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Descriptor modelling toward control of two links pneumatic robot manipulator: a T-S multimodel approach

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Abstract

This paper presents the first steps towards a robust model-based controller design for two-link manipulators using a Takagi Sugeno multimodel descriptor form. Due to the circumstances that this form is more similar to a given original nonlinear equation as a Takagi Sugeno multimodel, it allows to reduce the conservatism of the controller design by using common matrix structures. The model-based control law here is equivalent to the well-known parallel distributed compensation scheme. The challenge of the investigated modelling and control problem in this case is the highly nonlinear dynamics of the dual-actuator drive powered by air-pressure that interacts with the dynamics of the robot manipulator.

Key words: Descriptor systems, Control oriented models, Robotic manipulators, Fluid Power System

1. Introduction

With the growing interest in modelling and control of systems with complex dynamics, nonlinear control theory becomes popular and several applications have been proposed, for instance in robotics [6, 7, 16]. For several

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years Takagi Sugeno (T-S) systems [19] are widely used in a context of modelling for control design [12, 20, 23, 8, 11, 2]. Indeed, it is well known that T-S multimodels is an universal approximator of continuous nonlinear systems by using, for instance, the sector nonlinearity approach [21] or a tensor product approach [10], [15]. Thus, by analogy to switched systems [3], a T-S multimodel can be viewed as a set of local LTI systems blended together by smooth switching functions. Also, linear descriptor systems, which differ from state-space representations, have generated a great deal of interest in control system design. Descriptor systems describe a wider class of systems including physical models and nondynamic constraints [9]. The interest of choosing a descriptor system rather than a classical state space representation, in particular for mechanical systems, is based on: 1. Reducing the number of rules \rightarrow reducing conservatism, see [5]. 2. Using the natural model structure due to the inertia matrix. Then, the use of a descriptor representation is justified to model a robot if the inertia matrix contains nonlinear terms, i.e. if the structure of the considered robot is geometrically variable, that is the case of the SCARA robot.

There exist a large number of papers on the stability analysis of T-S multimodels based on the state-space representation and also some applications in the control design process e.g. [4, 14, 18]. But in contrast, stability analysis of T-S multimodels based on descriptor representation as weighted combinations of linear descriptor systems (from now called *T-S descriptor systems*) have not been investigated until [22] and recently applied in biomechanics [5] and in robotics [16]. The purpose of this work is to propose a robust multimodel controller design based on a first derived descriptor form of a two-link robot manipulator powered by pneumatic actuators as a reasonable example of a highly nonlinear complex system.

This paper is organized as follows: First, for a class of serial manipulators, the so-called SCARA-type robot manipulators [17], a nonlinear state-space model is formulated by the Newton-Euler equation of the manipulator arm coupled with a reduced model of the actuators which contains two bounded uncertainties. Then the state-space model combined with dynamic constraints is transformed into a T-S descriptor form [22]. It is shown by simulation results that this proposed model is capable to represent the coupled nonlinearities. Finally, in a prospective way, a global controller is designed based on the previously derived model as a robust parallel distributed compensation (PDC) [12].

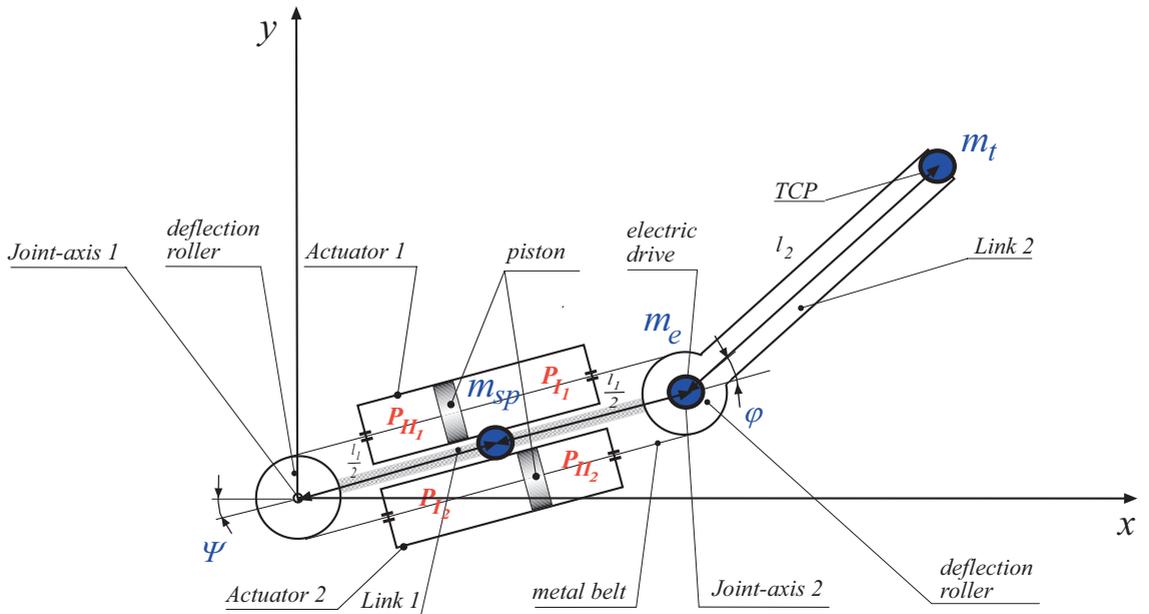


Figure 1: Two-link manipulator with a dual servo-pneumatic actuator as main drive (ψ) and an electric direct-drive (φ)

2. Physical System Modelling

In the following the dynamic model of a SCARA-type robot manipulator is presented. The robot arm is powered by an actuator mechanism based on a tangential feed as the main drive and an electric direct-drive as the actuator of the second axis presented in Fig. 1. The main drive consists of two rodless cylinders with pistons, two deflection rollers and a metal belt as a friction-locked connection between the rollers and the pistons (also called Belt-motor [13]). The forces on the pistons caused by opposed pressure differences in Actuator 1 and 2 are transmitted by the belt to the deflection rollers, where the linear motion of the pistons is converted to angular motion of the robot link 1. We assume that the normal strain of the belt is negligibly small (infinitely stiff) and the tangent stiffness is pure elastic.

2.1. Mechanical model of the manipulator dynamics

Consider the two-link planar arm shown in Fig. 1 that represents a reduced mechanical system of the manipulator. The robot arm has two vertical revolute joints, both joint axes are orthogonal to the xy-plane. Here,

the masses of the actuator cylinders and the mass of link 1 are combined to m_{sp} . Let m_e be the mass of the rotor of the electric drive, and m_t the mass of the tool and vertical drive in the tool center point (TCP, see Fig. 1), whereby the mass of link 2 is small in comparison to m_t . The distance of the centers of mass of m_{sp} and m_e to the Joint-axis 1 is defined by $l_1/2$, and l_1 as the length of link 1. The distance of the center of mass of m_t to the joint-axis 2 is defined by l_2 as the length of link 2. The equation of motion for this mechanical system can be written in the known matrix form [17] which represents the joint space dynamics model:

$$\mathbf{H}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau} \quad (1)$$

where $\mathbf{q} = [\psi \ \varphi]^T$ is the vector of generalized coordinates with ψ as the angle of link 1 from the x-axis, and with φ as the angle between link 1 and link 2. The inertia matrix

$$\mathbf{H}(\mathbf{q}) = \begin{bmatrix} H_{11}(\mathbf{q}) & H_{12}(\mathbf{q}) \\ H_{21}(\mathbf{q}) & H_{22}(\mathbf{q}) \end{bmatrix} \quad (2)$$

depends on the current arm configuration with:

$$H_{11}(\mathbf{q}) = I_{L1} + m_{sp} \left(\frac{l_1}{2} \right)^2 + m_e l_1^2 + I_{L2} \\ + m_t (l_1^2 + l_2^2 + 2 l_1 l_2 \cos \varphi) ,$$

$$H_{12}(\mathbf{q}) = H_{21}(\mathbf{q}) = I_{L2} + m_t l_2 (l_2 + l_1 \cos \varphi) ,$$

$$H_{22} = I_{L2} + m_t l_2^2 .$$

Note that, for this application, the assumption holds (the links are made of carbon-fiber-reinforced plastic) that the moments of inertias relative to the centers of mass of link 1 and link 2 as I_{L1} and I_{L2} can be neglected against the other terms, such as $(m_e l_1^2)$. So we set in the following considerations $I_{L1} = I_{L2} = 0$. The actuation torques in (2) are represented by $\boldsymbol{\tau} = [\tau_1 \ \tau_2]^T$. Finally, the vector

$$\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} h_1(\mathbf{q}, \dot{\mathbf{q}}) \\ h_2(\mathbf{q}, \dot{\mathbf{q}}) \end{bmatrix} \quad (3)$$

in (1) represents the centrifugal effects and Coriolis effects with $h_1(\mathbf{q}, \dot{\mathbf{q}}) = m_t l_1 l_2 \sin \varphi (-\dot{\varphi}^2 - 2 \dot{\psi} \dot{\varphi})$ and $h_2(\mathbf{q}, \dot{\mathbf{q}}) = m_t l_1 l_2 \sin \varphi \dot{\psi}^2$.

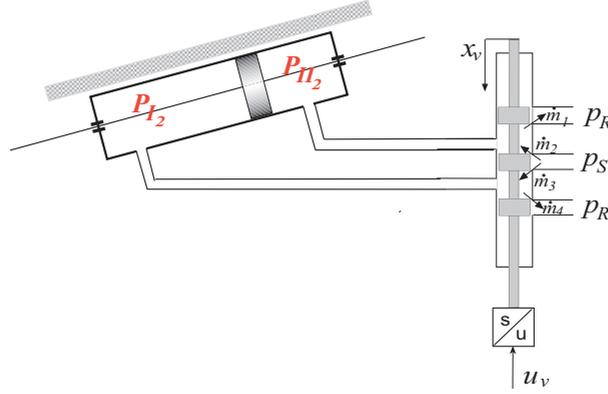


Figure 2: One cylinder of the dual-actuator with the accessory servovalve

2.2. Dual-Actuator model

The physical description of the main drive (see Fig.1) is based on the continuity and enthalpy balance equation of the gas flow through the actuator and the constitutive relations of the air. Whereby the most important aspect of the actuator dynamics that interact with the robot arm is described by the pressure evolution of each chamber. Assuming isentropic behavior of the gas flow, the pressure evolution in the chambers of actuator j ($j = 1, 2$, see Fig.1) is modeled by the equations:

$$\begin{aligned} \dot{p}_{I_j} (V_{0I} + A_K (x_K - x_{K0})) = \\ \kappa [R_g T_I (\dot{m}_2 - \dot{m}_1) - p_I A_K \dot{x}_K] \end{aligned} \quad (4a)$$

and

$$\begin{aligned} \dot{p}_{II_j} (V_{0II} - A_K (x_K - x_{K0})) = \\ \kappa [-R_g T_{II} (\dot{m}_4 - \dot{m}_3) + p_{II} A_K \dot{x}_K] . \end{aligned} \quad (4b)$$

where p_I, p_{II} [N/m^2] as the pressure in actuator chambers I, II , x_K [m] as the actuator piston displacement and \dot{m}_n [kg/s] as the mass flow over the servovalve control edges $n = 1, 2, 3, 4$ (see Fig. 2).

The *model parameters* are: x_{K0} [m] as the start position of the actuator piston, $V_{0I,0II}$ [m^3] as the start volume of the actuator chamber I, II , A_K [m^2] as the actuator piston area, R_g as the gas constant [$\frac{\text{J}}{\text{kg}\cdot\text{K}}$], and κ [-] as the adiabatic exponent of the gas. The two variable volume chambers in each

actuator cylinder are connected to a four-way servovalve. The configuration for one actuator cylinder in combination with the servovalve is shown in Fig. 2. In this case the servovalve controls the air-mass-flow $\dot{m}_1, \dots, \dot{m}_4$ by adjusting four orifice areas between the constant pressure supply p_s , the exhaust pressure p_R , and the chamber pressures $p_{I,II}$. The four orifice areas are continuously controlled by the servovalve voltage u_v . The equations for the mass flows are:

$$\dot{m}_1 = -\alpha_{D1} A_1(x_v) \psi \left(\frac{p_R}{p_{II}} \right) p_I \sqrt{\frac{2}{R_g T_{II}}} \quad (5a)$$

$$\dot{m}_2 = \alpha_{D2} A_2(x_v) \psi \left(\frac{p_{II}}{p_S} \right) p_S \sqrt{\frac{2}{R_g T_S}} \quad (5b)$$

$$\dot{m}_3 = -\alpha_{D3} A_3(x_v) \psi \left(\frac{p_I}{p_S} \right) p_S \sqrt{\frac{2}{R_g T_S}} \quad (5c)$$

$$\dot{m}_4 = \alpha_{D4} A_4(x_v) \psi \left(\frac{p_R}{p_I} \right) p_I \sqrt{\frac{2}{R_g T_I}} \quad (5d)$$

with the servovalve displacement x_v and the orifice areas of the control edges for $n = 2, 4$:

$$A_n(x_v) = \begin{cases} x_v \pi d_n & \text{for } x_v \geq x_{\ddot{u}_n} \\ 0 & \text{for } x_v < x_{\ddot{u}_n} \end{cases} \quad (6a)$$

and for $n = 1, 3$:

$$A_n(x_v) = \begin{cases} x_v \pi d_n & \text{for } x_v \leq -x_{\ddot{u}_n} \\ 0 & \text{for } x_v > -x_{\ddot{u}_n} \end{cases}, \quad (6b)$$

and the flow function:

$$\psi \left(\frac{p_a}{p_b} \right) = \begin{cases} \psi_0 \sqrt{1 - \left(\frac{\frac{p_a}{p_b} - p_{krit}}{1 - p_{krit}} \right)^2} & \text{for } \frac{p_a}{p_b} \geq p_{krit} \\ \psi_0 & \text{for } \frac{p_a}{p_b} < p_{krit} \end{cases}. \quad (6c)$$

The not yet defined model parameters are:

α_{D_n} as the flow coefficient of the control edge
 $n = 1, \dots, 4$,

- d_n as the effective diameter of the control edge n [m],
- R_g as the gas constant [$\frac{\text{J}}{\text{kg}\cdot\text{K}}$],
- κ as the adiabatic exponent of the gas [-],
- p_{krit} as the critical pressure ratio [-],
- ψ_0 as the maximum value of the flow function ψ , and
- $x_{\ddot{u}_n}$ as the valve overlap of the control edge n [m].

3. Reduced physical model in T-S multimodel descriptor form

3.1. Reduced actuator model in descriptor form

The detailed physical description of the servo-pneumatic actuators is reduced by simplification of the mass flow relations (5) and by linearization of the switching functions for zero valve overlap ($x_{\ddot{u}_n} = 0$ for $n = 1, \dots, 4$). The influence of the model reductions on the differential equation of the chamber pressures p_I, p_{II} is condensed in two bounded uncertainties as $\Delta \mathbf{B}$ and $\Delta \mathbf{a}$. The remaining nonlinear terms are transferred into T-S multimodel relations using a sector nonlinearity approach [21]. It leads to the following descriptor form:

$$\sum_{j=1}^2 \nu_j^P(x_K) \mathbf{E}_j^P \dot{\mathbf{x}}^P = \sum_{i=1}^8 h_i^P(\dot{x}_K, p_I, p_{II}) [\mathbf{A}_i^P \mathbf{x}^P + \mathbf{B}_i^P x_v] + \Delta \mathbf{B} x_v + \Delta \mathbf{a} \quad (7)$$

with the state vector

$$\mathbf{x}^P = [p_I, p_{II}]^T \quad (8a)$$

and

$$\mathbf{E}_j^P = \begin{bmatrix} \frac{V_{0I} + A_K(*_1 - x_{K0})}{\kappa} & 0 \\ 0 & \frac{V_{0II} - A_K(*_1 - x_{K0})}{\kappa} \end{bmatrix}, \quad (8b)$$

$*_1 \in \{\underline{x}_K, \bar{x}_K\}$

$$\mathbf{A}_i^P = \begin{bmatrix} A_K *_2 & 0 \\ 0 & A_K *_2 \end{bmatrix}, \quad *_2 \in \{\underline{\dot{x}}_K, \bar{\dot{x}}_K\}, \quad (8c)$$

$$\mathbf{B}_i^P = \begin{bmatrix} R_g T_I \alpha_D \pi d (\tilde{a}_{02} + \tilde{b}_{02} *_3) \\ -R_g T_{II} \alpha_D \pi d (\tilde{a}_{03} + \tilde{b}_{03} *_4) \end{bmatrix}, \quad (8d)$$

$$*_3 \in \{\underline{p}_I, \bar{p}_I\}, \quad *_4 \in \{\underline{p}_{II}, \bar{p}_{II}\}.$$

and the uncertainty terms

$$\Delta \mathbf{a} = \begin{bmatrix} \Delta p_I \\ \Delta p_{II} \end{bmatrix}, \quad (8e)$$

$$\Delta \mathbf{B} = \begin{bmatrix} R_g T_I \alpha_D \pi d \Delta \tilde{a}_2 \\ -R_g T_{II} \alpha_D \pi d \Delta \tilde{a}_3 \end{bmatrix}. \quad (8f)$$

3.2. Model of the manipulator dynamics in T-S multimodel descriptor form

The equations (1),(2) can be directly written in the following "exact" T-S multimodel descriptor form

$$\sum_{j=1}^2 \nu_j^R(\varphi) \mathbf{E}_j^R \dot{\mathbf{x}}^R = \sum_{i=1}^4 h_i^R(\varphi, \dot{\varphi}, \ddot{\varphi}) \mathbf{A}_i^R \mathbf{x}^R + \mathbf{B}^R \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (9)$$

with

$$\mathbf{x}^R = \begin{bmatrix} \psi, \varphi, \dot{\psi}, \dot{\varphi} \end{bmatrix}^T, \quad (10a)$$

$$\mathbf{E}_j^R = \begin{bmatrix} \mathbf{I}_{2 \times 2} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & \begin{bmatrix} a + 2b *_5 & c + b *_5 \\ c + b *_5 & c \end{bmatrix} \end{bmatrix}, \quad (10b)$$

$$*_5 \in \{\underline{f}_1, \bar{f}_1\},$$

$$\mathbf{A}_i^R = \begin{bmatrix} \mathbf{0}_{2 \times 2} & \mathbf{I}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & \begin{bmatrix} 2b *_6 & b *_6 \\ -b *_7 & 0 \end{bmatrix} \end{bmatrix}, \quad (10c)$$

$$*_6 \in \{\underline{g}_1, \bar{g}_1\}, \quad *_7 \in \{\underline{g}_2, \bar{g}_2\},$$

and

$$\mathbf{B}^R = \begin{bmatrix} \mathbf{0}_{2 \times 2} \\ \mathbf{I}_{2 \times 2} \end{bmatrix}, \quad (10d)$$

whereby

$$\begin{aligned}
f_1 &= \cos \varphi \quad , \\
g_1 &= \dot{\varphi} \sin \varphi \quad , \\
g_2 &= \dot{\psi} \sin \varphi \quad , \\
a &:= m_{sp} \left(\frac{l_1}{2} \right)^2 + m_e l_1^2 + m_t (l_1^2 + l_2^2) \quad , \\
b &:= m_t l_1 l_2 \quad , \\
c &:= m_t l_2^2 \quad .
\end{aligned}$$

Based on the assumptions that, first, the piston area of the actuators are equal $A_{K_1} = A_{K_2} =: A_K$, and second, the mass flow control of each actuator caused by opposed pressure differences $p_{L_j} = p_{I_j} - p_{II_j}$, $j = 1, 2$ (see Fig. 1) the magnitudes are

$$p_{L_1} = p_{L_2} =: p_L \quad , \quad (11)$$

the driving torque of joint 1 can be written as:

$$\tau_1 = p_{L_1} A_{K_1} \frac{d_r}{2} + p_{L_2} A_{K_2} \frac{d_r}{2} = p_L A_K d_r \quad , \quad (12)$$

with d_r as the roller diameter. In this consideration it is assumed that the electric actuator of joint 2, see Fig. 1, behaves as an ideal torque-controlled generator, which gives a proportional relation between the torque τ_2 and the control voltage u_m established by the motor constant k_m .

$$\tau_2 = k_m u_m \quad (13)$$

3.3. Complete dynamic model in descriptor form

The complete dynamic model is based on a combination of the actuator model in descriptor form (7) and the robot arm dynamics in descriptor form (9). It was derived by using the following algebraic constraints between the translational motion of the actuator piston and the rotational motion of the deflection roller:

$$x_K = \psi \frac{d_r}{2} \quad , \quad \dot{x}_K = \dot{\psi} \frac{d_r}{2} \quad . \quad (14)$$

With the assumption that the piston areas are $A_K = A_{K_1} = A_{K_2}$ and the mass flow control of each actuator is caused by opposed pressure difference in both actuators the chamber pressures are:

$$p_I = p_{I_1} = p_{I_2} \quad , \quad p_{II} = p_{II_1} = p_{II_2} \quad .$$

The driving torque in joint 1 is

$$\begin{aligned} \tau_1 &= (p_{I_1} - p_{II_1}) A_{K_1} \frac{d_r}{2} + (p_{I_2} - p_{II_2}) A_{K_2} \frac{d_r}{2} \\ &= (p_I - p_{II}) A_K d_r \quad . \end{aligned} \quad (15)$$

So the complete dynamic model can be presented as:

$$\begin{aligned} \sum_{k=1}^4 \tilde{\nu}_k(\psi, \varphi) \mathbf{E}_k \dot{\mathbf{x}} = \\ \sum_{l=1}^{32} h_l(\varphi, \dot{\varphi}, \psi, \dot{\psi}, p_I, p_{II}) [\mathbf{A}_l \mathbf{x} + (\mathbf{B}_l + \Delta \mathbf{B}) \mathbf{u}] + \Delta \tilde{\mathbf{a}} \end{aligned} \quad (16)$$

with $\mathbf{E}_k \in \mathbb{R}^{6 \times 6}$, $\mathbf{A}_l \in \mathbb{R}^{6 \times 6}$, $\mathbf{B}_l \in \mathbb{R}^{6 \times 2}$, $\Delta \tilde{\mathbf{a}} \in \mathbb{R}^{6 \times 1}$, the input vector

$$\mathbf{u} = [x_v, \tau_1]^T \quad , \quad (17a)$$

and the state vector

$$\mathbf{x} = [\psi, \varphi, \dot{\psi}, \dot{\varphi}, p_I, p_{II}]^T \quad . \quad (17b)$$

The left hand side weighting functions $\tilde{\nu}_k$ in (16) are obtained from all combinations of ν_j^P in (7) and ν_i^R in (9) for i and $j = 1, 2$ and the right hand side weighting functions h_l in (16) from all combinations of h_j^P in (7) and h_i^R in (9) for $i = 1, \dots, 4$ and $j = 1, \dots, 8$. In detail the matrices are:

$$\mathbf{E}_1 = \begin{bmatrix} \mathbf{E}_1^R & \mathbf{0}_{4 \times 2} \\ \mathbf{0}_{2 \times 4} & \mathbf{E}_1^P \end{bmatrix} \quad , \quad \mathbf{E}_2 = \begin{bmatrix} \mathbf{E}_2^R & \mathbf{0}_{4 \times 2} \\ \mathbf{0}_{2 \times 4} & \mathbf{E}_1^P \end{bmatrix} \quad , \quad (18a)$$

$$\mathbf{E}_3 = \begin{bmatrix} \mathbf{E}_1^R & \mathbf{0}_{4 \times 2} \\ \mathbf{0}_{2 \times 4} & \mathbf{E}_2^P \end{bmatrix} \quad , \quad \mathbf{E}_4 = \begin{bmatrix} \mathbf{E}_2^R & \mathbf{0}_{4 \times 2} \\ \mathbf{0}_{2 \times 4} & \mathbf{E}_2^P \end{bmatrix} \quad , \quad (18b)$$

$$\mathbf{A}_l = \begin{bmatrix} \mathbf{0}_{2 \times 2} & \mathbf{I}_{2 \times 2} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & \begin{bmatrix} 2b^*_{*6} & b^*_{*6} \\ -b^*_{*7} & 0 \end{bmatrix} & \begin{bmatrix} A_K d_r & -A_K d_r \\ 0 & 0 \end{bmatrix} \\ \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} & \begin{bmatrix} A_K^*_{*8} & 0 \\ 0 & A_K^*_{*8} \end{bmatrix} \end{bmatrix}, \quad (18c)$$

with $*_8 \in \{\dot{x}_K, \bar{x}_K\}$ where $\dot{x}_K = \dot{\psi} \frac{d_r}{2}$, $\bar{x}_K = \bar{\psi} \frac{d_r}{2}$ and

$$\mathbf{B}_l = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ b_5(*_3) & 0 \\ b_6(*_4) & 0 \end{bmatrix}, \quad \Delta \mathbf{B} = \begin{bmatrix} \mathbf{0}_{4 \times 2} & \\ \Delta b_5 & 0 \\ \Delta b_6 & 0 \end{bmatrix} \quad (18d)$$

with

$$\begin{aligned} b_5(*_3) &= R_g T_I \alpha_D \pi d (\tilde{a}_{02} + \tilde{b}_{02} *_3) \quad , \\ b_6(*_4) &= -R_g T_{II} \alpha_D \pi d (\tilde{a}_{03} + \tilde{b}_{03} *_4) \quad , \\ \Delta b_5 &= R_g T_I \alpha_D \pi d \Delta \tilde{a}_2 \\ \Delta b_6 &= -R_g T_{II} \alpha_D \pi d \Delta \tilde{a}_3 \end{aligned}$$

and finally

$$\Delta \tilde{\mathbf{a}} = \begin{bmatrix} \mathbf{0}_{4 \times 1} \\ \Delta p_I \\ \Delta p_{II} \end{bmatrix}. \quad (18e)$$

4. Experimental validation of the actuator model

The proposed actuator model in descriptor form (7) is validated by comparison of the simulated pressure evolution with experimental data measured in each chamber of the dual-actuator, see Fig. 2. For the simulation purpose the nominal model (7) with $\Delta \mathbf{B} = 0$ and $\Delta \mathbf{a} = 0$ was transferred into the Matlab/Simulink framework using the S-function application programming interface (API). At each integration step n a system of linear descriptor equations of the form:

$$\mathbf{E}^P(n) \dot{\mathbf{x}}^P(n) = \mathbf{A}^P(n) \mathbf{x}^P(n) + \mathbf{B}^P(n) x_v$$

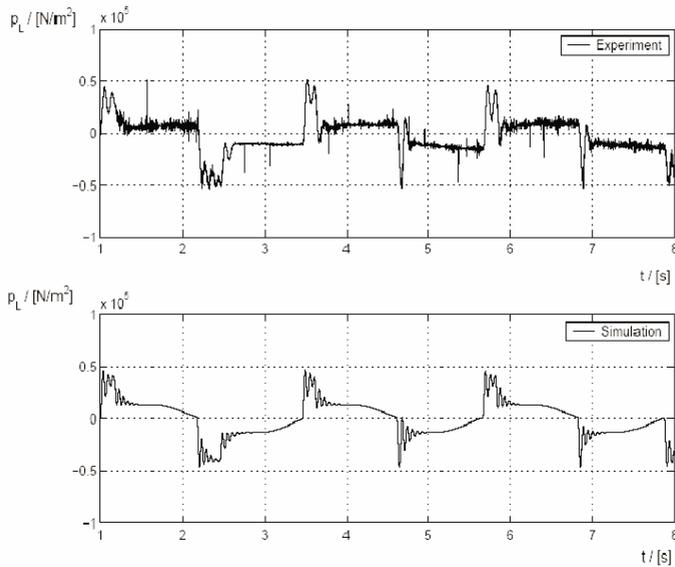


Figure 3: Difference pressure evolution $p_L = p_I - p_{II}$ in one cylinder, see (Fig.2)

with for instance $\mathbf{E}^P(n) = \sum_{j=1}^2 \nu_j^P(x_K(n)) \mathbf{E}_j^R$ is solved, see (7). The difference pressure evolution $p_L = p_I - p_{II}$ in one cylinder is shown in Fig. 3. For this, the actuator is excited in closed loop by a sequence of step signals with a variation of the cylinder positions over the whole stroke. The investigated cylinder has a dimension of $V_0 = 9 \cdot 10^5 [\text{N}/\text{m}^2]$ as half swept chamber volume and a piston area $A = 2.8 \cdot 10^{-3} [\text{m}^2]$.

5. Model-based controller design concept

The basic idea behind the investigated controller concept is that the model base of the *feedback loop controllers* will be increased so a less accurate knowledge of the system model is required in the *feedforward compensation*. In this paper we consider the special case of a structure without a common feedforward compensation as an appropriate starting point.

By defining $\tilde{\mathbf{x}} = [\mathbf{x}^T \ \dot{\mathbf{x}}^T]^T$, the descriptor system can be rewritten as:

$$\tilde{\mathbf{E}} \dot{\tilde{\mathbf{x}}} = \sum_{l=1}^{32} \sum_{k=1}^4 h_l \tilde{\nu}_k \left[\tilde{\mathbf{A}}_{lk} \tilde{\mathbf{x}} + (\tilde{\mathbf{B}}_l + \Delta \tilde{\mathbf{B}}) \mathbf{u} \right] + \Delta \tilde{\mathbf{a}} \quad (19)$$

with $h_l = h_l(\varphi, \dot{\phi}, \psi, \dot{\psi}, p_I, p_{II})$ and $\tilde{v}_k = \tilde{v}_k(\psi, \varphi)$ where:

$$\begin{aligned} \tilde{\mathbf{E}} &= \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, & \tilde{\mathbf{A}}_{lk} &= \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{A}_l & -\mathbf{E}_k \end{bmatrix}, \\ \tilde{\mathbf{B}}_l &= \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_l \end{bmatrix}, & \Delta\tilde{\mathbf{B}} &= \begin{bmatrix} \mathbf{0} \\ \Delta\mathbf{B} \end{bmatrix}, & \Delta\tilde{\mathbf{a}} &= \begin{bmatrix} \mathbf{0} \\ \Delta\tilde{\mathbf{a}} \end{bmatrix}. \end{aligned}$$

We propose a modified parallel distributed compensation scheme (PDC):

$$\mathbf{u} = \sum_{l=1}^{32} \sum_{k=1}^4 h_l \tilde{v}_k \tilde{\mathbf{F}}_{lk} \dot{\mathbf{x}} \quad (20)$$

with $\tilde{\mathbf{F}}_{lk} = [\mathbf{F}_{lk} \ \mathbf{0}]$ to stabilize the multimodel descriptor system (19). The controller design problem here is to determine the local feedback gains using the local models $\{\tilde{\mathbf{A}}_{lk}, \tilde{\mathbf{B}}_l, \Delta\tilde{\mathbf{B}}, \Delta\tilde{\mathbf{a}}\}$ that globally stabilize the multimodel descriptor system with uncertainties by (20). We currently solve this problem using a novel LMI (Linear Matrix Inequalities) condition derived in [1].

6. Simulation results

We will now consider the results of the simulation with the global descriptor model of the plant (19), that is operated in closed loop. For the model validation purpose a fixed linear state-space controller for the whole operating space is used. It is worth to note that this controller is not able to handle the dominate nonlinearities in the plant.

The validation process is exemplified by a reference trajectory shown in Fig. 5 that defines the desired tool center position (TCP), see Fig. 1, in the xy-plane of the working space. The corresponding joint angles in Fig. 4 have been calculated by the well-known inverse kinematics of a two-link planar arm [17]. The simulation model of the dual actuators is based on the descriptor form (7), but in the first step without uncertainties. The influence of the air-pressure evolution in the actuator cylinders that causes a weakly damped dynamics is recognized in Fig. 5 where the desired trajectory distinctly differs from the actual path. The large tracking error at the corners of the rectangle is an indication that the linear controller is not able to handle the dynamics of the dual-actuator pneumatic drive that interacts with the dynamics of the robot arm. This fact is also a motivation for a nonlinear control design, for instance based on (20).

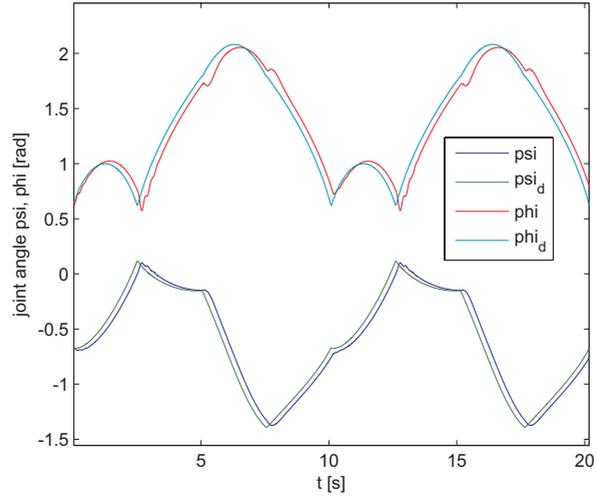


Figure 4: Time history of the reference joint positions $\psi_d(t), \varphi_d(t)$ and simulated joint positions $\psi(t), \varphi(t)$ relate to Fig. 5.

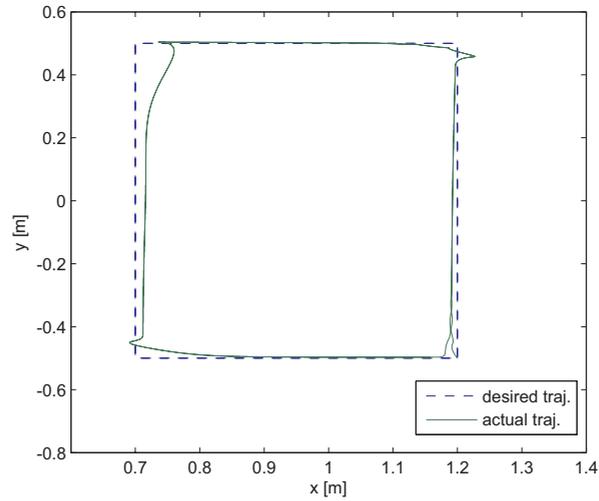


Figure 5: The reference (desired) rectangle trajectory of the TCP, see Fig. 1, and the actual simulated TCP-position. Starting position: $x = 1.2$ m, $y = -0.5$ m

7. Conclusion

In this paper the possibilities of the use of a T-S descriptor form to describe the dynamics of a two-link manipulators with dual-actuators were analysed. A complete manipulator model using the combination of the actuator models in descriptor form and the robot arm dynamics in descriptor form was derived for a model-based control concept. We must insist on the fact that this paper is a first step dealing with T-S descriptor modelization and a further paper will present the whole T-S multimodel control study. We think this is an appropriate starting point for a robot control concept that systematically take into consideration the influence of model reduction by structured uncertainties.

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