Dynamics modeling of a Two-wheeled vehicle using Jourdain's principle

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Abstract— This paper describes a modeling technique for developing the motion equation of a motorcycles vehicle. Based on the Jourdain's principle approach, the derived model presents a nice analytical formulation with recursive Jacobian matrices computation. The synthesized model takes in consideration the main forces and moments affecting the behavior of the motorcycle and allowing the simulation of 11DOF.

Index Terms— Motorcycle dynamics, Jourdain's principle.

I. INTRODUCTION

Riding a two-wheeled vehicle is not simple and intuitive like driving a car vehicle. Involved accelerations, critical equilibrium, aerodynamic effects, trajectory anticipation are, among others, many points that make driving a motorbike to be a complicated exercise.

To predict the behavior of such a system, one method is to model it. In the study of mechanical systems, modeling usually leads to a set of differential algebraic equations (DAE) derived from laws of mechanics. In this field, two approaches are proposed: the analytical approach and the numerical one. In this later, many tools were developed such MSC ADAMS [1], simMechanics [2], Dymola [3], etc.. The alternative approach is the development of symbolic equations of motion. It is very time consuming method, however, it offers more flexibilities in the system description. In this case, using a dedicated packages implemented in a computer algebra software, like TsiProPac [4] or MBsymba [5], is highly recommended.

Many works have been addressed the modeling of the two-wheeled vehicles. Indeed, the first study on the stability and balance of a bicycle vehicle were performed by Whipple [6]. This bicycle is represented by two bodies linked via the steering mechanism. Wheels are modeled as circular thin disks where the gyroscopic effect is taken into account. After Whipple, other work has been undertaken [7], [8], [9]. In all these studies, the tire-road interaction has been neglected. In 1971, Sharp integrated tires' efforts [10]. The objective of this study is to examine the effect on stability of a variation in the vehicle's parameters. His model is represented as a set of two rigid bodies connected by the steering mechanism, which can simulate 4 Degrees of Freedom (DOF).

So far, in all these models, the longitudinal velocity was assumed to be constant and the vehicle suspension movements were ignored. It was interesting to examine how these factors influenced the frequency characteristics of the different stability modes. These aspects have been investigated in a further work of Sharp in 1974 [11]. In 1978,

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Weir made the first approach to stabilize a motorcycle by including a human in the simulation loop [12]. In 1994 Sharp published a paper, in which, the motorcycle is represented by a set of four rigid bodies to simulate 8DOF. The originality of this model lies in the integration of flexibilities and the rider tilting motion [13].

In the same vein, a model incorporating the suspension travels and a more complex representation of the tire-road interaction is presented in [14]. The advances made by this model have allowed to explore the dynamics of such a vehicle for a large motion around equilibrium. In this case, the interaction between the in-plane and the out-of-plane motion is best described. More recently, the project FastBike presents a nonlinear model of a motorcycle described as a set of 6 bodies and simulating 11DOF. Each body is identified by its natural coordinates and the assembly is done by introducing the geometric constraints [15]. Finally, a modeling technique based on the recursive Newton-Euler approach is adapted to derive a motorcycle dynamics model intended for a two-wheeled simulator application [16].

In the present paper, a modeling technique based on the Jourdain's principle is used to develop the motion equation of a motorcycles vehicle.

II. JOURDAIN'S PRINCIPLE

A. Preliminaries

Consider a mechanical system of n bodies described by m natural coordinates. We assume that the system dynamics will be expressed into a moving reference frame $\Re_v$. The configuration of the moving reference $\Re_v$ with respect to (w.r.t) the inertial reference $\Re_o$ is given by the position vector $\bar{r}_{ov} = [w, y, z]^T$ and orientation vector $\bar{q}_v = [\varphi, \theta, \psi]^T$. The position of a body i w.r.t the inertial reference $\Re_o$ expressed in this latter is given by:

$$\bar{r}_{oi}^o = \bar{r}_{ov}^o + \bar{r}_{vi}^o \qquad (1)$$

where, $\bar{r}_{vi}^o = \mathcal{R}_{o\to v}\bar{r}_{vi}^v$. In this equation, $\mathcal{R}_{o\to v}$ is the rotation transformation matrix and $\bar{r}_{vi}^v$ is the position of the body i w.r.t the moving reference $\Re_v$. Its linear velocity is obtained by a direct differentiation $\bar{v}_{oi}^o = \frac{d}{dt}\bar{r}_{oi}^o$ as following:

$$\bar{v}_{oi}^o = \bar{v}_{ov}^o + \bar{\omega}_{ov}^o \times \bar{r}_{vi}^o + \dot{\bar{r}}_{vi}^o \qquad (2)$$

Again, a second differentiation yields to the body acceleration $\bar{a}_{oi}^o = \frac{d}{dt}\bar{v}_{oi}^o$ like:

$$\bar{a}_{oi}^o = \bar{a}_{ov}^o + \bar{\epsilon}_{ov}^o \times \bar{r}_{vi}^o + \bar{\omega}_{ov}^o \times (\bar{\omega}_{ov}^o \times \bar{r}_{vi}^o + 2\dot{\bar{r}}_{vi}^o) + \ddot{\bar{r}}_{vi}^o \quad (3)$$

In the other hand, the body angular velocity is defined by:

1

$$\bar{\omega}_{oi}^o = \bar{\omega}_{ov}^o + \mathcal{R}_{o \to v}\bar{\omega}_{vi}^v \qquad (4)$$

and its angular acceleration as:

$$\bar{\epsilon}_{oi}^o = \frac{d}{dt}\bar{\omega}_{oi}^o = \bar{\epsilon}_{ov}^o + \bar{\omega}_{ov}^o \times \mathcal{R}_{o \to v}\bar{\omega}_{vi}^v + \dot{\bar{\omega}}_{vi}^o \qquad (5)$$

Since we aim to express the vehicle dynamics in $\Re_v$ reference, one can characterize the kinematics of the moving reference by its velocities expressed in the moving reference frame $[\bar{v}_{ov}^v, \bar{\omega}_{ov}^v]$ instead of the inertial basis $[\bar{v}_{ov}^o, \bar{\omega}_{ov}^o]$. For this, let writing:

$$\bar{v}_{ov}^o = \mathcal{R}_{o \to v}\bar{v}_{ov}^v \to \bar{a}_{ov}^v = \bar{\omega}_{ov}^v \times \bar{v}_{ov}^v + \dot{\bar{v}}_{ov}^v \qquad (6)$$

and:

$$\bar{\omega}_{ov}^o = \mathcal{R}_{o \to v}\bar{\omega}_{ov}^v \to \bar{\epsilon}_{ov}^v = \dot{\bar{\omega}}_{ov}^v \qquad (7)$$

By projecting equations (3) and (5) into the $\Re_v$ reference and by using (6) and (7) we can define the linear and angular acceleration of the each body i as following:

$$\bar{a}_{oi}^v = \dot{\bar{v}}_{ov}^v + \dot{\bar{\omega}}_{ov}^v \times \bar{r}_{vi}^v + \ddot{\bar{r}}_{vi}^v + \bar{\omega}_{ov}^v \times (\dot{\bar{r}}_{vi}^v + \bar{v}_{oi}^v) \qquad (8)$$

and:

$$\bar{\epsilon}_{oi}^v = \dot{\bar{\omega}}_{ov}^v + \bar{\omega}_{ov}^v \times \bar{\omega}_{vi}^v + \dot{\bar{\omega}}_{vi}^v \qquad (9)$$

Equation (8) and (9) are useful to deduce Jacobian matrices, as it will be demonstrated in the next sub-section.

B. Jourdain's principle

Suppose that the kinematics of the mechanical system is characterized by k generalized speeds $\bar{\vartheta} = [\bar{v}_{ov}^v, \bar{\omega}_{ov}^v, \dot{\bar{q}}_s]$ corresponding to m natural coordinates $\bar{\chi} = [\bar{r}_{ov}^o, \bar{q}_v, \bar{q}_s]$. Consequently, the linear and angular velocity of each body can be expressed as followings (henceforth, all vectors are expressed in $\Re_v$ reference unless otherwise stated):

$$\bar{v}_{oi} = \bar{v}_{oi}(\bar{\vartheta}, \bar{\chi}) \qquad \bar{\omega}_{oi} = \bar{\omega}_{oi}(\bar{\vartheta}, \bar{\chi})$$

Then, the linear acceleration is expressed by:

$$\bar{a}_{oi} = \frac{\partial \bar{v}_{oi}(\bar{\vartheta}, \bar{\chi})}{\partial \bar{\vartheta}}\dot{\bar{\vartheta}} + \frac{\partial \bar{v}_{oi}(\bar{\vartheta}, \bar{\chi})}{\partial \chi}\dot{\bar{\chi}}$$

and the same equation for $\bar{\omega}_{oi}$ can be deduced. The partial derivatives are called Jacobian matrices. Let the position and the angular rates of the i-th body w.r.t the $\Re_v$ to be:

$$\dot{\bar{r}}_{vi}^v = \bar{\alpha}_i\dot{\bar{q}}_s \qquad \bar{\omega}_{vi}^v = \bar{\beta}_i\dot{\bar{q}}_s \qquad (10)$$

Now, it is easy to construct the i-th Jacobian matrix as shown in the following two equations:

$$\frac{\partial \bar{v}_{oi}}{\partial \bar{\vartheta}} = \begin{bmatrix} \mathcal{I}_3 & -\tilde{r}_{vi} & \bar{\alpha}_i \end{bmatrix} \qquad (11)$$

$$\frac{\partial \bar{\omega}_{oi}}{\partial \bar{\vartheta}} = \begin{bmatrix} 0_3 & \mathcal{I}_3 & \bar{\beta}_i \end{bmatrix}$$

Now, all kinematics quantities are defined, then we introduce the principle of the virtual power called also Jourdain's principle. Jourdain's principle states that the virtual power done by motion compatible constraint efforts $\bar{F}_{i,c}/\bar{M}_{i,c}$ is null [17], then:

$$\sum_n \left\{ \delta\bar{v}_{oi}^T \bar{F}_{i,c} + \delta\bar{\omega}_{oi}^T \bar{M}_{i,c} \right\} = 0 \qquad (12)$$

Next, the Newton/Euler dynamics principle allows to express the constraint efforts w.r.t to the external applied efforts $\bar{F}_{i,e}/\bar{M}_{i,e}$ as:

$$m_i\bar{a}_{oi} = \bar{F}_{i,c} + \bar{F}_{i,e}$$
$$\mathcal{I}_i\bar{\epsilon}_{oi} + \bar{\omega}_{oi} \times \mathcal{I}_i\bar{\omega}_{oi} = \bar{M}_{i,c} + \bar{M}_{i,e}$$

By replacing this equations into (12), and after some algebraic manipulations, we deduce the dynamics motion expression as following:

$$\mathcal{M}\dot{\bar{\vartheta}} = \bar{Q} \qquad (13)$$

where $\mathcal{M}$ is the mass matrix obtained by the direct calculation of Jacobian matrices:

$$\mathcal{M} = \sum \left\{ m_i \frac{\partial \bar{v}_{oi}^T}{\partial \bar{\vartheta}} \frac{\partial \bar{v}_{oi}}{\partial \bar{\vartheta}} + \frac{\partial \bar{\omega}_{oi}^T}{\partial \bar{\vartheta}} \mathcal{I}_i \frac{\partial \bar{\omega}_{oi}}{\partial \bar{\vartheta}} \right\} \qquad (14)$$

and the vector $\bar{Q}$ of the generalized efforts can be split into two parts $\bar{Q} = \bar{Q}_e - \bar{Q}_r$, $\bar{Q}_e$ is the generalized efforts vector associated to the external applied efforts and $\bar{Q}_r$ is the generalized efforts vector associated to the acceleration residual terms in acceleration equations (8,9). Hence:

$$\bar{Q}_r = \sum m_i \frac{\partial \bar{v}_{oi}^T}{\partial \bar{\vartheta}} \left(\frac{\partial \bar{v}_{oi}}{\partial \bar{\chi}}\dot{\bar{\chi}} \right) \qquad (15)$$

$$+ \frac{\partial \bar{\omega}_{oi}^T}{\partial \bar{\vartheta}} \left(\frac{\partial \bar{\omega}_{oi}}{\partial \bar{\chi}}\dot{\bar{\chi}} + \bar{\omega}_{oi} \times \mathcal{I}_i\bar{\omega}_{oi} \right) \qquad (16)$$

$$\bar{Q}_e = \sum \left\{ \frac{\partial \bar{v}_{oi}^T}{\partial \bar{\vartheta}} \bar{F}_{i,e} + \frac{\partial \bar{\omega}_{oi}^T}{\partial \bar{\vartheta}} \bar{M}_{i,e} \right\} \qquad (17)$$

III. VEHICLE KINEMATICS

In this section, a direct application of the Jourdain's principle (section II) to the motorcycle modeling is exposed. For this, the kinematics of each body w.r.t the moving reference $\Re_v$ is demonstrated.

At first, we highlight the different motion reference frames used in the current modeling work. As shown in figure (1), the moving reference $\Re_v$ is obtained after two successive rotations w.r.t the inertial reference $\Re_O$. First, a yaw rotation ψ around $\Re_o$ z-axes is done to give an intermediate reference $\Re_\psi$. Next, $\Re_v$ is obtained by a roll rotation φ around $\Re_\psi$ x-axes. The origin v of $\Re_v$ is defined by $\bar{\chi}_v = [\bar{r}_{ov}^o, \bar{q}_v]$.

The motorcycle is considered as a set of six bodies (Fig.1). The main body G_r, the front assembly G_f including the handlebar, steering column and the higher fork part, the unsprung front assembly G_u, the rear swing arm G_s and

the two wheels G_{Rf}, G_{Rr}. The mass of the front wheel is included in the unsprung front assembly, only its rotation will be considered as an independent DOF. The rider motion is not taken into account and is considered to be rigidly attached to the main body G_r.

In order to completely define the configuration of the motorcycle, a set of 11DOF has been considered. A full characterization of motorcycle's kinematics includes :

- the longitudinal, lateral and vertical position (x, y, z) of the main body,
- the roll, pitch and yaw orientation (φ, θ, ψ) of the main body,
- the steering angle δ,
- the front suspension travel λ_f and the swing arm rotation μ,
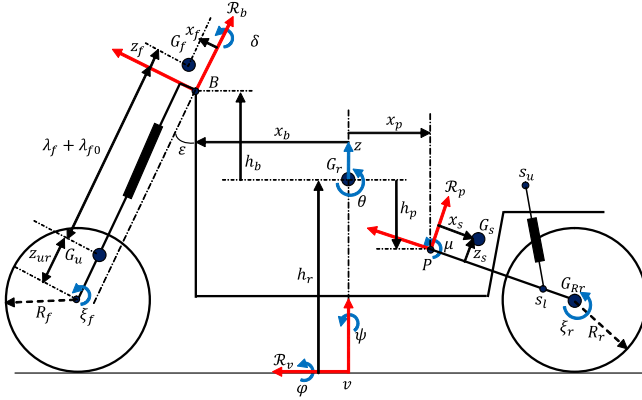- and finally, the front and rear tires spinning, ξ_f and ξ_r.

Fig. 1. The motorcycle geometrical description

In the following subsections, the kinematics of each body is presented.

A. Main body G_r

The main body G_r has 1DOF w.r.t $\Re_v$ which represents the vertical displacement z. Then, its position vector is:

$$\bar{r}_{vG_r}^v = \begin{bmatrix} 0 & 0 & z + h_r \end{bmatrix}^T \rightarrow \dot{\bar{r}}_{vG_r}^v = \dot{z}\bar{e}_z$$

From equation (10), we conclude that:

$$\bar{\alpha}_{G_r} = \begin{bmatrix} \bar{e}_z & 0_{3,6} \end{bmatrix} \qquad \bar{\beta}_{G_r} = 0_{3,7}$$

B. Front body G_f

The front body G_f is belonging to the fixed-body reference frame $\Re_b$ and has 2DOF w.r.t the main body G_r. $\Re_b$ frame is obtained by a three successive rotations namely, pitch θ about the y-axis of $\Re_v$, caster ϵ about the same axes and the steer δ about the $\Re_b$ z-axis. The position vector of the front body w.r.t $\Re_v$ is:

$$\bar{r}_{vG_f}^v = \bar{r}_{vG_r}^v + \mathcal{R}_{v \rightarrow g}(\bar{r}_{G_r B}^v + \mathcal{R}_{g \rightarrow b}\bar{r}_{BG_f}^b)$$

Knowing that vectors $\bar{r}_{BG_f}^b$ and $\bar{r}_{G_r B}^v$ are constant, we can write :

$$\dot{\bar{r}}_{vG_f}^v = \dot{\bar{r}}_{vG_r}^v + \bar{\omega}_{vg}^v \times \bar{r}_{G_r G_f}^v + \bar{\omega}_{gb}^v \times \bar{r}_{BG_f}^v$$

where $\bar{\omega}_{vg}^v = \dot{\theta}\bar{e}_y$ and $\bar{\omega}_{gb}^v = \dot{\delta}\bar{e}_\delta$. In this case, $\bar{\alpha}_{G_f} = \bar{\alpha}_{G_r} + \bar{\alpha}'_{G_f}$ where:

$$\bar{\alpha}'_{G_f} = \begin{bmatrix} 0_{3,1} & \bar{e}_y \times \bar{r}_{G_r G_f}^v & \bar{e}_\delta \times \bar{r}_{BG_f}^v & 0_{3,4} \end{bmatrix}$$

In the other hand, the angular rates is $\bar{\omega}_{vG_f}^v = \bar{\omega}_{vg}^v + \bar{\omega}_{gb}^v$, and hence:

$$\bar{\beta}_{G_f} = \begin{bmatrix} 0_{3,1} & \bar{e}_y & \bar{e}_\delta & 0_{3,4} \end{bmatrix}$$

It can be seen that the advantage of this method, is that the computation of Jacobian matrices can be obtained recursively. At each step, only the contribution of the considered body is calculated.

C. Front unsprung body G_u

The front unsprung body has 1DOF w.r.t G_f representing the suspension travel λ_f. Its position vector is:

$$\bar{r}_{vG_u}^v = \bar{r}_{vG_f}^v + \mathcal{R}_{v \rightarrow g}\mathcal{R}_{g \rightarrow b}\bar{r}_{G_f G_u}^b$$

Herein, $\bar{r}_{G_f G_u}^b = (\lambda_f + \lambda_{f0})\bar{e}_z$ where λ_{f0} is the initial fork suspension length. By differentiation of the position equation, we can write :

$$\dot{\bar{r}}_{vG_u}^v = \dot{\bar{r}}_{vG_f}^v + (\bar{\omega}_{vg}^v + \bar{\omega}_{gb}^v)\bar{r}_{G_f G_u}^v + \dot{\lambda}_f \bar{e}_\delta$$

Consequently, $\bar{\alpha}_{G_u} = \bar{\alpha}_{G_f} + \bar{\alpha}'_{G_u}$, where:

$$\bar{\alpha}'_{G_u} = \begin{bmatrix} 0_{3,1} & \bar{e}_y \times \bar{r}_{G_f G_u}^v & \bar{e}_\delta \times \bar{r}_{G_f G_u}^v & \bar{e}_\delta & 0_{3,4} \end{bmatrix}$$

Finally, since there is no rotation between the two front bodies: $\bar{\beta}_{G_u} = \bar{\beta}_{G_f}$.

D. Swing arm body G_s

The swing arm body is belonging to the fixed-body reference frame $\Re_p$ and has 2DOF w.r.t the main body G_r. $\Re_p$ frame is obtained by two successive rotations, pitch θ and swing arm pitch μ around the y-axis of $\Re_v$. Its position vector is expressed by::

$$\bar{r}_{vG_s}^v = \bar{r}_{vG_r}^v + \mathcal{R}_{v \rightarrow g}(\bar{r}_{G_r P}^g + \mathcal{R}_{g \rightarrow p}\bar{r}_{PG_s}^p)$$

and the corresponding position rate is :

$$\dot{\bar{r}}_{vG_s}^v = \dot{\bar{r}}_{vG_r}^v + \bar{\omega}_{vg}^v \times \bar{r}_{G_r G_s}^v + \bar{\omega}_{gp}^v \times \bar{r}_{PG_s}^v$$

where $\bar{\omega}_{gp}^v = \dot{\mu}\bar{e}_y$. Then, $\bar{\alpha}_{G_s} = \bar{\alpha}_{G_r} + \bar{\alpha}'_{G_s}$, where:

$$\bar{\alpha}'_{G_s} = \begin{bmatrix} 0_{3,1} & \bar{e}_y \times \bar{r}_{G_r G_s}^v & 0_{3,2} & \bar{e}_y \times \bar{r}_{PG_s}^v & 0_{3,2} \end{bmatrix}$$

In the other hand, the angular rates is $\bar{\omega}_{vG_s}^v = \bar{\omega}_{vg}^v + \bar{\omega}_{gp}^v$, and hence:

$$\bar{\beta}_{G_s} = \begin{bmatrix} 0_{3,1} & \bar{e}_y & 0_{3,2} & \bar{e}_y & 0_{3,2} \end{bmatrix}$$

E. Front and rear wheels body

The two wheels has one independent generalized velocity w.r.t to their respective previous body (it means G_u for the front wheel and G_s for the rear one), which consist simply on tire rotation around wheel spin axes $\bar{e}_{yR}$. It follows that $\bar{\omega}_{vG_{Rr}}^v = \bar{\omega}_{vG_s}^v + \dot{\xi}_r \bar{e}_y$ this leads to :

$$\bar{\beta}_{G_{Rr}} = \bar{\beta}_{G_s} + \begin{bmatrix} 0_{3,6} & \bar{e}_{yR,r} \end{bmatrix}$$

The same reasoning can be followed for the front wheel:

$$\bar{\beta}_{G_{Rf}} = \bar{\beta}_{G_u} + \begin{bmatrix} 0_{3,5} & \bar{e}_{yR,f} & 0_{3,5} \end{bmatrix}$$

At now, the whole motorcycle kinematics is defined. Matrix mass $\mathcal{M}$ and the first part of generalized efforts $\bar{Q}_r$ (equations 14, 15) are fully defined. In the next section, we look for the development of the generalized efforts vector associated to external applied efforts.

IV. VECTOR OF GENERALIZED EFFORTS

To express generalized efforts vector associated to external applied efforts, one can make use of equation (16). First, it is essential two establish the set of all efforts exerted on each body. This method may be a tedious one especially one the number of applied efforts is much high. So, it is most convenient to find the virtual power done by each effort and find the associated generalized effort.

As an example, consider a body i under a force vector $\bar{F}_e$ applied at a point $j \in i$ and a moment vector $\bar{F}_e$ around an axes belonging to this body. The resulting virtual power can be expressed as:

$$\delta P = \delta \bar{v}_{oj}^T \bar{F}_e + \delta \bar{\omega}_i^T \bar{M}_e = \delta \bar{\vartheta}^T \bar{Q}$$

By replacing $\delta \bar{v}_{oj}$ and $\delta \bar{\omega}_i$ by their Jacobians, we obtain the following equation:

$$\bar{Q} = \left(\frac{\partial \bar{v}_{oj}}{\partial \bar{\vartheta}} \right)^T \bar{F}_e + \left(\frac{\partial \bar{\omega}_{oi}}{\partial \bar{\vartheta}} \right)^T \bar{M}_e \quad (18)$$

The only difference w.r.t equation (16) is the use of the Jacobian related to the force point application velocity ($\bar{v}_{oj}$) instead of that of the body center of mass $\bar{v}_{oi}$. In the following, we use equation (17) for each external force/moment exerted on the motorcycle vehicle.

A. Front and rear suspension

The front suspension force $\bar{F}_{sf}$ is applied between the the front body G_f and the front unsprung body G_u. The contribution of this force in the total generalized effort is given by using equation (17), like following:

$$\bar{Q}_{sf} = \frac{\partial}{\partial \bar{\vartheta}} \left(\bar{v}_{oG_f} - \bar{v}_{oG_u} \right)^T \bar{F}_{sf} \quad (19)$$

By replacing each Jacobian by its expression and after some algebraic manipulations we get:

$$\bar{Q}_{sf} = \begin{bmatrix} 0_{3,3} \\ (\lambda_f + \lambda_{f0}) \tilde{e}_\delta^T \\ -\bar{\alpha}_{G_u}'^T \end{bmatrix} \bar{F}_{sf} \quad (20)$$
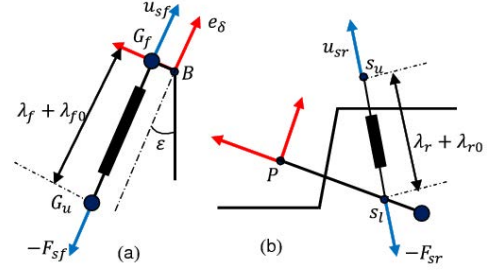

Fig. 2. Suspensions configuration, (a) front, (b) rear

Knowing that the force suspension vector can be written as $\bar{F}_{sf} = F_{sf} \bar{u}_{sf}$, where, $\bar{u}_{sf} = \bar{e}_\delta$ is the unit vector, then:

$$\bar{Q}_{sf} = -F_{sf} \frac{\partial \dot{\lambda}_f}{\partial \vartheta} \quad (21)$$

Doing the same for the rear suspension, we conclude :

$$\bar{Q}_{sr} = \begin{bmatrix} 0_{8,1} \\ (\bar{e}_y \times \bar{r}_{Ps_l}^v)^T \bar{u}_{sr} \\ 0_{2,1} \end{bmatrix} F_{sr} \quad (22)$$

Equation (21) shows that the rear suspension force can be viewed as a stiffness torque about the swing arm pivot P.

B. Gravity forces

Each body is subject to the gravity force created by its own mass. The contribution of this force in $\bar{Q}$ is directly given by:

$$Q_g = \sum_n m_i \left(\frac{\partial v_{oi}}{\partial \vartheta} \right)^T \bar{g} \quad (23)$$

C. Rider action

The rider's torque τ_r exerted on the motorcycle's handlebar is considered to be the most important input for the vehicle driving. This torque acts around the steering axis and hence jointly on the main body G_r and front body G_f. The contribution of this torque in $\bar{Q}$ is defined by:

$$\bar{Q}_{\tau_r} = \frac{\partial}{\partial \bar{\vartheta}} \left(\bar{\omega}_{G_f} - \bar{\omega}_{G_r} \right)^T \tau_r \bar{e}_\delta = \begin{bmatrix} 0_{8,1} \\ \tau_r \\ 0_{4,1} \end{bmatrix} \quad (24)$$

D. Propulsion and braking torques

For the front wheel, braking force acts between the wheel rim and unsprung assembly G_u which contains braking disk. For the rear wheel, the braking torque is exerted between the rear wheel body and the swing arm assembly. In addition to the propulsive torque, the contribution of these torques is expressed by:

$$\bar{Q}_{DB} = \frac{\partial}{\partial \bar{\vartheta}} \left(\bar{\omega}_{G_{Rf}} - \bar{\omega}_{G_u} \right)^T \tau_{bf} \bar{e}_{yR,f} +$$

$$\frac{\partial}{\partial \bar{\vartheta}} \left(\bar{\omega}_{G_{Rr}} - \bar{\omega}_{G_s} \right)^T (\tau_{br} + \tau_D) \bar{e}_{yR,r}$$

After simplification we get:

$$\bar{Q}_{DB} = \begin{bmatrix} 0_{11,1} \\ \tau_{bf} \\ \tau_{br} + \tau_D \end{bmatrix} \qquad (25)$$

In the next section, we expose the tire-road interaction to fully defining the vector of the generalized efforts Q.

V. TIRES KINEMATICS AND DYNAMICS

A. Tires kinematics

To describe the tire motion, a new reference frame $\Re_T(C, \bar{e}_{xT}, \bar{e}_{yT}, \bar{e}_{zT})$ is introduced at the contact point C. $\bar{e}_{zT}$ is the normal vector to the road surface. Vector $\bar{e}_{xT}$ is obtained as $\bar{e}_{xT} = \bar{e}_{yR} \times \bar{e}_{zT}$ and $\bar{e}_{yT}$ completes the reference axis.
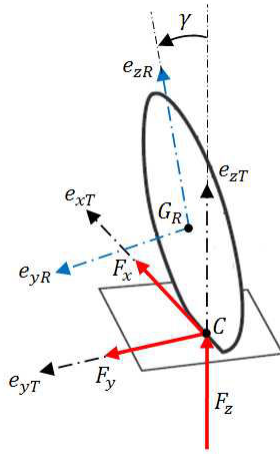

Fig. 3.   Tire reference frame and camber

As shown in figure 3, the camber angle is an important variable in the motorcycle safety studies. This variable is expressed by the following equation:

$$\sin\gamma = \bar{e}_{yR}^T \bar{e}_{zT} \qquad (26)$$

The position of the contact point w.r.t the inertial frame is expressed by:

$$\bar{r}_{oC}^o = \bar{r}_{oG_R}^o - R_{rm}\bar{e}_{zR}^o - R_{cr}\bar{e}_{zT}^o \qquad (27)$$

Where, R_{rm} is the wheel rim radius, R_{cr} is the dynamic tire crown radius and $\bar{e}_{zR} = \bar{e}_{xT} \times \bar{e}_{yR}$. By differentiating this equation, one obtains the contact point velocity vector, as:

$$\bar{v}_{oC}^o = \bar{v}_{oG_R}^o - R_{rm}\dot{\bar{e}}_{zR}^o - \dot{R}_{cr}\bar{e}_{zT}^o \qquad (28)$$

Knowing that the contact point C is belonging always to the road surface (Fig.4) then, R_{cr} et $\dot{R}_{cr}$ are obtained by performing a product scalar of equations (26,27) by $\bar{e}_{zT}^o$. Once the tire's kinematics is established, the equivalent tire forces/moments wrench at the center of each wheel is given by the following expressions:

$$\bar{F}_T = F_x\bar{e}_{xT} + F_y\bar{e}_{yT} + F_z\bar{e}_{zT}$$
$$\bar{M}_T = M_x\bar{e}_{xT} + M_y\bar{e}_{yT} + M_z\bar{e}_{zT} + \bar{F}_T \times \bar{r}_{CG_R}$$

where $F_x = F_x(\kappa)$ is the longitudinal force, $F_y = F_y(\zeta, \gamma)$ is the lateral force and $F_z = F_z(\delta_z)$ is the vertical force. M_x is the tire torque about $\bar{e}_{xT}$, M_y is the rolling resistance torque and $M_z = M_z(\zeta, \gamma)$ is the alignment torque. This forces and torques are obtained from the longitudinal slip κ, the lateral slip ζ and δ_z is the vertical tire deformation.
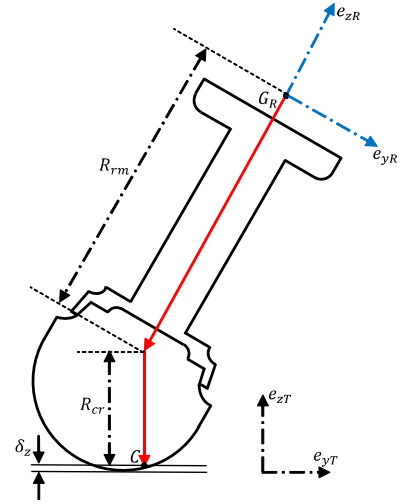

Fig. 4.   Wheel and tire geometry: rim and crown

Applying equation(17), we can deduce the contribution of tires efforts as expressed in the following equation:

$$\bar{Q}_T = \sum_j \left(\frac{\partial \bar{v}_{oG_{Rj}}}{\partial \bar{\vartheta}}\right)^T \bar{F}_T + \left(\frac{\partial \bar{\omega}_{oG_{Rj}}}{\partial \bar{\vartheta}}\right)^T \bar{M}_T \qquad (29)$$

With $\bar{Q}_T$, the vector of the generalized efforts $\bar{Q}_e$ is fully defined, and:

$$\bar{Q}_e = \bar{Q}_{sf} + \bar{Q}_{sr} + \bar{Q}_g + \bar{Q}_{\tau_r} + \bar{Q}_{DB} + \bar{Q}_T \qquad (30)$$

VI. SIMULATION

To test the behavior of the developed motorcycle model, a linear PID controller is synthesized for the longitudinal speed regulation. This controller acts like an acceleration/braking torque directly applied on the rear wheel. The control torque is saturated by the maximum torque admissible by the engine at the rear tire axis. For this test, we drive the motorcycle from an initial speed of 10 km/h to a desired velocity of 72 km/h. Figure (5) shows the control driving torque as computed by the PID controller and the longitudinal speed profile. Figure (6) describes the resulting longitudinal acceleration and the front and rear suspension travels. Finally, figure (7) represents the vertical force F_z applied at the contact point of each tire. the two hiccups in the speed vs time curve (and in the other figures) are due to the gear

changes introduced by a simple logic switching (transmission model is not described in this paper).
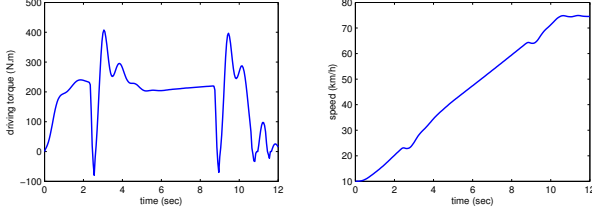

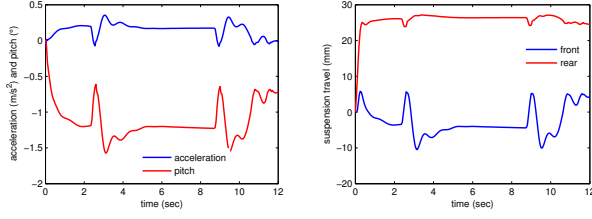Fig. 5.   Driving torque and the longitudinal speed

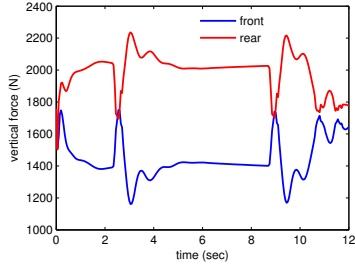Fig. 6.   Longitudinal acceleration and suspensions travel

Fig. 7.   Vertical force F_z

Although we have no experimental data, these simulations seems to be very correct in comparison with a previously published work in literature. The lateral dynamics simulation is very hard to carried out. Indeed, the motorcycle vehicle is highly unstable and a suitable controller must be synthesized.

VII. CONCLUSION

Based on the virtual power principle, called also Jourdain's principle, a motorcycle dynamic model is proposed. The development method is chosen for its simplicity and its fine analytical derivation with an acceptable calculation burden. A first test is presented to show the ability of the proposed model to simulate the longitudinal/vertical motion.Future works will focus on the lateral mode simulation by implementing a suitable controller especially for trajectory tracking like cornering, slalom and lane changes.

VIII. ACKNOWLEDGMENT

IX. APPENDIX

- $\bar{e}$ variables with over bar are vectors.
- $\mathcal{M}$ calligraphic variables are matrices.
- $\bar{e}_x = [1,0,0]$, $\bar{e}_y = [0,1,0]$, $\bar{e}_z = [0,0,1]$,
- for an arbitrary rotation ϕ, we have:
  - $\mathcal{R}_{\phi,x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$
  - $\mathcal{R}_{\phi,y} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$
  - $\mathcal{R}_{\phi,z} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- $\mathcal{R}_{v\to g} = \mathcal{R}_{\theta,y}$,
- $\mathcal{R}_{g\to b} = \mathcal{R}_{\epsilon,y}\mathcal{R}_{\delta,z}$,
- $\bar{e}_\delta = \mathcal{R}_{v\to g}\mathcal{R}_{g\to b}e_z$,
- $\bar{e}^v_{yR,f} = \mathcal{R}_{v\to g}\mathcal{R}_{g\to b}\bar{e}_y$,
- if $\bar{x} = [x,y,z]^T$, $\tilde{x} = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}$

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