Problem Title Description. Problem Name: Saruman of Many Colours Author's Name: Colouring Blocks Problem Code: CBLOCKS Alphabet: I

Problem: You have N Uruk-hai which you want to brand in the manner as described by string c. You can choose any K consecutive Uruk-hai and brand them with the same colour. Minimize the number of brandings you perform, or if it is impossible to get the string c, then output -1.

Solution: The solution to this problem uses a greedy approach along with a **necessary-and-sufficient-condition** argument.

First construct a frequency array F as follows. For each set of consecutive characters that are the same in c, add its count to the array. That means, for example, if c = "aacccd", the frequency array would be [231].

Given this frequency array, it is clearly infeasible if $K > \max(F)$, since what was coloured last would have K consecutive same-colour elements. (The **necessary** condition part).

Now, if $K <= \max(F)$, claim that it is not only feasible, but also the minimum required steps is actually $\sum ceil(F[i]/K)$. To prove this, we need to (a) show feasibility and show how to do it in these many steps and (b) prove that it can't be done in less.

Assuming feasibility, part (b) is easy to see. Each element of F corresponds to a set of consecutive elements of the same colour. At any point we can colour only K of them. Hence we need atleast $F[i]/K$ steps to colour just this consecutive block. Hence, the whole string cannot be coloured in less than $\sum ceil(F[i]/K)$ steps.

Recall that we have assumed $\max(F) >= K$. Let that element be the jth element of F. Now, we greedily colour the string from left upto the jth block using $ceil(F[i]/K)$ steps for each block. We also greedily colour the string from right to the jth block in the same manner. Since $F[j] >= K$, we know that the right-side colouring is not going to interfere with a block on the left-side. Finally, we colour the jth block greedily ensuring that it doesn't spill over into the right or left. Thus we have *found* a way of colouring the string using exactly $\sum ceil(F[i]/K)$ steps.

Time Complexity: O(N) per test-case.