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Contribution to the Constrained Output Feedback Control

Souad Bezzaoucha, Benoît Marx, Didier Maquin, José Ragot

Abstract—In this paper, a Takagi-Sugeno model is used to represent the nonlinear behaviour of an actuator with saturation constraint. The control design is based on an output feedback controller (static or dynamic) depending on the saturation levels. Stabilization conditions are derived using the Lyapunov method and expressed in terms of linear matrix inequalities. The proposed representation describes the saturation model during the controller design in order to obtain a controller which parameters depend on the saturation bounds. The design of stabilizing controllers for T-S nonlinear systems with sector nonlinearity approach is used to represent the nonlinear behaviour of an actuator with sector nonlinearity. A Takagi-Sugeno (T-S) formalism is envisaged for the global representation of nonlinear systems by the interpolation of a set of linear models. Each submodel contributes to the global behaviour of the system through a weighting function $h_i(\xi(t))$ [11]. The T-S structure is given by

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{n} h_i(\xi(t)) (A_i x(t) + B_i u(t)) \\
y(t) &= \sum_{i=1}^{n} h_i(\xi(t)) (C_i x(t) + D_i u(t))
\end{align*}
\]

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the control input and $y(t) \in \mathbb{R}^q$ is the system output. $\xi(t) \in \mathbb{R}^q$ is the decision variable assumed to be measurable (as the system output) or known (as the system input). The weighting functions $h_i(\xi(t))$ satisfy the so-called convex sum property

\[
\sum_{i=1}^{n} h_i(\xi(t)) = 1, \quad 0 \leq h_i(\xi(t)) \leq 1, \quad i = 1, \ldots, n
\]

In the remaining of the paper, the following lemma is used.

Lemma 1: For any matrices $X$, $Y$ and $G = G^T > 0$, the following inequality holds

\[
X^T Y + Y^T X \leq X^T G X + Y^T G^{-1} Y
\]

The following notations are used throughout the paper: a block diagonal matrix with the square matrices $A_1, \ldots, A_n$ on its diagonal is denoted $\text{diag}(A_1, \ldots, A_n)$. The smallest and largest eigenvalues of the matrix $M$ are respectively denoted $\lambda_{\text{min}}(M)$ and $\lambda_{\text{max}}(M)$. The saturation function for...
a signal $v(t)$ is defined as (4), where $v_{max}$ and $v_{min}$ denote the saturation levels.

$$sat(v(t)) := \begin{cases} v(t) & \text{if } v_{min} \leq v(t) \leq v_{max} \\ v_{max} & \text{if } v(t) > v_{max} \\ v_{min} & \text{if } v(t) < v_{min} \end{cases}$$ (4)

### III. Problem statement

A first contribution of this work is to model the nonlinear actuator saturation using the Takagi-Sugeno representation. For that, it is proposed to re-write the $j^{th}$ component of the saturated control input (denoted $u_{sat}^j(t)$, for $j = 1, \ldots, n_u$) under a particular form of the $j^{th}$ component of the $j^{th}$ input vector components (7), as follows:

$$u_{sat}^j(t) = \begin{cases} \text{sat}(u_j(t)), & f = 1, \ldots, n_u \\ \sum_{i=1}^{n_u} \mu_i(t) \left( \lambda_i^j u_j(t) + \gamma_i^j \right) \end{cases}$$ (5)

with: $\lambda_i^j = 0, \lambda_j^i = 1, \lambda_j^i = 0; \gamma_i^j = u_{min}^j, \gamma_2^j = 0, \gamma_3^j = u_{max}^j$

and the weighting functions

$$\begin{cases} 
\mu_i^j(t) = \frac{1 - \text{sign}(u_j(t) - u_{min}^j)}{n_u} & \\
\mu_2^j(t) = \frac{\text{sign}(u_j(t) - u_{min}^j) - \text{sign}(u_j(t) - u_{max}^j)}{2} & \\
\mu_3^j(t) = \frac{1 + \text{sign}(u_j(t) - u_{max}^j)}{2} 
\end{cases}$$ (6)

Based on the convex sum property (2) of the weighting functions (7), equation (5) can be written in order to have the same activation functions for all the input vector components:

$$u_{sat}^j(t) = \sum_{i=1}^{n_u} \mu_i(t) \left( \lambda_i^j u_j(t) + \gamma_i^j \right)$$ (8)

For $n_u$ inputs, $3^{n_u}$ submodels are obtained. It is important to note that the saturated control is directly expressed in terms of the control variable $u(t)$. Equation (8) is equivalent to

$$u_{sat}(t) = \sum_{i=1}^{3^{n_u}} \mu_i(t) (\Lambda_i u(t) + \Gamma_i)$$ (9)

where the global weighting functions $\mu_i(t)$, the matrices $\Lambda_i \in \mathbb{R}^{n_u \times n_u}$ and vectors $\Gamma_i \in \mathbb{R}^{n_u \times 1}$ are defined as follow

$$\begin{cases} 
\mu_i(t) = \prod_{j=1}^{n_u} \mu_i^j(u_j(t)) \\
\Lambda_i = \text{diag}(\lambda_1^i, \ldots, \lambda_{n_u}^i) \\
\Gamma_i = \left( \gamma_1^i, \ldots, \gamma_{n_u}^i \right)^T 
\end{cases}$$ (10)

The indexes $\sigma_i^j(i = 1, \ldots, 3^{n_u})$ and $j = 1, \ldots, n_u$, equal to 1,2 or 3, indicate which partition of the $j^{th}$ input ($\mu_1^j, \mu_2^j$ or $\mu_3^j$) is involved in the $i^{th}$ submodel.

The relations between the submodel number $i$ and the $\sigma_i^j$ indices are given by the following equation:

$$i = 3^{n_u-1} \sigma_1^1 + 3^{n_u-2} \sigma_2^1 + \ldots + 3^0 \sigma_1^{n_u} - (3^1 + 3^2 + \ldots + 3^{n_u-1})$$

For more details, see [1].

Let us now consider a T-S nonlinear system subject to actuator saturation represented by the following state equatation

$$\begin{cases} 
\dot{x}(t) = \sum_{j=1}^{3^{n_u}} \mu_i(t) h_j(\xi(t)) (A_j x(t) + B_j \mu_{sat}(t)) \\
y(t) = \sum_{j=1}^{3^{n_u}} \mu_i(t) h_j(\xi(t)) (C_j x(t) + D_j \mu_{sat}(t)) 
\end{cases}$$ (11)

According to the T-S writing of the saturation developed above, equation (11) can be written as

$$\begin{cases} 
\dot{x}(t) = \sum_{i=1}^{3^{n_u}} \sum_{j=1}^{n_u} \mu_i(t) h_j(\xi(t)) (A_j x(t) + B_j (\Lambda_i u(t) + \Gamma_i)) \\
y(t) = \sum_{j=1}^{3^{n_u}} \mu_i(t) h_j(\xi(t)) (C_j x(t) + D_j (\Lambda_i u(t) + \Gamma_i)) 
\end{cases}$$ (12)

### IV. Static output feedback controller: a descriptor approach

The objective is to design a stabilizing static output feedback control ensuring the stability of the system, even in the presence of control input saturation. The solution is obtained by representing the saturation as a T-S system and by solving an optimization problem under LMI constraints. In order to highlight the interest of considering the saturation when computing the controller, the controller design is envisaged without (section IV-A) and with (section IV-B) a priori taking into account the input saturation. A comparison of the obtained results is made in section VI.

#### A. Nominal control law

In order to stabilize the system at the origin, let us consider a linear output feedback controller given by (13)

$$u(t) = \sum_{j=1}^{n_u} h_j(\xi(t)) K_j y(t)$$ (13)

To reduce the number of LMI to obtain the gains $K_j$, a descriptor approach is applied. This approach is well known to avoid the coupling terms between the feedback gains and the Lyapunov matrices. As a consequence, the number of LMI decreases and relaxed conditions are obtained [2]. The control law (13) and the output signal of system (1) are written as follows

$$\begin{cases} 
0. \dot{u}(t) = \sum_{j=1}^{n_u} h_j(\xi(t)) K_j y(t) - u(t) \\
0. y(t) = \sum_{j=1}^{n_u} h_j(\xi(t)) (C_j x(t) + D_j u(t)) - y(t) 
\end{cases}$$ (14)

With the augmented state $x_\alpha(t) = (\begin{smallmatrix} x^T(t) \\ u^T(t) \\ y^T(t) \end{smallmatrix})^T$ and from equations (1) and (14), we can write

$$E^x \dot{x}_\alpha(t) = \sum_{j=1}^{n_u} h_j(\xi(t)) \mathcal{A}_j^T x_\alpha(t)$$ (15)

with

$$E^x = \begin{pmatrix} I_{n_u} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 
\end{pmatrix}, \quad \mathcal{A}_j^T = \begin{pmatrix} A_j & B_j & 0 \\
0 & -I_{n_u} & K_j \\
C_j & D_j & -I_m 
\end{pmatrix}$$ (16)
Let us consider the following Lyapunov function

$$V(t) = x_u^T(t)(P^s)^TPx_u(t)$$

with the condition

$$(E^s)^TP = P^TE^s \geq 0$$

and where the matrix $P$ is taken as $\text{diag}(P_1, P_2, P_3)$ with $P_1 = P_2^T = P_3^T > 0$. One can note that $V(t)$ is quadratic in the system state, since (17) reduces to $V(t) = x^T(t)P_3x(t)$. The time derivative of the Lyapunov function is given by

$$\dot{V}(t) = \sum_{j=1}^{n} h_j(\xi(t))x^T_u(t)((A_j^s)^TP + P^T A_j^s)x_u(t)$$

To ensure the stability of (15), developing (19) with (16), the conditions to solve with regard to $P_1$, $P_2$, and $P_3$, and $R_j$, $j = 1, \ldots, n$, for the nominal case, are the following

$$
\begin{pmatrix}
A_j^s P_1 + P_1 A_j & P_1 B_j & C_j^T P_3 \\
* & -P_2 - P_2^T & R_j + D_j^T P_3 \\
* & * & -P_3 - P_3^T
\end{pmatrix} < 0, \quad j = 1, \ldots, n
$$

The output feedback gains are deduced from $K_j = (P_2^T)^{-1}R_j$.

B. Saturated T-S control law

The objective is to adapt the gains $K_j$ of the controller (13) to the saturation limits and guarantee the stability of the closed loop system. The system (12) is written as a descriptor system using the augmented state $x_u(t)$ already defined

$$E^s x_u(t) = \sum_{i=1}^{3n_u} \sum_{j=1}^{n} \mu_i(t)h_j(\xi(t))(A_j^s x_u(t) + B_j^s)$$

with $E^s$ given by (16) and

$$A_j^s = \begin{pmatrix} A_j & B_j \Lambda_j & 0 \\ 0 & -I_{n_u} & K_j^\epsilon \\ C_j & D_j \Lambda_j & -I_m \end{pmatrix}, \quad B_j^s = \begin{pmatrix} B_j \Gamma_j \\ 0 \\ D_j \Gamma_j \end{pmatrix}$$

**Theorem 1:** There exists a static feedback controller (13) for a saturated input system (12) such that the system state converges toward an origin-centered ball of radius bounded by $\bar{\beta}_s$ if there exists matrices $P^s = (P_1^s)^T > 0, P_2^s > 0, P_3^s, R_j^s, \Sigma_j^s = (\Sigma_j^s)^T > 0, \Sigma_j^{3s} = (\Sigma_j^{3s})^T > 0$, solutions of the following optimization problem (for $i = 1, \ldots, 3n_u$ and $j = 1, \ldots, n$)

$$\min_{P_1^s, P_2^s, P_3^s, R_j^s, \Sigma_j^s, \Sigma_j^{3s}} \beta_s$$

s.t. (24)(see next page) with

$$
\Gamma_j^T B_j^s \Sigma_j^s + \Gamma_j^T \Sigma_j^{3s} D_j \Gamma_j < \bar{\beta}_s
$$

The gains of the controller are given by $K_j^s = ((P_2^s)^T)^{-1}R_j^s$

**Proof:** Let us define the quadratic Lyapunov function

$$V(t) = x_u^T(t)(E^s)^TP^sx_u(t)$$

with the condition

$$(E^s)^TP^s = (P^s)^T E^s \geq 0$$

From (16) and (27), the matrix $P^s$ is chosen as follows, with $P_1^s = (P_1^s)^T > 0$

$$P^s = \text{diag}(P_1^s, P_2^s, P_3^s)$$

From equations (21) and (26), the time derivative of the Lyapunov function is given by

$$\dot{V}(t) = \sum_{i=1}^{3n_u} \sum_{j=1}^{n} \mu_i(t)h_j(\xi(t))(\Sigma_j^s)^T P^sx_u(t) + x_u^T(t)(P^s)^T \Sigma_j^s + x_u^T(t)((A_j^s)^TP^s + (P^s)^T A_j^s)x_u(t)$$

Using Lemma 1, it follows that

$$(\Sigma_j^s)^T P^sx_u(t) + x_u^T(t)(P^s)^T \Sigma_j^s \leq \Gamma_j^T D_j^T \Sigma_j^{3s} D_j \Gamma_j + \Gamma_j^T B_j^s \Sigma_j^s B_j \Gamma_j + x_u^T(t)(\Sigma_j^s)^T P^s + (P^s)^T \Sigma_j^s + \text{diag}(P_1^s (\Sigma_j^s)^{-1} P_1^s, 0, P_3^s (\Sigma_j^{3s})^{-1} P_3^s)$$

Using (30), (29) is bounded as follows

$$\dot{V}(t) \leq \sum_{i=1}^{3n_u} \sum_{j=1}^{n} \mu_i(t)h_j(\xi(t))(\Gamma_j^T B_j^s \Sigma_j^s + \Gamma_j^T \Sigma_j^{3s} D_j \Gamma_j + x_u^T(t)(\Sigma_j^s)^T P^s + (P^s)^T \Sigma_j^s + \text{diag}(P_1^s (\Sigma_j^s)^{-1} P_1^s, 0, P_3^s (\Sigma_j^{3s})^{-1} P_3^s))x_u(t)$$

Let us define

$$\omega_{ij} = (\Sigma_j^s)^T P^s + (P^s)^T \Sigma_j^s + \text{diag}(P_1^s (\Sigma_j^s)^{-1} P_1^s, 0, P_3^s (\Sigma_j^{3s})^{-1} P_3^s)$$

$$\epsilon^s = \min_{i=1:3n_u, j=1:n} \lambda_{\min}(\omega_{ij})$$

$$\delta^s = \max_{i=1:3n_u, j=1:n} (\Gamma_j^T B_j^s \Sigma_j^s + \Gamma_j^T \Sigma_j^{3s} D_j \Gamma_j)$$

Since $\Sigma_j^s$ and $\Sigma_j^{3s} > 0$, from equation (31) with the convex sum property (2), $V(t) < -\epsilon^s \| x_u \|^2 + \delta^s$. It follows that $V(t) < 0$ for

$$\omega_{ij} < 0$$

and $\| x_u \|^2 > \frac{\delta^s}{\epsilon^s}$

which means that $x_u(t)$ is uniformly bounded and converges to the origin-centered ball of radius $\sqrt{\frac{\delta^s}{\epsilon^s}}$ according to Lyapunov stability theory [16].

The objective is now to minimize the radius $\sqrt{\frac{\delta^s}{\epsilon^s}}$. Firstly $\delta^s$ (34) is bounded by $\beta_s$ from LMIs (25). Secondly, it can be shown that $1/\epsilon^s < \beta_s$. From (24), with a Schur’s complement and the variable change

$$R_j^s = (P_2^s)^T K_j^s$$

it follows that

$$-\omega_{ij} > (1/\beta_s) I, \quad i = 1, \ldots, 3n_u, j = 1, \ldots, n$$

meaning that all eigenvalues of $(-\omega_{ij})$, including $\epsilon^s$, are bigger than $1/\beta_s$. Thus $1/\epsilon^s < \beta_s$ and the radius $\sqrt{\frac{\delta^s}{\epsilon^s}}$ is bounded by $\beta_s$.

**Remark 1:** It is important to highlight that the saturation may cause a performance degradation of the nonlinear system and even destabilize it. However, with the proposed
From (42) and (40), the matrix with the condition and (38), it follows
feedback control is proposed in the following section.

V. DYNAMIC OUTPUT FEEDBACK CONTROLLER: A
DESCRIPTOR APPROACH

The objective is now to design a stabilizing dynamic output feedback control ensuring the stability of the system, even in the presence of control input saturation. The solution is obtained by representing the saturation as a T-S system and by solving an optimization problem under LMI constraints.

A. Nominal control law

In order to stabilize the system (1) at the origin, let us consider a dynamic output feedback controller defined by:

\[
\begin{align*}
\dot{x}_c(t) &= \sum_{j=1}^{n} h_j(\xi(t)) (A_j^r x_c(t) + B_j y(t)) \\
u(t) &= \sum_{j=1}^{n} h_j(\xi(t)) (C_j^r x_c(t) + D_j y(t))
\end{align*}
\] (38)

Once again, the descriptor approach is applied. Considering the augmented state vector \( x_a(t) = (x^T(t) \quad x_c^T(t) \quad u^T(t) \quad y^T(t) )^T \), from equations (1) and (38), it follows

\[
E^d \dot{x}_a(t) = \sum_{j=1}^{n} h_j(\xi(t)) \varphi^d j x_a(t)
\] (39)

with \( E^d = \text{diag}(I_{2n_t}, 0_{n_a+m}) \) and

\[
\varphi^d j = \begin{pmatrix}
A_j & 0 & B_j & 0 \\
0 & A_j^c & 0 & B_j^c \\
0 & C_j & -I_{n_a} & D_j^c \\
C_j & 0 & D_j & -I_m
\end{pmatrix}
\] (40)

Let us consider the following Lyapunov function

\[
V(t) = x_a^T(t) (E^d)^T P x_a(t)
\] (41)

with the condition

\[
(E^d)^T P = P^T E^d \geq 0
\] (42)

From (42) and (40), the matrix \( P \) is chosen as

\[
P = \begin{pmatrix}
P_{11} & 0 & 0 & 0 \\
0 & P_{22} & 0 & 0 \\
0 & 0 & P_{33} & 0 \\
P_{41} & P_{42} & P_{43} & P_{44}
\end{pmatrix}
\] (43)

with \( P_{11} = P_{11}^T > 0, P_{22} = P_{22}^T > 0, P_{33} > 0 \)

The time derivative of the Lyapunov function is given by

\[
\dot{V}(t) = \sum_{j=1}^{n} h_j(\xi(t)) x_a^T(t) ((\varphi^d j)^T P + P^T \varphi^d j) x_a(t)
\] (44)

To ensure the stability of (39), the conditions to satisfy, for the nominal case, are given by (46) (for \( j = 1, \ldots, n \)) with

\[
\begin{align*}
A_j^r &= P_{11}^{-1} A_j \\
B_j^c &= P_{12}^{-1} B_j \\
C_j^r &= (P_{33}^T)^{-1} C_j \\
D_j^c &= (P_{33}^T)^{-1} D_j
\end{align*}
\] (45)

B. Saturated T-S control law

The objective is now to design the dynamic controller (38) to guarantee the stability of the saturated system (12) according to the saturation limits. Considering the previous augmented state vector \( x_a(t) \), from equations (12) and (38), the closed loop system is given by:

\[
E^d \dot{x}_a(t) = \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_i(t) h_j(\xi(t)) (\varphi^d j)^T x_a(t) + \varphi^d j
\] (47)

with \( E^d = \text{diag}(I_{2n_t}, 0_{n_a+m}) \) and

\[
\varphi^d j = \begin{pmatrix}
A_j & 0 & B_j & 0 \\
0 & A_j^c & 0 & B_j^c \\
0 & C_j & -I_{n_a} & D_j^c \\
C_j & 0 & D_j & -I_m
\end{pmatrix}
\] (48)

Theorem 2: There exists a dynamic feedback controller (38) for a saturated input system (12) such that the system state converges toward an origin-centered ball of radius bounded by \( \beta_d \) if there exists \( P_{11} = (P_{11}^T)^T > 0, P_{22} = (P_{22}^T)^T > 0, P_{33} > 0, P_{41}, P_{42}, P_{43}, P_{44}, A_j^r, B_j^c, C_j^r, D_j^c, \Sigma_{1d}^j, \Sigma_{2d}^j, \Sigma_{3d}^j > 0 \), solutions of the following optimization problem (for \( i = 1, \ldots, n \) and \( j = 1, \ldots, n \))

\[
\min_{\beta_d} \Gamma_i^T B_j^T \Sigma_{1d}^j B_j \Gamma_i + \Gamma_i^T D_j^T \Sigma_{2d}^j D_j \Gamma_i < \beta_d
\] (51)

The gains of the controller (38) are given by

\[
\begin{align*}
A_j^r &= (P_{11}^T)^{-1} A_j \\
B_j^c &= (P_{12}^T)^{-1} B_j \\
C_j^r &= ((P_{33}^T)^{-1}) C_j \\
D_j^c &= ((P_{33}^T)^{-1}) D_j
\end{align*}
\] (52)

static controller its stability is ensured.

Note that if the submodels are initially unstable, the proposed approach is not suitable for this case since the LMI condition can not be fulfilled. In order to improve the obtained results and relax the stabilization constraint, the dynamic output feedback control is proposed in the following section.
Proof: Let us define the Lyapunov function $V(t) = x_t^T(t)(E^d)^TP^d x(t)$ with $E^d$ defined as (40) and the condition $(E^d)^TP^d = (P^d)^TE^d \geq 0$

To satisfy this condition, the matrix $P^d$ is then chosen

$$P^d = \begin{pmatrix} P_{11}^d & 0 & 0 & 0 \\ 0 & P_{22}^d & 0 & 0 \\ 0 & 0 & P_{33}^d & 0 \\ P_{41}^d & P_{42}^d & P_{43}^d & P_{44}^d \end{pmatrix}$$

with $P_{11}^d = (P_{11}^d)^T > 0$, $P_{22}^d = (P_{22}^d)^T > 0$ and $P_{33}^d > 0$. Applying the same development as for the static controller with the variable changes

$$\begin{cases} \tilde{x}_j = P_{j1}^d A_j \\ \tilde{c}_j = P_{j3}^d C_j \end{cases}$$

and defining $e^d$ and $\delta^d$

$$e^d = \min_{1 \leq i, 1 \leq j, 1 \leq a} \lambda_{\min}(-2Q_{ij}^d)$$

$$\delta^d = \max_{1 \leq i, 1 \leq j} \Gamma_i (B_j^T \Sigma_{ij}^d B_j + D_j^T \Sigma_{ij}^d D_j) \Gamma_i$$

with $Q_{ij}^d$ defined in the same way as $Q_{ij}$ was.

The stabilizing conditions are linearized and given by (50). As the weighting functions satisfy (2) and $\Sigma_{ij}^d, \Sigma_{ij}^d > 0$, if (50) holds, and $x_i \|x_i\| > \frac{\delta^d}{\rho^d}$, then $\dot{V}(x(t)) < 0$, implying that $x_i(t)$ converges to an origin centered ball of radius $\sqrt{\frac{\delta^d}{\rho^d}}$. Similarly to what is done in the proof of theorem 1, the radius of the ball is bounded by $\tilde{\beta}^d$ due to (50) and (49).

VI. NUMERICAL EXAMPLE

This section is devoted to the comparison of the results obtained with the nominal controller and with the proposed controller, both in the static (13) and dynamic cases (38).

Let us consider system (11) with $n = 2$, $D_1 = D_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and

$$\begin{pmatrix} A^T P_{11} + P_{11} A_j + C^T P_{41} + P_{41} C_j \\ C^T P_{22} \end{pmatrix} \begin{pmatrix} P_{11} B_j + C^T P_{43} + P_{43} C_j \\ B_j \end{pmatrix} - \begin{pmatrix} C^T P_{44} - P_{41} \end{pmatrix} \begin{pmatrix} B_j \\ -P_{43} \quad -P_{44} \end{pmatrix} < 0 \tag{46}$$

$$Q_{ij}^d = A^T P_{11}^d + P_{11}^d A_j + C^T P_{41}^d + (P_{41}^d)^T C_j \quad \text{and} \quad Q_{ij}^d = -P_{33}^d - (P_{33}^d)^T D_j \quad \text{and} \quad Q_{ij}^d = -P_{33}^d - (P_{33}^d)^T D_j \quad \text{with} \quad P_{33}^d > 0.$$
For the proposed example, the control goal is to ensure the state trajectory convergence to the origin in spite of the input saturation. One can observe from the depicted figures that the results are slightly better for the dynamic controller with respect to the nominal closed-loop system without saturation. One can observe from the depicted figures that the results are slightly better for the dynamic controller with respect to the nominal closed-loop system without saturation.

The solution of this problem is based on the Lyapunov theory using the descriptor approach and is expressed in terms of LMI. Two output feedback controllers were synthesized, a static controller and a dynamic one. The dynamic controller was considered in order to increase the number of degrees of freedom in the design and improve the control performances.

VII. CONCLUSIONS

Using the T-S approach to describe both the nonlinearities and the input saturation, a nonlinear system with saturated actuator(s) can be represented with a T-S model. This unified representation allows to deal with these difficulties and to synthesize an output feedback controller which gains depend on the saturation bounds. The solution of this problem is based on the Lyapunov theory using the descriptor approach and is expressed in terms of LMI. Two output feedback controllers were synthesized, a static controller and a dynamic one. The dynamic controller was considered in order to increase the number of degrees of freedom in the design and improve the control performances.

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