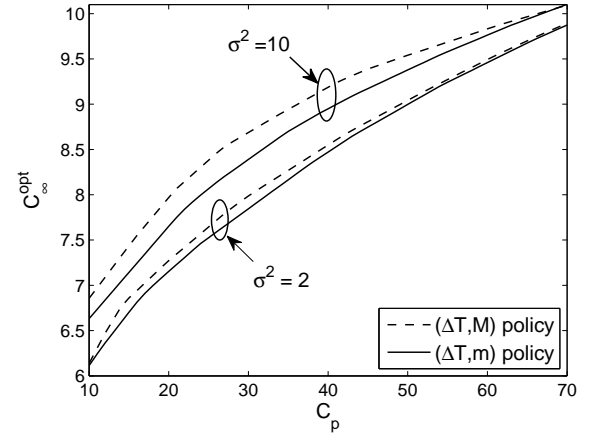
- (1) Variable inspection cost: C_i varies from 2 to 20 with unit step equals 1, $C_c = 100$, $C_p = 30$, and $C_d = 50$.
- (2) Variable preventive replacement cost: $C_i = 5$, $C_c = 100$, C_p varies from 10 to 70 with unit step equals 1, and $C_d = 50$.
- (3) Variable cost rate of system inactivity: $C_i = 5$, $C_p = 30$, $C_c = 100$, and C_d varies from 10 to 150 with unit step equals 1.

These maintenance costs are applied to two configurations of degradation process: $\alpha = \beta = 0.5$ (almost deterministic degradation $\sigma^2 = 2$, $m = 1$) and $\alpha = \beta = 0.1$ (highly variable degradation $\sigma^2 = 10$, $m = 1$). The other parameters of model are chosen by $L = 30$ and $M_s = 15$. The numerical results are illustrated in the Fig. 3.

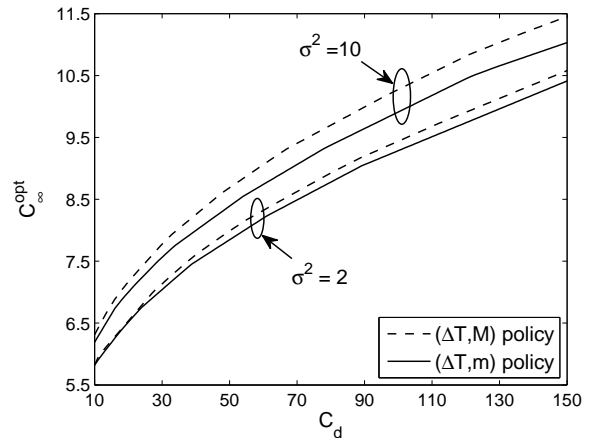
Fig. 3a, Fig. 3b and Fig. 3c show the evolution of the optimal long-run expected maintenance cost rate for the previous case studies. In each figure, the solid lines and



(a) Variable inspection cost



(b) Variable preventive replacement cost



(c) Variable cost rate of system inactivity

Fig. 3. Comparison of $(\Delta T, m)$ policy and $(\Delta T, M)$ policy the dashed lines reflect respectively the $(\Delta T, m)$ policy and the $(\Delta T, M)$ policy. And the lower line corresponds to the policy with higher performance.

6.2 Analysis and discussions on numerical results

The numerical results show that the $(\Delta T, m)$ policy is always more profitable than the $(\Delta T, M)$ policy. This

phenomenon can be explained by the fact that the conditional MRL contains more information on the system state compared to degradation level, hence making a preventive maintenance decision relied on the conditional MRL can lead to more saving than making a preventive maintenance decision based on the degradation level. Moreover, one can remark that the advantage of $(\Delta T, m)$ policy is clearer when the variance of degradation process becomes more important. The reason is that, for the DTDS model, not only the failure time due to degradation but also the one due to shock are more dispersive when the variance of degradation process increases (see Huynh (2011)). So considering the amount of information incorporated into each conditional indicator, the higher the variance the more the degradation level is less advantageous in characterizing the functional behavior of system.

Nevertheless, from practical viewpoint, the $(\Delta T, m)$ is not always pertinent compared to $(\Delta T, M)$ policy. One can see in the Fig. 3 that the optimal expected maintenance cost rates of both considered policies are more or less identical when the variance of degradation is low, the preventive replacement is expensive, and/or the inspection and the system inactivity correspond to low costs. For these situations, the $(\Delta T, M)$ policy is preferable, because the MRL prediction takes time but less useful. This shows the necessary of developing analytical models to make a choice on the use of condition MRL or degradation level for maintenance decision-making.

7. CONCLUSION

The present paper provides a case study to evaluate the advantage of constructing a MRL indicator for CBM decision-making. A condition-based approach is used to calculate the MRL of the DTDS model. Based on this model, the maintenance cost model of a CBM policy implementing MRL-based decision is developed and compared to the one implementing degradation-based decision. The theoretical and numerical results show that the MRL is a more general and effective reliability indicator for CBM compared with the degradation level. However, the choice to resort to MRL prediction has to be cautious and should rely for each configuration on precise quantification step.

In spite of very promising results, the study in this paper is based on a rather strong assumption: perfect and direct monitoring, and Markovian system. Our future work aims to relax this assumption by considering jointly the noisy measurements, the environmental covariates and the non-stationary non-Markovian failure mechanisms of systems. Also, a MRL-based CBM policy with dynamic decision structures will be considered in order avoid more efficiently inopportune interventions.

REFERENCES

- Ahmad, R. and Kamaruddin, S. (2012). An overview of time-based and condition-based maintenance in industrial application. *Computers & Industrial Engineering*, 63(1), 135–149.
- Banjevic, D. (2009). Remaining useful life in theory and practice. *Metrika*, 69(2), 337–349.
- Bebbington, M., Lai, C., and Zitikis, R. (2008). Reduction in mean residual life in the presence of a constant competing risk. *Applied Stochastic Models in Business and Industry*, 24(1), 51–63.
- Bérenghier, C., Grall, A., Dieulle, L., and Roussignol, M. (2003). Maintenance policy for a continuously monitored deteriorating system. *Probability in the Engineering and Informational Sciences*, 17(02), 235–250.
- Huynh, K. (2011). *Quantification de l'apport de l'information de surveillance dans la prise de décision en maintenance*. Ph.D. thesis, Université de technologie de Troyes, France.
- Huynh, K., Barros, A., Bérenghier, C., and Castro, I. (2010). Value of condition monitoring information for maintenance decision-making. In *Proc. Ann. Reliability & Maintainability Symp*, 543–548.
- Huynh, K., Barros, A., Bérenghier, C., and Castro, I. (2011). A periodic inspection and replacement policy for systems subject to competing failure modes due to degradation and traumatic events. *Reliability Engineering & System Safety*, 96(04), 497–508.
- Jardine, A., Lin, D., and Banjevic, D. (2006). A review on machinery diagnostics and prognostics implementing condition-based maintenance. *Mechanical systems and signal processing*, 20(7), 1483–1510.
- Lehmann, A. (2009). Joint modeling of degradation and failure time data. *Journal of Statistical Planning and Inference*, 139(5), 1693–1706.
- Li, W. and Pham, H. (2005). An inspection-maintenance model for systems with multiple competing processes. *Reliability, IEEE Transactions on*, 54(2), 318–327.
- Navarro, J. and Rychlik, T. (2010). Comparisons and bounds for expected lifetimes of reliability systems. *European Journal of Operational Research*, 207(1), 309–317.
- Peng, Y., Dong, M., and Zuo, M. (2010). Current status of machine prognostics in condition-based maintenance: a review. *The International Journal of Advanced Manufacturing Technology*, 50(1–4), 1–17.
- Sikorskaa, J., Hodkiewicz, M., and Ma, L. (2011). Prognostic modelling options for remaining useful life estimation by industry. *Mechanical Systems and Signal Processing*, 25(5), 1803–1836.
- Singpurwalla, N. (1995). Survival in dynamic environments. *Statistical Science*, 10(1), 86–103.
- Tijms, H. (2003). *A first course in stochastic models*. Wiley, New York.
- Van Noortwijk, J. (2009). A survey of the application of gamma processes in maintenance. *Reliability Engineering & System Safety*, 94(1), 2–21.
- Van Noortwijk, J., Van der Weide, J., Kallen, M., and Pandey, M. (2007). Gamma processes and peaks-over-threshold distributions for time-dependent reliability. *Reliability Engineering & System Safety*, 92(12), 1651–1658.
- Wang, H. (2002). A survey of maintenance policies of deteriorating systems. *European Journal of Operational Research*, 139(3), 469–489.
- Wang, W. and Zhang, W. (2005). A model to predict the residual life of aircraft engines based upon oil analysis data. *Naval Research Logistics (NRL)*, 52(3), 276–284.
- Ye, Z., Tang, L., and Xu, H. (2011). A distribution-based systems reliability model under extreme shocks and natural degradation. *IEEE Transactions on Reliability*, 60(1), 246–256.