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# Lower Fractional Transformation Based H-infinity

## Output Stabilization for Takagi-Sugeno Fuzzy

### Models

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**Abstract** – In this paper, a global stabilization and robust  $H_\infty$  Dynamic Output Feedback Controller (DOFC) design methodology for Takagi-Sugeno (T-S) fuzzy systems is proposed. Based on a standard robust control structure, one rewrites an original nonlinear model as an extended nonlinear model with exogenous inputs. The latter contains control objectives whose are classically introduced in terms of linear weighting functions in robust control theory. Then, in order to point out the interconnection between the extended nonlinear model and the designated DOFC, unsolvable stability conditions are derived using Linear Fractional Transformation (LFT) and  $H_\infty$  tools. Thus, based on the universal approximator properties of T-S modeling, the obtained nonlinear stability conditions are transformed into Linear Matrix Inequalities (LMI) to allow the design of a dedicated Full Parallel Distributed Compensation (FPDC) DOFC. In that case, when a solution is tractable from the proposed LMI conditions, the synthesized controller guarantees the prescribed stability performances. Finally, a numerical example is used to illustrate the validity of the designed approach.

**Keywords:** Takagi-Sugeno fuzzy models, robust control, Linear Fractional Transformation (LFT), Linear Matrix Inequalities (LMI),  $H_\infty$ .

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## 1 Introduction

In the past few years, considerable attention has been devoted to the stability and controller design of fuzzy control systems. Among nonlinear control theory, the Takagi-Sugeno (T-S) fuzzy model [1] is becoming popular since it is able to universal approximations of a wide class of nonlinear systems [2, 3]. Moreover, some recent studies have shown their practicability; see e.g [4, 5]. Thus, the T-S fuzzy models are intensively used for analyzing stability problems related to fuzzy systems [6, 7]. Several corresponding control schemes have been developed, in the literature, to solve the problem of the T-S fuzzy models stabilization and control since early nineties (see for instance [8, 9] and references therein). The typical approach for controller design is carried out via the so-called Parallel Distributed Compensation (PDC) method [10, 11].

Since, stability and performance are two essential problems in control theory. Many improved results have been proposed such as state feedback [11, 12, 13] and robust static output feedback controller design [14]. Note that these approaches remain to static state feedback which leads to lower computational cost when implementing real time systems. Nevertheless, as the compensation to the nonlinear dynamics of the controlled system remains on a simple state feedback, it can be shown that static state feedback are less powerful when specified performances are desired with respect to the transient response [15, 16].

To improve the closed-loop dynamics control law's performances, robust control based on Dynamic Output Feedback Controller (DOFC) has been extensively studied in various kinds of linear systems (Linear Time Invariant (LTI), Linear Parameter Varying (LPV), Linear Time Varying (LTV)...) [15, 16, 17]. These techniques are often based on the  $H_\infty$  criteria [16, 18] and on the Linear Fractional Transformation (LFT) paradigm [21]. Dealing with T-S

fuzzy models, DOFC design has been proposed in [20, 27, 28]. Let us point out that, in the author's best knowledge, LFT based approaches are scarcely used in T-S fuzzy approaches.

In this paper, one proposes an  $H_\infty$  / LFT based DOFC design methodology for T-S fuzzy models. This one can be summarized as follows: after introducing the control objectives in terms of linear weighting functions, an extended nonlinear model with exogenous inputs can be obtained from the nonlinear model. Then, nonlinear T-S fuzzy output feedback dynamic control law can be obtained using a finite set of Linear Matrix Inequalities (LMI) with  $H_\infty$  criteria optimization [16].

This paper is organized as follows: After the nonlinear control problem statement, a recall of T-S modeling, lower LFT representations and the proposed nonlinear T-S control design methodology will be presented in section 2. Finally, the efficiency of the proposed approach is illustrated through a numerical example in section 3.

## 2 Nonlinear control problem statement

Let us consider the following class of nonlinear systems:

$$\begin{cases} \dot{x}(t) = A(x(t))x(t) + B(x(t))u(t) \\ y(t) = C(x(t))x(t) \end{cases} \quad (1)$$

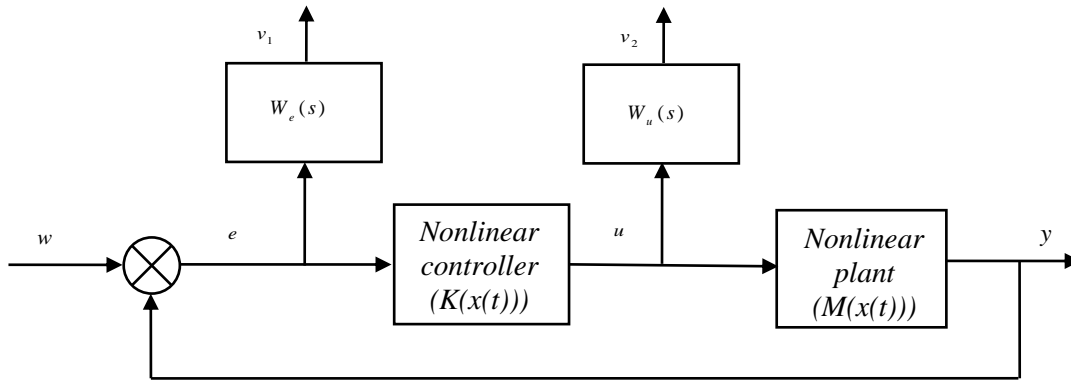
where  $x(t) \in \mathfrak{R}^n$  is the state vector,  $u(t) \in \mathfrak{R}^m$  is the control vector,  $y(t) \in \mathfrak{R}^q$  is the measured output vector,  $A(\cdot) \in \mathfrak{R}^{n \times n}$ ,  $B(\cdot) \in \mathfrak{R}^{n \times m}$  and  $C(\cdot) \in \mathfrak{R}^{q \times n}$  are the state, input and output nonlinear matrices, respectively.

The objective is to design a DOFC given by the following nonlinear state space representation:

$$\begin{cases} \dot{x}^c(t) = A^c(x(t))x^c(t) + B^c(x(t))y(t) \\ u(t) = C^c(x(t))x^c(t) + D^c(x(t))y(t) \end{cases} \quad (2)$$

where  $x^c(t) \in \mathfrak{R}^{n_c}$  is the controller state vector,  $A^c(\cdot) \in \mathfrak{R}^{n_c \times n_c}$ ,  $B^c(\cdot) \in \mathfrak{R}^{n_c \times q}$ ,  $C^c(\cdot) \in \mathfrak{R}^{m \times n_c}$  and  $D^c(\cdot) \in \mathfrak{R}^{m \times q}$  are the controller matrices.

To design such controller, performance control objectives can be introduced in terms of linear weighting functions. This can be achieved using the standard control structure depicted in figure 1 [22, 23].



**Figure 1** – Standard control structure diagram

Within this structure,  $w(\cdot) \in \mathfrak{R}^q$  represent an exogenous input vector,  $e(\cdot) \in \mathfrak{R}^q$  is the error vector and  $v(\cdot) = [v_1(\cdot)^T, v_2(\cdot)^T]^T \in \mathfrak{R}^{q+m}$  is the controlled output vector. The different objectives are specified in terms of linear weighting functions  $W_e(s)$  and  $W_u(s)$  on the sensitivity ( $S(s)$  transfer) and controls inputs ( $KS(s)$  transfer) respectively (figure 1). The weighting functions are defined in order to remove the fast variation of the error signal  $e(t)$  and to bound variations of the control signal  $u(t)$ . The choice of these weighting functions is done following the methodology presented in [22, 23].

Thus, the linear tracking weighting function is given by the following linear state space representation:

$$\begin{cases} \dot{x}_e(t) = A_e x_e(t) + B_e e(t) \\ v_1(t) = C_e x_e(t) + D_e e(t) \end{cases} \quad (3)$$

In the same way, the linear control weighting function is given as follows:

$$\begin{cases} \dot{x}_u(t) = A_u x_u(t) + B_u u(t) \\ v_2(t) = C_u x_u(t) + D_u u(t) \end{cases} \quad (4)$$

where  $x_e(\cdot) \in \mathfrak{R}^q$  and  $x_u(\cdot) \in \mathfrak{R}^m$  are the  $W_e$  and  $W_u$  state vector respectively.  $A_e \in \mathfrak{R}^{n_e \times n_e}$ ,  $B_e \in \mathfrak{R}^{n_e \times q}$ ,  $C_e \in \mathfrak{R}^{q \times n_e}$ ,  $D_e \in \mathfrak{R}^{q \times q}$ ,  $A_u \in \mathfrak{R}^{n_u \times n_u}$ ,  $B_u \in \mathfrak{R}^{n_u \times m}$ ,  $C_u \in \mathfrak{R}^{m \times n_u}$  and  $D_u \in \mathfrak{R}^{m \times m}$  are constant matrices.

Combining (1), (3) and (4), an extended nonlinear model with exogenous inputs can be written as follows:

$$\begin{cases} \dot{\tilde{x}}(t) = \tilde{A}(x(t)) \tilde{x}(t) + \tilde{B}_1 w(t) + \tilde{B}_2(x(t)) u(t) \\ v(t) = \tilde{C}_1(x(t)) \tilde{x}(t) + \tilde{D}_{11} w(t) + \tilde{D}_{12} u(t) \\ y(t) = \tilde{C}_2(x(t)) \tilde{x}(t) \end{cases} \quad (5)$$

where  $\tilde{x}(t) = [x_e^T(t) \quad x_u^T(t) \quad x^T(t)]^T$  is the extended state vector,  $\tilde{C}_2(\tilde{x}(t)) = [0 \ 0 \ C(x(t))]$ ,

$$\tilde{A}(\tilde{x}(t)) = \begin{bmatrix} A_e & 0 & -B_e C(x(t)) \\ 0 & A_u & 0 \\ 0 & 0 & A(x(t)) \end{bmatrix}, \quad \tilde{B}_1 = \begin{bmatrix} B_e \\ 0 \\ 0 \end{bmatrix}, \quad \tilde{B}_2(\tilde{x}(t)) = \begin{bmatrix} 0 \\ B_u \\ B(x(t)) \end{bmatrix}, \quad \tilde{D}_{11} = \begin{bmatrix} D_e \\ 0 \end{bmatrix},$$

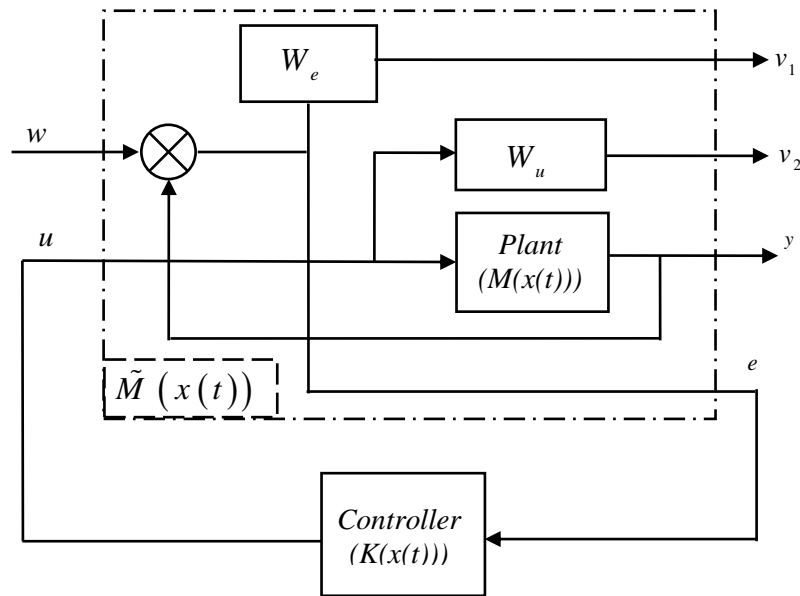
$$\tilde{D}_{12} = \begin{bmatrix} 0 \\ D_u \end{bmatrix} \text{ and } \tilde{C}_1(\tilde{x}(t)) = \begin{bmatrix} C_e & 0 & -D_e C(x(t)) \\ 0 & C_u & 0 \end{bmatrix}.$$

Let us recall that a convenient way to run to the closed loop formulation from (5) and (2) is to consider the well-known LFT tools. Note that, since the striking pioneer work of Redheffer [21], this tool play an important role in robust control system design [15, 24]. The LFT algebra arises naturally when one describes a so-called “well-posed” feedback system as shown by the block diagram depicted in figure 2 where  $\tilde{M}(x(t))$  and  $K(x(t))$  are the system’s matrices realizations of (5) and (2) respectively defined as:

$$\tilde{M}(x(t)) = \left( \begin{array}{c|cc} \tilde{A}(x(t)) & \tilde{B}_1 & \tilde{B}_2(x(t)) \\ \tilde{C}_1(x(t)) & \tilde{D}_{11} & \tilde{D}_{12} \\ \tilde{C}_2(x(t)) & 0 & 0 \end{array} \right) \quad (6)$$

and

$$K(x(t)) = \left( \begin{array}{c|c} A^c(x(t)) & B^c(x(t)) \\ \hline C^c(x(t)) & D^c(x(t)) \end{array} \right) \quad (7)$$



**Figure 2.** Lower LFT diagram

From figure 2, the resulting input/output relation can be represented as  $v(t) = F_l(\tilde{M}(x(t)), K(x(t)))w(t)$ , where  $F_l(\cdot)$  is said to be the lower LFT of  $\tilde{M}(x(t))$  on  $K(x(t))$ . Then, the corresponding closed loop system's matrix realization  $F_l(\tilde{M}(x(t)), K(x(t)))$  is given using the Redhefer star product of  $\tilde{M}(x(t))$  and  $K(x(t))$  defined by [15, 21]:

$$F_l(\tilde{M}(x(t)), K(x(t))) = \left( \begin{array}{c|c} A_{cl}(x(t)) & B_{cl} \\ \hline C_{cl}(x(t)) & D_{cl} \end{array} \right) = \left( \begin{array}{cc|c} \tilde{A}(x(t)) + \tilde{B}_2(x(t))D^c(x(t))\tilde{C}_2(x(t)) & \tilde{B}_2(x(t))C^c(x(t)) & \tilde{B}_1 \\ \hline B^c(x(t))\tilde{C}_2(x(t)) & A^c(x(t)) & 0 \\ \hline \tilde{C}_1(x(t)) + \tilde{D}_{12}D^c(x(t))\tilde{C}_2(x(t)) & \tilde{D}_{12}C^c(x(t)) & \tilde{D}_{11} \end{array} \right) \quad (8)$$

Once the closed-loop dynamics (8) being defined, the goal is now to provide conditions ensuring to find a DOFC of the form (2) so that the closed loop system is stable. To address this problem, the direct Lyapunov methodology can be employed. Thus, let us consider the following candidate quadratic Lyapunov function:

$$V(\tilde{x}(t)) = \tilde{x}(t)^T P \tilde{x}(t) \geq 0, \quad P = P^T > 0 \quad (9)$$

The problem is to design a nonlinear DOFC (2) stabilizing the closed loop nonlinear system (8) and satisfying the following  $H_\infty$  constraints:

$$\sup_{\|w(t)\|_2 \neq 0} \frac{\|v(t)\|_2}{\|w(t)\|_2} = \|F_l(\tilde{M}(x(t)), K(x(t)))\|_\infty < \gamma \quad (10)$$

where  $\|\cdot\|_2$  is defined as the  $L_2$  norm and  $\gamma$  is a positive scalar performance level to be minimized to ensure the best dynamics of the closed-loop system.

Then (10) can be rewritten as:



$$\int_0^{\infty} \left( v(t)^T v(t) - \gamma^2 w(t)^T w(t) \right) dt < 0 \quad (11)$$

In the sequel, when there is no ambiguity, the time variable  $t$  and the parameters of the nonlinear matrices  $\square(x(t))$  will be omitted for more clarity of the mathematical expressions.

The decrease of (9) can be constrained by (11), it yields:

$$\dot{\tilde{x}}^T P \tilde{x} + \tilde{x}^T P \dot{\tilde{x}} + v^T v - \gamma^2 w^T w < 0 \quad (12)$$

From (8), we have  $\dot{\tilde{x}} = A_{cl} \tilde{x} + B_{cl} w$  and  $v = C_{cl} \tilde{x} + D_{cl} w$ , so (12) can be rewritten as:

$$\begin{aligned} \tilde{x}^T \left( A_{cl}^T P + P A_{cl} \right) \tilde{x} + w^T B_{cl}^T P \tilde{x} + \tilde{x}^T P B_{cl} w + \tilde{x}^T C_{cl}^T v \\ + w^T D_{cl}^T v + v^T C_{cl} \tilde{x} + v^T D_{cl} w - v^T v - \gamma^2 w^T w < 0 \end{aligned} \quad (13)$$

That is to say:

$$\begin{pmatrix} \tilde{x} \\ w \\ v \end{pmatrix}^T \begin{pmatrix} A_{cl}^T P + P A_{cl} & (*) & (*) \\ B_{cl}^T P & -\gamma^2 I & (*) \\ C_{cl} & D_{cl} & -I \end{pmatrix} \begin{pmatrix} \tilde{x} \\ w \\ v \end{pmatrix} < 0 \quad (14)$$

It is obvious that (14) holds if:

$$\begin{pmatrix} A_{cl}^T P + P A_{cl} & (*) & (*) \\ B_{cl}^T P & -\gamma^2 I & (*) \\ C_{cl} & D_{cl} & -I \end{pmatrix} < 0 \quad (15)$$

Let us consider  $L = P^{-1} = \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} > 0$  and  $L^{-1} = P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix} > 0$  such that:

$$L = L_1 L_2 = \begin{bmatrix} Q_1 & I \\ Q_2^T & 0 \end{bmatrix} \begin{bmatrix} I & P_1 \\ 0 & P_2^T \end{bmatrix}^{-1} \quad (16)$$

Since  $LL^{-1} = I$ , we have the following constraint  $Q_2 P_2^T = I - Q_1 P_1$  and the  $P > 0$  condition is

equivalent to  $\begin{bmatrix} Q_1 & I \\ I & P_1 \end{bmatrix} > 0$  [19].

Multiplying respectively left and right by  $\begin{pmatrix} L & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}$  and  $\begin{pmatrix} L & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}^T$ , one obtains:

$$\begin{pmatrix} L_2^T L_1^T A_{cl}^T + A_{cl} L_1 L_2 & (*) & (*) \\ B_{cl}^T & -\gamma^2 I & (*) \\ C_{cl} L_1 L_2 & D_{cl} & -I \end{pmatrix} < 0 \quad (17)$$

then, multiplying left by  $\begin{pmatrix} L_2^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}^T$  and right by  $\begin{pmatrix} L_2^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}$ , one obtains:

$$\begin{pmatrix} L_1^T A_{cl}^T L_2^{-1} + L_2^{-T} A_{cl} L_1 & (*) & (*) \\ B_{cl}^T L_2^{-1} & -\gamma^2 I & (*) \\ C_{cl} L_1 & D_{cl} & -I \end{pmatrix} < 0 \quad (18)$$

Substituting (8) and (16) in (18), an equivalent nonlinear inequality (NLI) can be obtained and given in the following extended form:

$$\begin{pmatrix} \Psi_{11}(x(t)) & (*) & (*) & (*) \\ \Psi_{21}(x(t)) & \Psi_{22}(x(t)) & (*) & (*) \\ \tilde{B}_1^T & \tilde{B}_1^T P_1 & -\gamma^2 I & (*) \\ \Psi_{41}(x(t)) & \Psi_{42}(x(t)) & \tilde{D}_{11} & -I \end{pmatrix} < 0 \quad (19)$$

where:

$$\begin{aligned} \Psi_{11}(x(t)) &= Q_1^T \tilde{A}(x(t))^T + \tilde{A}(x(t)) Q_1 + Q_1^T \tilde{C}_2(x(t))^T D^c(x(t))^T \tilde{B}_2(x(t))^T \\ &\quad + \tilde{B}_2(x(t)) D^c(x(t)) \tilde{C}_2(x(t)) Q_1 + Q_2 C^c(x(t))^T \tilde{B}_2(x(t))^T + \tilde{B}_2(x(t)) C^c(x(t)) Q_2^T \end{aligned}$$

,

$$\begin{aligned} \Psi_{21}(x(t)) &= \tilde{A}(x(t))^T + \tilde{C}_2(x(t))^T D^c(x(t))^T \tilde{B}_2(x(t))^T + P_1^T \tilde{A}(x(t)) Q_1 + P_2 B^c(x(t)) \tilde{C}_2(x(t)) Q_1 \\ &\quad + P_2 A^c(x(t)) Q_2^T + P_1^T \tilde{B}_2(x(t)) D^c(x(t)) \tilde{C}_2(x(t)) Q_1 + P_1^T \tilde{B}_2(x(t)) C^c(x(t)) Q_2^T, \\ \Psi_{22}(x(t)) &= \tilde{A}(x(t))^T P_1 + P_1^T \tilde{A}(x(t)) + \tilde{C}_2(x(t))^T D^c(x(t))^T \tilde{B}_2(x(t))^T P_1 \\ &\quad + P_1 \tilde{B}_2(x(t)) D^c(x(t)) \tilde{C}_2(x(t)) + \tilde{C}_2(x(t))^T B^c(x(t))^T P_2^T + P_2 B^c(x(t)) \tilde{C}_2(x(t)) \\ , \quad \Psi_{41}(x(t)) &= \tilde{C}_1(x(t)) Q_1 + \tilde{D}_{12} D^c(x(t)) \tilde{C}_2(x(t)) Q_1 + \tilde{D}_{12} C^c(x(t)) Q_2^T \quad \text{and} \\ \Psi_{42}(x(t)) &= \tilde{C}_1(x(t)) + \tilde{D}_{12} D^c(x(t)) \tilde{C}_2(x(t)). \end{aligned}$$

**Remark 1:** The nonlinear model (1), with the tracking and control performances (3) and (4), is stable via the output feedback control law (2) under  $H_\infty$  performances (10), if there exist the matrices  $P_1 = P_1^T > 0$ ,  $P_2 = P_2^T > 0$ ,  $Q_1 = Q_1^T > 0$ ,  $Q_2 = Q_2^T > 0$ ,  $A^c(x(t))$ ,  $B^c(x(t))$ ,  $C^c(x(t))$ ,  $D^c(x(t))$  regulars with appropriate dimensions and a scalar  $\gamma > 0$  such that the NLI (19) is fulfilled.

Notice that, there are no direct tools to solve this NLI. Therefore, in the next section, one proposes to use a fuzzy T-S modeling approach in order to provide LMI stability conditions able to overcome this problem.

### 3 LMI stability formulation based on T-S modelling

Let us recall that, over the past two decades, there has been rapidly growing interest in approximating a nonlinear system by a T-S fuzzy model [1, 6, 25]. The fuzzy modelling approach provides a powerful tool for modelling complex nonlinear systems. Unlike conventional modelling approaches, where a single model is used to describe the global behaviour of a system, T-S modelling approach considers local LTI systems combined by membership functions to describe the global behaviour of the nonlinear system [1]. Indeed, a continuous time nonlinear system of the form (1) can be approximated by a T-S fuzzy model constituted by the  $r$  following IF-THEN rules:

Plant Rule  $i$ : **IF**  $z_1(t)$  is  $M_1^i$  and ... and  $z_p(t)$  is  $M_p^i$ , **THEN**

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases}, i = 1, 2, \dots, r \quad (20)$$

where  $z_1(t), \dots, z_p(t)$  are the premise variables,  $M_j^i$  ( $j = 1, \dots, p$ ) are fuzzy sets,  $x(t) \in \mathfrak{R}^n$  is the state vector,  $u(t) \in \mathfrak{R}^m$  is the input vector,  $y(t) \in \mathfrak{R}^q$  is the controlled output vector,  $A_i \in \mathfrak{R}^{n \times n}$ ,  $B_i \in \mathfrak{R}^{n \times m}$  and  $C_i \in \mathfrak{R}^{q \times n}$  are constant matrices and  $r$  is the number of IF-THEN rules.

Then, using the barycentric defuzzification method [8], the fuzzy model is inferred as:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r h_i(z(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^r h_i(z(t))(C_i x(t)) \end{cases} \quad (21)$$

with  $z(t) = [z_1(t), \dots, z_p(t)]^T$ ,  $w_i(z(t)) = \prod_{j=1}^p M_j^i(z(t))$  and  $h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^r w_i(z(t))}$  where

$M_j^i(z_j(t))$  is the fuzzy membership grade of  $z_j(t)$  in  $M_j^i$ .

Note that,  $w_i(z(t)) \geq 0$ ,  $\sum_{i=1}^r w_i(z(t)) > 0$ . Therefore, the following convex property holds:

$$h_i(z(t)) \geq 0 \text{ and } \sum_{i=1}^r h_i(z(t)) = 1.$$

Note that (21) can obviously be rewritten, when considering the weighting functions (3) and (4), in the form of an exogenous input (5). That is to say, by the following exogenous T-S model:

$$\begin{cases} \dot{\tilde{x}}(t) = \sum_{i=1}^r h_i(z(t)) (\tilde{A}_i \tilde{x}(t) + \tilde{B}_{2i} u(t)) + \tilde{B}_1 w(t) \\ v(t) = \sum_{i=1}^r h_i(z(t)) \tilde{C}_{1i} \tilde{x}(t) + \tilde{D}_{11} w(t) + \tilde{D}_{12} u(t) \\ y(t) = \sum_{i=1}^r h_i(z(t)) \tilde{C}_{2i} \tilde{x}(t) \end{cases} \quad (22)$$

where  $\tilde{A}_i$ ,  $\tilde{B}_1$ ,  $\tilde{B}_{2i}$ ,  $\tilde{C}_{1i}$ ,  $\tilde{C}_{2i}$ ,  $\tilde{D}_{11}$  and  $\tilde{D}_{12}$  are the LTI system state matrices defined as:

$$\tilde{A}_i = \begin{bmatrix} A_e & 0 & -B_e C_i \\ 0 & A_u & 0 \\ 0 & 0 & A_i \end{bmatrix}, \quad \tilde{B}_1 = \begin{bmatrix} B_e \\ 0 \\ 0 \end{bmatrix}, \quad \tilde{B}_{2i} = \begin{bmatrix} 0 \\ B_u \\ B_i \end{bmatrix}, \quad \tilde{C}_{1i} = \begin{bmatrix} C_e^T & 0 \\ 0 & C_u^T \\ -C_i^T & D_e^T \end{bmatrix}^T, \quad \tilde{D}_{11} = \begin{bmatrix} D_e \\ 0 \end{bmatrix} \text{ and}$$

$$\tilde{D}_{12} = \begin{bmatrix} 0 \\ D_u \end{bmatrix}.$$

Let us also recall that the most commonly used control scheme for T-S fuzzy model is the so-called Parallel Distributed Compensation (PDC) [11]. The meaning of such control laws is the use of the same membership functions as the one used by the T-S model to be stabilized. Thus, after checking (19) with the fuzzy blending of (22), one can propose the following PDC control law using a summation structure chosen in order to fully compensate the nonlinear stability condition (19):

$$\begin{cases} \dot{x}^c(t) = \sum_{i=1}^r \sum_{j=1}^r \sum_{k=1}^r h_i h_j h_k A_{ijk}^c x^c(t) + \sum_{j=1}^r \sum_{k=1}^r h_j h_k B_{jk}^c y(t) \\ u(t) = \sum_{j=1}^r \sum_{k=1}^r h_j h_k C_{jk}^c x^c(t) + \sum_{k=1}^r h_k D_k^c y(t) \end{cases} \quad (23)$$

where  $A_{ijk}^c$ ,  $B_{jk}^c$ ,  $C_{jk}^c$  and  $D_k^c$  are constant matrices with appropriate dimensions to be synthesized.

Thus, substituting (22) and (23) in (19), the NLI can be rewritten as:

$$\sum_{i=1}^r \sum_{j=1}^r \sum_{k=1}^r h_i h_j h_k \left( \begin{array}{ccc} \Psi_{11}^{ijk} & (*) & (*) \quad (*) \\ \Psi_{21}^{ijk} & \Psi_{22}^{ijk} & (*) \quad (*) \\ \tilde{B}_1^T & \tilde{B}_1^T P_1 & -\eta I \quad (*) \\ \tilde{C}_{1i} Q_1 + \tilde{D}_{12} D_k^c \tilde{C}_{2i} Q_1 + \tilde{D}_{12} C_{ik}^c Q_2^T & \tilde{C}_{1i} + \tilde{D}_{12} D_k^c \tilde{C}_{2i} & \tilde{D}_{11} \quad -I \end{array} \right) < 0 \quad (24)$$

where  $\Psi_{11}^{ijk} = Q_1^T \tilde{A}_i^T + \tilde{A}_i Q_1 + Q_1^T \tilde{C}_{2i}^T D_k^c \tilde{B}_{2j}^T + \tilde{B}_{2j} D_k^c \tilde{C}_{2i} Q_1 + Q_2 C_{ik}^c \tilde{B}_{2j}^T + \tilde{B}_{2j} C_{ik}^c Q_2^T$ ,

$$\Psi_{21}^{ijk} = \tilde{A}_i^T + \tilde{C}_{2i}^T D_k^c \tilde{B}_{2j}^T + P_1^T \tilde{A}_i Q_1 + P_1^T \tilde{B}_{2j} D_k^c \tilde{C}_{2i} Q_1 + P_1^T \tilde{B}_{2j} C_{ik}^c Q_2^T + P_2 B_{jk}^c \tilde{C}_{2i} Q_1 + P_2 A_{ijk}^c Q_2^T,$$

$$\text{and } \Psi_{22}^{ijk} = \tilde{A}_i^T P_1 + P_1^T \tilde{A}_i + \tilde{C}_{2i}^T D_k^c \tilde{B}_{2j}^T P_1 + P_1 \tilde{B}_{2j} D_k^c \tilde{C}_{2i} + \tilde{C}_{2i}^T B_{jk}^c P_2^T + P_2 B_{jk}^c \tilde{C}_{2i}.$$

Using the following change of variables:  $G_{1ik} = D_k^c C_{2i} Q_1 + C_{ik}^c Q_2$ ,  $G_{2jk} = P_1^T \tilde{B}_{2j} D_k^c + P_2 B_{jk}^c$ ,

$$G_{3ijk} = P_1^T (\tilde{A}_i + \tilde{B}_{2j} D_k^c \tilde{C}_{2i}) Q_1 + P_2 B_{jk}^c \tilde{C}_{2i} Q_1 + P_1^T \tilde{B}_{2j} C_{ik}^c Q_2^T + P_2 A_{ijk}^c Q_2^T \quad \text{and} \quad \gamma^2 = \eta, \quad \text{the}$$

inequality (24) becomes:

$$\sum_{i=1}^r \sum_{j=1}^r \sum_{k=1}^r h_i h_j h_k \Upsilon_{ijk} < 0 \quad (25)$$

$$\text{with } \Upsilon_{ijk} = \left( \begin{array}{ccc} A_i Q_1 + Q_1 A_i^T + B_{2j} G_{1ik} + G_{1ik}^T B_{2j}^T & (*) & (*) \quad (*) \\ A_i^T + G_{3ijk} + C_{2i}^T D_k^c B_{2j}^T & P_1 A_i + A_i^T P_1 + G_{2jk} C_{2i} + C_{2i}^T G_{2jk}^T & (*) \quad (*) \\ B_{1i}^T & B_{1i}^T P_1 & -\eta \quad (*) \\ C_{1i} Q_1 + D_{12i} G_{1ik} & C_{1i} + D_{12} D_k^c C_{2i} & D_{11} \quad -I \end{array} \right).$$

Note that, a convenient way to obtain LMI stability conditions from (25) is to search the decision variables considering that each  $\Upsilon_{ijk} < 0$ , for all combination of  $i, j, k = 1, \dots, r$  [10, 11]. Obviously, these conditions are conservative. In order to relax LMI stability conditions, one proposes to extend the well-known relaxation scheme introduced in [6] to the case of a triple summation structure as appearing in (25). Let us consider the following proprieties [6]:

$$\sum_{i=1}^r \sum_{j=1}^r h_i(z) h_j(z) \varphi_{ij} = \sum_{i=1}^r h_i^2(z) \varphi_{ii} + \sum_{i=1}^r \sum_{\substack{j=1 \\ j < i}}^r h_i(z) h_j(z) (\varphi_{ij} + \varphi_{ji}) \quad (26)$$

Therefore, applying twice (26) on (25) one has:

$$\sum_{i=1}^r h_i^3 \Upsilon_{iii} + \sum_{i=1}^r \sum_{\substack{j=1 \\ j < i}}^r h_i h_j^2 (\Upsilon_{ijj} + \Upsilon_{jii}) + \sum_{i=1}^r \sum_{j=1}^r \sum_{\substack{k=1 \\ k < j}}^r h_i h_j h_k (\Upsilon_{ijk} + \Upsilon_{ikj}) < 0 \quad (27)$$

Obviously (27) holds if the conditions summarized in the following theorem are satisfied.

**Theorem 1:** The T-S fuzzy model with exogenous inputs given by (22) is stable via the control law (23) with  $H_\infty$  quadratic performances, if there exist matrices  $P_1 = P_1^T > 0$ ,  $Q_1 = Q_1^T > 0$ ,  $D_k^c$ ,  $G_{1ik}$ ,  $G_{2jk}$ ,  $G_{3ijk}$  regulars with appropriate dimensions and a scalar  $\eta > 0$  such that the following LMIs are satisfied:

$$\text{for } i = 1, \dots, r, \quad \Upsilon_{iii} < 0 \quad (28)$$

$$\text{for } i = 1, \dots, r, \quad j = 1, \dots, r \quad \text{and} \quad j < i, \quad \Upsilon_{ijj} + \Upsilon_{jii} < 0 \quad (29)$$

$$\text{for } i = 1, \dots, r, \quad j = 1, \dots, r, \quad k = 1, \dots, r \quad \text{and} \quad k < j, \quad \Upsilon_{ijk} + \Upsilon_{ikj} < 0 \quad (30)$$

with  $\Upsilon_{ijk}$  defined in (25).

**Remark 2:** For a particular numerical example, when there exist a solution to theorem 1 from LMIs (28), (29) and (30), the fuzzy controller matrices defined in (23) stabilizing the fuzzy system (21) with respect to the performances objectives introduced by (3) and (4) under  $\gamma^2 = \eta$ , are obtained using the bijective change of variables  $C_{ik}^c = (G_{1ik} - D_k^c C_{2i} Q_1) Q_2^{-1}$ ,

$$A_{ijk}^c = P_2^{-1} \left( G_{3ijk} - P_1^T \left( \tilde{A}_i + \tilde{B}_{2j} D_k^c \tilde{C}_{2i} \right) Q_1 - P_2 B_{jk}^c \tilde{C}_{2i} Q_1 - P_1^T \tilde{B}_{2j} C_{ik}^c Q_2^T \right) Q_2^{-T} \quad \text{and}$$

$$B_{jk}^c = P_2^{-1} \left( G_{2jk} - P_1^T \tilde{B}_{2j} D_k^c \right).$$

**Remark 3:** Note that other relaxations are available in the literature, see e.g. [29, 30, 31]. The objective of this study is not dwelling on the subject of relaxing more and more the proposed stability condition. Nevertheless, In case where is it required by any practical application, these relaxation schemes can obviously be employed together with condition (25) without loss of generality.

## 4 Simulation results

To illustrate the efficiency of the above proposed controller design methodology, let us consider the following numerical example:

$$\begin{cases} \dot{x}(t) = A(x(t))x(t) + B(x(t))u(t) \\ y(t) = Cx(t) \end{cases} \quad (31)$$

where  $x(t) = [x_1^T(t), x_2^T(t)]^T$  is the state vector,  $u(t)$  is the input vector,  $y(t) = x_1(t)$  is the

measured output vector and  $A(x(t)) = \begin{bmatrix} \sin^2(x_1(t)) & 3 \\ 1 & 1 \end{bmatrix}$ ,  $B(x(t)) = \begin{bmatrix} 1 + \sin^2(x_1(t)) \\ 1 \end{bmatrix}$  and

$$C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T.$$

The desired performance specifications on control system are defined in terms of frequency low-pass filter  $W_e$  and high-pass filter  $W_u$  (respectively defined in (3) and (4)) to remove the fast variation of the error signal  $e(t)$  and to bound variations of the control signal  $u(t)$ . In order to show the influence of these weighting functions on the whole nonlinear system



behavior, two cases are considered with different parameters (bandwidth, cut-off frequency and static gain). These are given by:

- **Case 1:**  $W_e : \begin{cases} \dot{x}_e = -0.5x_e + 0.5e \\ v_1 = 0.9x_e \end{cases}, W_u : \begin{cases} \dot{x}_u = -1000x_u - 99.49u \\ v_2 = 99.49x_u + 10u \end{cases}$  (32)

- **Case 2:**  $W_e : \begin{cases} \dot{x}_e = -0.0025x_e - 0.4985e \\ v_1 = -0.4985x_e + 0.1e \end{cases}, W_u : \begin{cases} \dot{x}_u = -1000x_u - 99.49u \\ v_2 = 99.49x_u + 10u \end{cases}$  (33)

Combining (31), (32) (or (33)) and considering  $\tilde{x} = [x_e^T \quad x_u^T \quad x_1^T \quad x_2^T]^T$ , for each cases,

the obtained nonlinear model with exogenous inputs can be represented as:

$$\begin{cases} \dot{\tilde{x}}(t) = \tilde{A}(x(t))\tilde{x}(t) + \tilde{B}_1 w(t) + \tilde{B}_2(x(t))u(t) \\ v(t) = \tilde{C}_1(x(t))\tilde{x}(t) + \tilde{D}_{11}w(t) + \tilde{D}_{12}u(t) \\ y(t) = \tilde{C}_2(x(t))\tilde{x}(t) \end{cases} \quad (34)$$

where  $\tilde{C}_2 = [0 \ 0 \ C]$ ,  $\tilde{A}(\tilde{x}(t)) = \begin{bmatrix} A_e & 0 & -B_e C \\ 0 & A_u & 0 \\ 0 & 0 & A(x(t)) \end{bmatrix}$ ,  $\tilde{B}_1 = \begin{bmatrix} B_e \\ 0 \\ 0 \end{bmatrix}$ ,  $\tilde{B}_2(\tilde{x}(t)) = \begin{bmatrix} 0 \\ B_u \\ B(x(t)) \end{bmatrix}$ ,

$$\tilde{D}_{11} = \begin{bmatrix} D_e \\ 0 \end{bmatrix}, \tilde{D}_{12} = \begin{bmatrix} 0 \\ D_u \end{bmatrix} \text{ and } \tilde{C}_1 = \begin{bmatrix} C_e & 0 & -D_e C \\ 0 & C_u & 0 \end{bmatrix}.$$

The nonlinear model (34) contains one nonlinearity ( $\sin^2(x_1(t))$ ). Thus, according to the well know sector nonlinearity approach [8], an exact T-S fuzzy representation of (34) can be obtained considering:

$$\sin^2(x_1) = (1)(\sin^2(x_1)) + (0)(1 - \sin^2(x_1)), h_1(x_1) = \sin^2(x_1) \text{ and } h_2(x_1) = 1 - h_1(x_1) \quad (35)$$

In that case, one has the following 2 rules T-S fuzzy model:

$$\begin{cases} \dot{\tilde{x}}(t) = \sum_{i=1}^2 h_i(x_1(t))(\tilde{A}_i \tilde{x}(t) + \tilde{B}_{2i} u(t)) + \tilde{B}_1 w(t) \\ v(t) = \tilde{C}_1 \tilde{x}(t) + \tilde{D}_{12} u(t) \\ y(t) = \tilde{C}_2 \tilde{x}(t) \end{cases} \quad (36)$$

$$\text{with } \tilde{A}_1 = \begin{bmatrix} A_e & 0 & -B_e C \\ 0 & A_u & 0 \\ 0 & 0 & A_1 \end{bmatrix}, \quad \tilde{A}_2 = \begin{bmatrix} A_e & 0 & -B_e C \\ 0 & A_u & 0 \\ 0 & 0 & A_2 \end{bmatrix}, \quad \tilde{B}_1 = \begin{bmatrix} B_e \\ 0 \\ 0 \end{bmatrix}, \quad \tilde{B}_{21} = \begin{bmatrix} 0 \\ B_u \\ B_1 \end{bmatrix}, \quad \tilde{B}_{22} = \begin{bmatrix} 0 \\ B_u \\ B_2 \end{bmatrix},$$

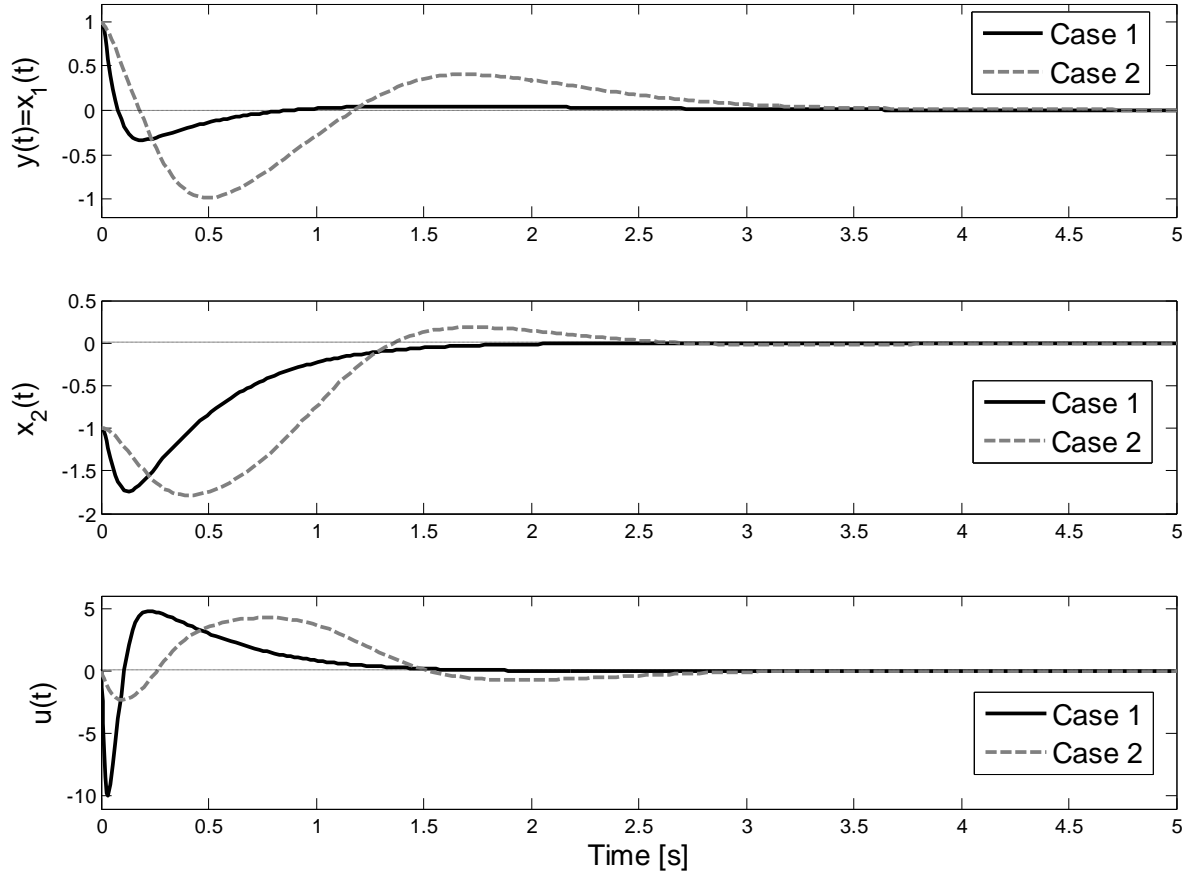
$$\tilde{C}_1 = \begin{bmatrix} C_e & 0 & -D_e C \\ 0 & C_u & 0 \end{bmatrix}, \quad \tilde{C}_2 = \begin{bmatrix} 0 \\ 0 \\ C \end{bmatrix}^T, \quad \tilde{D}_{12} = \begin{bmatrix} 0 \\ D_u \end{bmatrix} \quad \text{and where } A_1 = \begin{bmatrix} 0 & 3 \\ 1 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

Then, the goal is now to design a DOFC controller such as:

$$\begin{cases} \dot{x}^c(t) = \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i h_j h_k A_{ijk}^c x^c(t) + \sum_{j=1}^2 \sum_{k=1}^2 h_j h_k B_{jk}^c y(t) \\ u(t) = \sum_{j=1}^2 \sum_{k=1}^2 h_j h_k C_{jk}^c x^c(t) + \sum_{k=1}^2 h_k D_k^c y(t) \end{cases} \quad (37)$$

The LMI conditions proposed in theorem 1 are solved via the Matlab<sup>®</sup> LMI Control Toolbox [26] twice for case 1 and case 2. For each of these two cases, the whole closed loop system has been simulated through Matlab/Simulink<sup>®</sup> with the initial state  $x(0) = [1 \quad -1]^T$ . The comparison of the performances is shown in figure 3 where it can be noticed that stabilization in case 1 is better than the one in case 2.



**Figure 3.** Stabilization, state and control signals, comparison between case 1 and case 2

For the reader's information, the solutions of theorem 1 in the case 1 are given by the matrices:

$$A_{111}^c \square A_{121}^c \square \begin{bmatrix} -1 & -46 & 63 & -1 \\ 3 & -79 & 79 & 1134 \\ 1 & 24 & -79 & -18643 \\ 0 & 0 & 0 & -87 \end{bmatrix}, A_{112}^c \square A_{122}^c \square \begin{bmatrix} -4 & 175 & -245 & -4 \\ 4 & -106 & 117 & 1134 \\ 2 & 2 & -49 & -18644 \\ 0 & 0 & 0 & -87 \end{bmatrix},$$

$$A_{211}^c \square A_{221}^c \square \begin{bmatrix} 0 & -50 & 63 & -1 \\ 3 & -79 & 79 & 1134 \\ 1 & 25 & -79 & 18643 \\ 0 & 0 & 0 & 87 \end{bmatrix}, A_{212}^c \square A_{222}^c \square \begin{bmatrix} -4 & 171 & -245 & 4 \\ 4 & -106 & 117 & 1133 \\ 2 & 3 & -49 & -18644 \\ 0 & 0 & 0 & -87 \end{bmatrix},$$

$$B_{11}^c \square B_{12}^c \square B_{21}^c \square B_{22}^c \square [159.4 \quad -5205 \quad -3739 \quad -27.1]^T,$$

$$C_{11}^c \square C_{12}^c \square C_{21}^c \square C_{22}^c \square [-0.0036 \quad 0.2387 \quad -0.3325 \quad 0.0064], D_1^c \square D_2^c \square 0,$$

$$P_1 \square \begin{bmatrix} 1.80 & 0 & -0.0001 & 0.0005 \\ 0 & 174631.80 & 0 & 0 \\ -0.0001 & 0 & 30345386.18 & -35325781.03 \\ 0.0005 & 0 & -35325781.03 & 43554724.33 \end{bmatrix}, \text{ and}$$

$$Q_1 \square \begin{bmatrix} 2.05 & 0.09 & 5.51 & -1.21 \\ 0.09 & 46.11 & 20.86 & 14 \\ 5.51 & 20.86 & 32.04 & -1.57 \\ -1.21 & 14 & -1.57 & 33.21 \end{bmatrix} \text{ with a minimal } H_\infty \text{ performance level } \gamma = 0.9 ,$$

## 5 Conclusion

In this paper, a general framework for Takagi-Sugeno (T-S) fuzzy models LFT based output feedback robust control has been proposed. Thus, adapted from classical linear robust control theory [15, 16, 17], an extended model, combining the nonlinear one and the considered weighting function has been derived. Then a bounded real lemma has been obtained in the general case of nonlinear systems. A way to solve the nonlinear control problem, a fuzzy T-S modeling approach is used to derive LMI stability conditions. Indeed, the proposed control methodology remains to  $H_\infty$ /LFT dynamic output feedback controller synthesis for Takagi-Sugeno systems. That one allows setting stability performances of the closed-loop system. Finally, simulation results and a comparison between two sets of weighting functions have been presented and conclude to the efficiency of the proposed nonlinear control synthesis.

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