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A two-stage production/inventory system with admission control of product returns [★]

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Abstract

We consider a two-stage production-inventory system. Each stage is made of an exponential production server and an output inventory. The downstream inventory faces a Poisson demand and each inventory receives product returns that can be either accepted or rejected. The objective is to find a production/admission policy minimizing the expected discounted costs over an infinite horizon. We consider linear holding, backorder, production, admission and rejecting costs. We show that the optimal policy is characterized by state-dependent base-stock levels and state-dependent disposal thresholds. We also obtain monotonicity results for these switching curves. Interestingly, our results can be transposed to an hybrid single-echelon system with a production facility and a remanufacturing facility, both with exponential leadtimes. Finally, we design heuristics to control production and disposal and we carry out a numerical study to investigate their performances.

Key words: Multi-echelon, Reverse logistics, Production/Inventory system, Optimal Control, Remanufacturing

1 Introduction

Products are more and more returned in supply chains. Customers can return products a short time after purchase due to take-back commitments of the supplier. The proportion of returns is particularly important in electronic business where customers can not touch a product before purchasing it. Customers might also return used products a long time after purchase. This type of return has increased in recent years due to new regulations encouraging waste reduction, especially in Europe. Some industries also encourage it for economical and marketing reasons.

Returns constitute a reverse flow, from the customer to the supplier, which complicates inventory control. A recent review of literature on reverse logistics is provided by Ilgin and Gupta [8]. Inventory management constitutes a part of this review, and we can see that an abundant literature is devoted to the inventory control of single echelon systems. In several situations, the structure of the optimal policy is similar to the case without returns. For instance, Fleischmann and Kuik [6] consider a single inventory with stochastic demand and stochastic independent returns. To model the returns, they consider a demand that can be both positive or negative. They show average cost optimality of an (s, S) policy.

In this paper, we consider a two-stage production /inventory system with returns of products at each stage and a disposal option upon arrival of returns. Before presenting in detail our model, we briefly review the related

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literature of inventory control. In particular, we focus on models with a disposal option and on multi-echelon models.

1.1 Disposal option

We first review the inventory control literature considering a disposal option for returns. Heyman [7] considers an inventory system with backorders, Poisson demand and Poisson returns. He assumes zero lead-times and linear costs for both procurement and remanufacturing. These strong assumptions imply that the optimal production policy is a make-to-order policy. The optimal disposal policy is a simple threshold policy: when the inventory level exceeds a certain disposal threshold R , every returned item is disposed upon arrival. He also provides an explicit expression for the optimal disposal threshold. Zerhouni et al. [21] relax the assumption of zero manufacturing lead-time. Only one item at a time can be manufactured and the manufacturing lead-time is exponentially distributed. For both lost sale and backorder cases, the optimal policy is an (R, S) policy stating to produce when the net inventory is below S and to dispose returned items when the net inventory exceeds R . The ordering of the two optimal parameters depends on the ordering of the linear manufacturing and remanufacturing costs. In the backorder case, they obtain an explicit expression for the optimal base-stock level when the difference $(S - R)$ is given. The disposal policy of [7] can be seen as a special case of the (R, S) policy where $S = 0$.

Ouyang and Zhu [10] relax the zero lead-time and fixed cost assumptions. The production lead-time is deterministic while the remanufacturing leadtime is neglected. They consider an (s_p, Q_p, s_d) where s_p is the reorder point, Q_p the order quantity and s_d the disposal threshold (returns are remanufactured if and only if the stock level is below s_d). This policy was first introduced in the paper of van der Laan et al. [14] which is presented in the next paragraph. Ouyang and Zhu derive analytical expressions for the average cost while van der Laan et al. use numerical methods. The optimal parameters can then be obtained by solving a difficult nonlinear integer programming. The (R, S) policy in [21] can be seen as an (s_p, Q_p, s_d) with $s_p = S - 1$, $Q_p = 1$ and $s_d = R$.

Van der Laan et al. [15] extend the model of Heyman in a different direction. They model explicitly the inventory of remanufacturable products and the remanufacturing process. The repair lead-time is exponential and demand/return processes are again independent Poisson processes. They introduce the (s_p, Q_p, N) policy where any return is disposed whenever the number of products waiting for repair equals N . The flow of repaired items is then the output process of an $M/M/N/N$ queue. Analytical expressions are also computed for the average cost. Van der Laan et al. [14] generalize the policies of [10] and [15] with the (s_p, Q_p, s_d, N) policy. They compare numerically the optimal policy belonging to this class with the optimal policy belonging to (s_p, Q_p, s_d) (respectively (s_p, Q_p, N)) class of policies.

Van der Laan et Salomon [16] consider a model with inter-occurrence times between demands (resp. returns) being Coxian-2 distributed. The demand process and return process are correlated. They compare an (s_p, Q_p, Q_r, s_d) push-disposal policy with an $(s_p, Q_p, s_r, S_r, s_d)$ pull-disposal policy to coordinate manufacturing and remanufacturing decisions. For the push-disposal policy, s_p is the manufacturing reorder point and Q_p the manufacturing order quantity. Returned products are remanufactured with batch size Q_r and the remanufactured items are disposed if the inventory position reaches s_d . For the pull-disposal policy, remanufacturing is initiated only when the inventory position is below s_r and the remanufacturable inventory is above S_r . Teunter and D. Vlachos [13] complement the numerical study of the above model.

Unlike [7] and [21], other papers do not guarantee that the optimal policy belongs to the class of policies under consideration.

1.2 Multi-echelon systems

In their seminal work, Clark and Scarf [1] studies a series inventory system with N stages, finite horizon, periodic review, linear holding and backorder cost, no setup cost and stochastic demand only on the downstream stage. They prove that a Base-stock echelon policy is optimal. These assumptions have been relaxed in several papers. For instance, Parker and Kapuscinski [11] add a constraint of capacity on the orders. In general, a base-stock echelon policy is not optimal when a capacity constraint is added. They show that a base-stock echelon policy is nearly optimal when the downstream echelon is not overloaded. In the contrary case, the Kanban policy is nearly optimal.

In production-inventory systems, replenishment is modelled in a different way than in classical inventory systems. Items are produced by servers one by one, or possibly by batches. Each unit, or batch, requires a random lead-time to be produced. Implicitly, in production-inventory systems, replenishments are capacitated. Veatch and Wein [18] consider a two-stage system with one exponential server at each stage. Otherwise, their assumptions are similar to [1]. Again, as in [11], they show on numerical experiments that the optimal policy is not a base-stock echelon policy. They investigate several policy (base-stock, Kanban, fixed buffer and conwip) and compare them to the optimal policy. Their conclusions are the same as those of [11] : the base-stock policy is generally the nearest optimal policy but when the downstream station is overloaded the Kanban policy is better. In another paper, Veatch and Wein [17] characterize the form of the optimal policy for a series system with N stages and generalize their results to assembly systems, disassembly systems and to the routing problem. The optimal policy is a state-dependent base-stock policy. It is optimal to produce at stage i when the inventory level at stage i is smaller than a base-stock level, depending on inventory levels at all other stages. Dallery and Liberopoulos [2] investigates a Kanban generalized policy. This policy is a mix between Kanban and base-stock echelon policy. They also provide a review of heuristic policies.

Fewer papers investigate the control of multi-echelon systems with product returns. DeCroix et al. [4] analyze a setting similar to [1] except that demand can be negative. They prove that a base-stock echelon is still optimal. They also propose a method to compute a near optimal policy, explain how to extend their model when returns occur at different stages and compare the base-stock echelon policy to fixed buffer policies. With the same type of modelling, DeCroix and Zipkin [5] characterize the optimal policy for an assembly system and DeCroix [3] analyzes a simple series model with only one return in any of the stages and evaluates whether the base-stock model is optimal. His conclusion is that the base-stock echelon policy is optimal if the return is made at the upstream station. Vercraene et al. [19] extend [18] for the case with Poisson returns at each stage. The structure of the optimal policy is similar to [18]. In an extensive numerical study, they compare the performances of base-stock, Kanban, fixed buffer and half-optimal policies.

1.3 Our contributions

To the best of our knowledge, we are not aware of papers dealing with a multi-stage inventory system with a disposal option upon arrival of returns . We fill this gap by considering an extension to [19] where items can be rejected or accepted when they are returned.

In Section 2, we detail the problem formulation. More precisely, each stage is made of an exponential production server and an output inventory. The downstream inventory faces a Poisson demand and each inventory receives product returns, according to Poisson processes, that can be either accepted or rejected. The objective is to find a production/admission policy minimizing the expected discounted costs over an infinite horizon. We consider linear holding, backorder, production, admission and rejecting costs. In Section 3, we show that the optimal policy is characterized by four switching curves: a state-dependent base-stock level and a state-dependent admission threshold for each stage. We also derive monotonicity results for the switching curves.

In Section 4, we show that our results can be extended to the case with two types of returns at each stage: one type has a disposal option while the other one does not have a disposal option. Then we explain how our results can be transposed to an hybrid single-stage system with a production facility and a remanufacturing facility where both facilities have exponential replenishment leadtimes.

In Section 5, we design several simple heuristics and study the performances of some of them. The production servers can be controlled according to classic multi-echelon policies (e.g. Kanban, base-stock echelon, fixed buffer or generalized Kanban). At stage 1, due to the symmetry between production control and admission control, we suggest to use admission controls that are similar to the production controls at the upstream server. At stage 2, it is less clear that we can adapt the production policy to the admission control policy and we propose new control policies. After a brief explanation on how to compute the policies, we compare the performance of some of them in Section 6.

2 Problem formulation

We consider a 2-stage production/inventory system in series which satisfies end-customer demand (see Figure 1). Station M_i produces items one by one. The production lead-time of station M_i is exponentially distributed with rate μ_i . Preemption is allowed and station M_i can be started or stopped at any time. Produced items

are stocked in a buffer B_i just after M_i . The end buffer M_2 sees customer demands arriving according to a Poisson process with rate λ . We assume that backorders are allowed. At time t , the on-hand inventory at B_i is denoted by $X_i(t)$. Note that $X_2(t)$ can be negative due to backorders. When buffer B_1 is empty, the production is blocked at station M_2 . Returns of products occur at buffer B_i according to an independent Poisson process with rate δ_i . Another way to see these two return flows is to consider a situation where there is a single flow of returns for the whole system (rate $\delta_1 + \delta_2$) and, after an inspection, returned products are routed to the Finished Good (FG) inventory with probability $\delta_2/(\delta_1 + \delta_2)$ and to the Work-In-Process (WIP) inventory with probability $\delta_1/(\delta_1 + \delta_2)$. To ensure the stability of queue lengths, we assume that the demand is smaller than the production capacity for each echelon: $\lambda < \mu_1 + \delta_1 + \delta_2$ (echelon 1) and $\lambda < \mu_2 + \delta_2$ (echelon 2). Unlike the system without disposal options [19], we don't have to assume that the total return rate is smaller than the demand rate.

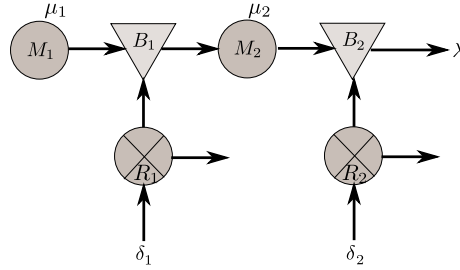


Fig. 1. Model

Upon arrival at buffer B_i , a returned product is either accepted with cost C_i^a or disposed (rejected) with cost C_i^d . When a return is accepted in buffer B_i , it can be used immediately as a new product (we neglect the remanufacturing lead-time). Let $C_i^r = C_i^a - C_i^d$ be the relative remanufacturing cost. The unit production cost is C_i^p . The system incurs in state (X_1, X_2) a cost rate $c(X_1, X_2) = h_1X_1 + h_2X_2^+ + bX_2^-$ where h_i is the inventory holding cost per unit of time at buffer B_i and b is the backorder cost per unit of time.

A production policy π specifies when to produce and when to accept returns, for each stage. The discounted expected cost over an infinite horizon of a policy π , with initial state (x_1, x_2) and discount rate $\alpha > 0$, is given by :

$$v^\pi(x_1, x_2) = E \left[\int_0^{+\infty} e^{-\alpha t} c(X_1(t), X_2(t)) dt \mid X_1(0) = x_1, X_2(0) = x_2, \pi \right] + E \left[\sum_{i=1}^{\infty} \sum_{j=1}^2 \left(e^{-\alpha T_j^p(i)} C_j^p + e^{-\alpha T_j^a(i)} C_j^a + e^{-\alpha T_j^d(i)} C_j^d \right) \mid X_1(0) = x_1, X_2(0) = x_2, \pi \right] \quad (1)$$

where $T_j^p(i)$, $T_j^a(i)$ and $T_j^d(i)$ respectively represent the i^{th} time instant at stage j when a product is produced, when a return item is accepted and when a return is disposed.

The objective is to minimize the expected discounted cost over an infinite horizon. This problem can be formulated as a continuous-time Markov Decision Process (MDP). Let v^* be the optimal value function: $v^*(x_1, x_2) = \min_{\pi} v^\pi(x_1, x_2) = v^{\pi^*}(x_1, x_2)$.

After uniformization of the MDP with rate $\tau = \lambda + \mu_1 + \mu_2 + \delta_1 + \delta_2 + \alpha$, we can transform the continuous-time MDP into a discrete time MDP[12]. The optimal value function can be shown to satisfy the following optimality equations:

$$v^* = Tv^* \quad \text{with} \quad Tv(x_1, x_2) = \frac{1}{\tau} \left(\begin{array}{l} x_1 h_1 + x_2^+ h_2 + x_2^- b + \lambda v(x_1, x_2 - 1) \\ + \mu_1 \min(v(x_1, x_2), v(x_1 + 1, x_2) + C_1^p) \\ + \mu_2 \min(v(x_1, x_2), v(x_1 - 1, x_2 + 1) + C_2^p) \\ + \delta_1 \min(v(x_1, x_2) + C_1^d, v(x_1 + 1, x_2) + C_1^a) \\ + \delta_2 \min(v(x_1, x_2) + C_2^d, v(x_1, x_2 + 1) + C_2^a) \end{array} \right) \quad (2)$$

3 Characterization of the Optimal Policy

In order to derive structural properties of the optimal policy, we will show that the optimal value function belongs to the following set of value functions V .

Definition 1 A value function v belongs to V if, for all x_1, x_2 , we have

- (a) $v(x_1 + 1, x_2) - v(x_1, x_2) \leq v(x_1 + 1, x_2 + 1) - v(x_1, x_2 + 1)$
- (b) (i) $v(x_1 + 1, x_2) - v(x_1, x_2 + 1) \leq v(x_1 + 2, x_2) + v(x_1 + 1, x_2 + 1)$
and
(ii) $v(x_1, x_2 + 1) - v(x_1 + 1, x_2) \leq v(x_1, x_2 + 2) - v(x_1 + 1, x_2 + 1)$
- (c) $v(x_1 + 1, x_2) - v(x_1, x_2) \leq v(x_1 + 2, x_2) - v(x_1 + 1, x_2)$
- (d) $v(x_1, x_2 + 1) - v(x_1, x_2) \leq v(x_1, x_2 + 2) - v(x_1, x_2 + 1)$

In Koole [9], property (a) is called supermodularity and denoted $Super(1, 2)$. Property (b) is called superconvexity and denoted $SuperC(1, 2)$. Finally (c) and (d) refer to the convexity of v in x_1 and x_2 and are denoted $Conv(1)$ et $Conv(2)$. Properties (a) and (b) imply together properties (c) and (d).

Lemma 2 If $v \in V$ then $Tv \in V$. Moreover v^* belongs to V .

Proof: Due to page constraints, we provide only a sketch of the proof. As operator T is a contraction mapping, the fixed point theorem in a Banach space [12] ensures that any sequence of value functions (v_n) defined as $v_{n+1} = Tv_n$ will converge to the optimal value function v^* , the unique solution of the optimality equations $v^* = Tv^*$.

The operator T is a linear combination of operators that propagate $Super(1, 2)$ and $SuperC(1, 2)$ (see [9]). As a result, if a value function v is in V , then the value function Tv is also in V . If we take a null value function v_0 , it is clear that $v_0 \in V$. By recursion, we conclude that $v^* \in V$. \square

We can now state our main theorem.

Theorem 3 The optimal policy is a state-dependent policy. There exists four switching curves $s_1(x_2)$, $s_2(x_1)$, $r_1(x_2)$ and $r_2(x_1)$ such that :

- Produce at M_1 if and only if $x_1 < s_1(x_2)$. Moreover $s_1(x_2) - 1 \leq s_1(x_2 + 1) \leq s_1(x_2)$.
- Produce at M_2 if and only if $x_2 < s_2(x_1)$. Moreover $s_2(x_1) - 1 \leq s_2(x_1 + 1) \leq s_2(x_1)$.
- Accept return at R_1 if and only if $x_1 < r_1(x_2)$. Moreover $r_1(x_2) - 1 \leq r_1(x_2 + 1) \leq r_1(x_2)$.
- Accept return at R_2 if and only if $x_2 < r_2(x_1)$. Moreover $r_2(x_1) + 1 \geq r_2(x_1 + 1) \geq r_2(x_1)$.

In addition, $s_1(x_2)$ and $r_1(x_2)$ can be ordered for each x_2 in the following way: $s_1(x_2) \leq r_1(x_2)$ if $C_1^r > C_1^p$, $s_1(x_2) = r_1(x_2)$ if $C_1^r = C_1^p$ and $s_1(x_2) \geq r_1(x_2)$ if $C_1^r < C_1^p$.

Proof: As $v^* \in V$, the four switching curves are well defined. $Conv(1)$ ensures that we can define the production threshold $s_1(x_2) = \min[x_1 | v(x_1 + 1, x_2) - v(x_1, x_2) + C_1^p > 0]$ and the disposal threshold $r_1(x_2) = \min[x_1 | v(x_1 + 1, x_2) - v(x_1, x_2) + C_1^r > 0]$. $SuperC(1, 2)$ ensures that we can define the production threshold $s_2(x_1) = \min[x_2 | v(x_1 - 1, x_2 + 1) - v(x_1, x_2) + C_2^p > 0]$. Finally $Conv(2)$ ensures that we can define the disposal threshold $r_2(x_1) = \min[x_2 | v(x_1, x_2 + 1) - v(x_1, x_2) + C_2^r > 0]$.

The monotonicity results on the switching curves are also implied by the fact that $v^* \in V$. For instance, $Super(1, 2)$ ensures that $s_1(x_2 + 1) \leq s_1(x_2)$ and $SuperC(1, 2)$ ensures that $s_1(x_2) - 1 \geq s_1(x_2 + 1)$. The other monotonicity results are obtained in a similar way. \square

In order to illustrate Theorem 3, we provide a numerical example in Figure 2. It describes the switching curves in function of the state of the system. We can see four curves because there are four decision operators with minimization in T , so four decisions to take to control optimally the system. The optimal policy is simple, we produce or accept returns in a facility if the system state is below the corresponding switching curve.

For a single-echelon system, the optimal policy is a simple (R, S) policy with S the base-stock level and R the disposal threshold [20].

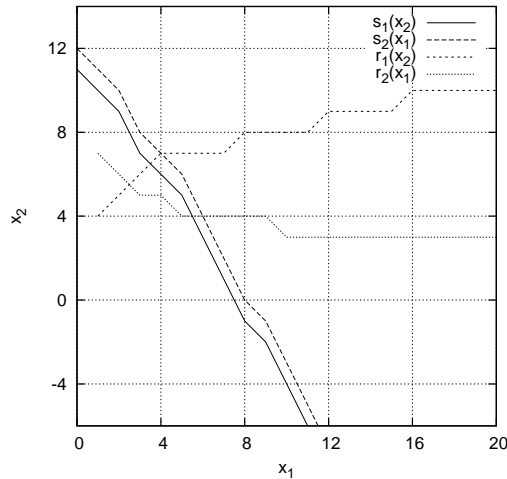


Fig. 2. Optimal policy ($\mu_1 = 0.5$, $\mu_2 = 0.7$, $\delta_1 = 0.1$, $\delta_2 = 0.6$, $\lambda = 1$, $h_1 = 1$, $h_2 = 2$, $b = 4$, $C_1^r = -1$, $C_2^r = 5$, $C_1^p = C_2^p = 0$)

4 Extension to forced returns

Consider an extension of our previous model where there is a Poisson return flow without disposal option (rate δ'_i) in addition to the Poisson return flow with disposal option (Figure 3). Returns without a disposal option will be called *forced returns* and returns with a disposal option will be called *optional returns*. It is straightforward to extend Theorem 3 to this new problem. So the structure of the optimal policy is exactly the same than the problem with disposal option only.

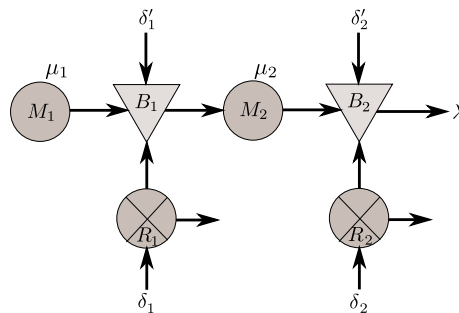


Fig. 3. Extension with forced returns

There are now three necessary and sufficient conditions for the stability of the queue lengths. First, the demand rate must be larger than the sum of forced return rates: $\lambda > \delta'_1 + \delta'_2$. Second, all forced returns at stage 1 must be treated by station M_2 : $\delta'_1 < \mu_2$. Third, each echelon j must be able to satisfy the demand: $\lambda < \mu_j + \sum_{i=j}^2 (\delta_i + \delta'_i)$ for $j = 1, 2$.

Interestingly, we can use this model with forced returns to model a hybrid production/remanufacturing system with positive production and remanufacturing leadtimes (Figure 4). It suffices to set $\delta_1 = \delta'_2 = \mu_1 = 0$ in the model with forced returns to take another viewpoint (consider return as production and vice versa). The optional returns at stage 2 (R_2) can be seen as a controllable production facility with production rate δ_2 and preemption allowed. This manufacturing facility produces new products for the finished good inventory B_2 . In parallel, forced returns occur with rate δ'_1 . These returns are put in a remanufacturable inventory (B_1). Then M_2 is a controllable remanufacturing facility with rate μ_2 . When a product has been remanufactured, it is placed in the finished good inventory B_2 and can be sold as a new product. The cost structure include linear production, remanufacturing, return and holding costs. The problem is then to jointly control the manufacturing server and the remanufacturing server in order to minimize expected discounted costs.

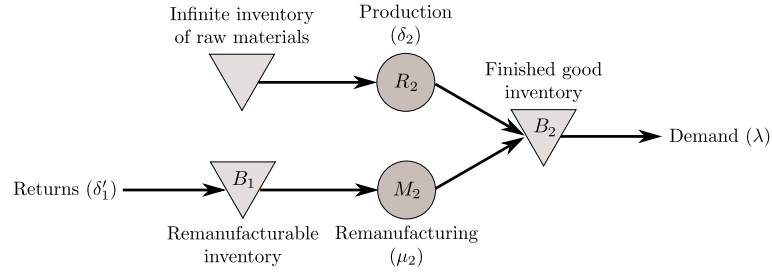


Fig. 4. Hybrid production/remanufacturing system

5 Heuristic policies

Since the optimal policy has a complex structure and is difficult to implement in industry (see Figure 2), we use, in this section, our theoretical results (see Theorem 3) to design simple and efficient policies. We propose the following controls for stations M_1 , M_2 and optional returns R_1 and R_2 .

Control of M_1

- Fixed buffer (FB_{s_1}) : produce when $x_1 < s_1^{FB}$
- Base-stock echelon (BS_{s_1}) : produce when $x_1 + x_2 < s_1^{BS}$
- Kanban (KB_{s_1}) : produce when $x_1 + x_2^+ < s_1^{KB}$

We propose to use classic policies for multi-echelon systems (see e.g. [18]): fixed buffer base-stock echelon and Kanban. The base-stock echelon (BS_{s_1}) states to produce when $x_1 + x_2 < s_1^{BS}$ while the Kanban policy (KB_{s_1}) states to produce when $x_1 + x_2^+ < s_1^{KB}$. We don't recommend to use a fixed buffer policy (FB_{s_1}) which states to produce when $x_1 < s_1^{FB}$. It is well known that it performs very poorly, most of the time.

Control of M_2 :

- Fixed buffer (FB_{s_2}) : produce when $x_2 < s_2^{FB}$

The main heuristic used in the literature to control M_2 is the fixed buffer policy FB_{s_2} which states to produce when $x_1 + x_2 < s_2^{FB}$. It might be interesting to explore new heuristics to have a better approximation of $s_2(x_1)$ since this switching curve might depend a lot on x_1 .

Control of R_1 :

- Fixed buffer (FB_{r_1}) : accept return when $x_1 < r_1^{FB}$
- Base-stock echelon (BS_{r_1}) : accept return when $x_1 + x_2 < r_1^{BS}$
- Kanban (KB_{r_1}) : accept return when $x_1 + x_2^+ < r_1^{KB}$

Due to the equivalence between the control of M_1 and the control of optional returns R_1 , we suggest to use similar policies to control R_1 and M_1 : base-stock echelon (BS_{r_1}) which states to accept return when $x_1 + x_2 < r_1^{BS}$ and Kanban (KB_{r_1}) which states to accept return when $x_1 + x_2^+ < r_1^{KB}$.

Control of R_2 :

- Fixed buffer (FB_{r_2}) : accept return when $x_1 < r_2^{FB}$
- Base-stock return (BR_{r_2}) : accept return when $x_1 + x_2 < r_2^{BR}$
- Base-stock return with priority (BP_{r_2}) : accept return when $x_1 + x_2 < r_2^{BP}$ or $x_2 < 0$

Fixed buffer (FB_{r_2}) which states to accept return when $x_2 < r_2^{FB}$ is a very simple heuristic because it considers only local informations. However, we see in section 3 that optimal $r_2(x_1)$ decreases. We have an intuition about that : if the server M_2 is fast, stock B_1 and stock B_2 could be seen as one global stock. In the limit case ($\mu_2 \rightarrow \infty$) the system is like the single echelon of [20]. Due to this observation a new heuristic called base-stock return (BR_{r_2} : accept return when $x_1 + x_2 < r_2^{BR}$) seems relevant. But, an other limit observation have to be noted : if we have a lot of product in B_1 the system could be seen as a single echelon with an infinite stock B_1 , it assures that $\lim_{x_1 \rightarrow \infty} r_2(x_1) = constant$. From this observation we propose an heuristic called base-stock return with Priority (BP_{r_2}) which states to accept return when $x_1 + x_2 < r_2^{BP}$ or $x_2 < 0$. The BP_{r_2} heuristic could be extend to a policy (obviously better) with a variable level of priority $x_2 < r_2^{BP}$. This heuristic would be similar to the generalized Kanban [2]. Here we chose 0 as priority level because we think it is the easiest to implement in industry.

Moreover we propose to try specific cases with $s_i = r_i$ ($i \in \{1, 2\}$) and with the same heuristic for the control of M_i and R_i . We call these cases equal policies. They are obviously worse but easier to implement in industry.

6 Numerical study

In this numerical study, we focus on the average cost criterion. The optimal policy is computed by a value iteration algorithm [12] based on the fact that any sequence $v_{n+1} = Tv_n$ converges to the optimal value function. We truncate the state space in three directions and increase the state space until the average cost is no more sensitive to the truncation level. Heuristic policies are also computed by a value iteration algorithm where we adapt the operator T to the heuristic under consideration.

For the heuristic policies, we want to find the parameters s_1, s_2, r_1, r_2 that minimize the average cost function $C(s_1, s_2, r_1, r_2)$ which is computed by the previous value iteration algorithm. This optimization problem is a non linear problem with integer variables that might be long to solve since evaluating a given policy might already take time. Therefore, we make the plausible assumption that the function $C(s_1, s_2, r_1, r_2)$ is unimodal. A function f is unimodal if for x, y and z on a line and y between x and z : $f(x)$ is finite and $f(x) \leq f(y)$ implies $f(y) \leq f(z)$. The assumption of unimodularity has been validated on several instances. Based on this assumption, we can solve the system with the maximal gradient with constant step method. This method is very efficient here because we can start the optimization with an approximate value of s_1, s_2, r_1 and r_2 resulting from the calculation of the optimal policy. Note that if we want to valid that a cost $C(s_1, s_2, r_1, r_2)$ is a minimum we need to check that all adjacent points of (s_1, s_2, r_1, r_2) are higher. With 4 dimensions we need to check $3^4 = 81$ points, so the computation time can be long.

We limit our study to a system with $\delta_1 = \delta'_2 = 0$ to have only one flow which could be forced or/and controllable from production and flow which could be controllable from return. Moreover we make the classical assumption that $h_1 = \lambda = 1$. Under these hypotheses we compute several instances.

As expected, the optimal policy respect the structure described in Section 3, the relative position between $s_1(x_2)$ and $r_1(x_2)$ is function of the sign of $C_1^r - C_1^p$ and when M_2 is fast, we have $r_2(x_1)$ near curve $x_1 + x_2 = constant$. Note that we do not find a clear explanation of the relative position between $s_2(x_1)$ and $r_2(x_1)$ (see Figure 5).

About heuristics, the first observation is that equal heuristics (i.e. heuristics with $r_i^r = s_i^r$) are obviously worse than others heuristics and even often very bad with a cost gap over $> 10\%$. Their implementation in industries is easier but the loss can be very important. The second observation is that, for control of M_1 , we obtain the same conclusion as [18]. The Kanban policy on M_1 is better than others when M_2 is overloaded with regard to μ_1 . Ordinary the best heuristics for M_1 is the base stock echelon policy. The fixed buffer policy is numerically always the worse. The optimal thresholds plot in Figure 5 show this trend. The last observation is that the control of R_2 is more complicated. We give an more specific study about this control in the following paragraph.

We compute instances with heuristic BS_{s_1} for the control of M_1 and FB_{s_2} for the control of M_2 given Figures 5 and 6. The first figure, shows the variation of production capacity in server M_2 . The rate μ_2 can vary on $]0.6, \infty[$. Here we start from 0.65 because compute near limit of stability increased the state space and the computation time. FB_{r_2} is the best heuristic when M_2 is overloaded (i.e. μ_2 near 0.6). But BR_{r_2} and BP_{r_2} are very efficient when utilization of M_2 is low (i.e. $\mu_2 \gg \lambda$). This phenomenon is expected because optimal curves $r_2(x_1)$ on the three other plots have this behavior. When $\mu_2 = 0.65$, $r_2(x_1)$ is near horizontal so FB_{r_2} threshold is a good approximation. But when μ_2 is bigger, $r_2(x_1)$ is near curve $x_1 + x_2 = constant$ so BR_{r_2} heuristic became the best. Note that $s_2(x_1)$ increases a lot when M_2 is overloaded. We do not have a clear intuition about this but a heuristic with threshold which increasing in x_1 could be tested in this case. Moreover, note that peaks and irregularity are due to discrete character of parameters s_i and r_2 .

The BP_{r_2} policy seems a good heuristic, but it is not always the case. For instance, if we have a lot of forced returns, then $r_2(x_1)$ became negative for all x_1 . So BP_{r_2} became a bad approximation of optimal threshold because r_2^{BP} can not be negative. To solve this problem we could relax the constraint that priority level is 0 and look for the optimal priority level. Figure 6 shows the variation of forced return in stock B_1 . The return rate δ'_1 can varies on $[0, 1[$. This figure shows clearly the limit of BP_{r_2} when optimal $r_2(x_1) < 0$. Nevertheless for $\delta'_1 < 0.7$ it is generally a good heuristic.

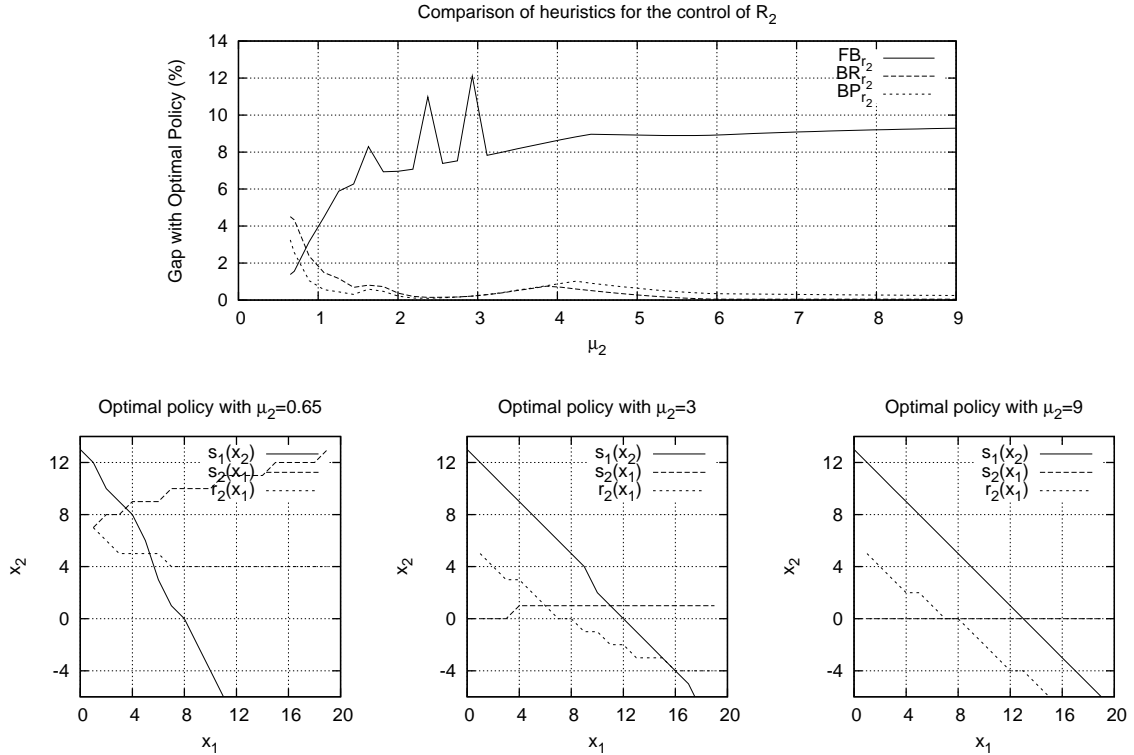


Fig. 5. Variation of μ_2 for the set of parameters : $\mu_1 = 0.6$, $\delta'_1 = 0$, $\delta_2 = 0.6$, $\lambda = 1$, $h_1 = 1$, $h_2 = 2$, $b = 4$, $C_2^r = 10$, $C_1^p = C_2^p = 0$

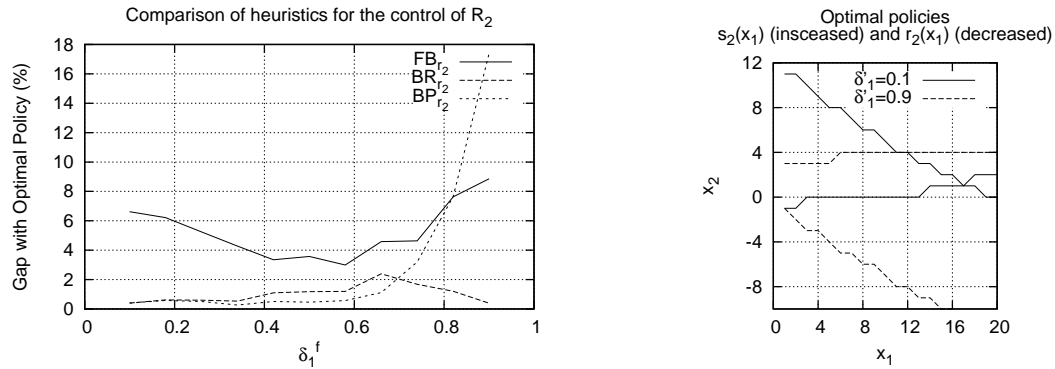


Fig. 6. Variation of δ_1^f for the set of parameters : $\mu_1 = 0$, $\mu_2 = 1.5$, $\delta_2 = 1$, $\lambda = 1$, $h_1 = 1$, $h_2 = 2$, $b = 4$, $C_2^r = 10$, $C_1^p = C_2^p = 0$

7 Conclusion

In this paper, we prove the structure of the optimal policy of a two-stage production/inventory system with returns at each stage that could be disposed or not. Due to the complexity of the optimal policy, we propose heuristics policies to simplify their implementation in industry. At the end, we make an numerical study to test their efficiency. Our results are complementary to those in the literature but it would be interesting to extend the numerical study and investigate efficient heuristics to control the downstream server.

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