Kinematic Analysis of Lower Mobility Cooperative Arms by Screw Theory

Philip Long, Stéphane Caro, Wisama Khalil

▶ **To cite this version:**

Philip Long, Stéphane Caro, Wisama Khalil. Kinematic Analysis of Lower Mobility Cooperative Arms by Screw Theory. 9th International Conference on Informatics in Control Automation and Robotics, Jul 2012, Rome, Italy. pp.208-285. hal-00786897

HAL Id: hal-00786897
https://hal.science/hal-00786897

Submitted on 23 Sep 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

KINEMATIC ANALYSIS OF LOWER MOBILITY COOPERATIVE ARMS BY SCREW THEORY

Philip Long[1], Stéphane Caro[1] and Wisama Khalil[1]

[1]*Institut de Recherche en Communications et Cybernétique de Nantes, UMR CNRS n° 6597 1 rue de la Noë, École Centrale de Nantes, 44321, Nantes, France.*
{*firstname.secondname*}*@irccyn.ec-nantes.fr*

Keywords: Cooperative Manipulators:Humanoid Robots: Screw theory

Abstract: This paper studies the kinematic modeling and analysis of two cooperative manipulators. Cooperative manipulators are defined as multi-arm systems working together on a common task. The task is defined as the transportation of an object in space. In this paper the cooperative system is the dual arm of the humanoid Nao robot, where the serial structure of each arm has five degrees of freedom. The mobility of the closed loop system is analyzed and the nature of the possible motion explored. The serial singular configurations of the system are considered. Furthermore the parallel singularities due to the system constraints and chosen actuation scheme are analyzed.

1 INTRODUCTION

The capability of dual independent arms when processing parts reduces the need for custom fixtures and permits the use of a simpler end effector. The system can then execute sophisticated tasks that may be difficult for a single arm system. For example, rather than using a large serial robot a cooperative system distributes a heavy load among several smaller robots. Similarly if the object is of an unwieldy, non-rigid or awkward composition, the single arm robot may struggle to manipulate it.

By using a cooperative system of two or more manipulators, both the location and the internal forces of the object can be controlled. The two principal approaches that avail of force sensors on the robot are: hybrid position/force control (Uchiyama and Dauchez, 1988) and impedance control (Bonitz and Hsia, 1996), (Sadati and Ghaffarkhah, 2008), (Caccavale et al., 2008).

Another approach is to formulate kinematic relations that create a task space describing the multi-arm system while grasping an object. The main methods are known as *Symmetric formulation* (Uchiyama and Dauchez, 1988) and *Task orientated formulation* (Chiacchio et al., 1996; Caccavale et al., 2000). Both create a cooperative task space of velocity variables, describing the object motion in space and the relative motion between the end effectors (i.e forces applied on object). On the other hand, the system can also be viewed as a redundantly actuated parallel manipulator. In this case kinematic constraint equations are derived that establish a relationship between the chosen independent and dependent joint variables (Yeo et al., 1999; Liu et al., 1999; Cheng et al., 2003; Özkan and Özgören, 2001). The dependent joint variables adopt values that ensure loop closure at each instant.

Most of the preceding work has been carried out with dual arm systems, where both arms are either of planar or 6-DOF spatial composition. Thus away from singularities, the grasped object has a mobility of 3 or 6 respectively. On the other hand the study of lower mobility cooperative manipulators has been limited. In (Yeo et al., 1999) the cooperation between a 5-DOF and 4-DOF robot is used in conjunction with a passive joint in order to execute a 4-DOF position/force task. In (Zielinski and Szynkiewicz, 1996) admissible path planning for two 5-DOF robots is explored. In (Bicchi et al., 1995) a generalized method based on the Jacobian matrix of each arm, and their constraint relations with the object, is formulated. Analysis of these matrices permits the calculation of the mobility, possible first order differential motions and manipulability velocity ellipsoids of general multiple limb robots. The scheme is validated on two cooperating 4-DOF SCARA robots.

Lower mobility systems suffer from three types of singularities, limb (serial) singularities, actuation and constraint (parallel) singularities (Amine et al., 2011). For a closed chain system, certain joints must be des-

ignated actuated and others passive. In (Liu et al., 1999) the presence of parallel singularities of a cooperative system with passive joints is explored. The issue of a valid selection of actuators, for cooperative manipulators is addressed in (Özkan and Özgören, 2001). In both cases an analysis of the Jacobian matrix is carried out. Conversely screw theory can be used to locate and understand parallel singularities in closed chain mechanisms (Zlatanov et al., 2002).

In this paper the cooperative system, defined by the two arms of Aldebaran NAO T14 humanoid robot and a grasped object, is examined. The system has been modeled as a closed chain mechanism (Sections 2 and 3). The originality of this paper lies in the use of screw theory techniques to analyze the system's mobility, singularities and motion type. The benefit of this approach is that special configurations such as the loss of stiffness, loss of DOF etc., can be determined without the complex derivation of the Jacobian matrices (or their inverses) (Sections 4 and 5).

2 DESCRIPTION OF THE SYSTEM

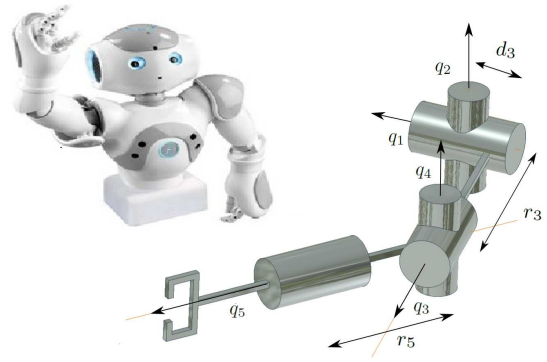

Figure 1: NAO T14 (Courtesy of Aldeberan-Robotics) with schematic representation of its arm.

The NAO T14 robot illustrated in Fig.1 is the experimental platform used. The Modified Denavit-Hartenberg (MDH) notation (Khalil and Kleinfinger, 1986) is used to describe the system as given in Table 1. The right arm consists of joints 1-5 and the left arm consists of joints 6-10. Once the object is grasped a closed loop is formed from the two arms, the object and the common robot torso. As illustrated in Fig. 2, link 5 of the closed chain is composed of link 5 , link 10 of the open chain and the object. Frame 10 is thus fixed on link 5. We introduce frame 11, that is equivalent to frame 10, but its antecedent is frame 5. The system has in this case only nine bodies. Hence

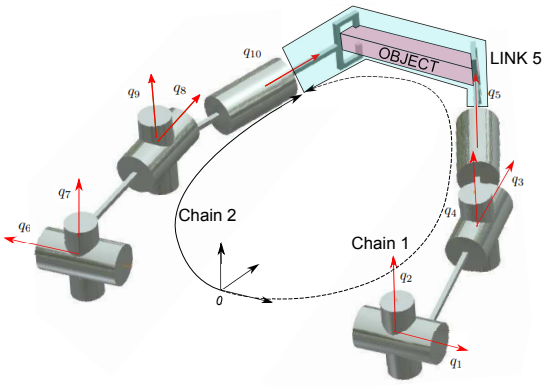

Figure 2: Closed Loop Formulation.

joint 10 is denoted as the cut joint. The table parameters are explained as:

- j is the joint number, $a(j)$ is its antecedent joint
- σ_j is the joint type: revolute (σ_j=0), prismatic (σ_j=1) or fixed (σ_j=2)
- $\mathbf{z}_j$ is the jth joint axis
- $\mathbf{u}_j$ is the common normal between $\mathbf{z}_{a(j)}$ and $\mathbf{z}_j$
- $\mathbf{x}_j$ is perpendicular to $\mathbf{z}_j$ and one of the succeeding axes, $\mathbf{z}_k$ such that $a(k) = j$
- γ_j is the angle between $\mathbf{x}_{a(j)}$ and $\mathbf{u}_j$ about $\mathbf{z}_{a(j)}$
- b_j is the distance between $\mathbf{x}_{a(j)}$ and $\mathbf{u}_j$ along $\mathbf{z}_{a(j)}$
- d_j is the distance between $\mathbf{z}_{a(j)}$ and $\mathbf{z}_j$ along $\mathbf{u}_j$
- α_j is the angle between $\mathbf{z}_{a(j)}$ and $\mathbf{z}_j$ about $\mathbf{u}_j$
- θ_j is the angle between $\mathbf{u}_j$ and $\mathbf{x}_j$ about $\mathbf{z}_j$
- r_j is the distance between $\mathbf{u}_j$ and $\mathbf{x}_j$ along $\mathbf{z}_j$
- $\gamma_j = b_j = 0$ when $\mathbf{x}_{a(j)}$ is perpendicular to $\mathbf{z}_j$

The parameters of frame 11 in Table 1 are defined by the grasp. Therefore, when the robot grasps the object, the transformation matrices of each serial arm are calculated. The parameters defining frame 11 with respect to frame 5 are calculated such that they yield ${}^{5}\mathbf{T_{11}}$.

$$ {}^{5}\mathbf{T}_{11} = \left({}^{0}\mathbf{T}_{5}\right)^{-1} {}^{0}\mathbf{T}_{10} \qquad (1) $$

${}^{i}\mathbf{T_j}$ is the 4×4 transformation matrix from frame i to frame j. The six parameters defining frame 11 can be found by solving the following equation:

$$ \begin{aligned} {}^{5}\mathbf{T}_{11} \quad = \quad & rot_z(\gamma_{11}) \cdot trans_z(b_{11}) \cdot rot_x(\alpha_{11}) \cdot \\ & trans_x(d_{11}) \cdot rot_z(\theta_{11}) \cdot trans_z(r_{11}) \end{aligned} \qquad (2) $$

where $rot_i(\theta)$ indicates a rotation of θ radians about the ith axis and $trans_i(l)$ a translation of l meters along the ith axis.

Table 1: MDH Parameters of NAO robot of the closed loop chain.

j	$a(j)$	$\sigma(j)$	$\gamma(j)$	b	d	α	θ	r
1	0	0	0	b_1	0	$-\frac{\pi}{2}$	θ_1	-r_1
2	1	0	0	0	0	$\frac{\pi}{2}$	θ_2	0
3	2	0	0	0	-d_3	$\frac{\pi}{2}$	θ_3	r_3
4	3	0	0	0	0	$-\frac{\pi}{2}$	θ_4	0
5	4	0	0	0	0	$\frac{\pi}{2}$	θ_5	r_5
6	0	0	0	b_1	0	$-\frac{\pi}{2}$	θ_6	r_1
7	6	0	0	0	0	$\frac{\pi}{2}$	θ_7	0
8	7	0	0	0	d_3	$\frac{\pi}{2}$	θ_8	r_3
9	8	0	0	0	0	$-\frac{\pi}{2}$	θ_9	0
10	9	0	0	0	0	$\frac{\pi}{2}$	θ_{10}	r_5
11	5	2	γ_{11}	b_{11}	d_{11}	α_{11}	θ_{11}	r_{11}

3 KINEMATIC CONSTRAINT EQUATIONS

The location, velocity and acceleration of the frame at the cut joint must be equivalent when calculated via either chain. This ensures a constant object grasp throughout the trajectory. In the closed loop formulation, some joints may be actuated and some others could be passive. Let $\mathbf{q_a}$ contain the joint variables that are actuated, $\mathbf{q_p}$ contain the passive joint variables and $\mathbf{q_c}$ contain the passive joint variable where the chain is considered to be cut. The passive and cut joint variables can be obtained in terms of the active joint variables using the following geometric constraint equations:

$$ {}^0\mathbf{T}_1\,{}^1\mathbf{T}_2\,{}^2\mathbf{T}_3\,{}^3\mathbf{T}_4\,{}^4\mathbf{T}_5\,{}^5\mathbf{T}_{11} = {}^0\mathbf{T}_6\,{}^6\mathbf{T}_7\,{}^7\mathbf{T}_8\,{}^8\mathbf{T}_9\,{}^9\mathbf{T}_{10} \quad (3) $$

The kinematic constraints are given by:

$$ \begin{bmatrix} \omega_{11} \\ \mathbf{v}_{11} \end{bmatrix} = {}^0\mathbf{J}_{11} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} = {}^0\mathbf{J}_{11}\,\dot{\mathbf{q}}_r \quad (4) $$

$$ \begin{bmatrix} \omega_{10} \\ \mathbf{v}_{10} \end{bmatrix} = {}^0\mathbf{J}_{10} \begin{bmatrix} \dot{q}_6 \\ \dot{q}_7 \\ \dot{q}_8 \\ \dot{q}_9 \\ \dot{q}_{10} \end{bmatrix} = {}^0\mathbf{J}_{10}\,\dot{\mathbf{q}}_l \quad (5) $$

where $\dot{\mathbf{q}}_r$ and $\dot{\mathbf{q}}_l$ contain the joint velocities of the right arm and the left arm, respectively. ${}^\mathbf{0}\mathbf{v_j}$ is the linear velocity and ${}^0\omega_j$ the angular velocity of frame j in frame 0, ${}^0\mathbf{J}_j$ is the kinematic Jacobian matrix of frame j w.r.t. frame 0. As frames 10 and 11 are coincident:

$$ \begin{bmatrix} \omega_{11} \\ \mathbf{v}_{11} \end{bmatrix} = \begin{bmatrix} \omega_{10} \\ \mathbf{v}_{10} \end{bmatrix} \quad (6) $$

by substitution from (4) and (5)

$$ \begin{bmatrix} {}^0\mathbf{J}_{11} & -{}^0\mathbf{J}_{10} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_r \\ \dot{\mathbf{q}}_l \end{bmatrix} = 0 \quad (7) $$

By rearranging the rows and columns of (7), a relationship is obtained between the passive joint velocities and the actuated joint velocities

$$ \begin{bmatrix} \mathbf{G}_a & \mathbf{G}_p & \mathbf{0} \\ \mathbf{G}_{ac} & \mathbf{G}_{pc} & \mathbf{G}_c \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_a \\ \dot{\mathbf{q}}_p \\ \dot{\mathbf{q}}_c \end{bmatrix} = 0 \quad (8) $$

that can be rewritten as

$$ \mathbf{J}_c\,\dot{\mathbf{q}} = 0 \quad (9) $$

From the first row of (8), we obtain:

$$ \dot{\mathbf{q}}_p = \mathbf{G}\,\dot{\mathbf{q}}_a \quad (10) $$

where

$$ \mathbf{G} = -\mathbf{G}_p^{-1}\,\mathbf{G}_a \quad (11) $$

From the second row of (8), we obtain:

$$ \dot{\mathbf{q}}_c = -\mathbf{G}_c^{-1}\,(\mathbf{G}_{ac} + \mathbf{G}_{pc}\,\mathbf{G})\,\dot{\mathbf{q}}_a \quad (12) $$

Upon differentiation of (8) with respect to time the acceleration constraints equation is expressed as:

$$ \begin{bmatrix} \mathbf{G}_a & \mathbf{G}_p & \mathbf{0} \\ \mathbf{G}_{ac} & \mathbf{G}_{pc} & \mathbf{G}_c \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_a \\ \ddot{\mathbf{q}}_p \\ \ddot{\mathbf{q}}_c \end{bmatrix} + \dot{\mathbf{J}}_c\,\dot{\mathbf{q}} = 0 \quad (13) $$

4 MOBILITY ANALYSIS

The DOF of the system, n is equal to the number of independent infinitesimal translations and orientations in the *task space*. In this section the DOF of the object when grasped firmly by the two arms is examined by various methods. Each serial arm has 5-DOF however once the object is firmly grasped by each arm, a closed chain is formed and the object DOF becomes four.

4.1 Chebychev-Grübler-Kutzbach

The CGK formula is used to determine the mechanism DOF from the number of joints, loops and constraint type. CGK is very easy to use but gives incorrect results for a number of mechanisms. l is the number of joints, c_j is the independent kinematic constraint equations for loop j. b is the number of independent loops j:

$$n = l - \sum_{j=1}^{b} c_j \tag{14}$$

$$n = 10 - 6 = 4 \tag{15}$$

4.2 Moroskine's method

The mobility of a parallel mechanism can be calculated exactly by using closed loop kinematic constraint equations.

$$n = l - c = l - rank(\mathbf{J}_c) \tag{16}$$

The constraint Jacobian matrix is defined from (9). The primary drawback is that rank($\mathbf{J}_c$) is difficult to obtain symbolically, therefore it is obtained numerically for random configurations.

$$\mathbf{J}_c = \begin{bmatrix} {}^0\mathbf{J}_{11} & -{}^0\mathbf{J}_{10} \end{bmatrix} \tag{17}$$

The DOF can be found as the number of independent joints before loop closure minus those that lose their independence after the loop is closed. the numerical calculation gives rank($\mathbf{J}_c$) = 6

$$n = 10 - 6 = 4 \tag{18}$$

4.3 Gogu's Method

To overcome the limitations of the CGK and Moroskine's method, Gogu (Gogu, 2007) proposed a scheme that determines the correct mobility yet does not require the costly calculation of the kinematic constraint equations. Instead the dimension of the operational space of each serial manipulator i, denoted as $dim(\mathbf{E}_i)$, is employed. The mobility is given by[1]:

$$l - (dim(\mathbf{E}_1) + dim(\mathbf{E}_2)) + dim(\mathbf{E}_1 \bigcap \mathbf{E}_2) \tag{19}$$

[1] v_j and ω_j stand for an instantaneous translation along or rotation around an axis j respectively.

In order to find $dim(\mathbf{E}_1 \bigcap \mathbf{E}_2)$, the operational spaces that can be generated by each serial arm are examined. Five directions are chosen from both $\mathbf{E}_r$ and $\mathbf{E}_l$ in order to minimize the intersection.

$$\mathbf{E}_r = \left.\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ v_x \\ v_y \\ v_z \end{bmatrix}\right\} \xrightarrow{5\ DOF} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ v_x \\ v_y \end{bmatrix} \tag{20}$$

$$\mathbf{E}_l = \left.\begin{bmatrix} \omega_x \\ \omega_z \\ v_x \\ v_y \\ v_z \end{bmatrix}\right\} \xrightarrow{5\ DOF} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ v_x \\ v_z \end{bmatrix} \tag{21}$$

$$\Rightarrow (\mathbf{E}_r \bigcap \mathbf{E}_l) = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ v_x \end{bmatrix} \tag{22}$$

$$dim(\mathbf{E}_r \bigcap \mathbf{E}_l) = 4 \tag{23}$$

The feasible motions would be given by (22).

4.4 Screw Theory

In the previous section the mobility is obtained as 4, however it must be stressed that in the case of 5 DOF serial arms, the previous method does not reveal the nature of these motions instead only their dimension is given. In order to elucidate the motion type, *screw theory* is used.

4.4.1 Mobility Analysis Based on Screw Theory

Screw theory can be used to analyze the instantaneous motions of complex mechanisms (Hunt, 1978; Ball, 1900). A screw of pitch λ is defined as:

$$\$_\lambda = \begin{bmatrix} \mathbf{s} \\ \mathbf{s} \times \mathbf{r} + \lambda \mathbf{s} \end{bmatrix} \tag{24}$$

$\mathbf{s}$ is the unit vector of the rotational axis of the screw. $\mathbf{r}$ is the vector from any point on the axis to the origin. A zero-pitch screw and an infinite-pitch screw are given respectively by:

$$\$_0 = \begin{bmatrix} \mathbf{s} \\ \mathbf{s} \times \mathbf{r} \end{bmatrix} \quad , \quad \$_\infty = \begin{bmatrix} \mathbf{0}_{3\times1} \\ \mathbf{s} \end{bmatrix} \tag{25}$$

For every screw system, consisting of n linearly independent screws, there exists a reciprocal screw system of dimension $6-n$. Two screws $\$_1$ and $\$_2$ are reciprocal if their instantaneous power is zero, namely,

$$\left(\begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{I}_3 \\ \mathbf{I}_3 & \mathbf{0}_{3\times3} \end{bmatrix} \$_1 \right)^T \$_2 = 0 \tag{26}$$

The following reciprocity conditions are defined from (26):

1. A $\$_0$ is reciprocal to a $\$_\infty$ if their axes are normal to each other;
2. A $\$_\infty$ is always reciprocal to another $\$_\infty$;
3. Two $\$_0$ are reciprocal to each other if their axes are coplanar;

A zero-pitch twist ε_0 corresponds to a pure rotation about its axis and an ∞-pitch twist ε_∞ corresponds to a pure translation along its axis. A zero-pitch wrench ζ_0 corresponds to a pure force along its axis and an ∞-pitch wrench ζ_∞ corresponds to a pure moment about its axis.

The twist system $\mathtt{T}^i$ and the wrench system $\mathtt{W}^i$ of a serial kinematic chain i composed of f joints are given by:

$$\mathtt{T}^i = \bigoplus_{j=1}^{f} \mathtt{T}_j \quad , \quad \mathtt{W}^i = \bigcap_{j=1}^{f} \mathtt{W}_j \tag{27}$$

The twist system $\mathtt{T}$ and the wrench system $\mathtt{W}$ of a parallel kinematic chain composed of m serial chains are given by:

$$\mathtt{T} = \bigcap_{i=1}^{m} \mathtt{T}^i \quad , \quad \mathtt{W} = \bigoplus_{i=1}^{m} \mathtt{W}^i \tag{28}$$

where $\mathbf{A} = \bigoplus_{j=1}^{f} \mathbf{B}_j$ means $\mathbf{A}$ is spanned by vectors $\mathbf{B}_{1\cdots f}$ and $\mathbf{A} = \bigcap_{j=1}^{f} \mathbf{B}_j$ means $\mathbf{A}$ is intersection of vectors of $\mathbf{B}_{1\cdots f}$

Table 1 shows that NAO T14 is composed of five revolute joints. From (27), the twist systems $\mathtt{T}^r$ and $\mathtt{T}^l$ of its right and left arms are defined as[2]:

$$\mathtt{T}^r = span(\varepsilon_{01}, \varepsilon_{02}, \varepsilon_{03}, \varepsilon_{04}, \varepsilon_{05}) \tag{29}$$

$$\mathtt{T}^l = span(\varepsilon_{06}, \varepsilon_{07}, \varepsilon_{08}, \varepsilon_{09}, \varepsilon_{010}) \tag{30}$$

The first two joints of each arm intersect and constitutes a universal (U)-joint. The last three joints also intersect and are equivalent to a spherial (S)-joint. Since the twist system of each arm is a 5-system, its reciprocal wrench system is a 1-system. Using the reciprocity condition 3, it can be shown that each arm applies one pure force (a zero-pitch wrench $\zeta_{0i}, i = 1,2$) to the object along an axis intersecting the U- and S-joint axes. Thus, the wrench systems of the two arms are defined as:

$$\mathtt{W}_r = span(\zeta_{01}) \quad , \quad \mathtt{W}_l = span(\zeta_{02}) \tag{31}$$

From (28), the constraint wrench system applied to the object is a 2-system given by:

$$\mathtt{W}^c = \mathtt{W}_r \oplus \mathtt{W}_l = span(\zeta_{01}, \zeta_{02}) \tag{32}$$

[2] r and l stand for right and left.

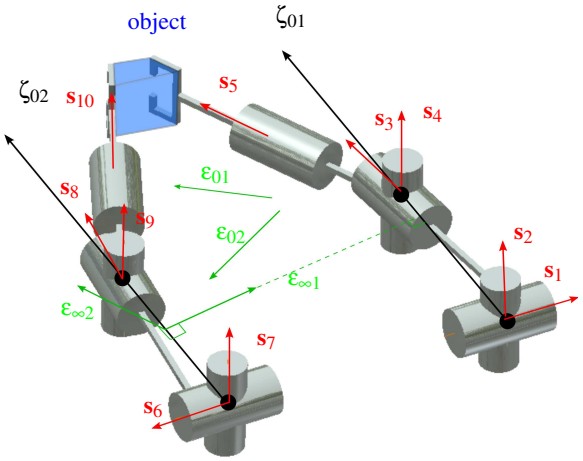

Figure 3: Reciprocal twists to parallel constraint forces (2T2R motion mode).

The object twist system is reciprocal to $\mathtt{W}^c$. Thus, it is a 4-system and the object has four DOF.

4.4.2 Motion Analysis Based on Screw Theory

When the constraint forces are parallel, i.e., $\zeta_{01} \parallel \zeta_{02}$, reciprocity condition 1 states that there are two independent ∞-pitch twists, $\varepsilon_{\infty 1}$ and $\varepsilon_{\infty 2}$, reciprocal (normal) to ζ_{01} and ζ_{02}. Reciprocity condition 3 states that there are also two independent zero-pitch twists, ε_{01} and ε_{02}, reciprocal(coplanar) to ζ_{01} and ζ_{02} as shown in Fig. 3. Therefore, the corresponding motion mode is 2T2R[3].

When $\zeta_{01} \nparallel \zeta_{02}$, but intersect, reciprocity condition 1 states there is one ∞-pitch twist $\varepsilon_{\infty 1}$ reciprocal(normal) to both ζ_{01} and ζ_{02} and condition 3 states there are three independent zero-pitch twists, ε_{01}, ε_{02} and ε_{03}, reciprocal (cooplanar) to both ζ_{01} and ζ_{02} as shown in Fig. 4. Consequently, the corresponding motion mode is 1T3R.

5 SINGULARITY ANALYSIS

This section deals with the singularity analysis of the NAO 14 when it firmly grasps an object. Limb singularities can be characterized by a loss of DOF locally, while a gain of DOF or a lack of stiffness is known as a parallel singular configuration (Amine et al., 2011),(Amine et al., 2012).

[3] T and R stand for Translation and Rotation, respectively.

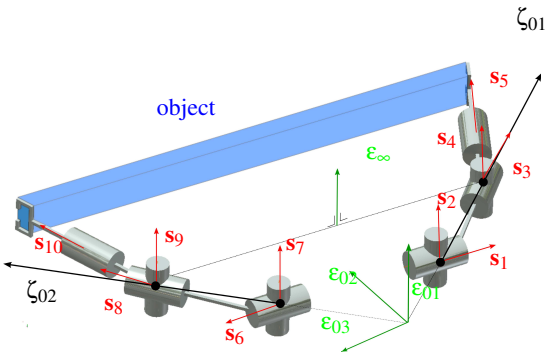

Figure 4: Reciprocal twists to intersecting constraint forces (1T3R motion mode).

5.1 Limb Singularities

A limb singularity is similar to the singularity of a serial manipulator. It occurs for the dual-arm system when the limb kinematic screw system (twist system) degenerates. Consequently, the grasped object loses one or more DOF in such a configuration. From (25) and (29) the kinematic Jacobian matrix of the right arm can be written as:

$$\mathbf{J}_r = \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_2 & \mathbf{s}_3 & \mathbf{s}_4 & \mathbf{s}_5 \\ \mathbf{s}_1 \times \mathbf{r}_1 & \mathbf{s}_2 \times \mathbf{r}_2 & \mathbf{s}_3 \times \mathbf{r}_3 & \mathbf{s}_4 \times \mathbf{r}_4 & \mathbf{s}_5 \times \mathbf{r}_5 \end{bmatrix} \tag{33}$$

Let the wrist center be the origin of the frame where vectors $\mathbf{r}_i$, $i = 1,\ldots,5$ are expressed. Equation (33) becomes

$$\mathbf{J}_r = \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_2 & \mathbf{s}_3 & \mathbf{s}_4 & \mathbf{s}_5 \\ \mathbf{s}_1 \times \mathbf{r}_1 & \mathbf{s}_2 \times \mathbf{r}_2 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \end{bmatrix} \tag{34}$$

The right arm reaches a limb singularity if and only if matrix $\mathbf{J}_r$ is rank deficient. The relations leading to rank deficiency are examined in Table 2. Condition 1,2,4 can be immediately ruled out as the axes in question are always orthogonal. Furthermore, the structure of the arm means the S-joint cannot lie on $\mathbf{s}_2$ thereby excluding condition 5, leaving two singular configurations.

Table 2: Rank deficiency of $\mathbf{J}_r$

No.	Config.	Value
1	$\mathbf{s}_1 \parallel \mathbf{s}_2$	-
2	$\mathbf{s}_3 \parallel \mathbf{s}_4$	-
3	$\mathbf{s}_3 \parallel \mathbf{s}_5$	$q_4 = 0 \pm \pi$
4	$\mathbf{s}_4 \parallel \mathbf{s}_5$	-
5	$\mathbf{s}_2 \times \mathbf{r}_2 = \mathbf{0}_3$	-
6	$\mathbf{s}_1 \times \mathbf{r}_1 = \mathbf{0}_3$	$q_2 = \text{atan}\left(\frac{-r_3}{d_3}\right)$

5.2 Parallel Singularities

The loss of stiffness due to a parallel singularity can be characterized by a degeneracy of screw system representing the wrenches. Examples are given in Table 3.

Table 3: Linear Dependence of Screws.

No.	ζ_λ	Condition
≥ 2	ζ_0	collinear axes
≥ 2	ζ_∞	parallel axes
≥ 3	ζ_0	intersect the same point
≥ 6	ζ_0	intersect the same line
1	ζ_∞	$\perp$ to plane formed by $\parallel \zeta_0$

5.2.1 Constraint Singularities

A constraint singularity occurs when the constraint wrench system (32) degenerates, i.e., when ζ_{01} and ζ_{02} are linearly dependent (condition 1 in Table 3). The closed loop system reaches such a configuration when the two S-joint centers lie on $\mathbf{s}_1$ and $\mathbf{s}_6$, from the geometric model:

$$q_2 = \text{atan}\left(\frac{-r_3}{d_3}\right) \quad \text{and} \quad q_7 = \text{atan}\left(\frac{r_3}{d_3}\right) \tag{35}$$

From Table 2 it is noted that when the closed loop system reaches a constraint singularity, both arms reach a limb singularity. Four wrenches forming a 3-system as described in Fig. 5 are applied on the object: the constraint wrenches ζ_{01} and ζ_{02} and the wrenches due to the serial singularity of each arm ζ_{0s1} and ζ_{0s2}. As a consequence, the object has 3-DOF in this configuration.

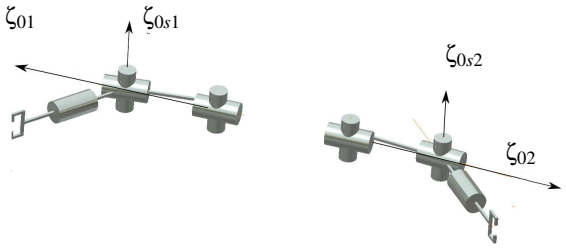

Figure 5: Constraint singularity of the dual arm system.

5.2.2 Actuation Singularities

In section 3 the system is modeled with actuated (independent) and passive (dependent) joint variables. In this section a selection criterion is given for a suitable actuated joints. Once an actuator is locked, it imposes a wrench on the object. The actuator wrench for joint

i is denoted as $\zeta^a_{\lambda j}$. This wrench is reciprocal to all the twists of the arm except the actuator twist itself and furthermore it should not lie in $\mathtt{W}^c$. The actuation wrench system $\mathtt{W}_a$ applied on the object is the span of actuation wrenches from both arms, namely,

$$\mathtt{W}^a = span(\mathtt{W}^a_r, \mathtt{W}^a_l) \tag{36}$$

The constraint wrenches and actuation wrenches of both arms should form a 6-system. This system is denoted as the global wrench system and is defined as:

$$\mathtt{W} = \mathtt{W}^c \oplus \mathtt{W}^a \tag{37}$$

An actuation singularity occurs when (37) degenerates while (32) does not. Since it is possible to actuate any four of the ten joints there are $\frac{10!}{4!(10!-4!)} = 210$ possible actuation schemes. By excluding non-symmetrical actuation schemes, one hundred actuation schemes remain. It should be noted that since all the joints in the arm can be actuated, a redundant actuation scheme can decrease the likelihood of actuation singularities.

Example 1: $\mathbf{q_a} = [q_1 \ q_2 \ q_6 \ q_7]$. Since it is generally preferable to actuate joints close to the base, the case when the base U-joints are actuated is examined. Using the reciprocity condition 3, the global wrench system is derived:

- A pure force $\zeta^a_{01} \parallel \mathbf{s}_2$ and intersecting $\mathbf{s}_3, \mathbf{s}_4, \mathbf{s}_5$ is coplanar to all twists except that generated by q_1
- A pure force $\zeta^a_{02} \parallel \mathbf{s}_1$ and intersecting $\mathbf{s}_3, \mathbf{s}_4, \mathbf{s}_5$ is coplanar to all twists except that generated by q_2
- A pure force $\zeta^a_{06} \parallel \mathbf{s}_6$ and intersecting $\mathbf{s}_8, \mathbf{s}_9, \mathbf{s}_{10}$ is coplanar to all twists except that generated by q_6
- A pure force $\zeta^a_{07}, \parallel \mathbf{s}_7$ and intersecting $\mathbf{s}_8, \mathbf{s}_9, \mathbf{s}_{10}$ is coplanar to all twists except that generated by q_7

The global wrench system as illustrated in Fig. 6, is then given by $\mathtt{W} = span(\zeta_{01} \ \zeta_{02} \ \zeta^a_{01} \ \zeta^a_{02} \ \zeta^a_{06} \ \zeta^a_{07})$. A line between point A and point B (the S-Joints centers of either arm) can be drawn. This line is intersected by all six constraint forces regardless of the configuration of the robot. Therefore this actuation scheme is not admissible, for any configuration the global wrench system degenerates, i.e. $\text{rank}(\mathtt{W}) = 5$, $\text{rank}(\mathtt{W}^c) = 2$, $\text{rank}(\mathtt{W}^a) = 4$, resulting in an uncontrollable closed chain mechanism.

Example 2: $\mathbf{q_a} = [q_1 \ q_2 \ q_3 \ q_7]$. To find the actuation wrench applied by q_3, the planes Π_{12}, Π_{45} spanned by ($\mathbf{s}_1$, $\mathbf{s}_2$) and ($\mathbf{s}_4$, $\mathbf{s}_5$) respectively, are examined. The actuation wrench

1. if Π_{12} and Π_{45} intersect, there is a pure force ζ^a_{03} acting along the intersection line formed by the two planes

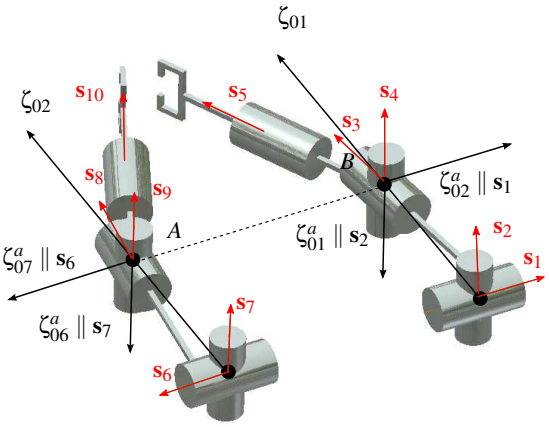

Figure 6: Non-Admissible Actuation scheme.

2. if Π_{12} and Π_{45} are parallel, there is a pure moment $\zeta^a_{\infty 3}$ acting around the line normal to both two planes

The global wrench system for case 1 is shown in Fig.7 and is $\mathtt{W} = span(\zeta_{01} \ \zeta_{02} \ \zeta^a_{01} \ \zeta^a_{02} \ \zeta^a_{03} \ \zeta^a_{07})$. In this case an actuation singularity occurs whenever the line ζ^a_{03} contains points A or B. Other such singularities can be obtained simply by observing when the wrench system formed by two or more wrenches degenerates (as shown in Table 3), for instance the case where ζ^a_{02} and ζ^a_{07} become collinear.

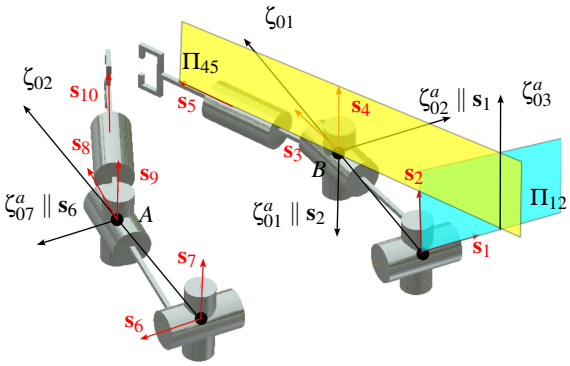

Figure 7: Admissible Actuation scheme.

6 CONCLUSIONS

In conclusion, this work has presented a comprehensive study of the two arms of the NAO robot when engaged in a cooperative task. The system is modeled

as a closed loop kinematic chain. Using the models of the system, an analysis of the structure of the robot is given. The degree of freedom of an object being simultaneously grasped by both arms was investigated. The singular configurations of the robot are also discussed. The singularities of the serial arm are straightforward, more interesting is the appearance of constraint singularities. The issue of actuation singularities, present due to the dependent joint variables of the closed chain, is investigated. By considering the wrenches exerted by the actuators, both an admissible and inadmissible actuation scheme was illustrated.

REFERENCES

Amine, S., Caro, S., Wenger, P., and Kanaan, D. (2011). Singularity analysis of the H4 robot using Grassmann–Cayley algebra. *Robotica*, 1(1):1–10.

Amine, S., Tale Masouleh, M., Caro, S., Wenger, P., and Gosselin, C. (2012). Singularity analysis of 3T2R parallel mechanisms using Grassmann–Cayley algebra and Grassmann geometry. *Mechanism and Machine Theory*.

Ball, S. R. (1900). *A Treatise on the Theory of Screws*. Cambridge Univ Pr.

Bicchi, A., Melchiorri, C., and Balluchi, D. (1995). On the mobility and manipulability of general multiple limb robots. *Robotics and Automation, IEEE Transactions on*, 11(2):215–228.

Bonitz, R. C. and Hsia, T. C. (1996). Internal force-based impedance control for cooperating manipulators. *Robotics and Automation, IEEE Transactions on*, 12(1):78–89.

Caccavale, F., Chiacchio, P., and Chiaverini, S. (2000). Task-space regulation of cooperative manipulators. *Automatica*, 36(6):879–887.

Caccavale, F., Chiacchio, P., Marino, A., and Villani, L. (2008). Six-DOF impedance control of dual-arm cooperative manipulators. *Mechatronics, IEEE/ASME Transactions on*, 13(5):576–586.

Cheng, H., Yiu, Y.-k., Member, S., and Li, Z. (2003). Dynamics and control of redundantly actuated parallel manipulators. *IEEE/ASME Transactions on Mechatronics*, 8(4):483–491.

Chiacchio, P., Chiaverini, S., and Siciliano, B. (1996). Direct and inverse kinematics for coordinated motion tasks of a two-manipulator system. *Journal of Dynamic Systems, Measurement, and Control*, 118:691.

Gogu, G. (2007). *Structural Synthesis of Parallel Robots: Part 1: Methodology*. Springer Verlag.

Hunt, K. (1978). *Kinematic geometry of mechanisms*. Cambridge Univ Press.

Khalil, W. and Kleinfinger, J. (1986). A new geometric notation for open and closed-loop robots. In *Robotics and Automation. Proceedings. 1986 IEEE International Conference on*, volume 3, pages 1174–1179. IEEE.

Liu, Y., Xu, Y., and Bergerman, M. (1999). Cooperation control of multiple manipulators with passive joints. *IEEE Transactions on Robotics and Automation*, 15(2):258–267.

Özkan, B. and Özgören, M. (2001). Invalid joint arrangements and actuator related singular configurations of a system of two cooperating scara manipulators. *Mechatronics*, 11(4):491–507.

Sadati, N. and Ghaffarkhah, A. (2008). Decentralized impedance control of nonredundant multi-manipulator systems. In *Networking, Sensing and Control, 2008. ICNSC 2008. IEEE International Conference on*, pages 206–211.

Uchiyama, M. and Dauchez, P. (1988). A symmetric hybrid position/force control scheme for the coordination of two robots. In *1988 IEEE International Conference on Robotics and Automation, Philadelphia, PA*, pages 350–356.

Yeo, H.-J., Suh, I. H., Yi, B.-J., and Oh, S.-R. (1999). A single closed-loop kinematic chain approach for a hybrid control of two cooperating arms with a passive joint: an application to sawing task. *IEEE Transactions on Robotics and Automation*, 15(1):141–151.

Zielinski, C. and Szynkiewicz, W. (1996). Control of two 5 dof robots manipulating a rigid object. In *Industrial Electronics, 1996. ISIE'96., Proceedings of the IEEE International Symposium on*, volume 2, pages 979–984. IEEE.

Zlatanov, D., Bonev, I., and Gosselin, C. (2002). Constraint singularities of parallel mechanisms. In *Robotics and Automation, 2002. Proceedings. ICRA'02. IEEE International Conference on*, volume 1, pages 496–502. IEEE.