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Concept and T–S tracking controller design of a CMC lower-limb rehabilitation device

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Abstract: We consider the kinematic concepts of a new lower-limb rehabilitation device in closed muscular chain. The proposed control structure is based on a trajectory generator and a continuous nonlinear tracking controller. The human efforts applied to this device are considered as external disturbances to the system's dynamics and as inputs to the trajectory generator and allow safe voluntary control of the system by the user. A H_∞ control structure based on a Takagi–Sugeno descriptor model is proposed to track the desired trajectories and to attenuate external disturbances. Stability conditions are given in terms of Linear Matrix Inequalities (LMIs) using a fuzzy Lyapunov function. Finally, simulation results of the proposed control structure for with new rehabilitation device during isokinetic movements illustrate the efficiency of the proposed approach.

Keywords: Closed muscular chain, Descriptors, Disturbances attenuation, Linear Matrix Inequalities (LMIs), Rehabilitation device, Takagi–Sugeno, Tracking controller.

1. INTRODUCTION

Technical assistance to functional rehabilitation has attracted great interest recently; see, for example, [1, 2] and references therein. Many rehabilitation devices have been designed. For instance, Lokomat is a device for gait rehabilitation of patients suffering from neuromuscular trauma [3]. Another example is an open muscular chain lower-limb rehabilitation device developed by our research centre and named Multi-Iso [4].

The design of a new device must start from specific needs expressed by clinicians in rehabilitation. For a specific muscular complex, several rehabilitation techniques may be considered. We are interested here in knee rehabilitation, and our interest naturally extends to the global lower limb musculoskeletal complex. For muscular deficiency, two rehabilitation techniques can be used: the first is named open muscular chain (OMC), which is characterised by strengthening an isolated muscle group; the second is named closed muscular chain (CMC), which is characterised by recruiting both the agonist and antagonist muscle groups that contribute to the movement [5, 6]. These two techniques are complementary and are used in various stages of rehabilitation protocols [7].

For technical reasons, it is difficult to design a device that is able to reproduce these two techniques simultaneously. The only existing such device is named MotionMaker, but it is a very complex and expensive robotic structure [8]. Consequently, it can be used in the context of research studies but is not appropriate for common rehabilitation centres. OMC devices are commonly used in clinical contexts for lower-limb rehabilitation with isokinetic devices such as Cybex, Biodex or Multi-Iso [9]. More recently, CMC rehabilitation has received particular attention because of its ability to stabilise the targeted joint during exercises similar to those used in daily life (such as walking, sitting or standing up). For instance, Moflex, Contrex LP or Erigo devices are CMC rehabilitation devices [10–12]. Previous studies in our research centre led to the development of Multi-Iso [4]. To cover a larger group of rehabilitation protocols, our aim is to design an isokinetic CMC rehabilitation apparatus and robust control

structure to ensure the safety of the users. Rehabilitation devices used for knee rehabilitation are controlled by classical control laws. Although the results obtained with these controllers are satisfying in terms of rehabilitation specifications, they are restrictive in terms of control performance, mainly because they do not theoretically guarantee good behaviour in the entire state space and do not ensure the rejection of external disturbances such as patient efforts. Thus, previous works have proposed employing a nonlinear control scheme based on switching controllers [4] or, more recently, a coupled trajectory generator with a Takagi–Sugeno (T–S) fuzzy controller [13, 14]. However, these studies were only concerned with stability analysis, and the tracking performance of the rehabilitation devices was not guaranteed. Consequently, safe behaviour was not guaranteed, because the controllers could not reject external disturbances or uncontrolled users’ efforts with the device.

We present the concept of a new CMC rehabilitation device, called Sys-Reeduc, and we propose a convenient tracking controller design that ensures safe behaviour for the users. In a preliminary study [15], the stability performance in simulation was obtained through major modeling simplifications (including linearisation and decoupling) that make it difficult to apply the synthesised control law to the real system. In this paper, after presenting the kinematic concepts of the new rehabilitation device, a nonlinear dynamic model is derived via the well-known Lagrange equations. Then, a tracking control plant, based on a trajectory generator added to a nonlinear H_∞ tracking control law, is proposed. A description of the trajectory generator is proposed for isokinetic cyclic rehabilitation protocols, which are the most commonly used protocols in lower limb rehabilitation. Afterward, the design of a nonlinear control law ensuring robust trajectory tracking is investigated.

Among nonlinear controllers, T–S fuzzy model-based approaches have become popular, because the model provides universal approximators of nonlinear systems [16, 17]. Therefore, in the past few decades, T–S fuzzy control has been the subject of many theoretical studies; see, for example, [17–19]. Moreover, this modeling approach has been successfully employed

for practical applications; see, for example, [20, 21]. The major interest of such approaches is that they allow extending some linear control techniques to nonlinear systems. In most cases, quadratic Lyapunov functions are employed to derive controller designs based on linear matrix inequalities (LMIs). Nevertheless, these approaches are conservative, because they require the existence of a common Lyapunov matrix for the whole set of LMIs. More recently, non-quadratic fuzzy Lyapunov approaches have been proposed to reduce the conservatism of LMI conditions [22–25]. Nevertheless, the major drawback of such approaches is that they require knowledge of the membership function time derivative's lower bounds, which are difficult to obtain in practice. Complementary to these approaches, tracking controller designs have been considered [26, 27].

In the present study, a tracking controller design is proposed based on T–S fuzzy descriptor modeling of the rehabilitation device [28, 29]. Indeed, although state space descriptors can be used to model algebraic systems such as singular systems [30–32], they are convenient for dealing with mechanical systems with time-varying inertia [20, 21]. Moreover, LMI-based descriptor stability conditions provide relaxed quadratic fuzzy Lyapunov approaches without requiring knowledge of the membership function derivative [33–35].

Note that, despite numerous works dealing with stability and stabilisation of T–S descriptor systems, to the best of the authors' knowledge, there are no available results, since the first quadratic result in [28], dealing with robust T–S descriptor tracking control problems. In this paper, for the control law synthesis, a H_∞ criterion is employed to guarantee the attenuation of uncontrolled human disturbances. Thus, a relaxed quadratic fuzzy Lyapunov-based tracking controller design methodology is provided, based on linear matrix inequality conditions for the considered class of perturbed nonlinear descriptors. Finally, simulation results of isokinetic movements will illustrate the efficiency of the proposed control approach.

2. CONCEPT OF SYS-REEDUC

2.1 Definitions, advantages and drawbacks of OMC and CMC rehabilitation techniques

Exercises in open muscular chains are defined by the contraction of the agonist muscle group that allows the movement of the considered segment. In this rehabilitation mode, for a lower limb, the foot is considered “free”, see Fig. 1. This approach is efficient and allows fast recovery when strengthening isolated muscle groups. However, the major drawback of OMC is that it causes constraints localised on the anterior cruciate ligament and on the patellar tendon that can generate significant pain during intensive use [5, 36]. Moreover, this technique may result in joint imbalance because of asymmetrical stretching of the targeted joint complex.

Exercises in CMC are defined by simultaneous contraction of both the agonist and antagonist muscles for a particular movement. In most cases, these exercises are possible when the foot is in contact with a support, see Fig. 2. Thus, all the muscles in a group contribute to the joint’s actuation. Consequently, the musculoskeletal stretching is applied in a symmetrical way and contributes to increasing the joint stability. Thus, patellar pains are reduced during CMC rehabilitation compared with OMC rehabilitation [6, 37]. Note that the benefit of CMC could be longer-lasting than that of OMC.

Finally, OMC and CMC rehabilitation techniques are complementary and together allow patients to reach specific rehabilitation goals relating to different pathologies. For instance, during long-term rehabilitation protocols, one can imagine proposing CMC exercises for stabilising the whole joint complex and alleviating constraints generated by the OMC in a first step. In a second step, when the patient’s clinical state makes it possible, rehabilitation in OMC could be proposed for a specific muscular strengthening while the joint balance achieved in CMC is maintained.

2.2 Kinematic concepts of Sys-Reeduc

Our objective is now to design an isokinetic CMC rehabilitation apparatus. By analogy with

robotic systems, the mechanical complex {lower limb, rehabilitation device} must constitute a closed kinematic chain (CKC). For each lower limb, a CKC, shown in Fig. 3, is proposed and is composed of the links C_1 , C_2 , C_3 , C_4 and C_5 , which are respectively the apparatus base, thigh, leg, foot and mobile foot's support. The joint between links i and j is denoted $L_{i,j}$.

According to the CKC depicted in Fig. 3, the design of a new device consists of defining the nature of the joints $L_{1,2}$, $L_{2,3}$, $L_{3,4}$, $L_{4,5}$ and $L_{5,1}$. In that way, we assume that the human body consists of a set of rigid polyarticulated segments. In the sagittal plane, the whole kinematic structure, presented in Fig. 4, is supposed to be composed of perfect joints such that:

- $L_{1,2}$ is the joint between the base and the thigh. The patient sits on a chair fixed to the base of the system. The trunk and the pelvis are also assumed to be fixed to the base. Then, $L_{1,2}$ is supposed to be a spherical joint corresponding to the coxo-femoral joint (hip).
- $L_{2,3}$ corresponds to the knee, which is the main joint we wish to rehabilitate. Its complex kinematics were the subject of a previous study [38] and will not be detailed in this paper. To simplify the specification of the Sys-Reeduc kinematic concept, the knee will be represented as a hinge joint around the main lower limb flexion–extension axis.
- The joints $L_{3,4}$ and $L_{4,5}$ can be reduced to a unique joint, $L_{3,5}$. This requires that the ankle axes and mobile support hinge joint, in the sagittal plane, must be perfectly aligned with those of the foot mobile support. This constraint allows removing the foot from the kinematic chain. Then, $L_{3,5}$ consists of two hinge joints. The first joint is the plantar flexion/dorsal flexion (L_{fd} ankle axis, which is collinear with the main lower limb flexion–extension axis). The second joint is the ankle internal–external rotation (L_{ie} ankle rotation in the tibia axis). Note that the adduction/abduction of the ankle was voluntarily omitted, because it has little influence on knee rehabilitation.
- The joint $L_{5,1}$, between the foot's mobile support and the base, is realised by a prismatic joint along the axis defined by the lower limb in complete extension.

Because the mechanical structure of Sys-Reeduc is symmetrical, Fig. 4 represents only the right lower limb. To simplify the presentation of the rehabilitation mode, only the right lower limb will be considered in the sequel. Moreover, according to the kinematics defined in Fig. 4, the internal–external ankle rotation L_{ie} (along the \bar{z}_s axis and denoted by the variable $q_2(t)$) and the translation $L_{5,1}$ (along the \bar{x} axis and denoted by the variable $q_1(t)$) are sufficient to stimulate the knee in its six degrees of freedom [38]. These two axes will therefore be motorised, and the plantar flexion/dorsal flexion will remain passive (free or forced by a mechanical brake). In terms of rehabilitation, the motorisation of the joint $L_{5,1}$ brings about a movement of the lower limb along the medial–lateral axis (flexion–extension movement). This makes it possible to rehabilitate the flexor muscular complex and limits the constraints applied to the cruciate ligaments. Moreover, the foot’s mobile support rotations allow the knee internal–external rotation and the plantar flexion/dorsal flexion. These lead to a selective and precise muscular constraint. Then, rehabilitation or muscular strengthening can be practiced in a specific way during a desired movement. For instance, lower limb movement with the foot in external rotation and in extension helps the stretching of medial hamstrings.

The efforts applied by the user to the device that may influence the dynamic behaviour of Sys-Reeduc are depicted in Fig. 5. Hence, $f_{p/x}(t)$ and $C_{p/z_s}(t)$ are respectively the force along the \bar{x} -axis and the torque along \bar{z}_s applied by the user.

Then, by considering the kinematics of Sys-Reeduc and the effort supplied by the user, we can define the type of muscle contraction occurring during a particular exercise. Indeed, as depicted in Fig. 6 for flexion–extension, we can define four quadrants in the effort–speed plan for the characterisation of the muscular exercise, as follows.

- In $Q_{1,1}$ and $Q_{1,3}$, $f_{p/x}(t)$ is applied in the device’s movement direction, and the contractile mode is therefore called “concentric”.
- In $Q_{1,2}$ and $Q_{1,4}$, $f_{p/x}(t)$ is applied in opposition to the device’s movement direction, and

the contractile mode is therefore called “eccentric”.

Fortunately, the observation depicted in Fig. 7 can also be made relative to the internal–external rotation movement of Sys-Reeduc. In that case, the four-quadrant diagram is characterised by:

- $Q_{2,1}$ and $Q_{2,3}$, where $C_{p/z_s}(t)$ is applied in the device’s movement direction, and the contractile mode is therefore called “concentric”;
- $Q_{2,2}$ and $Q_{2,4}$, where $C_{p/z_s}(t)$ is applied in opposition to the device’s movement direction, and the contractile mode is therefore called “eccentric”.

3. CONTROL STRUCTURE DESIGN FOR SYS-REEDUC

3.1 Generic control structure

A generic control structure for rehabilitation devices is presented in Fig. 8. This two-level structure is based on the following components.

- A trajectory generator, which provides the desired trajectory $x_d(t)$ to be tracked by the continuous system state $x(t)$. This trajectory generator, detailed in the next section, is based on the use of a state machine [4, 39] and allows voluntary movements activated from the measurement of the effort $\varphi(t)$ applied by the patient to the device.
- A closed-loop rehabilitation device plant, which is used to guarantee the tracking of the desired trajectory, $x_d(t)$, by the dynamic system consisting of the device and the user. To ensure safe behaviour, this closed-loop system must ensure the tracking error convergence of the system regardless of the patient’s efforts on the device. To achieve this goal, $\varphi(t)$ is considered as an external disturbance to be attenuated or, in the better case, to be rejected. Note that, without the trajectory generator, the closed-loop rehabilitation device plant by itself does not allow the voluntary control of the system by the patient.

3.2 Trajectory generator design

A rehabilitation exercise can be viewed as a succession of n elementary movement phases denoted ϕ_i for $i=1,\dots,n$. Hence, a discrete state machine can be used to represent these exercises, as depicted in Fig. 9. In this case, the device's current position $q_1(t)$, $q_2(t)$ and the external effort $\varphi(t)$, consisting of $f_{p/x}(t)$ and $C_{p/z_s}(t)$, can be considered as inputs. The trajectory generator provides as outputs the desired state, denoted $x_d(t)$, consisting of the desired positions $[q_{1d}(t) \ q_{2d}(t)]$ and velocities $[\dot{q}_{1d}(t) \ \dot{q}_{2d}(t)]$, to be tracked by the device.

Therefore, it is necessary to define the whole movement characteristics for each elementary phase and the conditions to ensure the validity of transitions between two phases. Elementary phases can be discriminated into two different major classes: passive and active phases.

A passive phase is characterised by an absence of voluntary control by the user on the elementary movement. In that case, independently of the user's efforts, the device achieves the goal assigned to the phase. For instance, a forced stop in the current position as well as any movements that do not depend on the applied effort can be considered as passive phases.

In an active phase, the device's movements must be actuated by the user. In the following, isokinetic movements are considered, because they are commonly used in lower limb rehabilitation protocols. Thus, during an isokinetic elementary phase, the velocities v_1 and v_2 are constant and the desired trajectory to be tracked by the device can be written as:

$$\begin{cases} \dot{q}_{1d} = \alpha_1 v_1 \\ \dot{q}_{2d} = \alpha_2 v_2 \\ q_{1d} = \alpha_1 v_1 t + q_{1d}(t_i) \\ q_{2d} = \alpha_2 v_2 t + q_{2d}(t_i) \end{cases}, \quad (1)$$

where t_i is the initial activation time of the i th phase and $\alpha_{k=1or2}$ are parameters required to set the contractile mode during the movement ($\alpha_k = 1$ for a concentric exercise, and $\alpha_k = -1$ for an eccentric one, see Figs. 6 and 7).

To define a cyclic isokinetic exercise, the discrete state machine, presented in Fig. 9 for flexion–extension, can be defined. The transition between phases depends on the force applied by the user on the mobile support as well as its current position.

At the beginning of a training session (Start), ϕ_0 is activated. In that state, the desired rehabilitation trajectory remains constant in its current position (the device is expected to be immobile). Then, the transition from ϕ_0 to ϕ_1 takes place if the user force $f_{p/x}(t)$ is greater than the force threshold $s_1 > 0$ (set by clinicians) and if the current position of the mobile support does not reach the upper limit of the operational space \bar{q}_1 . When ϕ_1 is activated, the transition to ϕ_0 occurs if $f_{p/x}(t)$ goes below the threshold s_1 or if the current position of the mobile support reaches the upper limit of the operational space. The transition from ϕ_0 to ϕ_2 occurs if $f_{p/x}(t)$ is lower than the threshold $s_2 < 0$ and if the current position of the support mobile does not reach the lower limit of the operational space \underline{q}_1 . Finally, the transition back to ϕ_0 occurs if the effort $f_{p/x}(t)$ is greater than the threshold $-s_1$ or if the current position of the support mobile reaches the lower limit of the operational space.

To illustrate the desired trajectory generation during a cyclic isokinetic exercise described by this state machine, a simulation was performed with the parameters $s_1 = 80$ N, $s_2 = -80$ N, $\bar{q}_1 = 1.2$ m, $\underline{q}_1 = 0.1$ m and $v_1 = 2$ m.s⁻¹. Figure 10 shows the results obtained with $f_{p/x}(t) = 500 \sin(6t)$. Note that the desired trajectory generation for internal–external rotation is not presented in this section, because it relies on the same principle as for flexion–extension. With the trajectory generator defined, the goal is now to ensure the tracking of these trajectories by the dynamic system (consisting of the device and the user). Therefore, the following section will present the proposed nonlinear tracking controller design methodology.

3.3 Continuous control level of Sys-Reeduc

The goal is now to provide a convenient controller design methodology that ensures the tracking of the desired trajectories provided by the trajectory generator. Note that, in that case, the external efforts, consisting of the forces and torques applied by the user, must be rejected or attenuated by the device's dynamics. In that context, a H_∞ -based controller design can be employed. Moreover, in view of the nonlinear dynamics of Sys-Reeduc, a Takagi–Sugeno tracking controller design will be proposed.

3.4 Dynamic modeling of Sys-Reeduc

The mechanical scheme of Sys-Reeduc is presented in Fig. 11. Note that the user is not included in this model because his/her movement cannot be artificially controlled. This justifies the use of a H_∞ -based design to attenuate these external disturbances to the device. Thus, to synthesise the control law, we consider that the device is to be controlled via the motors' torques while the patient applies external effort. $C_{M1}(t)$ and $C_{M2}(t)$ are the motors' torques allowing the movements associated with the degrees of freedom $q_1(t)$ and $q_2(t)$, respectively.

Recall that $\varphi(t) = [f_{p/x}(t) \ C_{p/z_s}(t)]^T$ denotes the vector of the efforts applied by the user on the device. The mechanical parameters used to model the Sys-Reeduc dynamics are defined in Table 1. These parameters were obtained from the design of the mechanical elements constituting Sys-Reeduc using the Catia software tools. Then, the dynamic model can be obtained using the well-known Lagrangian equations given by:

$$\frac{d}{dt} \left(\frac{\partial E_c}{\partial \dot{q}_i} \right) - \frac{\partial (E_c - E_p)}{\partial q_i} = \Gamma_i, \quad (2)$$

where q_i and Γ_i are respectively the coordinates and the generalised efforts associated with the i^{th} degree of freedom, and E_c and E_p are respectively the kinetic and the potential energy.

The mobile support's centre of gravity coordinates can be written in the frame $(o, \bar{x}, \bar{y}, \bar{z})$ as:

$$\overline{OG}_{/x} = q_1(t) + \beta \cos(\alpha) + b \cos(q_2(t)), \quad (3)$$

$$\overline{OG}_{/y} = h - \beta \sin(\alpha) + d \cos(q_2(t)), \quad (4)$$

$$\overline{OG}_{/z} = l \sin(q_2(t)), \quad (5)$$

with $b = l \sin(\alpha)$ and $d = l \cos(\alpha)$.

Thus, its velocity can be written in the frame $(o, \bar{x}, \bar{y}, \bar{z})$ as:

$$V_{G/x} = \dot{q}_1(t) - b\dot{q}_2(t) \sin(q_2(t)), \quad (6)$$

$$V_{G/y} = -d\dot{q}_2(t) \sin(q_2(t)), \quad (7)$$

$$V_{G/z} = l\dot{q}_2(t) \cos(q_2(t)). \quad (8)$$

Therefore, the kinetic energy can be expressed as:

$$E_C = \frac{1}{2}(M + m) \left[\dot{q}_1^2(t) + l^2 \dot{q}_2^2(t) - 2b\dot{q}_1(t)\dot{q}_2(t) \sin(q_2(t)) \right] + \frac{1}{2} J \dot{q}_2^2(t), \quad (9)$$

and the potential energy as:

$$E_p = mgh - mg\beta \sin(\alpha) + mgd \cos(q_2(t)), \quad (10)$$

where $g = 9.81 \text{ m.s}^{-2}$ is the gravitational constant.

Finally, substituting (9) and (10) in (2) and considering the generalised efforts $\Gamma_1(t) = C_{M1}(t) - f_{p/x}(t)$ and $\Gamma_2(t) = C_{M2}(t) - C_{p/z_s}(t)$, the dynamic model of Sys-Reeduc is expressed by the following motion equation:

$$M(q)\ddot{q}(t) + C(q, \dot{q})\dot{q}(t) + G(q)q(t) = Ru(t) + S\varphi(t), \quad (11)$$

where $M(q) = \begin{bmatrix} M + m & -(M + m)l \sin(\alpha) \sin(q_2(t)) \\ -(M + m)l \sin(\alpha) \sin(q_2(t)) & (M + m)l^2 + J \end{bmatrix}$ is the inertia matrix,

$C(q, \dot{q}) = \begin{bmatrix} 0 & -(M + m)l \sin(\alpha) \dot{q}_2(t) \cos(q_2(t)) \\ 0 & 0 \end{bmatrix}$ is the Coriolis matrix,

$G(q) = \begin{bmatrix} 0 & 0 \\ 0 & -mgl \cos(\alpha) \sin_c(q_2(t)) \end{bmatrix}$ is the gravitational effect, $R = \begin{bmatrix} 1/a & 0 \\ 0 & 1 \end{bmatrix}$ is the input

matrix, $S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the disturbance matrix, $q(t) = [q_1(t) \ q_2(t)]^T$ denotes the generalised

coordinates, and $u(t) = [C_{M1}(t) \ C_{M2}(t)]^T$ is the input vector.

3.5 T–S fuzzy descriptor modeling of Sys-Reeduc

We now propose a descriptor-based controller design. State space descriptors can be used to model algebraic systems such as singular systems [30–32]. Moreover, these are a convenient way to deal with mechanical systems with time-varying inertia and reduce the computational complexity of T–S-based LMI problems [20, 21]. By considering the following state vector of the system, $x(t) = [q_1(t) \ q_2(t) \ \dot{q}_1(t) \ \dot{q}_2(t)]^T$, (11) can be rewritten as the following state space descriptor:

$$E(x(t))\dot{x}(t) = A(x(t))x(t) + Bu(t) + H\varphi(t), \quad (12)$$

where $E(x(t)) = \begin{bmatrix} I & 0 \\ 0 & M(q) \end{bmatrix}$, $A(x(t)) = \begin{bmatrix} 0 & I \\ -G(q) & -C(q, \dot{q}) \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ R \end{bmatrix}$ and $H = \begin{bmatrix} 0 \\ S \end{bmatrix}$.

The goal is now to rewrite (12) as a T–S fuzzy descriptor of the form:

$$\sum_{k=1}^e v_k(z(t)) E_k \dot{x}(t) = \sum_{i=1}^r h_i(z(t)) A_i x(t) + Bu(t) + H\varphi(t), \quad (13)$$

where: e and r are the numbers of fuzzy sets for the left and right hand sides of the state equation (13), respectively; $z(t)$ is the premise vector depending on the state variables;

$v_k(z(t)) \geq 0$ for $k=1, \dots, e$ and $h_i(z(t)) \geq 0$ for $i=1, \dots, r$ are membership functions that

satisfy the convex sum property, that is, $\sum_{k=1}^e v_k(z(t)) = 1$ and $\sum_{i=1}^r h_i(z(t)) = 1$; E_k , A_i , B_i and

H_i are constant matrices, each defining $e \times r$ linear time invariant (LTI) descriptors that

compose the T–S fuzzy descriptor.

The sector nonlinearity approach [17] is a convenient way to rewrite a nonlinear descriptor

such as (12) as one of its T–S representations (13). Using this approach, the obtained T–S fuzzy model exactly matches the nonlinear model on a compact set of the state space. Hence, from (12), one can consider the nonlinear functions, included in $E(x(t))$ and $A(x(t))$, given by:

$$\omega(q_2(t)) = \sin(q_2(t)) \in [-1, 1], \quad (14)$$

$$\mu_1(q_2(t), \dot{q}_2(t)) = \dot{q}_2(t) \cos(q_2(t)) \in [-\bar{q}_2, \bar{q}_2], \quad (15)$$

$$\mu_2(q_2(t)) = \frac{\sin(q_2(t))}{q_2(t)} \in [\varpi, 1], \quad (16)$$

where $\varpi = -0.2172$ and $\bar{q}_2 \equiv 100 \text{ tr.min}^{-1} \equiv 10.47 \text{ rad.s}^{-1}$ is the maximal speed of the internal–external rotation set from product design specification.

To obtain each LTI model and the membership functions composing a T–S model matching (12), the following sector nonlinearity transformations are employed:

$$\omega(\cdot) = (1) \underbrace{\frac{\omega(\cdot) + 1}{2}}_{v_1(q_2)} + (-1) \underbrace{\frac{1 - \omega(\cdot)}{2}}_{v_2(q_2)}, \quad (17)$$

$$\mu_1(\cdot) = (\bar{q}_2) \underbrace{\frac{\mu_1(\cdot) + \bar{q}_2}{2\bar{q}_2}}_{w_{11}(\dot{q}_2, q_2)} + (-\bar{q}_2) \underbrace{\frac{\bar{q}_2 - \mu_1(\cdot)}{2\bar{q}_2}}_{w_{12}(\dot{q}_2, q_2)}, \quad (18)$$

$$\mu_2(\cdot) = (1) \underbrace{\frac{\mu_2(\cdot) - \varpi}{1 - \varpi}}_{w_{21}(q_2)} + (\varpi) \underbrace{\frac{1 - \mu_2(\cdot)}{1 - \varpi}}_{w_{22}(q_2)}. \quad (19)$$

From equations (17), (18) and (19), the membership functions are given by:

$$v_1(q_2) = \frac{\omega(q_2) + 1}{2}, \quad (20)$$

$$v_2(q_2(t)) = \frac{1 - \omega(q_2(t))}{2}, \quad (21)$$

$$h_1(\cdot) = w_{11}(\cdot) w_{21}(\cdot) = \frac{(\mu_1(\cdot) + \bar{q}_2)(\mu_2(\cdot) - \varpi)}{2\bar{q}_2(1 - \varpi)}, \quad (22)$$

$$h_2(\cdot) = w_{11}(\cdot)w_{22}(\cdot) = \frac{(\mu_1(\cdot) + \bar{q}_2)(1 - \mu_2(\cdot))}{2\bar{q}_2(1 - \varpi)}, \quad (23)$$

$$h_3(\cdot) = w_{12}(\cdot)w_{21}(\cdot) = \frac{(\bar{q}_2 - \mu_1(\cdot))(\mu_2(\cdot) - \varpi)}{2\bar{q}_2(1 - \varpi)}, \quad (24)$$

$$h_4(\cdot) = w_{12}(\cdot)w_{22}(\cdot) = \frac{(\bar{q}_2 - \mu_1(\cdot))(1 - \mu_2(\cdot))}{2\bar{q}_2(1 - \varpi)}, \quad (25)$$

and the LTI matrices by:

$$E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & M+m & -(M+m)l \sin \alpha \\ 0 & 0 & -(M+m)l \sin \alpha & (M+m)l^2 + J \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & M+m & (M+m)l \sin \alpha \\ 0 & 0 & (M+m)l \sin \alpha & (M+m)l^2 + J \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & (M+m)l \bar{q}_2 \sin \alpha \\ 0 & mgl \cos \alpha & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & (M+m)l \bar{q}_2 \sin \alpha \\ 0 & mgl \varpi \cos \alpha & 0 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -(M+m)l \bar{q}_2 \sin \alpha \\ 0 & mgl \cos \alpha & 0 & 0 \end{bmatrix}$$

$$\text{and } A_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -(M+m)l \bar{q}_2 \sin \alpha \\ 0 & mgl \varpi \cos \alpha & 0 & 0 \end{bmatrix}.$$

Consequently, considering $z(t) = [q_2 \quad \dot{q}_2]^T$, a T-S fuzzy model of Sys-Reeduc can be expressed as:

$$\sum_{k=1}^2 v_k(q_2) E_k \dot{x}(t) = \sum_{i=1}^4 h_i(q_2, \dot{q}_2) A_i x(t) + Bu(t) + H\varphi(t). \quad (26)$$

3.6 LMI-based tracking controller design for T-S descriptors

With the T-S descriptor model of Sys-Reeduc from the previous section, we now provide a convenient controller design methodology ensuring the tracking of desired trajectories

provided by the trajectory generator depicted in Section 3.2.

In the sequel, to simplify the mathematical expressions, we will consider the notations

$$X_h = \sum_{i=1}^r h_i(z(t)) X_i, \quad Y_{hv} = \sum_{k=1}^e \sum_{i=1}^r v_k(z(t)) h_i(z(t)) Y_{ik},$$

$$Z_{hhv} = \sum_{k=1}^e \sum_{i=1}^r \sum_{j=1}^r v_k(z(t)) h_i(z(t)) h_j(z(t)) Z_{ijk}, \text{ and so on.}$$

As usual, a star (*) in an expression indicates a transposed quantity. Moreover, when there is no ambiguity, the time t will be omitted.

For the sake of generality, the following LMI results will be developed for the general class of T-S fuzzy descriptors described by (13) and rewritten with the above-defined notations as:

$$E_v \dot{x}(t) = A_h x(t) + B_h u(t) + H_h \varphi(t), \quad (27)$$

where $A_h \in \mathbb{R}^{n \times n}$, $B_r \in \mathbb{R}^{n \times m}$, $H_h \in \mathbb{R}^{n \times p}$, $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $\varphi(t) \in \mathbb{R}^p$.

Consider the tracking control plant given by Fig. 12. This plant contains a reference model that is required to write the closed loop dynamics given by the state space representation:

$$\dot{x}_r(t) = A_r x_r(t) + B_r x_d(t), \quad (28)$$

with $A_r \in \mathbb{R}^{n \times n}$ a Hurwitz matrix, $B_r \in \mathbb{R}^{n \times n}$, $x_r(t) \in \mathbb{R}^n$ the reference state vector and $x_d(t) \in \mathbb{R}^m$ the desired trajectory to be tracked.

Note that (28) allows setting the dynamics of the tracking control trajectory. A convenient way to choose it will be proposed in the simulation results section.

The goal is now to write the closed loop dynamics of the tracking controller plant. Consider the Parallel Distributed Compensation (PDC) control law [40] defined by:

$$u(t) = -K_{hv} Z_{11}^{-1} (x(t) - x_r(t)), \quad (29)$$

where K_{hv} and $Z_{11} > 0$ are the gain matrices to be synthesised.

As is classically the case for descriptors [28], by considering the extended state vector

$$x^*(t) = \begin{bmatrix} x^T(t) & \dot{x}^T(t) \end{bmatrix}^T, \text{ (27) can be rewritten, for instance, as:}$$

$$E^* \dot{x}^*(t) = A_{hv}^* x^*(t) + B_h^*(t) u(t) + H_h^* \varphi(t), \quad (30)$$

$$\text{where } E^* = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, A_{hv}^* = \begin{bmatrix} 0 & I \\ A_h & -E_v \end{bmatrix}, B_h^* = \begin{bmatrix} 0 \\ B_h \end{bmatrix} \text{ and } H_h^* = \begin{bmatrix} 0 \\ H_h \end{bmatrix}.$$

In the same way, let us consider the extended reference state vector

$$x_r^*(t) = \begin{bmatrix} x_r^T(t) & \dot{x}_r^T(t) \end{bmatrix}^T. \quad (28) \text{ can be rewritten as:}$$

$$E^* \dot{x}_r^*(t) = A_r^* x_r^*(t) + B_r^* x_d(t), \quad (31)$$

$$\text{where } A_r^* = \begin{bmatrix} 0 & I \\ A_r & -I \end{bmatrix} \text{ and } B_r^* = \begin{bmatrix} 0 \\ B_r \end{bmatrix}.$$

Let us define $e^*(t) = x^*(t) - x_r^*(t)$ as the tracking trajectory error. We can write:

$$E^* \dot{e}^*(t) = E^* \dot{x}^*(t) - E^* \dot{x}_r^*(t), \quad (32)$$

and (29) can be rewritten:

$$u(t) = -K_{hv}^* e^*(t), \quad (33)$$

$$\text{where } K_{hv}^* = \begin{bmatrix} K_{hv} Z_{11}^{-1} & 0 \end{bmatrix}.$$

Combining (30), (31), (32) and (33), the closed loop dynamics can be expressed by:

$$\tilde{E} \dot{\tilde{e}}(t) = \tilde{A}_{hhv} \tilde{e}(t) + \tilde{H}_h \tilde{\phi}(t), \quad (34)$$

$$\text{with } \tilde{e}(t) = \begin{bmatrix} e^*(t) \\ x_r^*(t) \end{bmatrix}, \tilde{\phi}(t) = \begin{bmatrix} \varphi(t) \\ x_d(t) \end{bmatrix}, \tilde{E} = \begin{bmatrix} E^* & 0 \\ 0 & E^* \end{bmatrix}, \tilde{A}_{hhv} = \begin{bmatrix} (A_{hv}^* - B_h^* K_{hv}^*) & A_{hv}^* - A_r^* \\ 0 & A_r^* \end{bmatrix} \text{ and}$$

$$\tilde{H}_h = \begin{bmatrix} H_h^* & -B_r^* \\ 0 & B_r^* \end{bmatrix}.$$

Now, the aim is to synthesise the matrices K_{hv} and Z_{11} that stabilise the closed loop system (34) as well as ensure the attenuation of the external disturbances $\tilde{\phi}(t)$ with respect to the trajectory-tracking error $e(t)$ [26, 27]. Thus, we consider the following H_∞ criterion:

$$\int_{t_0}^{t_f} e^T(t) e(t) dt \leq \eta^2 \int_{t_0}^{t_f} \tilde{\phi}^T(t) \tilde{\phi}(t) dt, \quad (35)$$

where η is the attenuation level to be minimised.

The main theoretical result is summarised in the following theorem.

Theorem: If the matrices K_{jk} , $Z_{11} = Z_{11}^T > 0$, $Z_{41} = Z_{41}^T > 0$, Z_{13ij} , Z_{14ij} , Z_{23ij} , Z_{24ij} , Z_{33ij} ,

Z_{34ij} , Z_{43ij} and Z_{44ij} and the scalar δ exist such that the following LMI conditions are

satisfied for all $i = 1, \dots, r$, $j = 1, \dots, r$ and $k = 1, \dots, e$ under the minimisation problem $\min_{\mathbb{R}^+} \delta$:

$$\begin{bmatrix} \Upsilon_{ij}^{(1,1)} & (*) & (*) & (*) & (*) & 0 & 0 \\ \Upsilon_{ij}^{(2,1)} & \Upsilon_{ij}^{(2,2)} & (*) & (*) & (*) & 0 & 0 \\ Z_{11} & Z_{11} & -I & 0 & 0 & 0 & 0 \\ \Upsilon_{ijk}^{(4,1)} & \Upsilon_{ijk}^{(4,2)} & 0 & \Upsilon_{ijk}^{(4,4)} & (*) & (*) & (*) \\ \Upsilon_{ij}^{(5,1)} & \Upsilon_{ij}^{(5,2)} & 0 & \Upsilon_{ijk}^{(5,4)} & \Upsilon_{ij}^{(5,5)} & 0 & (*) \\ 0 & 0 & 0 & H_i^T & 0 & -\delta I & 0 \\ 0 & 0 & 0 & -B_r^T & B_r^T & 0 & -\delta I \end{bmatrix} \leq 0, \quad (36)$$

where $\Upsilon_{ij}^{(1,1)} = Z_{13ij}^T + Z_{13ij}$, $\Upsilon_{ij}^{(2,1)} = Z_{33ij} + Z_{23ij}^T$, $\Upsilon_{ij}^{(2,2)} = Z_{43ij}^T + Z_{43ij}$,

$$\Upsilon_{ijk}^{(4,1)} = Z_{14ij}^T + 2A_i Z_{11} - E_k Z_{13ij} - B_i K_{jk} - E_k Z_{33ij} - A_r Z_{11}^T + Z_{33ij},$$

$$\Upsilon_{ijk}^{(4,2)} = Z_{34ij}^T + A_i Z_{11} - E_k Z_{23ij} - B_i K_{jk} + A_i Z_{41} - E_k Z_{43ij} - A_r Z_{41} + Z_{43ij},$$

$$\Upsilon_{ijk}^{(4,4)} = -Z_{14ij}^T E_k^T - E_k Z_{14ij} - Z_{34ij}^T E_k^T - E_k Z_{34ij} + Z_{34ij}^T + Z_{34ij}, \quad \Upsilon_{ij}^{(5,1)} = A_r Z_{11}^T - Z_{33ij} + Z_{24ij}^T,$$

$$\Upsilon_{ijk}^{(5,2)} = Z_{44ij}^T + A_r Z_{41} - Z_{43ij}, \quad \Upsilon_{ijk}^{(5,4)} = -Z_{34ij} - Z_{24ij}^T E_k^T - Z_{44ij}^T E_k^T + Z_{44ij}^T \quad \text{and} \quad \Upsilon_{ij}^{(5,5)} = -Z_{44ij}^T - Z_{44ij},$$

then the asymptotic stability of the closed loop fuzzy system (34) is ensured and the H_∞

tracking control performance (35) is guaranteed with an external disturbance attenuation level

$$\eta = \sqrt{\delta}.$$

Proof: Let us consider the candidate Lyapunov function:

$$V(\tilde{e}(t)) = \tilde{e}^T(t) \tilde{E} \tilde{Z}_{hhv}^{-1} \tilde{e}(t) > 0, \quad (37)$$

with \tilde{Z}_{hhv} a non-singular C^1 matrix satisfying $\tilde{E} \tilde{Z}_{hhv}^{-1} = \tilde{Z}_{hhv}^{-T} \tilde{E} > 0$ and defined as:

$$\tilde{Z}_{hhv} = \begin{bmatrix} Z_{11} & 0 & Z_{21hh} & 0 \\ Z_{13hh} & Z_{14hh} & Z_{23hh} & Z_{24hh} \\ Z_{31hh} & 0 & Z_{41} & 0 \\ Z_{33hh} & Z_{34hh} & Z_{43hh} & Z_{44hh} \end{bmatrix}, \quad (38)$$

where $Z_{11}^T = Z_{11} > 0$, $Z_{21hh}^T = Z_{31hh} > 0$ and $Z_{41}^T = Z_{41} > 0$.

Let us consider $\tilde{Q} = \text{diag}[I \ 0 \ 0 \ 0]$, then the H_∞ criterion (35) can be rewritten as:

$$\int_{t_0}^{t_f} \tilde{e}^T(t) \tilde{Q} \tilde{e}(t) dt \leq \eta^2 \int_{t_0}^{t_f} \tilde{\phi}^T(t) \tilde{\phi}(t) dt. \quad (39)$$

Thus, the closed-loop descriptor (34) is stable and ensures the H_∞ performance η if:

$$\dot{V}(\tilde{e}(t)) + \tilde{e}^T(t) \tilde{Q} \tilde{e}(t) - \eta^2 \tilde{\phi}^T(t) \tilde{\phi}(t) \leq 0. \quad (40)$$

Considering (34) and (37), (40) is verified if:

$$\begin{bmatrix} \tilde{A}_{hhv}^T \tilde{Z}_{hhv}^{-1} + \tilde{Z}_{hhv}^{-T} \tilde{A}_{hhv} + \tilde{Q} & \tilde{Z}_{hhv}^{-T} \tilde{H}_h \\ \tilde{H}_h^T \tilde{Z}_{hhv}^{-1} & -\eta^2 I \end{bmatrix} \leq 0. \quad (41)$$

By pre- and post-multiplying (41) by $\begin{bmatrix} \tilde{Z}_{hhv}^T & 0 \\ 0 & I \end{bmatrix}$ and $\begin{bmatrix} \tilde{Z}_{hhv} & 0 \\ 0 & I \end{bmatrix}$, respectively, we obtain:

$$\begin{bmatrix} (\tilde{Z}_{hhv}^T \tilde{A}_{hhv}^T + \tilde{A}_{hhv} \tilde{Z}_{hhv} + \tilde{Z}_{hhv}^T \tilde{Q} \tilde{Z}_{hhv}) & \tilde{H}_h \\ \tilde{H}_h^T & -\eta^2 I \end{bmatrix} \leq 0. \quad (42)$$

After developing (42) in its extended form with the matrices defined in (34), (38) and (39), we apply the Schur complement [41] and the proof is completed. ■

4. SIMULATION RESULTS

The theoretical results in the previous section ensure the tracking performance of a general class of T-S fuzzy descriptors. In this part, our goal is to show, by simulation, the efficiency of the proposed tracking controller design methodology in the dynamic model of Sys-Reeduc given in (11). Following the tracking control plant depicted in Fig. 12 and considering the parameters required to solve the LMI problem (36), we must first choose an adequate reference model of the form (28). This reference model can be used to set the dynamics of the trajectory tracking inputs, and its influence on the LMI solution cannot be neglected. In the Sys-Reeduc application, the goal is to force the state $x(t)$ to track the reference state $x_r(t)$. According to (28), the reference model contains an input vector $x_d(t)$, which must be close

to $x_r(t)$. In this study, the reference model is chosen such that each transfer between an input variable $x_{dp}(s)$ and its associated reference state variable $x_{rp}(s)$, for $p = 1, \dots, n$, corresponds to a low-pass filter given in the frequency domain by:

$$\frac{x_{rp}(s)}{x_{dp}(s)} = \frac{1}{1 + \kappa s}, \quad (43)$$

where κ is the time constant and s the Laplace variable.

According to (43), a state-space reference model can be written by considering $A_r = -\frac{1}{\kappa} \text{eye}(4, 4)$ and $B_r = \frac{1}{\kappa} \text{eye}(4, 4)$. Obviously, the smaller κ is, the closer the reference state $x_r(t)$ dynamics will be to the desired reference $x_r(t)$. Indeed, decreasing the time constant leads to downgrade the H_∞ performance as the LMI problem is compelled. To illustrate this phenomenon, the minimal attenuation level η has been evaluated, using the Matlab LMI control toolbox [42], for several time constants κ , and the results are presented in Table 2.

In the present application, a good compromise could be $\kappa = 0.1$ s, leading to $\eta = 3.08$. The results computed from the LMI conditions (36) lead to the design of the PDC tracking control law given by:

$$u(t) = -\sum_{k=1}^2 \sum_{i=1}^4 v_k(q_2(t)) h_i(q_2(t), \dot{q}_2(t)) K_{ik} Z_{11}^{-1} (x(t) - x_r(t)), \quad (44)$$

$$\text{where } K_{11} = \begin{bmatrix} -33.2690 & -0.8128 & 660.0514 & 14.2361 \\ -7.9455 & -13.3568 & 157.6020 & 244.4651 \end{bmatrix},$$

$$K_{12} = \begin{bmatrix} -33.2693 & -0.1037 & 660.0595 & 0.3839 \\ 7.9549 & -13.3562 & -157.8613 & 244.5319 \end{bmatrix},$$

$$K_{21} = \begin{bmatrix} -33.2691 & -0.0054 & 660.0563 & 1.0741 \\ -7.9455 & -13.3567 & 157.6019 & 244.4642 \end{bmatrix},$$

$$K_{31} = \begin{bmatrix} -33.2690 & -0.8128 & 660.0514 & 14.2363 \\ -7.9455 & -13.2399 & 157.6019 & 243.4855 \end{bmatrix},$$

$$K_{32} = \begin{bmatrix} -33.2693 & -0.1037 & 660.0595 & 0.3840 \\ 7.9550 & -13.2390 & -157.8627 & 243.5470 \end{bmatrix},$$

$$K_{41} = \begin{bmatrix} -33.2691 & -0.0054 & 660.0563 & 1.0748 \\ -7.9455 & -13.2397 & 157.6022 & 243.4810 \end{bmatrix},$$

$$K_{42} = \begin{bmatrix} -33.2694 & 0.7037 & 660.0644 & -12.7777 \\ 7.9550 & -13.2392 & -157.8621 & 243.5496 \end{bmatrix}$$

$$\text{and } Z_{11} = \begin{bmatrix} -0.3900 & 0 & -1.5669 & 0.0001 \\ 0 & -0.3880 & 0 & -1.5692 \\ -1.5669 & 0 & 20.4455 & -0.0032 \\ 0.0001 & -1.5692 & -0.0032 & 20.3726 \end{bmatrix}.$$

Figures 13 and 14 illustrate the tracking performance of the closed loop rehabilitation device plant. The simulations are realised for a lower limb isokinetic extension with velocity $\dot{q}_1 = 1 \text{ m.s}^{-1}$ between $t=1 \text{ s}$ and $t=2 \text{ s}$ from the initial position $q_1(0) = 0.1 \text{ m}$ and for a sinusoidal internal–external rotation $q_2 = (\pi/4)\cos(2t)$. The efforts $f_{p/x}(t)$ and $C_{p/z_s}(t)$ have been set as non-physiological external disturbances such that they outperform common user capabilities:

$$f_{p/x}(t) = 500\sin(6t) + 250\sin(30t) + 125\sin(60t) + \text{rand}(t), \quad (45)$$

and

$$C_{p/z_s}(t) = 20\sin(5t). \quad (46)$$

Note that in (45), the function $\text{rand}(t)$ is used to simulate a noise measurement in the force applied by the patient. This function is set with a Gaussian disturbed signal of variance equal to 1000 and a sample time equal to 0.01 s.

These simulation results illustrate the efficiency of the performance in trajectory tracking obtained by using the proposed control law. Indeed, Figures 13 and 14 show that the H_∞ control law (44) successfully attenuates the external disturbances and compensates them via the input signal. In these simulations, the trajectory generator is not taken into account. Figure 15 illustrates the performance of the whole control structure design for Sys-Reeduc,

depicted in Fig. 8, including the isokinetic trajectory generator proposed in Section 3.2. This simulation shows that the trajectory generation is successfully realised while the tracking performance of the device is ensured.

5. CONCLUSIONS

After discussing the advantages and drawbacks of the OMC and the CMC rehabilitation techniques, the concept of a new CMC lower-limb rehabilitation device has been presented. The proposed control structure consists of a trajectory generator and a continuous level ensuring the tracking control stability of the closed loop mechanical system. For the sake of generality, the principle of the trajectory generator was described as a parameterised discrete-state machine. LMI-based tracking controller design methodology was developed for a class of T-S fuzzy descriptors suitable to represent the nonlinear dynamics of Sys-Reeduc. Moreover, a H_∞ criterion has been employed to attenuate the user's efforts that are considered, for the closed loop continuous dynamics, as external disturbances. Note that attenuating these disturbances provides a safe behaviour to the user of the rehabilitation device. Indeed, even if the user-rehabilitation device interaction is attenuated by the continuous level, the trajectory generator allows voluntary movements to be performed on the basis of the measurement of this interaction. One of the interests of such a control structure is the possibility of parameterising a particular trajectory generator that, for instance, motivates the user to provide an appropriate effort during rehabilitation movements. Finally, the proposed controller synthesis has been validated in simulation on Sys-Reeduc and has shown the efficiency of the proposed control plant. Ongoing realisation of the first prototype of Sys-Reeduc will enable us to develop and validate new rehabilitation protocols and to evaluate accurately the benefits of CMC versus OMC rehabilitation.

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Table I: Numerical parameters of the dynamical model of the rehabilitation device Sys-Reeduc

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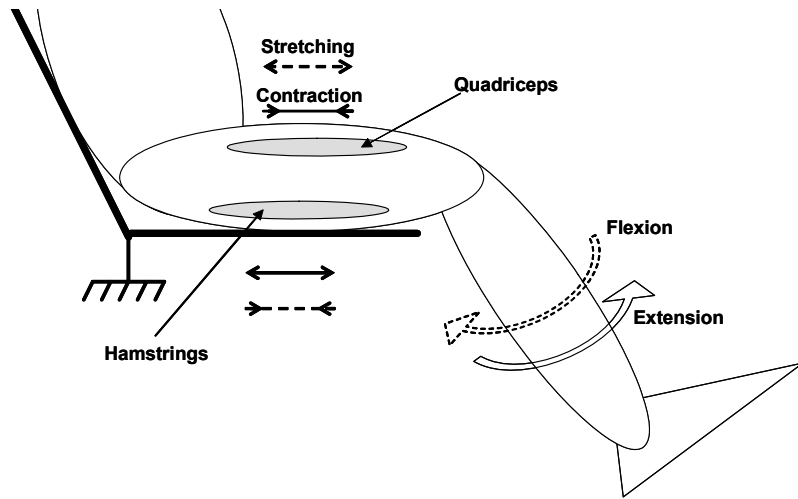


Fig. 1. Exercise in OMC.

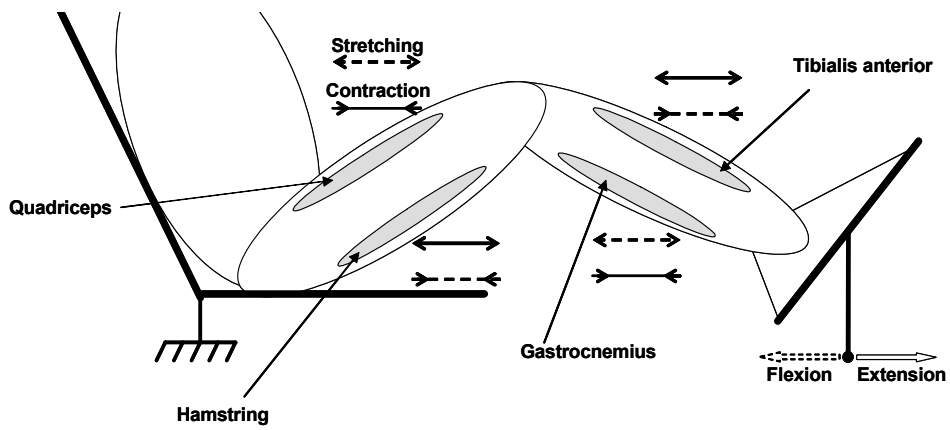


Fig. 2. Exercise in CMC.

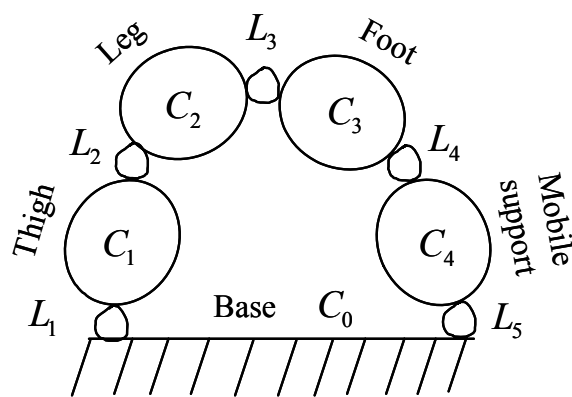


Fig. 3. Kinematic chain of a lower-limb CMC rehabilitation device.

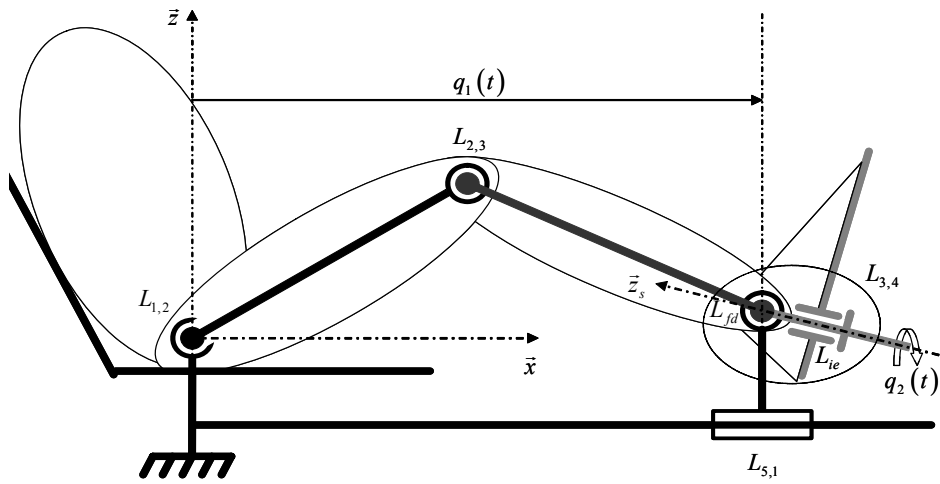


Fig. 4. Kinematic concepts of Sys-Reeduc in the sagittal plane.

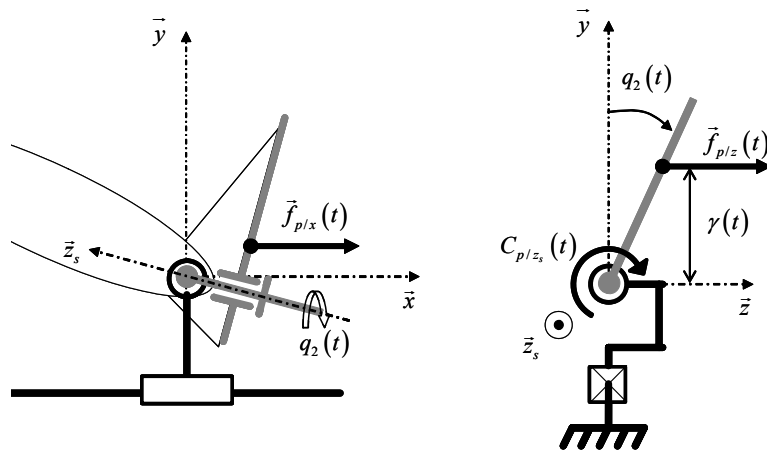


Fig. 5. External efforts that influence Sys-Reeduc's dynamics.

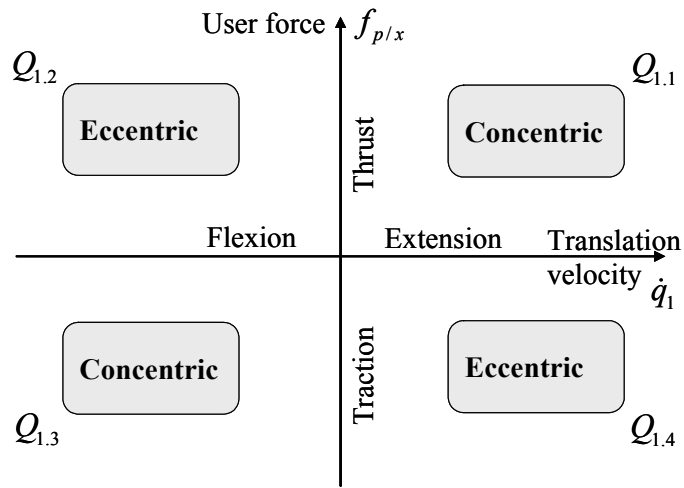


Fig. 6. External efforts that influence Sys-Reeduc's flexion-extension.

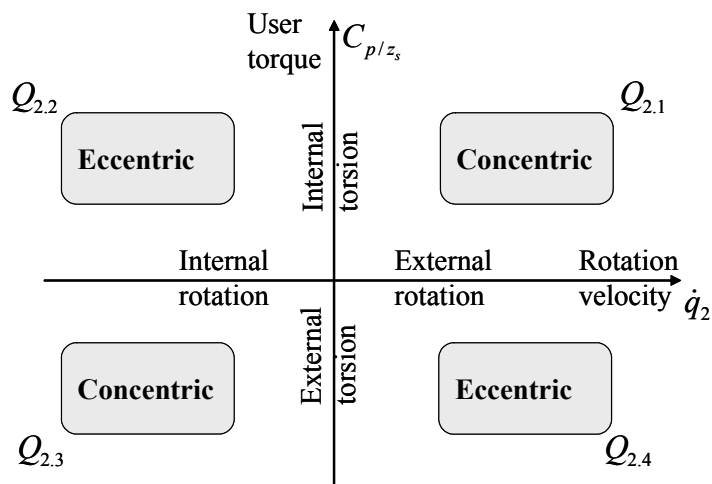


Fig. 7. External efforts that influence Sys-Reeduc's internal-external rotation.

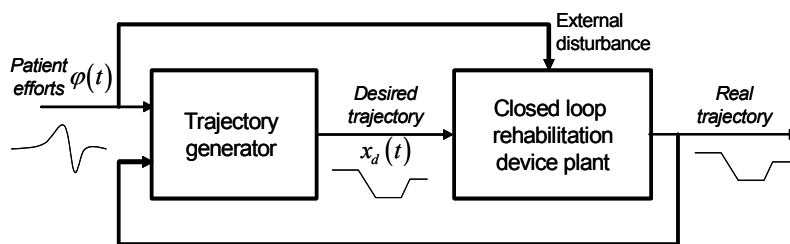


Fig. 8. Generic control structure for rehabilitation devices.

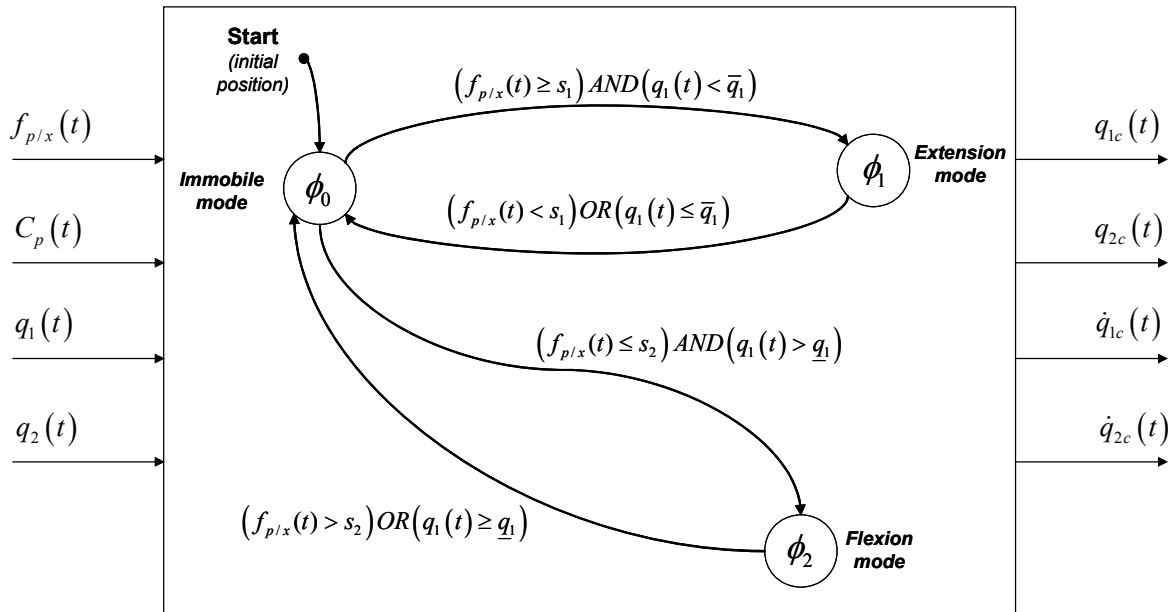


Fig. 9. Trajectory generator for Sys-Reeduc

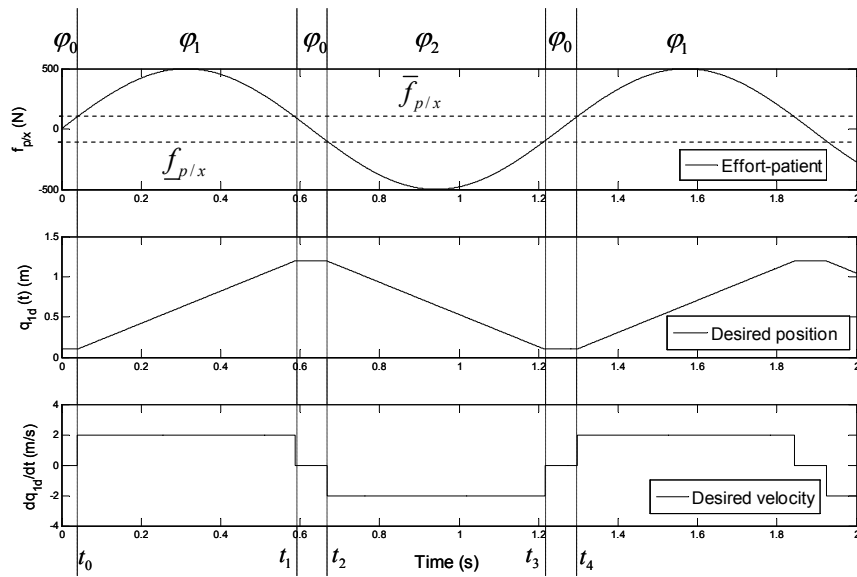


Fig. 10. Desired trajectory generated during isokinetic exercises.

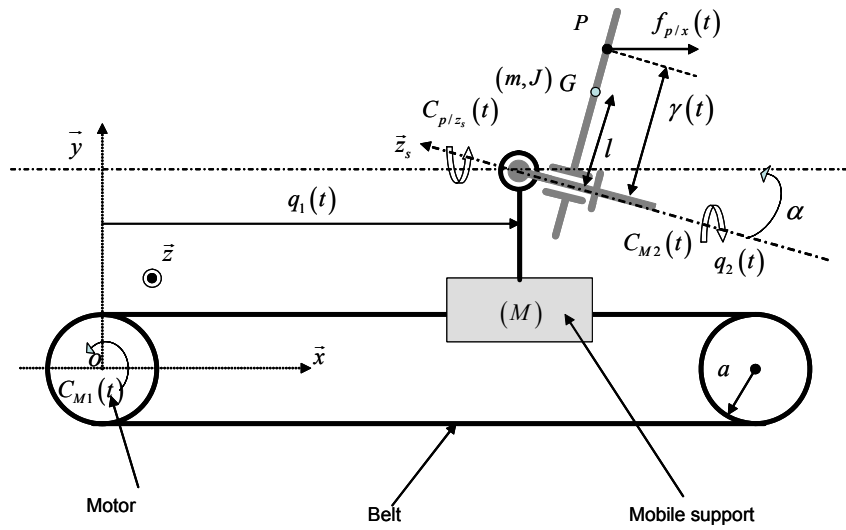


Fig. 11. Mechanical principle for the rehabilitation device.

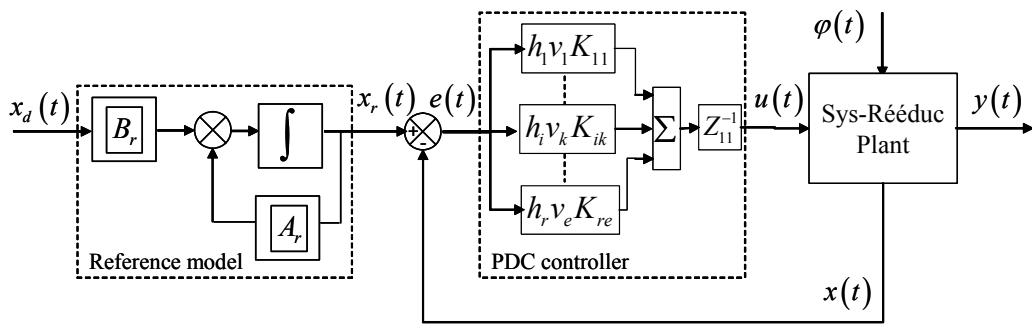


Fig. 12. T-S trajectory tracking control plant.

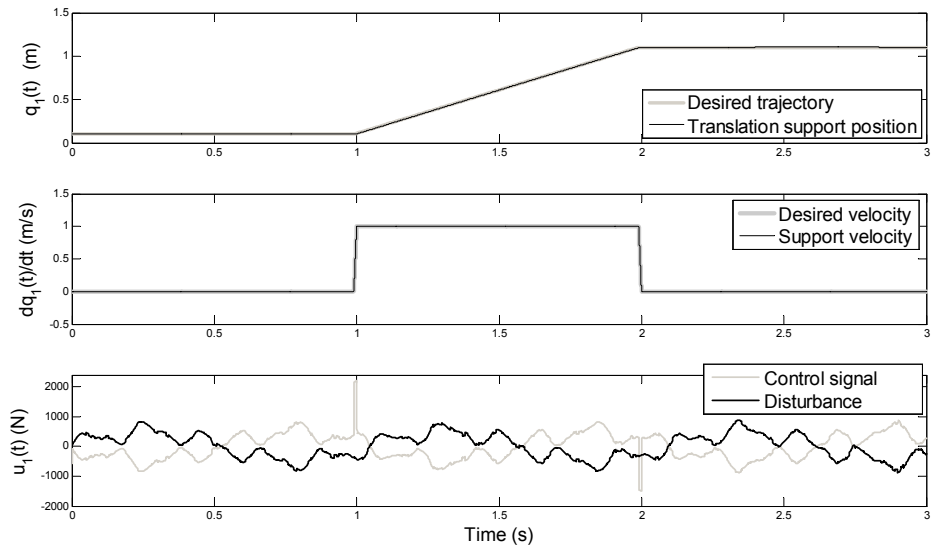


Fig. 13. Tracking trajectory for translation of the mobile support.

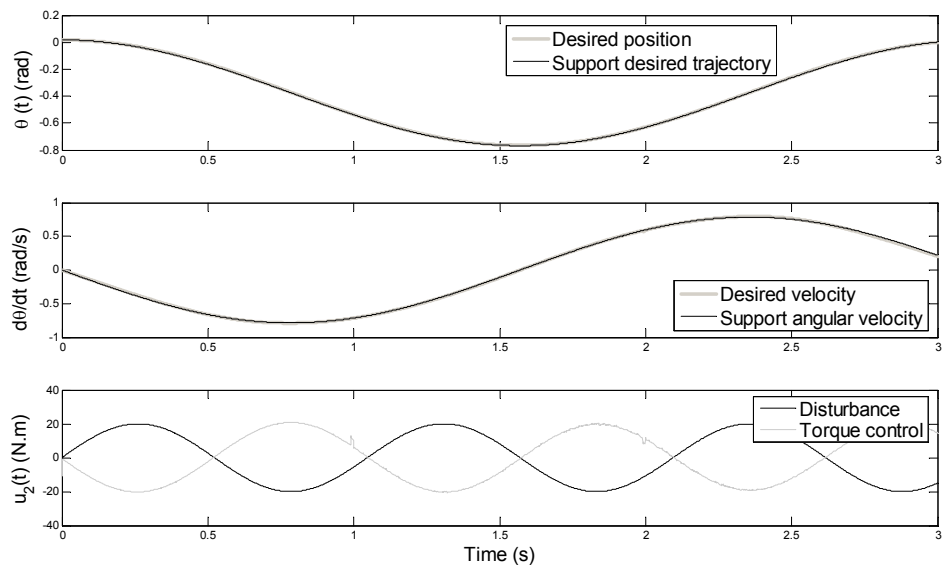


Fig. 14. Tracking trajectory for rotation of the mobile support.

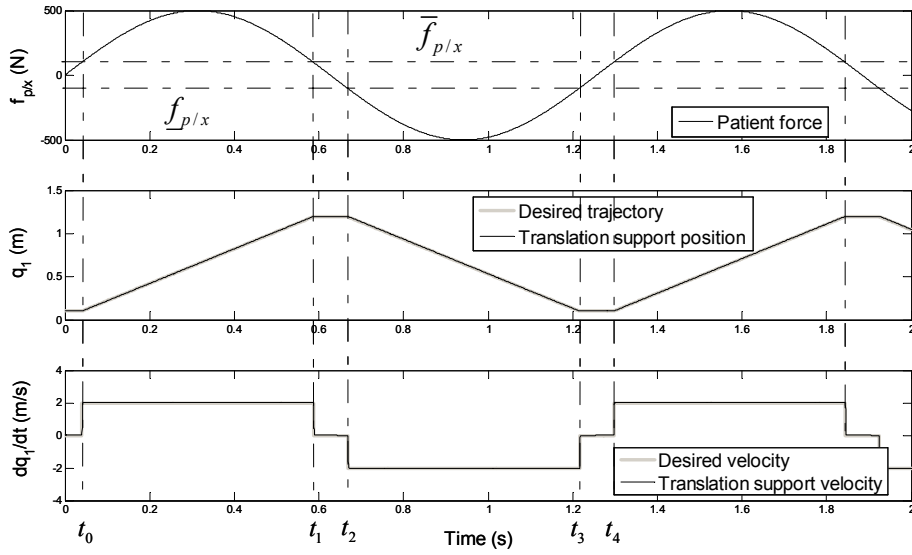


Fig. 15. Simulation of the whole Sys-Reeduc control structure.

TABLE 1

NUMERICAL PARAMETERS OF THE DYNAMIC MODEL OF THE REHABILITATION DEVICE SYS-REEDUC

Param.	Designation	Value
M	Mobile support mass (in translation)	14 kg
m	Mobile support mass (in rotation)	4 kg
J	Mobile support inertia along the \bar{z}_s axis (in rotation)	0.26 kg.m ²
a	Ray of the pulley-belt carrying out the translation along the \bar{x} axis	0.025 m
l	Distance between the rotation axis and the gravity centre of the support	0.05 m
α	Angle between the base horizontal axis and the rotation axis of the mobile support	20°

TABLE 2

EVOLUTION OF THE ATTENUATION RATE AS κ CHANGES

κ	1/400	1/100	1/50	1/30	1/20	1/10
η	19.98	9.97	7.03	5.43	4.41	3.08