

## Probability Experiments

### Solution Sketch:

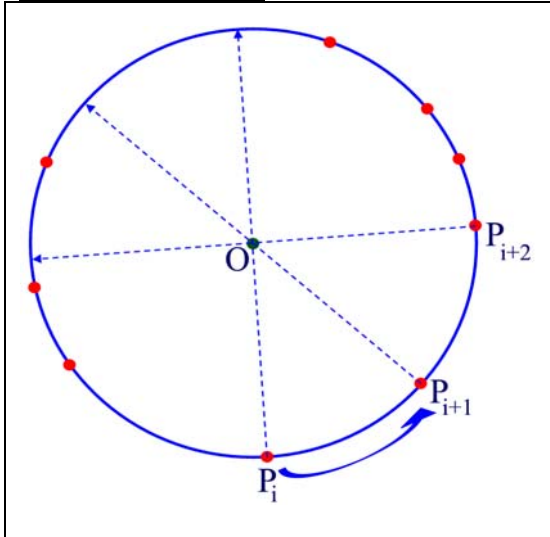


Figure 1: In the above figure  $L_i=5$ ,  $L_{i+1}=4$ ,  $L_{i+2}=4$ .

**Problem Type:** Computational Geometry, Combinatorics.

The problem that we have to solve here is: Given  $n$  points on the boundary of a circle, how many acute triangles do they form. Although the solution when described is pretty straight forward, it does not come into mind straight away. Here  $n$  can be as high as 20000 and the value of  $r$  has no real importance. So the complexity cannot be  $O(n^2)$  but it should be  $O(n \log(n))$ . Also we cannot directly find the acute triangles very easily. So we should find the right angled triangles and obtuse triangles first. We sort all the points in the ascending order of  $\theta$ . Suppose we call the  $i$ -th point on the circle  $P_i$ . From  $P_i$  we look towards the center  $O$  and say that at this point the number of points straight on the other side or on right is  $L_i$ . So  $P_i$  with any two of the  $L_i$  points can form an obtuse triangle. So the total

number of obtuse triangles formed is  $\frac{L_i(L_i - 1)}{2}$ . Similarly we can find it for all the  $n$  points and

add them up to find the total number of obtuse triangles. So the total number is:  $\sum_{i=1}^n \frac{L_i(L_i - 1)}{2}$ .

Normally finding  $L_i$  for all  $n$  points will require  $O(n^3)$  complexity. But if we use a sliding window that keeps track how many points are within  $\theta$  and  $\theta+180$  then  $L_i$  can be found with time complexity of  $O(n)$ . But the complexity of sorting the points was  $O(n \log(n))$ .