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To cite this version:

Lucie Baudouin, Simon Neild, Aude Rondepierre, David Wagg. Robust Measurement Feedback Control of an Inclined Cable. IFAC Workshop on Control of Systems Governed by Partial Differential Equations (CPDE), Sep 2013, Paris, France. 6p. hal-00784727

HAL Id: hal-00784727
https://hal.science/hal-00784727
Submitted on 4 Feb 2013

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Robust Measurement Feedback Control of an Inclined Cable

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Abstract: Considering the partial differential equation model of the vibrations of an inclined cable, we are interested in applying robust control technics to stabilize the system with measurement feedback when it is submitted to external disturbances. This paper focuses indeed on the construction of a standard linear infinite dimensional state space system and an $H_\infty$ feedback control of vibrations with partial observation of the state. The control and observation are performed using an active tendon.

Keywords: partial differential equations, robust control, inclined cable, state space model, measurement feedback.

1. INTRODUCTION

Inclined cables are critical components in many structures used in civil engineering - for instance in cable stayed bridges. Nonetheless, cable structures are not restricted to this range of applications and can also be found e.g. in telescopes or spacecraft, Smrz et al. (2011). One of the major issues related to such structures involving cables is the control of vibrations induced by any exterior perturbation.

Since cables are very flexible and lightly damped, cable structure systems usually have a range of dynamic problems. Their modelling is therefore very important in predicting and controlling the response to excitation. A good review about vibration suppression in civil structures and many references on this topic can be found in Song et al. (2006) and Preumont (1997).

In Section 2, we will first recall a linear PDE modeling of an inclined cable before detailing the standard state space model of infinite dimension describing the system, where the control and measurement of the vibrations of the inclined cable come from and active tendon connected to the cable, see Figure 1. Using a support motion in the cable axial direction is a natural choice of active control since the installation of the proper device can be done with small modification of the cable anchorage (see Fujino and Susumpow (1994)). We hope to obtain good results when considering robust control with partial observation using an active tendon since the collocation of actuator and sensor has proved great effectiveness in active damping of cables, as in for example Bossens and Preumont (2001), Song et al. (2006). In a second step, we will detail the state space model of infinite dimension we can derive from the PDE model and that we can use to perform the robust control of the inclined cable.

The $H_\infty$-control question for an infinite dimensional system with measurement feedback will be described in Section 3. In mechanics, the usual study of the vibrations of inclined cables is made through the consideration of a few structural modes. An important aspect of this work lies in the fact that we consider, as far as we can, the complete PDE model of the in-plane vibrations. Here, we will show that we can apply a robust control strategy based on modern control tools for distributed parameter systems.
presented in Bensoussan and Bernhard (1993) and van Keulen (1993). We will indeed prove the $H_{\infty}$-control of the inclined cable, robust with respect to specific exterior perturbations, by the means of the possible resolution of Ricatti equations.

Among the numerous possibilities for modelling the motions of inclined cables with small sag, we adopt the PDE presented in Wagg and Neild (2010) using the derivation from Warnitchai et al. (1995).

The cable, which is of length $\ell$, is supported at end points $a$ and $b$ and the direction of the chord line from $a$ to $b$ is defined as $x$, see Figure 1. The angle of inclination of the chord line relative to the horizontal is denoted $\theta$ and we set $\rho$ the density of the cable, $A$ the cross-sectional area, $E$ Young’s modulus and $g$ gravity. We then define $\rho = \rho g \cos \theta$ as the distributed weight perpendicular to the cable cord. The cable equilibrium sag position and the chord line both lie in the $xz$ plane.

We assume that there is no significant dynamic response along the $x$-axis (the axial vibrations are usually excluded from models since the frequency of oscillations is much faster and of smaller amplitude than that in other directions) and that the sag is small in comparison to the length of the cable.

Each dynamic variable is the sum of a quasi-static component ($q$) and a modal component ($m$):

- $u = u_q(x, t) + u_m(x, t)$ is the dynamic axial displacement of the cable;
- $v$ is the dynamic out-of-plane transverse displacement;
- $w = w_q(x, t) + w_m(x, t)$ is the dynamic in-plane transverse displacement;
- $T_s$ is the static tension of the cable and is assumed to be constant (w.r.t. $x$ and $t$); $T_i = T_q + T_m$ is the dynamic tension of the cable and is assumed not to depend on $x$;
- $w_s(x) = \frac{gA}{2Ts}(x - x^2)$ is the static in-plane displaced shape of the cable.

As one can read (and find details about the modeling of the inclined cable movement) in Wagg and Neild (2010), since we neglect the axial inertial force ($\partial_t u = 0$) we have the following nonlinear equation about the dynamic tension:

$$T_d = AE \left[ \partial_x u + \frac{1}{2} (\partial_x v)^2 + \frac{1}{2} (\partial_x w)^2 + \frac{dw_s}{dx}(x) \right].$$

In this current derivation, because of the intrinsic linearity of the setting we have to fit with, we will not consider the non-linearities that arise usually in modelling the vibrations of an inclined cable. For example, Warnitchai et al. (1995) use a detailed non-linear PDE model which is decomposed into the first few vibration modes from which the precise non-linear coupling between in-plane and out-of-plane vibrations can be seen. Neither do we consider a finite element modelling approach as in Preumont (1997), where one can find an introduction to active tendon control of cables.

We will actually work with a linearized equation of $T_d$:

$$T_d(t) = AE \left[ \partial_x u(x, t) + \frac{dw_s}{dx}(x) \partial_x w(x, t) \right].$$

Considering that this tension due to the dynamics, is small compared to the static tension (i.e. $T_d \ll T_s$), the equation of motion for the dynamic analysis of this inclined cable, are then given by, for all $(x, t)$ in $(0, \ell) \times (0, \infty)$,

$$\begin{align*}
\rho \ddot{A} \partial_x v(x, t) &= T_s \partial_{xx} v(x, t), \\
\rho \ddot{A} \partial_x w(x, t) &= T_s \partial_{xx} w(x, t) + T_d(t) \frac{d^2 w_s}{dx^2}.
\end{align*}$$

2. MODELING OF A CONTROLLED INCLINED CABLE

We are interested in the modeling of the robust feedback control of the vibrations of the cable subjected to perturbations that take the form of in-plane oscillations, using an active tendon as a control/measurement device. We use an infinite dimensional state space approach of the robust control, and it implies a linear model of the system. It is the reason why we linearized the dynamic tension $T_d$. Nevertheless, it also means that we lose the (nonlinear) coupling between $v$ and $w$. Therefore, the fact that the control and perturbations will only act in the sag plane does not allow us to consider the out-of-plane motion $v$ anymore. It won’t appear in the construction of our state space model since it satisfies a conservative wave equation that could only be influenced by coupling nonlinearities we don’t deal with here.

2.1 Partial differential equation model of the inclined cable

One can read in Wagg and Neild (2010) the details about how the decompostion of each displacement $(u, v, w)$ into quasi-static and modal components helps to simplify the analysis of the inclined cable vibration. Anyway, imposing the boundary conditions corresponding to the support motion

$$\begin{align*}
qu(0, t) &= 0, \\
v(0, t) &= 0, \\
w(0, t) &= 0, \\
u(\ell, t) &= u_s(t), \\
v(\ell, t) &= v_s(t), \\
w(\ell, t) &= w_s(t), \quad \forall t \in (0, \infty),
\end{align*}$$

we can calculate the following quasi-static components corresponding to the motion of the cable without taking into account any dynamic response. We obtain

$$\begin{align*}
u_q(x, t) &= \frac{Eq}{E} u_s(t) \left( x - \frac{\ell}{7} \right) - \frac{gA}{2T_s} u_b(t) \left[ \frac{x}{7} - \left( \frac{x}{7} \right)^2 \right] \\
v_q(x, t) &= \frac{\lambda^2 E_q}{4E} u_b(t) \left[ \frac{x}{7} - 2 \left( \frac{x}{7} \right)^2 + \frac{4}{3} \left( \frac{x}{7} \right)^3 \right] \\
w_q(x, t) &= \frac{T_q(t)}{AE} \frac{\ell}{\ell - 2x} \partial_x w_m
\end{align*}$$

for $(x, t) \in (0, \ell) \times (0, \infty)$. $E_q = E/(1 + \lambda^2/12)$ being the equivalent modulus of the cable and $\lambda^2 = E \rho^2 A^3 / T_s^3$, Irvine’s parameter.

Then, assuming that the modal axial displacement is small ($u_m = 0$), the modal dynamic tension satisfies

$$T_m = \frac{gA^2 E}{2T_s} (\ell - 2x) \partial_x w_m$$

and the in-plane modal displacement is solution of the following PDE on $(0, \ell) \times (0, \infty)$:

$$\rho \ddot{A} \partial_x (w_q + w_m) + T_s \partial_{xx} w_m = T_m \frac{gA}{T_s}.$$
submitted to homogeneous Dirichlet boundary conditions
\[ w_m(t, 0) = 0, \quad w_m(t, \ell) = 0, \quad \forall t \in (0, \infty) \]
and initial conditions equal to zero. Since we can calculate easily \( \partial_t w_m \) from (1), we obtain the self-contained equation:
\[
\partial_t w_m = \frac{T_s}{\rho A} \partial_{xx} w_m - \frac{\rho^2 A^2 E}{2\rho T_s^2} (\ell - 2x) \partial_x w_m - \frac{x}{\ell} \omega_0^2 w_m + \frac{\rho E_0 A^2}{2T_s^2} \left[ \frac{x}{\ell} - \left( \frac{x}{\ell} \right)^2 \right] w_m'' + \frac{\rho^2 E_0 A^2}{2T_s^2} \left[ \frac{x}{\ell} - \left( \frac{x}{\ell} \right)^2 \right] w_m''
\]
\[ (2) \]
\[ \text{with the measurement output} \]
\[ T_d = T_q + T_m = \frac{AE_0}{\ell} \omega_0^2 \rho T_s^2 (\ell - 2x) \partial_x w_m. \]
\[ (4) \]
\[ 2.2 \text{ Measurement and control of the cable} \]

We are considering an inclined cable as in Figure 1 perturbed by in-plane oscillations \((u, \omega)\) and connected at its bottom end with an active tendon. As we can read in Preumont and Bossens (2000), an active tendon consists of a displacement actuator (e.g. piezoelectric) collocated with a force sensor. Here, a small discussion is necessary. A tendon is principally meant to have an axial movement, that corresponds to a control \(u\), which is then an additive displacement term to the perturbation \(w\) in equation (2). But we can also prove that if we only consider this inertial control, we only have access to the symmetric modes of vibration. And because of the linear framework, we loose the complementary parametric control (see Preumont (1997)). To overcome this, we will consider that the control also acts through the in-plane bottom displacement as a term \(w\), which will be an added to the perturbation \(w\).

Besides, the force sensor allows to define the observation we will measure, in order to build our feedback, as the dynamic tension \(T_q\).

Let us now incorporate these facts into the PDE model. Since the perturbations of the system \(u\) and \(w\) will be proportional to \(\cos(\omega t)\), we can assume that \(u_0(t) = -\omega_0^2 \rho u(t)\) and \(u_0''(t) = -\omega_0^2 \rho u(t)\) and obtain the following state equation
\[
\partial_t w_m = \frac{T_s}{\rho A} \partial_{xx} w_m - \frac{\rho^2 A^2 E}{2\rho T_s^2} (\ell - 2x) \partial_x w_m + \frac{\rho E_0 A^2}{2T_s^2} \left[ \frac{x}{\ell} - \left( \frac{x}{\ell} \right)^2 \right] w_m'' - \omega_0^2 \frac{\rho E_0 A^2}{2T_s^2} \left[ \frac{x}{\ell} - \left( \frac{x}{\ell} \right)^2 \right] w_m'' + \frac{\rho^2 E_0 A^2}{2T_s^2} \left[ \frac{x}{\ell} - \left( \frac{x}{\ell} \right)^2 \right] w_m''
\]
\[ (3) \]

A linear infinite-dimensional state space model derived from the PDE model presented above will be used in the sequel. It will take the usual shape (Doyle et al. (1996))
\[
\begin{cases}
X'(t) = AX(t) + B_1 W(t) + B_2 U(t), & \forall t \geq 0 \\
X(0) = 0, \\
Z(t) = C_1 X(t) + D_{12} U(t), \\
Y(t) = C_2 X(t) + D_{21} W(t).
\end{cases}
\]
\[ (5) \]

In order to fit in this formalism, the following notations are introduced:
- The state vector is \(X = (w_m, \partial_t w_m)\);
This section is devoted to recalling the $H_\infty$-robust control theory proved in Bensoussan and Bernhard (1993) and/or van Keulen (1993) that we want to apply on the PDE model we derived. This result gives an equivalence between the $H_\infty$-robust control with measurement-feedback of an infinite-dimensional system and the solvability of two Riccati equations. We will explain later how this theorem can be applied to the model we consider, following the assumptions recalled below. For a survey of the $H_\infty$-control theory with state-feedback in the infinite-dimensional case, one can also read van Keulen et al. (1993). The main results in all these articles are a generalization of the finite-dimensional regular $H_\infty$-control problem (see for instance Doyle et al. (1996) and Tadmor (1990)) presented in a standard state-space approach.

Let us now make precise what is meant by $H_\infty$-optimal control (or robust control) with measurement feedback. The description (5) of the system we consider implies that, as in Figure 2, the plant $P$ is to be controlled under the cost function (related to the output $Z$)

$$J_0(U,W) = \int_0^\infty (|C_1 X|^2 + |D_{12} U|^2) \, dt$$

and the partial observation $Y = C_2 X + D_{21} W$. The point is to construct a feedback controller $K = (L, M, N, \bar{R})$ of the form

$$\begin{cases} 
\Phi'(t) = (A + M)\Phi(t) + NV(t), \\
\Phi(0) = 0, \\
U(t) = L\Phi(t) + RY(t).
\end{cases}$$

(6)

where $\Phi$, the adjoint state, depends on the measurement $Y$ and leads to the control $U$. The coupled system is then as follows:

$$\begin{cases} 
X' = (A + B_2 RC_2) X + B_2 L \Phi + B_1 N W \\
\Phi' = (A + M) \Phi + NC_2 X + ND_{21} W
\end{cases}$$

and introduces an operator $\Lambda = \begin{pmatrix} A + B_2 RC_2 & B_2 L \\
NC_2 & A + M \end{pmatrix}$.

The goal is to find a dynamic measurement-feedback controller $K$ that exponentially stabilizes this system (meaning that $A$ is exponentially stable, and also yield to a finite cost $J_0(\Phi + RY, W)$) and ensures that the influence of the disturbances on the “to be controlled output”, i.e. the ratio

$$\rho(K) = \sup_{W \in \mathcal{W}} \frac{J_0(L\Phi + RY, W)}{\|W\|_{\mathcal{W}}}$$

is smaller than some specific bound.

The main result we apply to the dynamic measurement feedback control of an inclined cable is the following.

**Theorem 1.** [Proof to be read in Bensoussan and Bernhard (1993) or van Keulen (1993)] Let $\gamma > 0$ and assume that the pair $A, B_1$ is stabilizable and that $A, C_1$ is detectable. The following assertions are equivalent:

(i) The $\gamma^2$-robustness property with partial observation hold for the system (5);

(ii) There exists an exponentially stabilizing dynamic output-feedback controller $K$ of the form (6) such that $A$ is exponentially stable and $\rho(K) < \gamma^2$;

(iii) There exist two nonnegative definite symmetric operators $P, \Sigma \in L(\mathcal{X})$ solutions of the Riccati and compatibility equations:

- $\forall X \in D(A), \forall X \in D(A^*)$
  $$(PA + A^*P + P(B_2 B_2^* - \gamma^{-2} B_1 B_1^*) + P(\Sigma)) X = 0$$

- $A - (B_2 B_2^* - \gamma^{-2} B_1 B_1^*) P$ generates an exponentially stable semigroup;

- $\forall X \in D(A^*)$, $PX \in D(A)$,
  $$(\Sigma A^* + \Sigma A + \Sigma(C_2^* C_2 - \gamma^{-2} C_1^* C_1) + B_1 B_1^*) X = 0$$

- and $A^* - (C_2^* C_2 - \gamma^{-2} C_1^* C_1) \Sigma$ generates an exponentially stable semigroup;

- $I - \gamma^{-2} \Sigma P$ is invertible and $P = (I - \gamma^{-2} \Sigma P)^{-1} \geq 0$.

Moreover, if the conditions (7)-(9) hold, then the feedback controller $K$ given by

$$\begin{align*}
M &= -(B_2 B_2^* - \gamma^{-2} B_1 B_1^*) P - P C_2 C_2, \\
N &= P C_2, \\
L &= -B_2^* P, \\
R &= 0
\end{align*}$$

(10)

gives an exponentially stable operator $A$ and guarantees that $\rho(K) < \gamma^2$. Finally, if the solutions to the Riccati equations exists, then they are unique. □

One can notice that the feedback controller $K$ given in (10) is actually sub-optimal and known as the central controller.

Since $B_1, B_2, C_1, C_2, D_{12}$ and $D_{21}$ are bounded operators well defined in the appropriate spaces we will be allowed to apply Theorem 1 if we can confirm that $(A, B_1)$ is stabilizable and $(A, C_1)$ is detectable.

The proof that $(A, B_1)$ is controllable (implying stabilizability) relies on the study of the controllability through $u_b$ and $w_b$ of the wave equation

$$\partial_t w_m - \frac{T_e}{\rho_A} \partial_{xx} w_m + \frac{q^2 A^2 E}{2 \rho T_e^2} (\ell - 2x) \partial_{xx} w_m$$

$$= -\omega_0^2 \frac{q E A^2}{2 T_e^2} \left[ x - \left( \frac{x}{\ell} \right)^2 \right] u_b + \omega_0^2 \frac{x}{t} w_b.$$  

(11)

The main difficulty comes from the fact that the control input has a prescribed shape in space. The control $u_b(t)$ alone is not sufficient because of the symmetry property of the function in $x$ in front of it (see the $c_i$ in section 4). We do not wish to give the proof here, but the additional...
control \( w_r(t) \) is used to overcome this restriction. One can consult Tucsnak and Weiss (2009) for details about observability results for the wave equation (even with a first order term), and to see the proof that observability is also true with a control acting only on the boundary of the domain. Therefore, we have \((A, B_1)\) stabilizable. Finally, since \( C_1 X = w_m \) on the whole domain, one proves easily an observability estimate and we obtain that \((A, C_1)\) is detectable. This closes the verification of the assumptions of Theorem 1.

Considering that we have now a well-posed robust control problem in infinite dimension, we would like to perform some numerical experiments to illustrate the results we can obtain.

4. A TRUNCATED MODEL FOR NUMERICAL DESIGN

4.1 Model of finite dimension

The goal of this section is to define an appropriate finite-dimensional model of the PDE system (5) whose state representation can be written as:

\[
\begin{pmatrix}
X_N(t) \\
Y_N(t) \\
Z_N(t)
\end{pmatrix} = \begin{pmatrix}
A_N & B_{2,N} & B_{1,N} \\
C_{2,N} & D_{21,N} & D_{12,N}
\end{pmatrix} \begin{pmatrix}
U(t) \\
W(t)
\end{pmatrix},
\]

where the operators of system (5) are replaced by real-valued matrices computed on a truncated basis of the N first eigenfunctions precisely defined below. \( X_N \in \mathbb{R}^{2N} \) is the state vector, \( Y_N \in \mathbb{R}^{N} \) is the measurement output vector, \( Z_N \in \mathbb{R}^{N+2} \) is the to-be-controlled output vector, \( W \in \mathbb{R}^{3} \) and \( U \in \mathbb{R}^{2} \) are still the exogenous perturbation and the control. The truncation of the PDE system can be seen as a way of coming back to the structural vibrations of the system.

In order to compute these objects, we choose to use everywhere the Hermitian base of \( L^2(0, \ell) \) given by the eigenfunctions of the (compact self-adjoint) operator \( \frac{T_s}{\rho A} \partial_x^2 \). The orthonormal base \((\phi_i)_{i \in \mathbb{N}}\) is indeed defined by:

\[
\phi_i(x) = \sqrt{\frac{2}{\ell}} \sin \left( \frac{i \pi x}{\ell} \right), \quad \omega_i = \frac{i \pi}{\ell} \sqrt{\frac{T_s}{\rho A}}
\]

and satisfies for all \( x \in (0, \ell) \) and \( i \in \mathbb{N}^* \),

\[
\frac{T_s}{\rho A} \partial_x \phi_i(x) = -\omega_i^2 \phi_i(x).
\]

This approach meets the Galerkin method used in (Wagg and Nelid, 2010, chap 7) and the point is that every \( y \in L^2(0, \ell) \) can be written

\[
y(x) = \sum_{i \geq 1} y_i \phi_i(x),
\]

\((y_i)_{i \in \mathbb{N}}\), being a sequence of real numbers satisfying

\[
y_i = y(y, \phi_i) := \int_0^\ell y(x) \phi_i(x) \, dx \quad \text{and} \quad \sum_{i \geq 1} y_i^2 < \infty.
\]

Given \( N \in \mathbb{N} \), we compute \( A_N, B_{1,N}, B_{2,N}, C_{1,N}, C_{2,N}, D_{12,N} \) and \( D_{21,N} \) using the truncated basis \( \{\phi_1, \ldots, \phi_N\} \). We make the assumption that the tuning parameters \( d = (d_i) \) are vectors of real numbers and recall that it is a weighting function of the disturbance signal \( W_{\text{mod}} \) that corresponds for instance to the “forgotten” non-linearities. We obtain:

\[
A_N = \text{block}_{i,j} \left( \begin{bmatrix}
-\omega_i^2 \delta_{ij} - \frac{\varrho}{\rho T_s} \delta_{ij} \\
0
\end{bmatrix} \right),
\]

where \( \delta_{ij} \) is the Kronecker symbol and

\[
a_{ij} = \left\langle \frac{\varrho A^2}{2T_s} (\ell - 2x) \partial_x, \phi_i \right\rangle.
\]

Using these matrices, we make the assumption that \( a_{ij} \) is non-zero if \( i \neq j \) and \( a_{ii} \) if \( i = j \). We can then perform some numerical experiments to illustrate the results we can obtain.

4.2 Numerical simulations

In the numerical simulations we use the parameter values given in Table 1 as chosen in Gonzalez-Buelga et al. (2008) to approximately match a typical full-scale bridge cable inclined at \( \theta = 20^\circ \) to the horizontal, of length 400m, mass per unit length 130kg.m\(^{-1}\) and tension 8000kN:

| Cable length | \( \ell \) | 1.98 m |
| Density \( \rho \) | 1.34 \times 10^3 \text{ kg.m}^{-3} |
| Cross sectional area \( A \) | 0.5 \times 10^{-6} \text{ m}^2 |
| Static tension \( T_s \) | 265 N |
| Steel Young’s modulus \( E \) | 200 \times 10^9 \text{ N.m}^{-2} |

Table 1. Cable characteristics

This gives the following parameters: \( E_g = 174 \times 10^9 \text{ N.m}^{-2} \), \( \lambda^2 = 1.74 \) and \( \varrho = \rho g \cos \theta = 12.35 \times 10^6 \text{ kg.s}^{-2}.\text{m}^{-2} \). These values correspond to an inclined steel cable experiment that could be used in a next step to implement our controllers. We also choose the perturbation frequencies
$\omega_u = \omega_m = 55.4\text{rad.s}^{-1}$, i.e. close to the second linear inplane natural frequency mode where the studies usually focus when studying the effect of the nonlinear modal coupling (not considered here).

We can illustrate the results of the closed loop control based on this truncated state model of the PDE modelling in several ways. Through the singular values of the frequency response, Figure 3 presents the attenuation of the first modes of vibrations for the case $N = 5$, with respect to the uncontrolled open loop. The $H_\infty$ optimal controller is computed using the hinfsyn Matlab function.

![Figure 3](image)

**Fig. 3.** Singular values of the frequency respond of the truncated model in open and closed loop.

### 5. DISCUSSION AND CONCLUSION

The goal of this article was to study the robust measurement feedback control of an infinite-dimensional state space model of an inclined cable. After dealing with several modelling issues, we were able to ensure the $H_\infty$-control of the system under the condition of solvability of two Riccati equations. This allowed us to perform some numerical computations on a truncated model in order to illustrate the action of the robust controller. We examined the possibility of connecting the inclined cable to an active tendon in order to bring active damping into the cable structure and as far as we know, there exists no such study of the robust control of an inclined cable when the partial differential equation model is used.

We can mention several new directions that we intend to pursue as future extensions of this study. Some of them concern the modeling of the controlled inclined cable. First we would propose a more sophisticated modeling of the output $Z_N$ (tuning the attenuation of the modes for instance) and of the $H_\infty$ channel $W \rightarrow Z_N$ but also to compare the control results we manage to obtain applied to the non-linear model of 2 or more modes that one can read in Wagg and Neild (2010). Then the next challenging step of this preliminary study would be to propose a multi-objective design problem, where the feedback controller has to respond favorably to several performance specifications: typically we could consider the active tendon control as a mixed $H_2/H_\infty$ synthesis problem, where the $H_\infty$ channel is used to enhance the robustness and the $H_2$ channel guarantees good performance despite e.g. measurement noise or uncertainty on the cable dynamic. Last but not least, improvements in the modeling of the inclined cable itself could be studied. Indeed when linearizing the infinite dimensional model, we lost the parametric control and the nonlinear coupling between $w_m$ and $v_m$ and therefore needed another control $w_c$ and neglected the out-of-plane motion $v_m$. Another objective would indeed be to find a way, maybe using a modified version of the dynamic tension that carries the coupling, to include some nonlinearities in the control loop in order to define a more complete system to stabilize. This will be considered in future studies.

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