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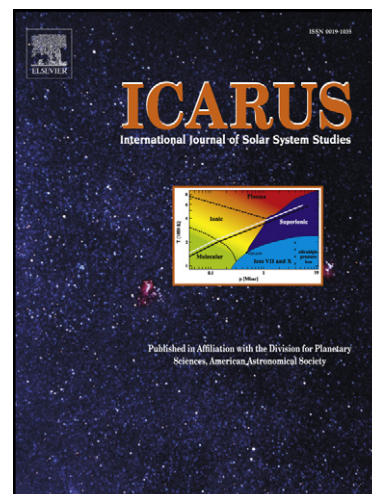
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Interior Structure Models of Solid Exoplanets Using Material Laws in the Infinite Pressure Limit

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Abstract

The detection of low-mass extrasolar planets has initiated growing interest in massive rocky bodies (super-Earths) for which no solar system analogue does exist. Here, we present a new model approach to investigate their interior structure and thermal state. We improve and extend previous interior models mainly in two areas: The first improvement is due to the consequent application of equations of state (EoS) that are compliant with the thermodynamics of the high-pressure limit and facilitate reinvestigating mass-radius relations for terrestrial-type exoplanets. **To quantify the uncertainty due to extrapolation, we compare a generalized Rydberg and a Keane EoS, which are both consistent with the physics of the thermodynamic limit. Furthermore, we consider a reciprocal K' EoS that fits the seismologically obtained Preliminary Reference Earth Model (PREM), thereby accounting for the mineralogical composition of the Earth.** As a result, the predicted planetary radii of terrestrial-type ex-

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oplanets of up to ten Earth masses would differ by less than 2 % between all three EoS, well within current observational limits. The second extension arises from the adoption of a mixing length formulation instead of the commonly used, more simplified parameterized approach to model convective heat transport in planetary mantles. In comparison to parameterized convection models, our results indicate generally hotter interiors with increasing planetary mass and a cumulative tendency to extended regimes of sluggish convection in the lowermost mantle. The latter is attributed to less efficient convective heat transport with increasing mantle pressures. An improved knowledge of the present thermal state is prerequisite to gain a better understanding of the pathways of internal evolution of terrestrial-type exoplanets. *Keywords:* super-Earths, interior structure, thermal state, mass-radius relationships, high-pressure physics

1. Introduction

Owing to the growing number of detected exoplanets, the field of comparative exoplanetology is rapidly expanding since the first discovery of a planet orbiting a star other than the Sun. More than 30 exoplanets with masses below 15 Earth masses (M_{\oplus}) have been discovered and without a doubt more will be detected by either space missions or ground-based surveys. One of the most prominent low-mass exoplanets is CoRoT-7b (Léger et al., 2009), which is supposed to be the first discovered rocky exoplanet or super-Earth (Queloz et al., 2009). Another example is GJ 1214b (Charbonneau et al., 2009), a planet with a total mass of $(6.55 \pm 0.98) M_{\oplus}$ and a planetary radius of $(2.68 \pm 0.13) R_{\oplus}$, implying a relatively low mean density without solar

12 system analogue (Rogers and Seager, 2010). Also worth mentioning is the
13 planetary system of Gliese 581 containing at least four planets: Including
14 Gliese 581d (Mayor et al., 2009), a low-mass exoplanet likely situated within
15 the habitable zone, a region where liquid water may exist on the planetary
16 surface, and Gliese 581e (Mayor et al., 2009), an exoplanet with the smallest
17 currently known minimum mass of about $2 M_{\oplus}$ orbiting a main sequence
18 star. These are only a few examples of the remarkable variety of low-mass
19 exoplanets detected so far.

20 Previous work by Valencia et al. (2006, 2007a,b), Sotin et al. (2007),
21 Fortney et al. (2007), Seager et al. (2007), and Grasset et al. (2009) have in-
22 vestigated mass-radius relations for silicate- and water-rich exoplanets using
23 different bulk compositions, equations of state (EoS), and other modeling
24 details. Uncertainties and discrepancies of such models mainly result from
25 the necessity to extrapolate an EoS to high pressures and, to a minor extent,
26 from the lack of knowledge about internal heat sources and rheological prop-
27 erties that would determine the present thermal state of exoplanet interiors.
28 The majority of the EoS used (e.g., Birch-Murnaghan, Vinet) represent semi-
29 empirical fits obtained from laboratory experiments and are not essentially
30 consistent with the thermodynamics of the high-pressure limit (Stacey and
31 Davis, 2004). Nevertheless, interior structure models for exoplanets have
32 been calculated based on the Vinet EoS (e.g., Valencia et al., 2007a) or the
33 Birch-Murnaghan EoS (e.g., Sotin et al., 2007). **As a consequence of the**
34 **lack of reliable experimental data mainly in the pressure range of**
35 **200 GPa to 10 TPa (Seager et al., 2007; Grasset et al., 2009), we**
36 **consider different EoS (generalized Rydberg, Keane, and recipro-**

37 cal K') that are consistent with the physics of the thermodynamic
38 limit. Hence, the present study is aimed at a comparison between ther-
39 modynamically consistent EoS to investigate how these would affect interior
40 structure models of massive terrestrial-type exoplanets. Another purpose of
41 this study is to apply the concept of mixing length to self-consistently model
42 the present thermal state of planetary interiors. Therefore, compared to pre-
43 vious models, the main differences are related to the EoS used in this study
44 and how radial temperature profiles of model planets are obtained.

45 In the following section, we describe how the interior structure models and
46 corresponding radial temperature profiles are constructed and relevant EoS
47 parameters are obtained. In section 3, we present our results for the imple-
48 mented EoS and quantify their effects on the structural models of exoplanet
49 interiors. Furthermore, radial temperature profiles calculated by using the
50 adopted mixing length approach are presented. Additionally, mass-radius re-
51 lations for silicate- and water-rich exoplanets are used to validate our model
52 approach. In section 4, we discuss our results obtained from the EoS com-
53 parison and the mixing length formulation by addressing main differences to
54 previous models and implications for the robustness of mass-radius relations.
55 Finally, conclusions are drawn and possible consequences for observational
56 thresholds of present and future missions will be summarized.

57 2. Method

58 We model the interior structure and calculate mass-radius relations for
59 several classes of low-mass exoplanets.

60 *2.1. Bulk Composition and Phase Transitions*

61 In the present study, we consider three principal classes of low-mass exo-
62 planets according to their bulk compositions: (a) Terrestrial planets such as
63 the Earth and Venus, for which an Earth-like bulk composition with an iron
64 core mass fraction of 32.5 wt.-% and a corresponding silicate mantle mass
65 fraction of 67.5 wt.-% are assumed. This class of exoplanets is of particular
66 interest because of the known potential to be life-sustaining. (b) Iron-rich
67 Mercury-type planets containing 70 wt.-% iron, which is concentrated in the
68 core, and a silicate mantle of 30 wt.-%. (c) Ocean planets are assumed to
69 contain at least 10 % H₂O by mass (Léger et al., 2004) and have bulk composi-
70 tions similar to that of a cometary core (50 wt.-% volatiles and 50 wt.-% rock
71 and iron). In the following, we consider an ocean planet similar to Jupiter’s
72 icy moon Ganymede, which is composed of 45 wt.-% water-ice, 48.5 wt.-%
73 silicate, and 6.5 wt.-% iron (Kuskov and Kronrod, 2001; Sohl et al., 2002).

74 Each rocky planet is subdivided into four chemically homogeneous shells.
75 The mantle consists of an upper olivine part composed of forsterite, un-
76 derlain by a perovskite shell and a lower post-perovskite part, whereas the
77 central core is assumed to be composed of pure iron. Therefore, two pressure-
78 induced phase transitions have been implemented into the model: (a) the
79 olivine to perovskite and (b) the perovskite to post-perovskite transition are
80 experimentally determined with Clapeyron slopes of $-0.0013 \text{ MPa K}^{-1}$ (Fei
81 et al., 2004) and $+13.3 \text{ MPa K}^{-1}$ (Tateno et al., 2009), respectively. We
82 have chosen a relatively large value for the Clapeyron slope of the perovskite
83 to post-perovskite transition in accordance with the seismic discontinuity
84 observed in the Earth’s D” region (Hernlund and Labrosse, 2007). **First**

85 principles calculations suggest that post-perovskite could dissoci-
 86 ate into CsCl-type MgO and cotunnite-type SiO₂ at pressures and
 87 temperatures prevalent in the deep interiors of rocky exoplanets
 88 (Umemoto et al., 2006). However, since only a small volume of the
 89 hot, lowermost mantle near the core-mantle boundary of Earth-
 90 like exoplanets would be affected by the small density increase at
 91 pressures above 1 TPa, it can be safely assumed that neglecting
 92 the putative dissociation of post-perovskite will not have a severe
 93 impact on the radial mass distribution.

94 In the case of ocean planets, an additional high-pressure water-ice layer
 95 is placed on top of the silicate mantle. We neglect low-pressure phase tran-
 96 sitions within the ice because of their minor effect on mass-radius relations
 97 (Seager et al., 2007). In addition to the core mass fraction, we finally have
 98 to specify the water mass fraction for ocean planets.

99 2.2. Numerical Model

100 We consider a spherically symmetric and fully differentiated planet in
 101 thermal steady state and perfect mechanical equilibrium. Under these as-
 102 sumptions, its depth-dependent interior structure is described by the follow-
 103 ing set of coupled differential equations for mass $m(r)$, acceleration of gravity
 104 $g(r)$, and pressure $P(r)$:

$$\frac{dm}{dr} = 4\pi r^2 \rho, \quad (1)$$

$$\frac{dg}{dr} = 4\pi G \rho - 2\frac{g}{r}, \quad (2)$$

$$\frac{dP}{dr} = -\rho g, \quad (3)$$

105 where r is the radial distance from the center of the planet, G is the
 106 gravitational constant, and ρ is the local density described in detail in the
 107 next paragraph.

108 **Within the core, we assume an adiabatic temperature distribu-**
 109 **tion given by**

$$\frac{dT}{dr} = -\frac{\gamma}{\Phi}gT, \quad (4)$$

110 where γ is the thermodynamic Grüneisen parameter and $\Phi=K_S/\rho$
 111 the seismic parameter. The latter involves the adiabatic bulk mod-
 112 ulus K_S and density ρ , which are both obtained from the local
 113 evaluation of an equation of state. The equation $K_S/K_T = 1 + \gamma\alpha T$
 114 relates the adiabatic bulk modulus K_S to the isothermal bulk mod-
 115 ulus K_T , where α is the thermal expansivity of a given material.
 116 No additional heat sources are assumed in the core. Neverthe-
 117 less, we evaluate a minimum heat flux conducted along the core
 118 adiabat across the core-mantle boundary (cmb) of $q_{cmb} = -k dT/dr$
 119 at $r = r_{cmb}$, where k is the thermal conductivity of the core taken
 120 constant at $35 \text{ W K}^{-1} \text{ m}^{-1}$ (Stacey and Davis, 2008).

121 **Within the silicate mantle, the following equations are consid-**
 122 **ered for heat flux $q(r)$ and temperature $T(r)$ as functions of the**
 123 **radial distance r :**

$$\frac{dq}{dr} = \epsilon\rho - 2\frac{q}{r}, \quad (5)$$

$$\frac{dT}{dr} = -\frac{q}{Nu_r k_c}, \quad (6)$$

124 where Nu_r is the dimensionless local Nusselt number and k_c
 125 is the thermal conductivity of the mantle as given by the model
 126 of Hofmeister (1999). For the sake of simplicity, we limit ourselves to
 127 radiogenic heating; accretional heating and secular cooling or tidal heating
 128 are therefore not explicitly taken into account. The specific heat production
 129 rate ϵ is taken to be constant and matches the present-day Earth-like value
 130 of $7.38 \times 10^{-11} \text{ W kg}^{-1}$ (Turcotte and Schubert, 2002).

131 The local Nusselt number Nu_r is a measure for the efficiency of radial
 132 heat transport in terms of the ratio of total heat flux to conductive heat flux
 133 and can be written as

$$Nu_r = \left(1 + \frac{k_v}{k_c}\right) \left[1 - \frac{k_v}{q} \left(\frac{dT}{dr}\right)_S\right]^{-1}, \quad (7)$$

134 where $(dT/dr)_S = -(\gamma g T)/\Phi$ is the adiabatic temperature gradient. The
 135 effective thermal conductivity k_v due to convection can be inferred from
 136 applying a mixing length formulation. The basic idea is that the heat transfer
 137 is primarily due to vertical motion of a fluid parcel. Sasaki and Nakazawa
 138 (1986) and Abe (1997) extended this concept for highly viscous fluids by
 139 considering that the Stokes' viscous drag is balanced by the buoyancy force
 140 operating on such a parcel. Mixing length formulations have been applied in
 141 planetary sciences by Senshu et al. (2002) to model the thermal history of
 142 early Mars and by Kimura et al. (2009) to investigate the thermal evolution
 143 of the Jovian moon Ganymede. Using this approach, k_v can be calculated
 144 according to

$$k_v = \frac{l^4 \rho^2 \alpha^2 g \Phi}{18 \gamma \eta} \left[\left(\frac{dT}{dr} \right)_s - \frac{dT}{dr} \right], \quad (8)$$

145 where l is the characteristic mixing length, α is the thermal expansivity
 146 of the mantle, and η is the local dynamic viscosity. Following Abe (1997)
 147 and Senshu et al. (2002), we treat the mixing length l as the distance from
 148 the nearest boundary of the convective layer. **The thermal expansiv-**
 149 **ity is evaluated locally using the definition of the thermodynamic**
 150 **Grüneisen parameter $\gamma = (\alpha K_S)/(\rho C_P)$, where C_P is the specific heat**
 151 **capacity of the lower mantle of about $1250 \text{ J kg}^{-1} \text{ K}^{-1}$.**

152 In the present study, we model a pressure- and temperature-
 153 dependent viscosity η defined as

$$\eta = \frac{\sigma}{2\dot{\epsilon}}, \quad (9)$$

154 where σ is the applied shear stress and $\dot{\epsilon}$ is the shear strain rate.
 155 A general flow law for steady state creep can be written as (e.g.,
 156 Ranalli, 2001)

$$\dot{\epsilon} = A d^m \sigma^p \exp\left(-\frac{E^* + PV^*}{RT}\right), \quad (10)$$

157 where d is the grain size, R is the universal gas constant, and A ,
 158 p , m , E^* , and V^* are flow constants related to the dominant creep
 159 mechanism. Seismic observations suggest that diffusion ($p = 1$)
 160 is the predominant creep mechanism in the lower mantle of the

161 Earth, while dislocation ($p \geq 3.5$) dominates the flow of the Earth's
 162 upper mantle (e.g., Karato et al., 1995). Therefore, we assume
 163 diffusion-controlled creep ($d \sim 10^{-3}$ m) in the MgSiO_3 lower mantle
 164 and dislocation-controlled creep at a constant strain rate of $\dot{\epsilon} =$
 165 10^{-15} s $^{-1}$ in the Mg_2SiO_4 upper mantle. We also assume a dry
 166 rheology and use activation parameters experimentally obtained
 167 by Karato and Wu (1993). To account for the depth dependence
 168 of the activation volume V^* , we approximate the pressure-induced
 169 shrinkage of the activation volume as a vacancy in the material
 170 (O'Connell, 1977). A similar approach was recently applied by Fu
 171 et al. (2010) to calculate the pressure-dependent activation volume
 172 of water-ice polymorphs.

173 We implicitly solve equations [1]-[3] in conjunction with equations [4]-[6]
 174 by numerical integration employing a BDF (backward differentiation formu-
 175 lae or Gear's method) routine. The algorithm starts the radial integration
 176 in the center of the model planet ($r = 0$) using central boundary conditions
 177 of $m(0) = 0$, $g(0) = 0$, $P(0) = P_c$, $q(0) = 0$, and $T(0) = T_c$ where P_c and T_c
 178 are educated initial guesses for the central pressure and temperature, respec-
 179 tively. Integration then proceeds outward through each shell until the total
 180 mass M_p of the planet is achieved. If necessary, this process will start over
 181 with iteratively adjusted central pressure P_c and temperature T_c . The algo-
 182 rithm stops integrating if the surface boundary conditions of $m(R_p) = M_p$,
 183 $P(R_p) = P_s$, and $T(R_p) = T_s$ are met at $r = R_p$.

184 2.3. Equation of State

185 The local density is calculated by

$$\rho(r) = f_{EoS}(P(r), T(r)), \quad (11)$$

186 where f_{EoS} is the equation of state (EoS), a unique function relating to
 187 each other density, pressure, and temperature of a given material in thermal
 188 equilibrium. Almost all EoS are semi-empirical fits to experimental data
 189 obtained by high-pressure experiments or seismological observations.

190 2.3.1. Isothermal Equation of State

191 In the upper silicate mantle we use a third-order Birch-Murnaghan EoS to
 192 calculate how the density of a given material behaves with increasing pressure
 193 at a constant reference temperature (Birch, 1952):

$$P = \frac{3}{2}K_0(x^{7/3} - x^{5/3}) \left[1 + \frac{3}{4}(K'_0 - 4)(x^{2/3} - 1) \right], \quad (12)$$

194 where $x = \rho/\rho_0$ is the compression ratio with respect to the ambient
 195 density ρ_0 ; K_0 and K'_0 denote the isothermal bulk modulus and its pressure
 196 derivation at ambient conditions, respectively.

197 This EoS is based on the expansion of Eulerian finite strain and is widely
 198 used in mineralogical and geophysical applications. Almost all high-pressure
 199 experiments are fitted to this EoS. However, extrapolation beyond 100 GPa
 200 is highly uncertain (Stacey and Davis, 2004). **Therefore, we have cho-**
 201 **sen to implement the following EoS for the lower mantle and the**
 202 **metallic core: (a) the generalized Rydberg EoS (Stacey, 2005),**
 203 **which is derived from the Rydberg interatomic potential function,**

204 (b) Stacey's reciprocal K' EoS (Stacey, 2000), which is directly ap-
 205 plicable to seismologically obtained data, and (c) the Keane EoS
 206 (Stacey and Davis, 2008) based on the importance of the deriva-
 207 tive of the bulk modulus in the limit of infinitely large pressure K'_∞
 208 (Keane, 1954).

209 Vinet et al. (1989) proposed a "universal" EoS based on an expression for
 210 the cohesive energy that varies only as a function of normalized interatomic
 211 separation. This EoS can be written as

$$P = 3K_0x^{\frac{2}{3}}(1 - x^{-\frac{1}{3}}) \exp \left[\frac{2}{3}(K'_0 - 1)(1 - x^{-\frac{1}{3}}) \right] \quad (13)$$

212 and is usually called the Vinet EoS. For materials under strong compres-
 213 sion, EoS based on an exponential repulsive potential, such as that of Vinet,
 214 are superior to finite strain theories (e.g., Hemley et al., 1990; Loubeyre et al.,
 215 1996). A comparison of different EoS with theoretically calculated values up
 216 to very high pressures (< 1 TPa) supports the usage of the Vinet EoS (Hama
 217 and Suito, 1996). **Later, it was shown by Stacey (1999, 2001) that the**
 218 **Vinet EoS is identical to the Rydberg EoS and does not satisfy the**
 219 **thermodynamic requirement of $K'_\infty \geq \frac{5}{3}$ when extrapolated to infi-**
 220 **nitely high pressures.** Following Stacey (2005), we obtain the generalized
 221 Rydberg EoS by adjusting the Vinet equation to arbitrary K'_∞ by writing

$$P = 3K_0x^{K'_\infty}(1 - x^{-\frac{1}{3}}) \exp \left[f(1 - x^{-\frac{1}{3}}) \right], \quad (14)$$

222 with $f = (3/2)K'_0 - 3K'_\infty + 1/2$.

223 An EoS already consistent with the thermodynamics of the high-
 224 pressure limit and compatible to data derived from seismic obser-
 225 vations is represented by the reciprocal K' relation (Stacey, 2000)

$$\frac{1}{K'} = \frac{1}{K'_0} + \left(1 - \frac{K'_\infty}{K'_0}\right) \frac{P}{K}, \quad (15)$$

226 where K and K' denote the local bulk modulus and its pressure
 227 derivative, respectively.

228 To obtain expressions for K/K_0 and ρ/ρ_0 in terms of the parameter P/K ,
 229 integration is done with respect to pressure (eq. 16) and density (eq. 17).

$$\frac{K}{K_0} = \left(1 - K'_\infty \frac{P}{K}\right)^{-\frac{K'_0}{K'_\infty}}, \quad (16)$$

$$\ln x = -\left(\frac{K'_0}{K'^2_\infty}\right) \ln \left(1 - K'_\infty \frac{P}{K}\right) + \left(1 - \frac{K'_0}{K'_\infty}\right) \frac{P}{K}. \quad (17)$$

230 The main advantage of equations [16] and [17] is that they can be directly
 231 fitted to the Preliminary Reference Earth Model (PREM) (Dziewonski and
 232 Anderson, 1981) to obtain the unknown zero pressure parameters ρ_0 , K_0 , K'_0 ,
 233 and K'_∞ for the lower mantle and the core.

234 Sufficiently similar to the reciprocal K' relationship is the Keane
 235 EoS

$$P = K_0 \left[\frac{K'_0}{K'^2_\infty} \left(x^{K'_\infty} - 1\right) - \left(\frac{K'_0}{K'_\infty} - 1\right) \ln x \right]. \quad (18)$$

236 In general, the Keane EoS can be applied more conveniently to
 237 laboratory data obtained from high-pressure experiments, whereas

238 **the reciprocal K' EoS is preferred if P/K is known from seismic**
 239 **observations.** Table 1 lists the parameters ρ_0 , K_0 , K'_0 , and K'_∞ used for
 240 each material and appropriate EoS.

241 *2.3.2. Thermal Correction according to Mie-Grüneisen-Debye*

242 To incorporate the effects of temperature into the EoS, we need to add
 243 a thermal pressure term. We use lattice dynamics to take into account tem-
 244 perature effects. In this approach, the crystal is viewed as if composed of a
 245 collection of harmonic oscillators. **The Helmholtz free energy can then**
 246 **be obtained by summing over all normal mode vibrational frequen-**
 247 **cies at a given volume.** The thermal pressure ΔP_{th} is determined by using
 248 the Mie-Grüneisen equation (e.g., Jackson and Rigden, 1996):

$$\Delta P_{th}(\rho, T) = \gamma \rho [E_{th}(\rho, T) - E_{th}(\rho, T_0)], \quad (19)$$

249 where the subscript 0 represents a reference state which is chosen to be the
 250 300 K isotherm, γ is the Grüneisen parameter, and E_{th} the internal thermal
 251 energy.

252 A simple but successful method to characterize the lattice vibrational
 253 modes is the Debye model which treats the solid as a continuous medium
 254 and parameterizes the vibrational spectrum in terms of a single characteristic
 255 temperature. **On that basis, the internal thermal energy E_{th} at a**
 256 **given temperature can be calculated as**

$$E_{th} = 9nk_B N_A T \left(\frac{T}{\theta_D} \right)^3 \int_0^{\theta_D/T} \frac{z^3}{\exp(z) - 1} \mathbf{d}z, \quad (20)$$

257 where n is the number of atoms in the formula unit of the cor-
 258 responding material, k_B is the Boltzmann constant, N_A is the Avo-
 259 gadro constant, and θ_D is the Debye temperature.

260 The Grüneisen parameter γ and the Debye temperature θ_D are assumed
 261 to be functions of density only. Their values are calculated for the
 262 upper silicate mantle as $\gamma = \gamma_0 x^{-\lambda}$ and $\theta_D = \theta_0 \exp[(\gamma_0 - \gamma)/\lambda]$. The
 263 logarithmic volume derivative of the thermodynamic Grüneisen pa-
 264 rameter λ is taken constant at a value of 3.2 for the upper olivine
 265 mantle (Katsura et al., 2009). Within the lower mantle and the metal-
 266 lic core, we use a formulation according to Al'tshuler et al. (1987) for the
 267 Grüneisen parameter $\gamma = \gamma_\infty + (\gamma_0 - \gamma_\infty)x^{-\beta}$ and for the Debye temperature
 268 $\theta_D = \theta_0 x^{\gamma_\infty} \exp[(1 - x^{-\beta})(\gamma_0 - \gamma_\infty)/\beta]$, respectively. The latter formulations
 269 describe the theoretically predicted material behavior under high pressure
 270 better. Table 2 summarizes the material parameters used for the thermal
 271 pressure correction.

272 For the pure iron core, additional terms are considered to account for
 273 electronic and anharmonic thermal pressure P_{el} and P_{anh} , respectively.

$$P_{el}(\rho, T) = \frac{3}{2} g^* n k_B N_A \rho e_0 x^{-g^*} T^2, \quad (21)$$

$$P_{anh}(\rho, T) = \frac{3}{2} m^* n k_B N_A \rho a_0 x^{-m^*} T^2. \quad (22)$$

274 The parameters e_0 , g^* , a_0 , and m^* have been obtained by fitting ab ini-
 275 tio electronic and anharmonic thermal pressures taken from Dewaele et al.
 276 (2006). For example, at a pressure of 300 GPa and a temperature of 6000 K,
 277 contributions from electronic and anharmonic pressure for pure iron are 15

278 and 3 GPa, respectively.

279 3. Results

280 Using the model approach above, we have computed interior structure
281 models of low-mass, terrestrial-type exoplanets of variable composition.

282 3.1. Interior Structure Models

283 **Figure 1 and 2 show the calculated interior structure for 1, 5,**
284 **and 10 M_{\oplus} exoplanets with Earth-like bulk composition. Com-**
285 **pared are the generalized Rydberg, the Keane, and the reciprocal**
286 **K' EoS. To verify the model approach, we have chosen the Preliminary**
287 **Reference Earth Model (PREM) (Dziewonski and Anderson, 1981), while a**
288 **reference geotherm proposed by Stacey and Davis (2008) is used**
289 **to characterize the present thermal state of the Earth.**

290 3.1.1. Earth Reference Comparison

291 **Panel (a)-(c) of Fig. 1 illustrates the density distribution, grav-**
292 **itational acceleration, and hydrostatic pressure as a function of**
293 **radial distance from the planet's center. A comparison between the**
294 **three Earth-sized models (1 M_{\oplus}) and PREM shows relatively good agree-**
295 **ment and therefore validates our model approach. The total mismatch of**
296 **the calculated planetary radius to PREM is below 0.5 % for all EoS**
297 **used.**

298 **Turning to panel (a) in particular, the core density discrepancy**
299 **between PREM and the 1 M_{\oplus} models using the generalized Ryd-**
300 **berg and the Keane EoS is due to the presence of a lighter element**

301 **than iron in the Earth's core. Therefore, the core radius is about**
302 **140 km smaller in these models if compared to the seismologically**
303 **observed core size.** For the $1 M_{\oplus}$ model using the reciprocal K' EoS, the
304 core density discrepancy in comparison to PREM is due to the EoS param-
305 eters which are obtained by fitting a "solidified" Earth's outer core. As seen
306 in panel (a), relative to PREM the model predicts slightly lower densities for
307 the inner and somewhat higher densities for the Earth's outer core, whereas
308 the predicted core size is close to that given by PREM.

309 The density distribution calculated within the mantle is in excellent agree-
310 ment between the models and PREM. **Only a small discrepancy is ob-**
311 **served for the lowermost mantle using the EoS fitted to laboratory**
312 **compression data. This offset is mainly attributed to the addi-**
313 **tion of magnesiowüstite in the Earth's lower mantle that is not**
314 **accounted for in the models. As a minor constituent, we expect it**
315 **to have only a negligible effect on the interior structure of massive**
316 **terrestrial exoplanets. However, due to a relatively high thermal**
317 **conductivity, it may have an impact on the efficiency of the heat**
318 **transport and thereby might affect the mantle convection pattern.**
319 A relatively small discrepancy is observed in the so-called transition region
320 of the Earth's upper mantle where consecutive phase transitions are occur-
321 ring, since those are not implemented in all three model calculations for the
322 sake of simplicity. Similar observations are valid for the radial distribution
323 of gravitational acceleration and hydrostatic pressure as illustrated in panel
324 (b) and (c), respectively.

325 **In panel (d) of Fig. 1 the radial temperature distributions calcu-**

326 lated for the generalized Rydberg, the Keane, and the reciprocal
327 K' model are compared to the reference geotherm. The tempera-
328 ture profiles of the Earth-sized models are in good agreement with the lat-
329 ter. **For all three models, the calculated upper mantle geotherm**
330 **is slightly colder when compared to the reference.** This difference
331 can be attributed to the simplifying assumption of an upper mantle com-
332 posed of olivine, thereby neglecting pressure-induced phase transitions to β
333 and γ spinel at shallow depth. **The discrepancy of the temperature**
334 **distribution within the lower mantle is attributed to principal un-**
335 **certainties of the calculated radial viscosity profile governed by**
336 **creep activation parameters and the actual grain size distribution**
337 **of lower mantle minerals. Overall, the comparison between the cal-**
338 **culated temperature profiles of the $1 M_{\oplus}$ models and a reference**
339 **geotherm indicates that the mixing length approach is consistent**
340 **with other estimates.**

341 3.1.2. EoS Comparison

342 The density as a function of radial distance from the planet's center is
343 shown in Fig. 2 panel (a). In all models of 5 and $10 M_{\oplus}$, the density increases
344 almost linearly within the dominant mantle regions composed of olivine, per-
345 ovskite, and post-perovskite, respectively. A pronounced density variation of
346 about 700 kg m^{-3} occurs at the phase transition boundary between the up-
347 per olivine mantle and the underlying perovskite layer. Due to similar elastic
348 properties of perovskite and post-perovskite, the phase transition from the in-
349 tervening perovskite layer to the lowermost post-perovskite mantle is hardly
350 visible. **For the models using the generalized Rydberg EoS, density**

351 variations ranging from 8090 ($5 M_{\oplus}$) to 9430 kg m^{-3} ($10 M_{\oplus}$) are
352 found at the core-mantle boundary, which represents a chemical
353 material transition from silicate to iron. In comparison, the mod-
354 els using the Keane EoS produce larger density variations at the
355 core-mantle boundary ranging from 8560 ($5 M_{\oplus}$) to 10,500 kg m^{-3}
356 ($10 M_{\oplus}$). Smaller density variations ranging from 6700 ($5 M_{\oplus}$) to
357 8000 kg m^{-3} ($10 M_{\oplus}$) are observed for the models using the recip-
358 rocal K' EoS. Within the iron cores, the density increases parabol-
359 ically with depth from the core-mantle boundary by about 30 % to
360 reach central densities of 20,600 and 24,900 kg m^{-3} for the models
361 using the generalized Rydberg EoS and by about 40 % to central
362 densities of 23,400 and 30,100 kg m^{-3} for the Keane models, respec-
363 tively. For the models using the reciprocal K' EoS, the density
364 increase amounts to about 35 %, reaching central density values of
365 20,400 and 25,800 kg m^{-3} .

366 Upon comparison of core density distributions predicted by the
367 different models, it is obvious that the difference in core den-
368 sities becomes much larger for more massive model planets. This is
369 related to (a) the extrapolation of different EoS and (b) the corre-
370 sponding increasingly pronounced density discrepancy within the
371 lower mantle. It is seen that the predicted central density for the 1
372 to $10 M_{\oplus}$ exoplanets increases tenfold from about 2 to 20 % compar-
373 ing the generalized Rydberg to the Keane EoS. This corresponds
374 to a core size uncertainty in the range of 5 % for the most massive
375 Earth-like exoplanets.

376 The difference in mantle density at the core-mantle boundary
377 increases from almost zero for the $1 M_{\oplus}$ models to slightly more
378 than 12 % in the case of the $10 M_{\oplus}$ planets when comparing the
379 models using the generalized Rydberg to those using the Keane
380 EoS. A similar comparison between the generalized Rydberg and
381 the reciprocal K' EoS models yield a maximum density discrepancy
382 at the core-mantle boundary of about 15 %. This can be attributed
383 to (a) the extrapolation of different EoS and (b) the method how
384 the material parameters for both the generalized Rydberg and the
385 reciprocal K' model are obtained. The reciprocal K' EoS represents
386 a fit to the Earth's lower mantle and, therefore, accounts for only
387 a small amount of post-perovskite within the lower mantle. Hence,
388 the mantles of model planets calculated by applying the reciprocal K' EoS
389 are predominantly composed of perovskite.

390 Since post-perovskite is expected to be the predominant mineral
391 phase within terrestrial-type exoplanets, the perovskite to post-
392 perovskite phase transition is explicitly incorporated in the models
393 using the generalized Rydberg and the Keane EoS. For example, for
394 a planet ten times as massive as the Earth, our calculations indicate that
395 about 43 % of the planet's radial extent would consist of post-perovskite.
396 Under this assumption, the models using fits to laboratory data
397 are more suitable to predict the deep mantle structure of low-mass
398 exoplanets, because the reciprocal K' EoS does not account for the
399 material parameter change from perovskite to the post-perovskite
400 phase. Nevertheless, we find that the predicted planetary radius

401 **for an $10 M_{\oplus}$ Earth-like exoplanet merely differs by less than 2 %**
 402 **between all three EoS, thereby constraining the maximum uncer-**
 403 **tainty due to the implemented EoS and compositional differences.**

404 For the gravitational acceleration shown in panel (b) of Fig. 2, we observe
 405 a linear increase for all planets from the center to the core-mantle boundary,
 406 where peak values are attained. The subsequent parabolic decrease merges
 407 into a roughly constant gravitational acceleration throughout the mantle.
 408 **The average surface gravity is found to be 20.0 ($5 M_{\oplus}$) and 27.7 m s^{-2}**
 409 **($10 M_{\oplus}$) for the models using the generalized Rydberg EoS, 20.4**
 410 **($5 M_{\oplus}$) and 28.9 m s^{-2} ($10 M_{\oplus}$) for the models using the Keane**
 411 **EoS, and 20.5 ($5 M_{\oplus}$) and 29.2 m s^{-2} ($10 M_{\oplus}$) for those using the**
 412 **reciprocal K' EoS, respectively.**

413 In panel (c) of Fig. 2 variations with radial distance from the planet's
 414 center of the hydrostatic pressure are compared for Earth-like exoplanets of
 415 5 and $10 M_{\oplus}$, respectively. A linear pressure increase through the mantle is
 416 observed in all models. For the models using the generalized Rydberg EoS,
 417 pressures of 645 and 1290 GPa are attained at the core-mantle boundary.
 418 **In comparison, for the models using the Keane and the reciprocal**
 419 **K' EoS we find higher pressures of 701, 1500 and 670, 1430 GPa at**
 420 **the core-mantle boundary, respectively.** Within the core, a parabolical
 421 increase of pressure with depth occurs in all models. **For the $5 M_{\oplus}$ planets,**
 422 **the central pressure is found to be 1920, 2170, and 1910 GPa using**
 423 **the generalized Rydberg, the Keane, and the reciprocal K' EoS,**
 424 **respectively. For the $10 M_{\oplus}$ planets, the corresponding central**
 425 **pressures are 3870, 4110, and 4750 GPa.**

426 Upon comparison of the resulting radial pressure distributions
 427 using generalized Ryberg and Keane EoS, we find that the Keane
 428 EoS predicts systematically higher pressures in the interior of mas-
 429 sive exoplanets. An increasingly pronounced central pressure dis-
 430 crepancy is attaining about 20 % for the $10 M_{\oplus}$ planets. This illus-
 431 trates how differently both EoS extrapolate to the high-pressure
 432 limit. A similar comparison between the central pressures cal-
 433 culated according to generalized Rydberg and reciprocal K' EoS
 434 yields an appreciable smaller pressure discrepancy reflecting the
 435 less dense iron alloy core represented by the reciprocal K' models.
 436 As shown in panel (c), both the central pressure and the pres-
 437 sure at the core-mantle boundary of an Earth-like exoplanets scale
 438 linearly with total mass.

439 Panel (d) of Fig. 2 illustrates the radial distribution of temperature for
 440 Earth-like exoplanets of 5 and $10 M_{\oplus}$, respectively. The temperature in-
 441 creases rapidly from an arbitrarily fixed surface value of 300 to about 1400 K
 442 within the topmost part of the upper mantle as a result of the mixing
 443 length approach. In this region, heat is predominantly transferred by con-
 444 duction, followed by an adiabatic temperature rise across the convecting
 445 mantle. We observe an increasingly super-adiabatic temperature rise toward
 446 the core-mantle boundary. This is interpreted in terms of a sluggish convec-
 447 tive regime within the lowermost mantle of more massive planets. **For the**
 448 **models using the generalized Rydberg EoS, the temperature at the**
 449 **core-mantle boundary is found to be 5510 K for $5 M_{\oplus}$ and 6650 K**
 450 **for $10 M_{\oplus}$. The corresponding central temperatures are 7970 and**

451 9790 K, respectively. Using the Keane EoS, the temperature at the
452 core-mantle boundary reaches 5770 and 7280 K, whereas temper-
453 atures of 9080 and 11,800 K would prevail in the center of 5 and
454 $10 M_{\oplus}$ planets, respectively. Applying the reciprocal K' EoS yields
455 temperatures at the core-mantle boundary of 6470 and 8190 K and
456 corresponding central temperatures of 9730 and 12,600 K, respec-
457 tively. These results differ from those obtained by using parameterized con-
458 vection models, which in general yield substantially colder interiors (Valencia
459 et al., 2006). Compared to Valencia et al. (2006), our results yield
460 a mean temperature value at the core-mantle boundary in excess
461 of 1900 ($5 M_{\oplus}$) and 2500 K ($10 M_{\oplus}$), respectively. In accordance with
462 previous studies (e.g., Seager et al., 2007), the pronounced differences in core
463 temperature do not much affect the overall structure of the corresponding
464 planetary interiors as shown in Fig. 2.

465 3.1.3. Mantle viscosity comparison

466 The radial temperature distribution is mainly controlled by the
467 mantle viscosity, which is treated in form of a temperature- and
468 pressure dependent Arrhenius law. Figure 3 panel (a) shows the
469 calculated mantle viscosity for 1, 5, and $10 M_{\oplus}$ exoplanets with
470 Earth-like bulk composition using the generalized Rydberg mod-
471 els for a case study. It can be seen that the $1 M_{\oplus}$ model has
472 the largest viscosity contrast within the mantle and the highest
473 peak viscosity ($\sim 5 \times 10^{23}$ Pa s) when compared to more massive
474 exoplanets. Large viscosity variations are situated in the upper
475 and the lowermost part of all mantles and can be attributed to

476 thermal boundary layers. Within the $1 M_{\oplus}$ model, we observe
477 the lowest viscosity at the core-mantle boundary, whereas for mas-
478 sive exoplanets the lowest viscosity appears in the upper mantle
479 and higher values are found at the core-mantle boundary due to a
480 more sluggish convective regime. Despite the increasing pressure
481 within more massive exoplanets, mantle viscosities stay relatively
482 constant over an extended mantle region due to the decrease in
483 activation volume and increase of temperature with depth. The
484 viscosity-controlling feedback mechanism involving temperature,
485 pressure, and activation volume seems closely related to the Tozer
486 effect (e.g., Tozer, 1972) as observed on Earth. A detailed analysis
487 of this effect, however, would involve the calculation of thermal
488 history of massive exoplanets.

489 *3.2. Mass-radius Relations and Scaling Laws*

490 Using our model approach, we investigate different types of low-mass
491 exoplanets, characterized by variable bulk compositions. We apply the gen-
492 eralized Rydberg EoS and calculate the planetary radius as a function of
493 mass for internally differentiated Earth-like, Mercury-type, and ocean plan-
494 ets. Figure 4 shows the resulting mass-radius relations for several classes
495 of low-mass exoplanets (dashed lines). Furthermore, the red line in Fig. 4
496 resembles the relation between mass and radius of a self-compressible, pure
497 silicate sphere. It divides fully differentiated dry planets from those contain-
498 ing a certain amount of volatiles. The curves indicating pure H_2O and iron
499 spheres confine the possible range of mass-radius relations of Earth-like and
500 ocean planets. Planets less dense than a pure water-ice sphere must have a

501 significant gas envelope and planets denser than a pure iron sphere are un-
502 likely to exist (Seager et al., 2007). Also included in Fig. 4 is the position of
503 the Earth, which perfectly fits the mass-radius curve for Earth-like planets.

504 Moreover, we show the relative position of the CoRoT-7b exo-
505 planet within its observational uncertainties. The nature of CoRoT-
506 7b has been intensely discussed due to a wide range of mass esti-
507 mates taken from the literature. The discovering paper of Léger
508 et al. (2009) published a planetary radius of $(1.68 \pm 0.09) R_{\oplus}$ which
509 was obtained by analyzing CoRoT lightcurves. Complementing
510 this study, Queloz et al. (2009) reported the first mass estimate
511 for CoRoT-7b of $(4.8 \pm 0.8) M_{\oplus}$ by employing the High Accuracy
512 Radial velocity Planet Searcher (HARPS). Later, Bruntt et al.
513 (2010) evaluated the same data sets and revised the planets radius
514 to $(1.58 \pm 0.10) R_{\oplus}$ and its total mass to $(5.2 \pm 0.8) M_{\oplus}$. Hatzes et al.
515 (2010) and Ferraz-Mello et al. (2010), taking into account the pos-
516 sible presence of more than two planets orbiting CoRoT-7, obtain
517 systematically higher masses of $(6.9 \pm 1.4) M_{\oplus}$ and $(8.0 \pm 1.2) M_{\oplus}$,
518 respectively. Using star spot modeling techniques, Boisse et al.
519 (2011) favor a planetary mass of $(5.7 \pm 2.5) M_{\oplus}$ for CoRoT-7b. Pont
520 et al. (2010) argue that CoRoT-7b may possess a relatively low
521 mass in the order of $1 - 4 M_{\oplus}$ and even questioning its existence.
522 Hatzes et al. (2011) have revisited previous mass estimates and sug-
523 gest a planetary mass of $(7.38 \pm 0.34) M_{\oplus}$ as currently most reliable
524 estimate for CoRoT-7b. Applying the mass-radius relationships
525 shown in Fig. 4, we find that CoRoT-7b is predominantly com-

526 posed of rock and iron, with an iron content ranging from Earth-
527 like (67.5 wt.-% silicate + 32.5 wt.-% iron) to Mercury-like (30 wt.-
528 % silicate + 70 wt.-% iron). Furthermore, we have added other
529 recently discovered transiting low-mass exoplanets such as Kepler-
530 10b (Batalha et al., 2011) that is similar to CoRoT-7b in terms of
531 bulk composition. Contrary, GJ 1214b (Charbonneau et al., 2009),
532 Kepler-11b, and Kepler-11f (Lissauer et al., 2011) rather resemble
533 hot gaseous planets due to their low average densities.

534 Finally, we perform a power law fit of $R/R_{\oplus} \propto (M/M_{\oplus})^{\beta}$ to the calculated
535 mass-radius relations and obtain scaling exponents for the three classes of
536 low-mass exoplanets. Table 3 summarizes the obtained scaling exponents β
537 for the considered low-mass exoplanets valid for a mass range from one to
538 ten times the mass of the Earth. These are in good agreement with scaling
539 laws previously proposed by Valencia et al. (2007a) and Sotin et al. (2007),
540 thereby underscoring the validity of our model approach.

541 4. Discussion and Conclusions

542 This study focuses on the comparison of different equations of
543 state (EoS). The models using the generalized Rydberg and the
544 Keane EoS are consistent with most recent findings obtained from
545 ab initio calculations. Hence, the main advantage is that they
546 account for a post-perovskite phase transformation in the lower-
547 most mantle. Recent ab initio calculations imply that this miner-
548 alogical phase of MgSiO_3 should be stable up to 1 TPa, but its
549 material parameters are subject to discussion and, in comparison

550 to perovskite, less well constrained due to the current limitations
551 of high-pressure experiments. In contrast, the model using the
552 reciprocal K' EoS essentially scales up the present Earth based
553 on seismological evidence, thereby representing a truly Earth-like
554 composition. The main advantage of that approach is that neither
555 high-pressure experiments nor ab initio calculations are involved
556 to a larger extent; the EoS parameters are essentially fitted to the
557 Earth as primary reference.

558 A major uncertainty in modeling the interior structure of low-mass exo-
559 planets arises due to the lack of experimental data to reliably parameterize
560 the EoS in the high-pressure range from 200 up to 10,000 GPa (Seager et al.,
561 2007; Grasset et al., 2009). To address this problem, we have used only EoS
562 that are compliant with the thermodynamics of the high-pressure limit to in-
563 vestigate implications on planetary mass and radius. **Although neither a**
564 **necessary nor a sufficient condition for an accurate EoS implemen-**
565 **tation at a given pressure range, the latter can be used to impose**
566 **an additional constraint for meaningful extrapolation to high pres-**
567 **ures in the absence of experimental and theoretical data. This**
568 **characteristics makes the extrapolation not only consistent with**
569 **the physics of thermodynamic limits but also more trustable.** It
570 should be pointed out, however, that due to the limitation of extrapolation
571 itself no model used so far is capable to account for possible mineralog-
572 ical phase transitions which may occur beyond the post-perovskite phase.
573 Therefore, the extrapolation approach yields only a lower bound of the den-
574 sity distribution within planets more massive than the Earth.

575 As expected, the reciprocal K' EoS yields an almost perfect fit to PREM
576 since the corresponding fit parameters are obtained from a wealth of seis-
577 mological observations. **For planets more massive than the Earth,**
578 **however, the generalized Rydberg and the Keane EoS are found to**
579 **be more suitable since the material parameters of post-perovskite**
580 **are explicitly taken into account.** Post-perovskite is expected to be the
581 predominant mineral phase assemblage present within the interiors of low-
582 mass Earth-like exoplanets. Despite the fact that different methods were
583 applied to fit these EoS to laboratory data or seismological observations, re-
584 spectively, the resultant total planetary radii only differ by a relatively small
585 amount for low-mass Earth-like exoplanets.

586 **To calculate the radial temperature structure within the model**
587 **planets we have applied a mixing length formulation instead of**
588 **parameterized convection models previously used by Valencia et al.**
589 **(2006). Contrary to the latter, the mixing length method uses local**
590 **parameters to describe the efficiency of the heat transport instead**
591 **of allocating a global Rayleigh number to convective mantle layers.**
592 This approach allows to self-consistently calculate radial temperature profiles
593 of massive exoplanets. Whereas Sotin et al. (2007) used fixed temperature
594 variations based on Earth-like values at boundary layers, Seager et al. (2007)
595 treated those as second-order effects and entirely neglect the influence of
596 temperature for planets more massive than the Earth. Compared to the
597 forementioned studies, the models presented in this study result in generally
598 hotter planets, primarily due to a steeper temperature gradient prevalent in
599 the deep interior that is mainly attributed to the pressure-induced increase of

600 viscosity at elevated lower mantle pressures. As a consequence, planets more
601 massive than the Earth will experience an increasing tendency to extended
602 sluggish convective regimes in the lowermost mantle because of the reduced
603 efficiency of convective heat transport with increasing mantle pressures.

604 **Furthermore, a possible metallization of some oxides at ultra-**
605 **high pressures could result in less efficient radiative heat transfer**
606 **(e.g., Umemoto et al., 2006) and generally reduced mantle viscosi-**
607 **ties (Karato, 2011), thereby affecting the thermal state of the low-**
608 **ermost mantle of Earth-like exoplanets. Here, we considered that**
609 **post-perovskite remains an electrical insulator and metallization**
610 **does not occur within the investigated pressure range up to 1.5 TPa.**
611 **This is supported by recent experiments of an MgSiO₃ analog im-**
612 **plying that the dissociation of MgSiO₃ may occur at higher pres-**
613 **ures than previously predicted (Grocholski et al., 2010). Ab initio**
614 **calculations suggest that at least MgO remains insulating before**
615 **metallizing at ultra-high pressures of about 21 TPa (Oganov et al.,**
616 **2003). For the metallic core, we have added terms to account**
617 **for the electronic and anharmonic thermal pressure in accordance**
618 **with ab initio calculations.** Especially the thermal excitation of electrons
619 of metals like iron cannot be neglected, because it amounts to about 20 % of
620 the total thermal pressure under Earth core conditions.

621 In the present study, scaling coefficients are obtained which are in good
622 agreement with those proposed previously for low-mass exoplanets. The dif-
623 ferent EoS used, the slightly different bulk compositions assumed, and the
624 different approaches to model radial temperature profiles are the main reason

625 for the small discrepancies seen in the scaling coefficients. **This similarity**
626 **is mainly attributed to the general insensitivity of mass-radius re-**
627 **lations to the present thermal state of planetary interiors as a con-**
628 **sequence of the increasingly close-packed matter due to elevated**
629 **pressures. This finding is in agreement with previous models by**
630 **Sotin et al. (2007) and Seager et al. (2007), thereby indicating the**
631 **robustness of mass-radius relations and their usage for the classifi-**
632 **cation of extrasolar planets in terms of bulk composition.**

633 According to the predictions of planet formation models and observa-
634 tional surveys, low-mass exoplanets beyond the solar system should be quite
635 abundant (e.g., Howard et al., 2010). The discovery of terrestrial-type exo-
636 planets relies to a large extent on current detection limits of ground-based
637 observational methods. For example, existing space-based telescopes like
638 CoRoT or Kepler, and future space missions under study such as PLATO,
639 are even capable to precisely measure the radius of relatively small planets
640 transiting their host stars. Complementary to this technique, precise radial
641 velocity observations provide the corresponding planetary mass. Our model
642 calculations using different EoS for low-mass exoplanets with fixed bulk com-
643 position indicate that the uncertainty in calculated planetary radius will be
644 substantially smaller than typical measurement uncertainties from transit
645 photometry. Moreover, Fig. 4 clarifies that planetary mass and radius im-
646 pose equally important constraints on model planets as massive as the Earth,
647 whereas in the upper mass range interior structure models are chiefly con-
648 strained by precise determinations of planetary radius. It also illustrates that
649 it is more difficult to distinguish a Mercury-like from an Earth-like planet

650 than an Earth-like from an ocean planet. **Furthermore, if cosmochem-**
651 **ical arguments are taken into account, it is possible to reduce or**
652 **eliminate principal non-uniquenesses in terms of bulk composition.**
653 **For example, the close proximity of CoRoT-7b and Kepler-10b to**
654 **their host stars may suggest that water-ice as a degenerative con-**
655 **stituent would be only minor, or even absent.** To interrelate this, for
656 planetary objects more massive than $10 M_{\oplus}$, typical ground-based surveys
657 with given measurement uncertainties of $\pm 10 \%$ would suffice to readily dis-
658 tinguish between the three principal classes of low-mass exoplanets. In the
659 intermediate mass range from 5 to $10 M_{\oplus}$, space telescopes like CoRoT and
660 Kepler with observational uncertainties of $\pm 5 \%$ are well suited to deliver
661 radius measurements precisely enough to distinguish between terrestrial-type
662 and ocean planets. For Earth-sized exoplanets, however, new space missions
663 such as, e.g., PLATO are needed in combination with equally precise mass
664 determinations to reliably deduce the bulk composition for classification pur-
665 poses.

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Table 1: Fit parameters for the EoS used.

| Material | ρ_0 [Mg m ⁻³] [GPa] | K_0 | K'_0 | K'_∞ | P range |
|---|---|--------|--------------------|----------------------|------------|
| <i>Generalized Rydberg</i> | | | | | |
| Iron ^a (ϵ -Fe) | 8.2694 | 149.4 | 5.650 ^g | 2.943 ^h | 17 – 197 |
| Post-perovskite ^b (MgSiO ₃) | 3.9776 | 204.03 | 4.201 | 2.561 ^h | 0 – 160 |
| Perovskite ^c (MgSiO ₃) | 4.1059 | 270.6 | 3.81 | 2.62975 ^h | 0 – 150 |
| Ice ^d (H ₂ O VII-X) | 1.2495 | 4.26 | 7.75 | – | 0 – 170 |
| <i>Keane</i> | | | | | |
| Iron ^a (ϵ -Fe) | 8.2694 | 164.7 | 5.650 ^g | 2.943 ^h | 17 – 197 |
| Post-perovskite ^b (MgSiO ₃) | 3.9776 | 197.66 | 4.818 | 2.561 ^h | 0 – 160 |
| Perovskite ^c (MgSiO ₃) | 4.1059 | 267.7 | 4.04 | 2.62975 ^h | 0 – 150 |
| <i>Reciprocal K'</i> | | | | | |
| Iron alloy ^e (solidified \oplus 's outer core) | 7.4883 | 169.82 | 4.9835 | 3.0 | 136 – 329 |
| Perovskite ^e (\oplus 's lower mantle) | 4.145 | 234.0 | 4.00 | 2.4107 | 23.8 – 136 |
| <i>Third-order Birch-Murnaghan</i> | | | | | |
| Olivine ^f (Mg ₂ SiO ₄) | 3.2137 | 127.4 | 4.2 | – | 0 – 14 |

References: ^aDewaele et al., 2006; ^bOganov and Ono, 2004; ^cOganov et al., 2001; ^dLoubeyre et al., 1999; ^eStacey and Davis, 2004; ^fKatsura et al., 2009; ^gfixed according to $K'_0 = 2(\gamma_0 + 0.95)$ (Isaak and Anderson, 2003); ^hfixed according to $K'_\infty = 2(\gamma_\infty + 1/6)$ (Stacey and Davis, 2004).

Table 2: Parameters used for the thermal correction according to Mie-Grüneisen-Debye.

| Material | γ_0 | γ_∞ | β | θ_0 [K] |
|---|------------|-----------------|--------------------|-------------------|
| <i>Generalized Rydberg & Keane</i> | | | | |
| Iron ^a (ϵ -Fe) | 1.875 | 1.305 | 3.289 ^e | 430 ^g |
| Post-perovskite ^b (MgSiO ₃) | 1.553 | 1.114 | 4.731 | 1100 ^h |
| Perovskite ^b (MgSiO ₃) | 1.506 | 1.14821 | 7.02469 | 1114 ^h |
| <i>Reciprocal K'</i> | | | | |
| Iron alloy ^c (solidified \oplus 's outer core) | 1.8345 | 1.3333 | 3.506 ^f | 430 ^g |
| Perovskite ^c (\oplus 's lower mantle) | 1.4545 | 1.0387 | 4.460 ^f | 1114 ^h |
| <i>Third-order Birch-Murnaghan</i> | | | | |
| Olivine ^d (Mg ₂ SiO ₄) | 1.31 | — | — | 760 |

No thermal correction has been calculated for the water-ice layer.

References: ^{a)} Dewaele et al., 2006; ^{b)} Ono and Oganov, 2005; ^{c)} Stacey and Davis, 2004; ^{d)} Katsura et al., 2009; ^{e)} assuming $\beta = \gamma_0/(\gamma_0 - \gamma_\infty)$; ^{f)} calculated according to $\beta = \lambda_0\gamma_0/(\gamma_0 - \gamma_\infty)$; ^{g)} Dubrovinsky et al., 2000; ^{h)} Tsuchiya et al., 2004.

Table 3: Scaling exponent β for low-mass exoplanets (1 - 10 M_{\oplus}).

| Composition | Valencia et al. (2007a) | Sotin et al. (2007) | this study |
|--------------|-------------------------|---------------------|------------|
| Earth-like | 0.262 | 0.274 | 0.267 |
| Ocean planet | 0.244 | 0.275 | 0.261 |
| Mercury-type | - | - | 0.269 |

Note: $R/R_{\oplus} \propto (M/M_{\oplus})^{\beta}$.

ACCEPTED MANUSCRIPT

Figure 1: Depth-dependent interior structure of $1 M_{\oplus}$ models and a reference for comparison: (a) the radial distribution of density, (b) the radial distribution of acceleration of gravity, (c) the radial distribution of hydrostatic pressure, and (d) the radial distribution of temperature. The different line styles indicate the chosen EoS, whereas the light gray dashed lines denote the reference model.

Figure 2: Depth-dependent interior structure of Earth-like exoplanets: (a) the radial distribution of density, (b) the radial distribution of acceleration of gravity, (c) the radial distribution of hydrostatic pressure, and (d) the radial distribution of temperature. From top to bottom the curves represent planets with 10 and 5 M_{\oplus} , respectively. The solid black lines correspond to the generalized Rydberg models, whereas the dashed black lines correspond to the Keane models and the solid gray lines to the reciprocal K' models.

Figure 3: (a) Mantle viscosity corresponding to the models using the generalized Rydberg EoS. (b) Creep activation volume V^* as a function of pressure. Note that V^* scales nonlinearly with pressure P that is given in units of Pa in the inset scaling law.

Figure 4: Mass-radius relationships of low-mass exoplanets ranging from 1 to 15 M_{\oplus} . The solid curves are homogeneous, self-compressible spheres of the following materials: water-ice (blue line), Mg-perovskite (red line), and ϵ -Fe (black line). The dashed curves denote differentiated planets of various bulk composition. The red dashed curve is for Earth-like planets with an iron core of 32.5 wt.-% and a 67.5 wt.-% silicate mantle. The blue curve is for ocean planets using the Jovian moon Ganymede as a type-example, resulting in a 45 wt.-% water-ice shell surrounding a 48.5 wt.-% silicate mantle, and a 6.5 wt.-% iron core. The black dashed curve is for iron-rich planets like Mercury composed of a 70 wt.-% iron core overlain by a 30 wt.-% silicate mantle. The triangle indicates the relative position of the Earth. The solid ellipses represent the first low-mass exoplanets with a measured planetary radius and mass according to their observational uncertainties.

Figure 1

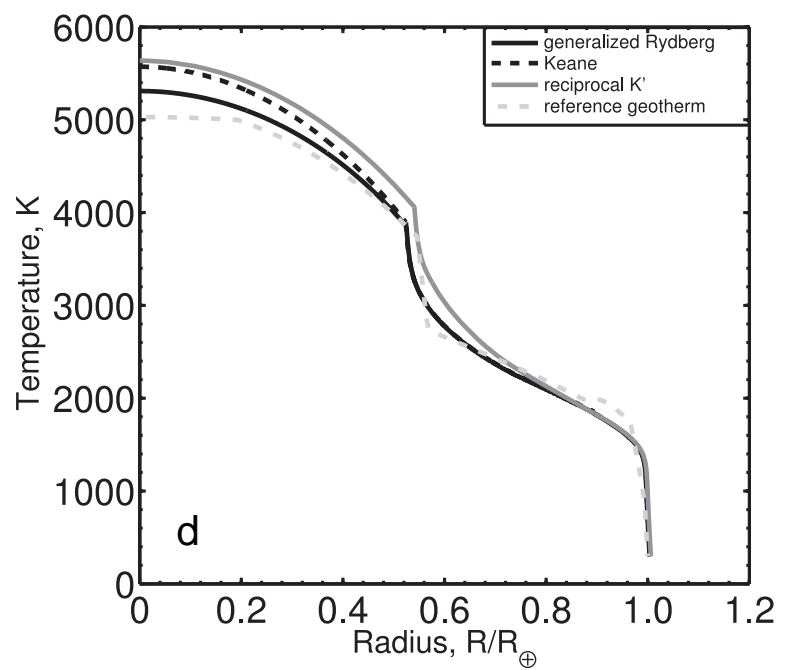
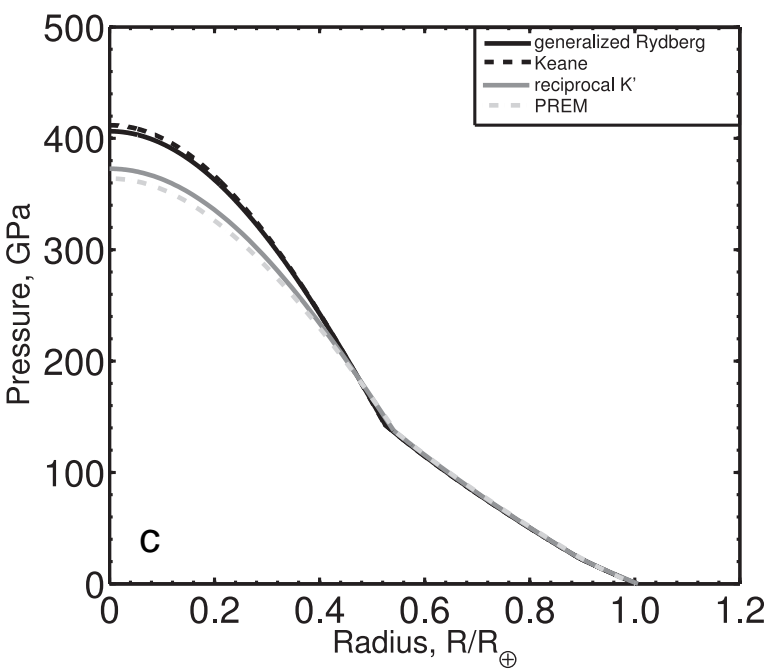
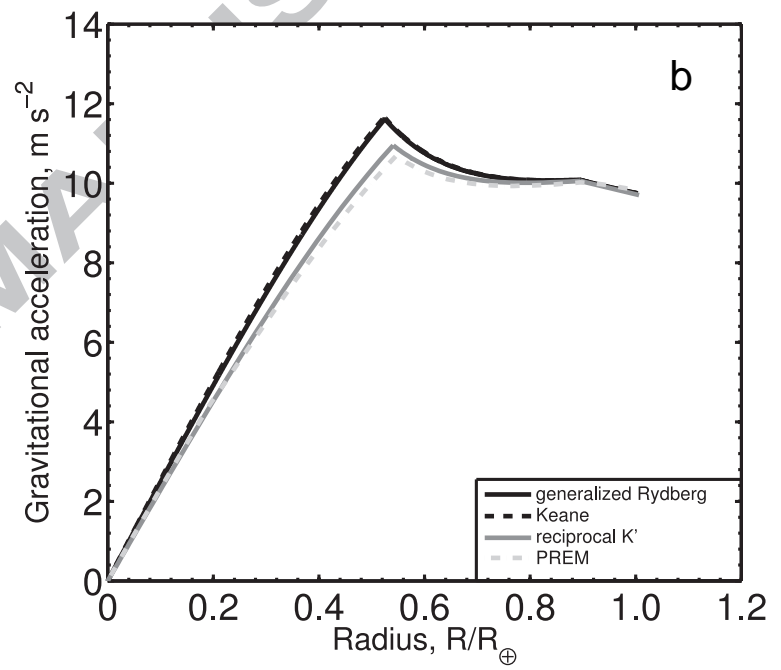
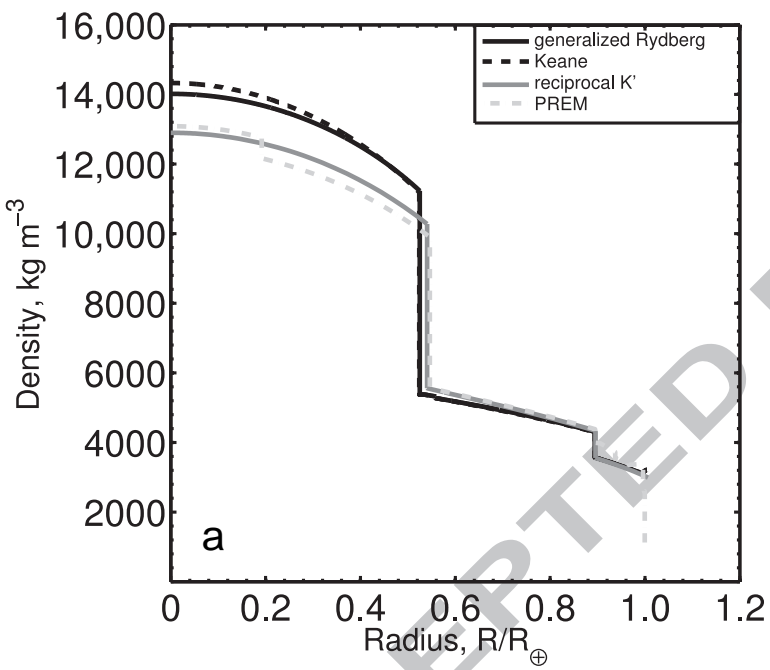


Figure 2

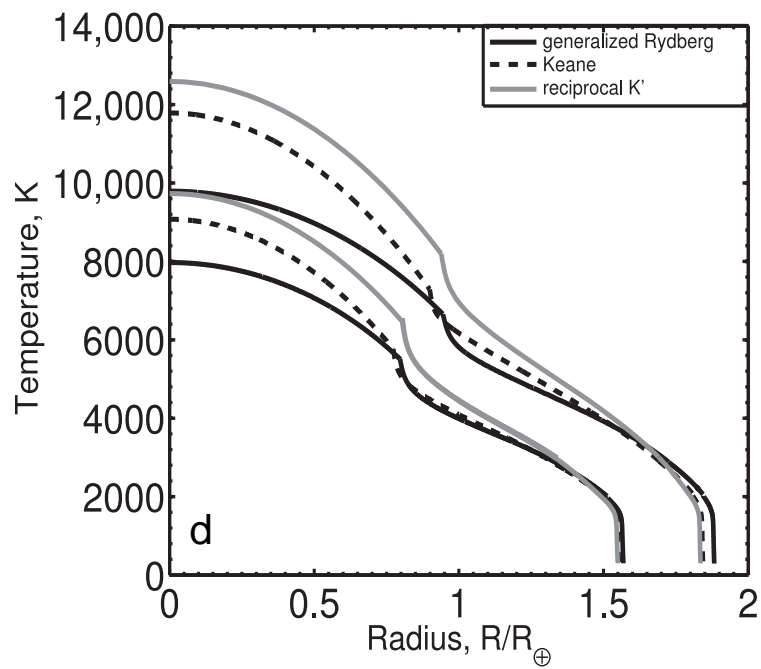
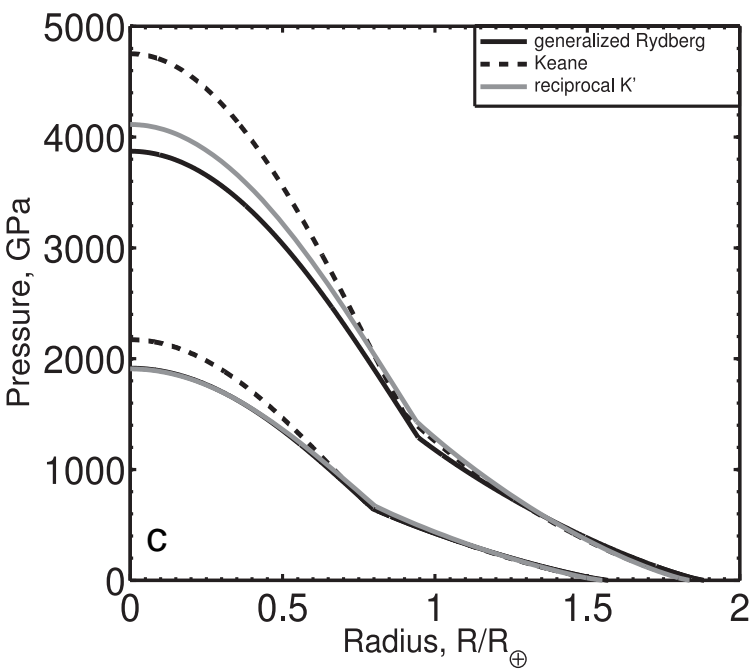
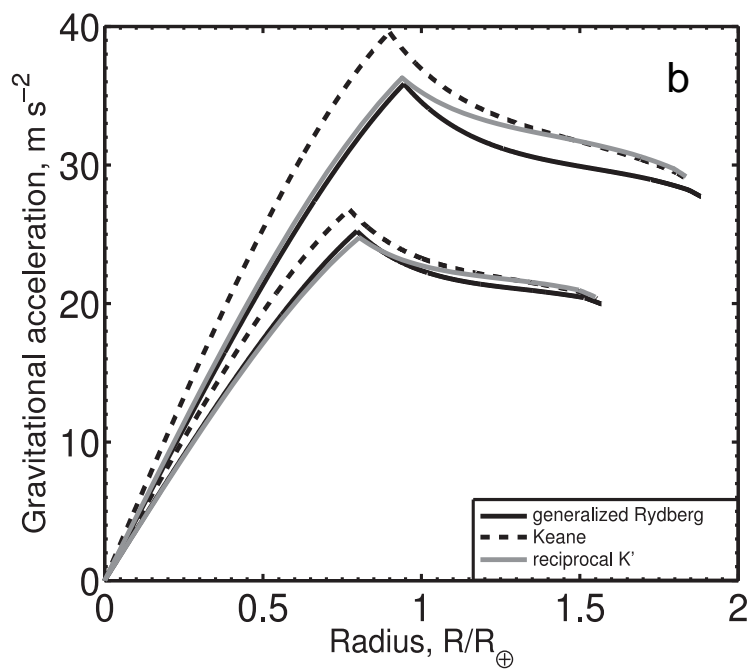
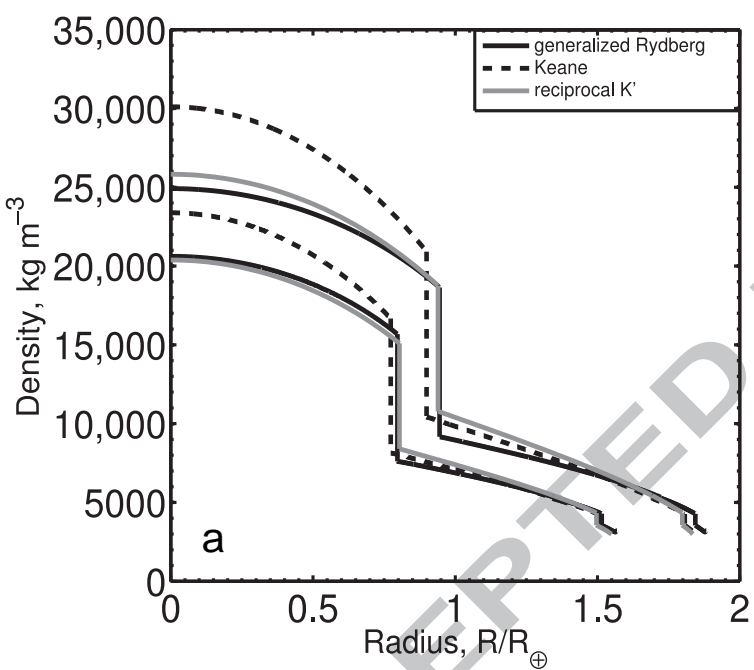


Figure 3

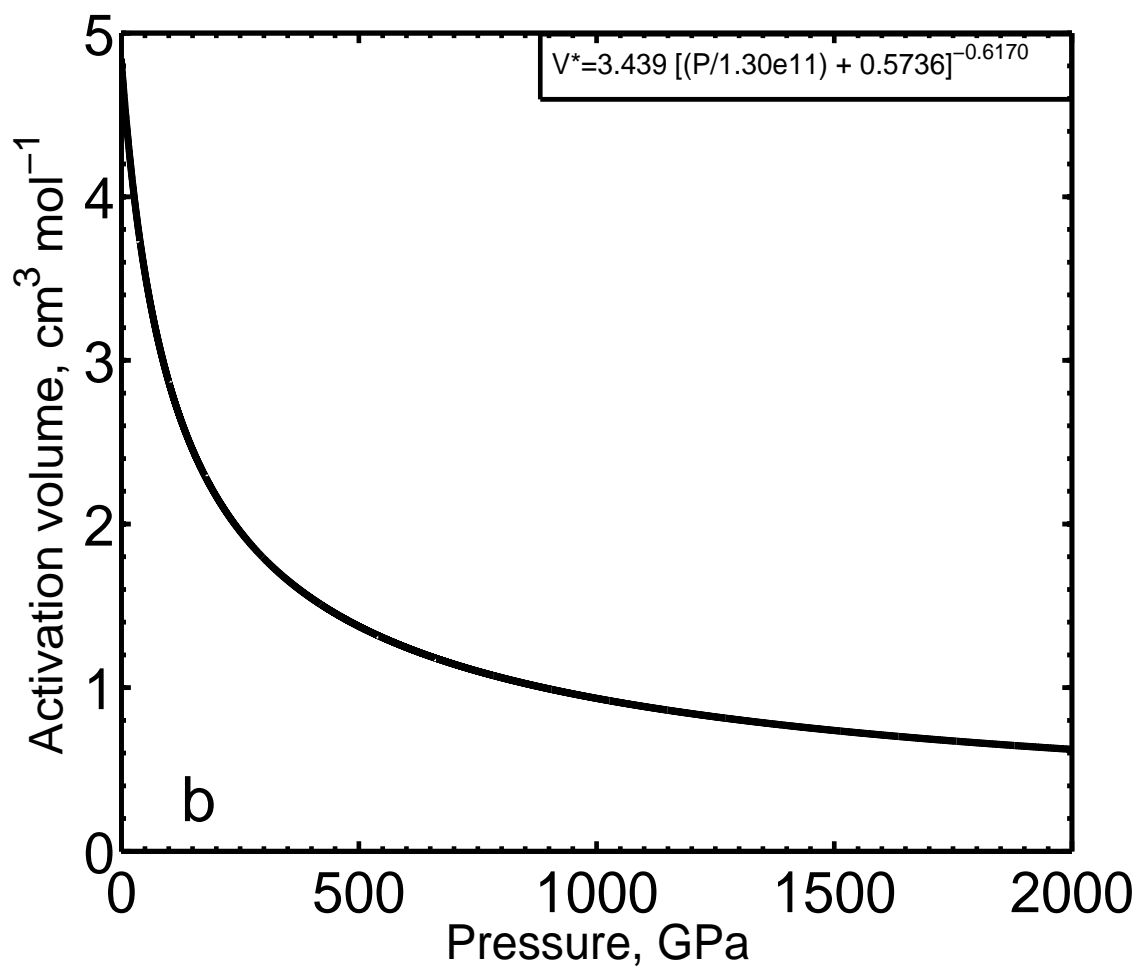
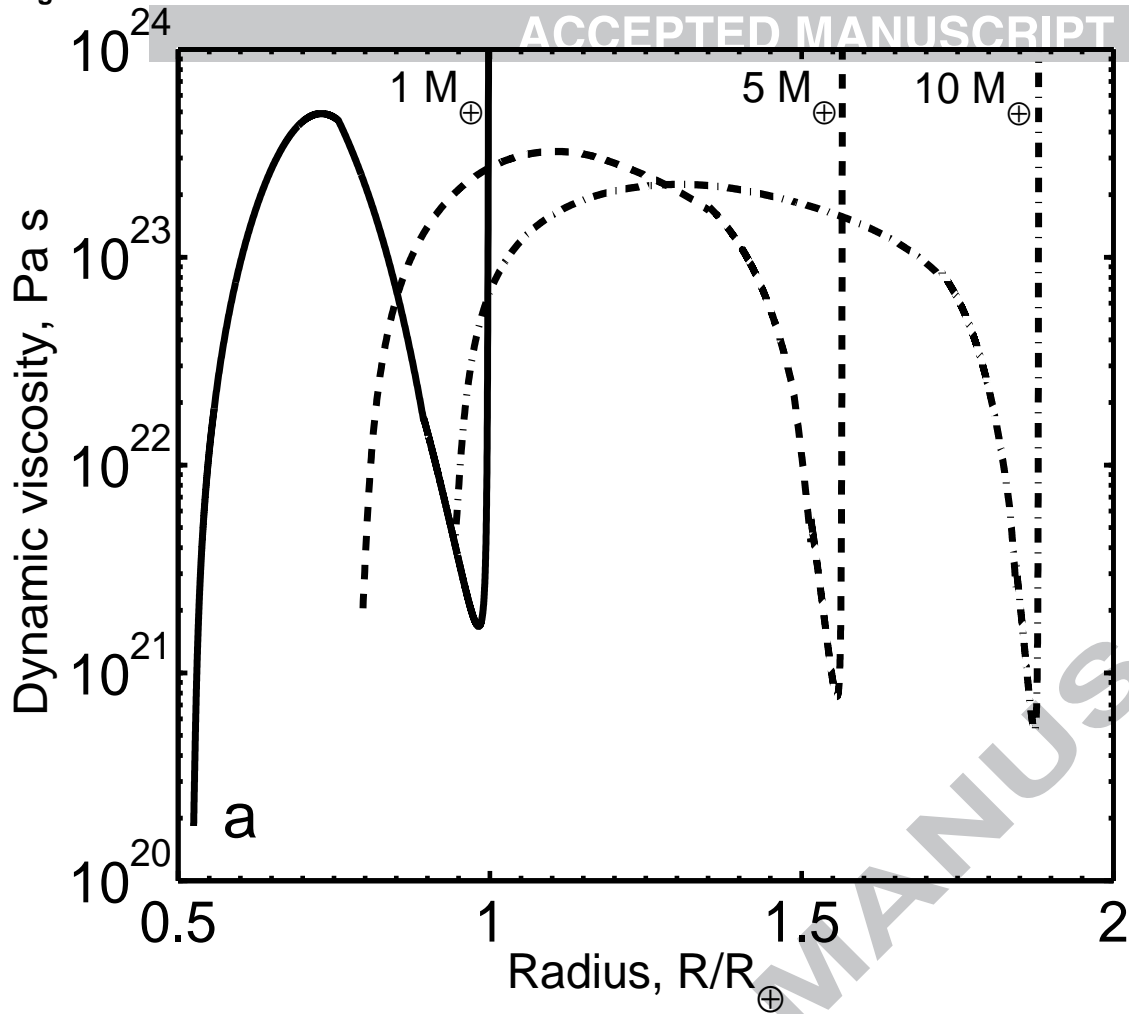
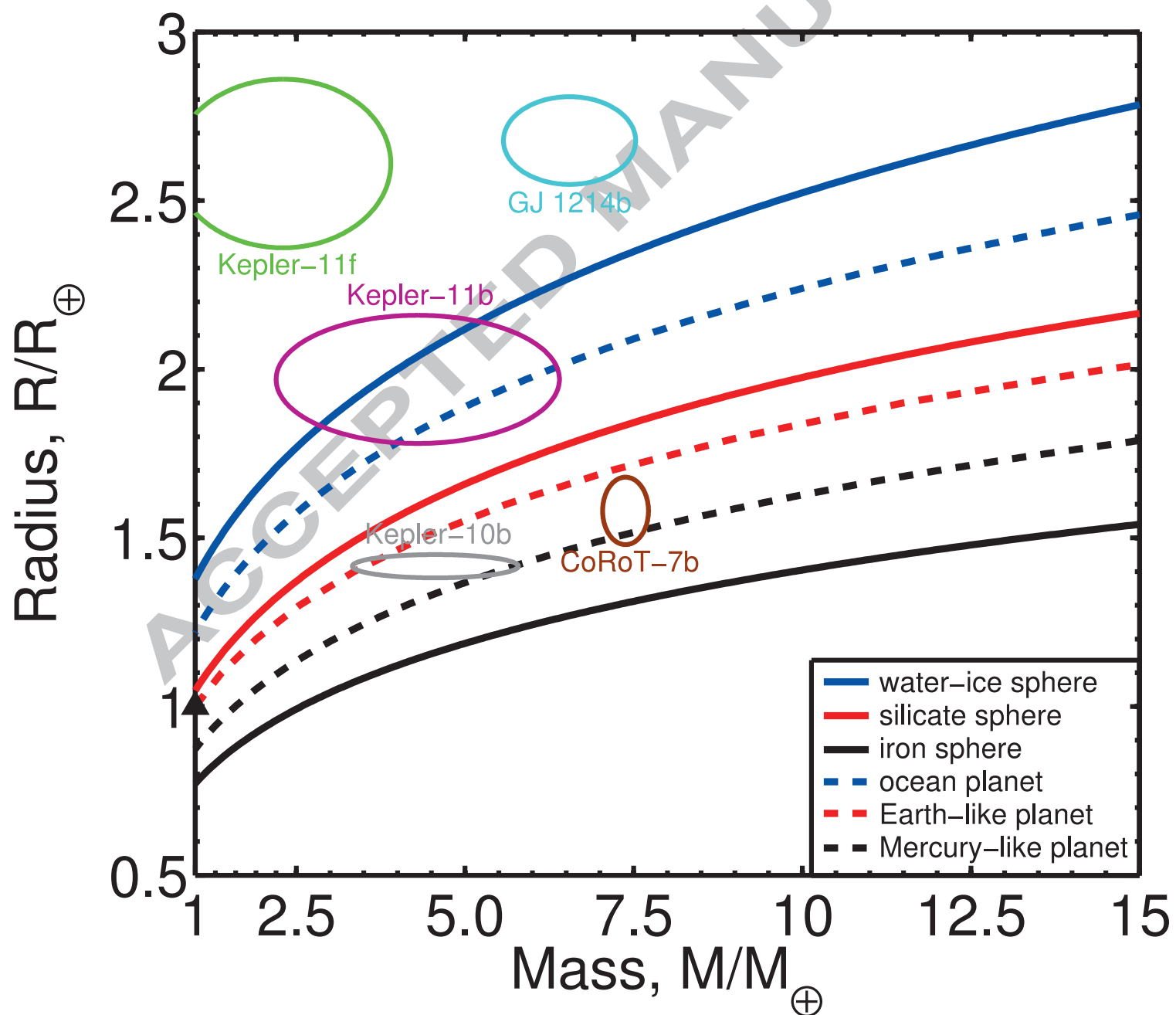


Figure 4



Research Highlights

Structural models of solid exoplanet interiors are constructed using equations of state for the radial density distribution, which are compliant with the thermodynamics of the high-pressure limit. Trade-offs in predicted radii of terrestrial-type exoplanets of up to ten Earth masses fall well within current observational limits. Deep exoplanet interiors are likely hotter than previously thought because of the pressure-induced, less vigorous convective heat transfer at depth.