

Problem Title Description. Problem Name: The Mirror of Galadriel Author's Name: Magical Strings Problem Code: MSTRINGS Alphabet: D

Problem: A string  $S$  is called magical if every substring of  $S$  has its reverse also as a substring of  $S$ . Determine if a given string  $S$  is magical.

Solution: The constraints were given for  $|S| \leq 10$ , which allowed for even  $O(|S|^4)$  solutions to pass. Thus a brute-force solution of fixing  $i, j$  and then searching in  $O(|S|^2)$  for  $\text{reverse}(S[i, \dots, j])$  in  $S$  would work.

However, we can prove that  $S$  is magical iff it is palindromic. This follows a **necessary-and-sufficient-condition** argument.

Since every substring has its reverse, and since there is only one string of length  $|S|$ , clearly  $\text{reverse}(S)$  has to be in  $S$ , and hence  $S$  has to be palindromic (the **necessary** part).

Now, assuming that  $S$  is palindromic, then reverse of any substring of it, say  $\text{reverse}(S_{i, \dots, j})$  is gonna be  $= S_{|S|-1-j, \dots, |S|-1-i}$ . And hence, the reverse of every substring is in the string too. (the **sufficient** part).

This gives a simple  $O(|S|)$  time solution to the problem, where you just check if  $S$  is palindromic or not.