A review of damage detection and health monitoring of mechanical systems from changes in the measurement of linear and non-linear vibrations

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A Review of Damage Detection and Health Monitoring of Mechanical Systems from Changes in the Measurement of Linear and Non-linear Vibrations

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1. Introduction

Damage are a main cause of structural failure and often occurs on structures. In the past decades, special attention was given to avoid the sudden failure of structural components by detection damage in structures in the early state. More specifically, structural health monitoring based on the vibration of structures has been at the focus of attention of many researchers in order to obtain very efficient tools of great importance for the civil, aeronautical and mechanical engineering communities.

This is why, in recent years, various developments of non-destructive techniques based on changes in the structural vibrations have been extensively published not only to detect the presence of damage but also to identify the location and the severity of the damage. Moreover, the need to be able to detect in the early stage the presence of damage in complex mechanical structures has led to the increase of non destructive techniques and new developments. Not only extension of techniques that are based upon structural linear vibration analysis, but also the emergence of non-linear methodology and analysis have been investigated.

It is generally admitted that Rytter [1] gave the four principal damage stages of structural health monitoring:

1- the determination of the presence of damage in the structure,

2- the determination of the damage location in the structure,

3- the quantification of the severity of the damage,

4- the prognosis of the remaining service life of the damaged structure.

If the last stage can be refer to the fields of structural design assessment or fatigue analysis, the first three states concern more specifically the problem of structural health monitoring techniques and the choice of the most appropriate method for the detection and identification of damages in structures.

At this point of the introduction, it must be said that the field of vibration-based structural health monitoring has been the subject of great interest with important comprehensive surveys covering various methodologies and applications. Comprehensive surveys have been proposed by Doebling et al. [2, 3] and Sohn et al. [4] for structural health monitoring and damage detection techniques in the civil and mechanical engineering communities. Friswell [5] presented a brief overview of the use of inverse methods in damage detection and location from measured vibration data. A review based on the detection of structural damage through changes in frequencies has been discussed by Salawu [6]. Different detection procedures to diagnose damage in rotors with various modeling of the cracked elements have been summarized by Wauer [7]. Dimarogonas [8] gave an extensive review about cracked structures including beam, rotor, shell and blades, plates and pipelines. Sabnavis et al. [9] proposed a review on cracked shaft detection and diagnostics. A review of vibration-based structural health monitoring with special emphasis on composite materials has been described by Montalvao et al. [10].

However few efforts have been dedicated to discuss advantages and limitations of linear and/or non-linear approaches based on vibrational measurements with theoretical developments. So, in addition of the first three stages defined by Rytter [1], the following questions need to be addressed:

a- Are the classical effective tool of non-destructive testing based on a linear analysis sufficient for detecting and identifying the location and severity of damage ?

b- If linear methods are not appropriate, how the non-linear measurements can be used for an efficient and robust detection and identification of damage ?

c-What is the significance of the non-linear vibrational measurements for mechanical damaged structures ?

d- What are the main advantages or limitations of linear and non-linear structural vibrational non-destructive techniques ?

e- Where next uncoming for future research and further developments ?

In order to propose some partial answers for the five previous questions and considering the previous reviews done in the field of structural mechanics, the decision has been made not to explore all the structural health monitoring techniques that are described in the literature but to give a broad view of the most efficient and well-known damage assessment techniques based on structural dynamic changes and the liner or/and non-linear vibrational measurements. Thus the three basic types of data used in the measurement of structural dynamics (i.e. time series, frequency domain, and modal model) will be investigated. Non-exhaustive advantages and limitations of the damage assessment techniques proposed in this review will be discussed for practical engineering applications. Moreover, special attention will be given to methods in which civil engineering structures and rotating machineries are used as case studies. For various linear and non-linear techniques, the basic theory, definitions and equations will be briefly presented: the objective is to give the reader the more important and principal informations for briefly but comprehensive review. Having in mind that some researchers will be more interested by specific cases of linear or non-linear techniques, various references are given for each damage detection method to allow the interested reader to find more details about the theory. Illustrative (non-exhaustive) examples with applications in civil, aeronautics and mechanical engineering communities will be discussed and references of papers will be given.

To situate this review into the broad literature survey, we recall that the damage assessment techniques can be divided into three categories [9]: modal testing, vibration-based methods with signal-based and model-based methods, and non-traditional methods. The most popular methods are probably modal testing in which changes in modal parameters such as changes in frequencies, modes shapes and response to specific excitation are used for the damage detection. Signal-based methods analyze the steady-state or transient vibrational measurements to detect damage and consider different known indicators of the presence of damage in mechanical structures. Model-based methods propose to correlate the experimental signature of damaged structures with analytical or numerical models to identify the damage parameters. The non-traditional methods concern more specific techniques based on genetic algorithms, neural networks, signal processing techniques such as the wavelet analysis or Wigner-Ville transforms.

In this review, the damage assessment techniques have been divided into three categories in regard to the following types of data available in the measurement of structural dynamics to detect damage: linear, non-linear and transient vibrational measurements. For these three categories, modal testing, signal-based methods and non-traditional methods based on wavelet transform will be discussed.

The present review is organized as follows: firstly, a description of linear analysis and non destructive techniques based on changes in modal parameters and linear vibrational measurements are discussed. Secondly, explanation is given of the most common uses of non-linear assessment techniques to detect the presence of damage by only considering the non-linear contributions due to the presence of damage. Then, damage detection for linear or non-linear transient vibrational signals is discussed. Finally, a non-exhaustive list of topics of interest for further research in the field of damage detection is briefly discussed.

2. Methods of damage detection using linear analysis

2.1. Direct use of modal parameters

Change in the natural frequencies

The change of natural frequencies can be considered as one of the prevalent damage detection method in structural assessment procedures. When a damage exists in a structure, the stiffness is reduced and consequently decreasing of the natural frequencies of the system can be observed. One of the most advantages of this detection technique is that frequency measurements can be quickly and easily conducted. Moreover, experimental techniques used for the determination of resonant frequencies are classical vibrational measurement techniques; thus allowing the vibrational measurements to be extensive with a great number of measurement points and a very cheap experimental procedure. Another advantage is that the frequency measurements can be extracted with a relative confident accuracy, and uncertainties in the measured frequencies can be easily estimated if the experimental measurements are done with a perfect control of the experimental conditions. Moreover, the knowledge of the global dynamical behavior of undamaged systems is very easy to obtain by using analytical developments or finite element models; thus allowing the measurement points to be adequately chosen for not only a quickly and efficient detection of the changes in frequencies but also the identification of the damage location and gravity.

According to Doebling et al. [2], the first article that proposed to detect damage by using vibration measurement was written by Lifshitz and Rotem [11]. They used the shifts in the natural frequencies via changes in the dynamic moduli to detect damage in elastomers.
Hearn and Testa [12] demonstrated that the change in the i^{th} natural frequency can be approximated by

$$\Delta\omega_i^2 = \frac{\left(\epsilon_{\mathbf{N}}\left(\Phi_i\right)\right)^T \mathbf{\Delta k_n} \left(\epsilon_{\mathbf{N}}\left(\Phi_i\right)\right)}{\Phi_i^T \mathbf{M} \Phi_i} \tag{1}$$

where $\mathbf{M}$ is the mass matrix. Φ_i defines the i^{th} mode shape vector and $\epsilon_{\mathbf{N}}\left(\Phi_i\right)$ the element deformation vector that is computed from the mode shapes. $\mathbf{\Delta k_n}$ is the change in the matrix stiffness due to the presence of damage. The previous expression assumes that the damage does not change the mass matrix. Hearn and Testa [12] also demonstrated that the ratio of the variations of the frequencies $\dfrac{\Delta\omega_i^2}{\Delta\omega_j^2}$ for two modes i and j is independent on the damage severity and so a function of the crack position only. So, this result allows the identification of the damage location. Hasan [13] illustrated this property for a damaged beam on elastic foundation.

Many other researchers have attempted to detect damage in structures by using changes in natural frequencies. Salawu proposed an intensive review in [6]. Some investigators [14–24] compared the natural frequencies of the undamaged and damaged structures (and the associated decreasing in frequencies).

For example, the Normalized Natural Frequencies that defines the ratio of damaged natural frequency to the undamaged natural frequency of the structure may be proposed and is given by

$$NNF_i = \frac{\omega_i^{cracked}}{\omega_i^{uncracked}} \tag{2}$$

where $\omega_i^{uncracked}$ and $\omega_i^{cracked}$ define the i^{th}-pulsation of the uncracked and cracked structure, respectively.

Others [25–28] proposed the percentage changes in the natural frequencies $\%C_i$

$$\%C_i = 100 \times \frac{\omega_i^{uncracked} - \omega_i^{cracked}}{\omega_i^{uncracked}} \tag{3}$$

For the two cases, to be able to detect the presence of damage, the natural frequencies of the undamaged structure need to be carefully estimated in order to be able to accuracy show if the measured frequencies are lower than expected. For the reader comprehension, it may be noted that the percentage change of the natural frequencies may be very low and inferior to 1% for small cracks or specific locations of the crack. So, uncertainties on the natural frequenices of the undamaged structures may hidden the small amount of frequency change. Classical results indicated that the crack has low effect if it is situated near a node of the modes of vibration. High decrease in the factor $\%C_i$ is shown if the crack is located where the bending moment of the i^{th} mode is greatest. Then, increase the crack depth, decrease the factor $\%C_i$, indicating loss of stiff for the damaged beam.

A method similar to the above consists to estimate the crack gravity and location by only considering the frequencies of the damaged structures (without comparison with the frequencies of the undamaged structure). In this case, the crack detection and identification require the knowledge of the material properties (the Young's modulus E and the density ρ for example) that are estimated by using the uncracked natural frequencies. This last approach can be considered to be equivalent with the procedures using the factors NNF and $\%C$ factors previously defined: these two factors are not affected by the parameters of material properties or uncertainties on the Young's modulus E and the density ρ. However, the undamaged frequencies are used and the material properties are implicitely consider.

Sinou [29] defined another indicator based on the changes in the ratio of frequencies. It is defined as follows

$$\%\Psi_{i,j}^{cracked} = 100 \times \left(\frac{\omega_i^{uncracked}}{\omega_j^{uncracked}} - \frac{\omega_i^{cracked}}{\omega_j^{cracked}} \right) \tag{4}$$

where $\omega_i^{uncracked}$ and $\omega_i^{cracked}$ correspond to the i^{th} pulsations of the uncracked and cracked structures, respectively. One of the advantages of this factor $\%\Psi_{i,j}^{cracked}$ is that the ratio of frequencies for the undamaged structures is generally known for academic structures such beams. For example, in the case of a simply supported beam this indicator can be rewritten as

$$\%\Psi_{2\alpha-a,2\beta-b}^{cracked} = 100 \times \left(\left(\frac{\alpha}{\beta} \right)^2 - \frac{\omega_{2\alpha-a}^{cracked}}{\omega_{2\beta-b}^{cracked}} \right) \tag{5}$$

with a and b are equal to 0 or 1, and $\alpha \in \aleph^*$ and $\beta \in \aleph^*$. For the reader comprehension, the pulsations $\omega_{2\alpha-a}$ (or $\omega_{2\beta-b}$) are associated with the vertical mode deflections if $2\alpha - a$ (or $2\beta - b$) are odd numbers. If $2\alpha - a$ (or $2\beta - b$) are even numbers, the pulsations $\omega_{2\alpha-a}$ (or $\omega_{2\beta-b}$) are associated with the horizontal mode deflections. In this case, the factors $\%\Psi_{2\alpha-a,2\beta-b}^{cracked}$ need only the knowledge of the pulsations of the cracked beam and do not change with the variations of the material properties like the Young modulus and the density. In the general case (academic beam or more complex structure such as civil engineering structures), this factor $\%\Psi_{i,j}^{cracked}$ indicates the relative effect of the damage for the j^{th} and i^{th} modes; if $\%\Psi_{i,j}^{cracked}$ is higher than 0, it can be concluded that the mode of i^{th} pulsation is more affected by the crack than the mode of j^{th} pulsation. The scenario is reversed if $\%\Psi_{i,j}^{cracked}$ is lower than 0 and the more affected mode corresponds to the j^{th} pulsation.

Messina et al. [30] proposed the Damage Location Assurance Criterion (DLAC) based on the changes in natural frequencies

$$DLAC\,(i) = \frac{|\Delta\omega_A^T \Delta\omega_B\,(i)\,|^2}{\left(\Delta\omega_A^T \Delta\omega_A\right)\left(\Delta\omega_B\,(i)^T\,\Delta\omega_B\,(i)\right)} \tag{6}$$

where $\Delta\omega_A$ is the experimental frequency shift vector and $\Delta\omega_B$ the theoretical change for damage that is situated at the i^{th} position. The values of the Damage Location Assurance Criterion (DLAC) vary between zero and unity. A value of zero indicates no correlation and a value of one an exact match. The damage location is obtained when the position of i gives the highest values for the Damage Location Assurance Criterion (DLAC).

Messina et al. [31] extended the Damage Location Assurance Criterion (DLAC) for the cases of multiple damage. The Multiple Damage Location Assurance Criterion (MDLAC) is given by

$$MDLAC\,(i) = \frac{|\Delta\omega_A^T \mathbf{S}\delta x_i|^2}{\left(\Delta\omega_A^T \Delta\omega_A\right)\left((\mathbf{S}\delta x_i)^T\,(\mathbf{S}\delta x_i)\right)} \tag{7}$$

where $\mathbf{S}$ defines the sensitivity matrix that contains the first order derivatives of n natural frequencies with respect to m damage variables x. The objective of the Multiple Damage Location Assurance Criterion (MDLAC) is to find the variable vector δx_i that makes the MDLAC equal to one. The sensitivity matrix is given by

$$\mathbf{S} = \begin{bmatrix} \dfrac{\partial\omega_1}{\partial x_1} & \dfrac{\partial\omega_1}{\partial x_2} & \cdots & \dfrac{\partial\omega_1}{\partial x_m} \\ \dfrac{\partial\omega_2}{\partial x_1} & \dfrac{\partial\omega_2}{\partial x_2} & \cdots & \dfrac{\partial\omega_2}{\partial x_m} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial\omega_n}{\partial x_1} & \dfrac{\partial\omega_n}{\partial x_2} & \cdots & \dfrac{\partial\omega_n}{\partial x_m} \end{bmatrix} \tag{8}$$

Koh and Dyke [32] used the Multiple Damage Location Assurance Criterion (MDLAC) for the detection of single or multiple damages for long-span civil engineering structures.

Identification based on the frequency contours methods

The identification of the crack parameters can be done by using the different factors and the combined effects of the crack in changes of frequencies of the damage structure.

Nikolakopoulos and Papadopoulos [33] proposed to present in contour graph form the dependency of the first two structural eigenfrequencies on crack depth and location. The intersection point of the superposed contour of the frequencies variations between the undamaged and damaged structures allows the identification of both the crack depth and location. The authors indicated that one contour line may correspond in a combination of different crack depths and locations. They validated the proposed methodology of damage identification by undertaking experimental tests on a clamped-clamped plane frame. They concluded that experimental results are in good agreement with the theory and can allow easy diagnosis of crack depth and position in frame structures.

Yang et al. [34] used the frequency contours method and the intersection of contours from different modes for the detection of damage in a simply supported beam. They noted that the intersecting point of the superposed three contours line that correspond to the measured frequency evolution and/or the corresponding factors caused by the presence of a damage indicates only one crack depth but can give two probable crack locations due to structural symmetry in the simply supported beam.

Dong et al. [35] proposed to use the evolution of mode shape (see section 2.1.) to avoid the non-uniqueness of the damage detection. Swamidas et al. [36] proposed to add an off-center mass in the initial structure to eliminate symmetrical solutions . Sinou [29] demonstrated that this technique is not sufficient in all cases and may be very difficult to use. In practical cases, it is not very clear and evident that the three contour lines have only one intersection for each case: due to experimental conditions or uncertainties, the three curves do not meet exactly, and the centroid of the three pairs of intersections is taken as the crack position and crack size [37, 38]. So Sinou [29] proposed to extend the methodology of the adding-mass by realizing two tests for the crack size and location identification. The first test consists of adding a mass at the right end of the symmetrical structure, and the second test at the other end. However, the author concluded that this methodology is limited for general structures due to uncertainties and ambient conditions in real cases. A second solution to avoid the non-uniqueness of the damage location consists on an appropriate use of resonances and antiresonances [39, 40]. More details on this methodology are given in the section 2.1.. One of the advantage of this solution is that no additional tests is required (contrary to the previous methodology) and antiresonances like resonances can be easily extracted with a relative confident accuracy.

Sinou [40] proposed an extension of the frequencies contour line method by considering not only the identification of the crack size and location but also the orientation of the front crack. The author indicated that the procedure works effectively only for measurement errors not exceeding 2% for small levels of damage. He also concluded that the quality of experimental data is an important key in order to achieve reliable results.

Change in damping

It appears natural to expect that an interesting indicator for damage detection would be the damping changes and dissipative effects due to the friction between crack surfaces. Modena et al. [41] indicated that one of the advantages of using changes in damping is that undetectable cracks by using changes in natural frequencies (due to uncertainties or little decrease of frequencies) can cause important changes in the damping factor allowing damage detection. In general, it is admitted that increasing the crack severity increases the damping factor.

However, experimental results with an edge fatigue crack of mode I at bending vibrations done by Bovsunovsky [42] revealed that the energy dissipation in a non-propagating crack is not caused by the friction between crack surfaces but must be attributed mainly to the elasto-plastic zone in the vicinity of crack tip. Moreover, he concluded that changes in energy dissipation may be used for the damage detection based on the prediction of changes in damping factor.

Kyriazoglou et al. [43] proposed the measurement of specific damping capacity (SDC) for the detection of damage in composite laminates. The SDC factor is defined by

$$SDC = \frac{\Delta U}{U} \tag{9}$$

where ΔU and U are the energy dissipated in one cycle and the total energy stored in that cycle. Experimental measurements and analysis of the SDC of composite beams vibrated in free-free flexure in their first mode tests were performed, the energy stored by the beam in bending being calculated from the mode shape maximum deflection at the center of the beam, and the energy dissipated in one cycle being the energy input. They demonstrated that damping properties strongly support the measurements and changes of SDC in order to detect successfully initial damage in composites before the damaged material includes catastrophic crack density and severe further damage mechanisms occur. One interesting observation is that larges changes in SDC are found for carbon fibre-reinforced laminates even if no detectable changes in the resonant frequencies can be observed. It also appears that the SDC factor is very sensitive to small cracks and changes in the crack depth.

Panteliou et al [44] showed that the damping factor increases with increasing the crack depth. They said that the identification of crack by using change in damping factor has the advantage to be relatively insensitive to boundaries conditions in comparison to the shifts in natural frequencies. Another advantage is that the However, it appears that a relative confident accuracy of changes in damping factor may be difficult to obtain for small cracks due to the uncertainties and experimental conditions.

On the same way, Leonard et al. [45] indicated that the modal damping value of a damage structure depends on vibration amplitudes that induces the opening or closing mechanism for the crack. For a cantilever beam, they demonstrated that the modal damping decrease when the amplitudes vibration are too small to generate an open crack. When the opening and closing of the crack introduced contact effects, the modal damping of a lot of modes is the most important. Consequently, using variations of modal damping to detect damage appears to be difficult due to the dependence of the evolution dependence of modal damping to the vibration amplitude. Moreover, uncertainties and shifts in the modal damping may be observed due the temperature rise.

Mode shapes and changes nodes position

Mode shapes approach has also received considerable attention in conjunction with changes in natural frequencies due to the fact that a mode shape is a unique characteristic and spatial description of the amplitude of a mechanical structure for each resonant frequency. So a local damage can cause changes in the mode shapes and the evolution of the spatial description of the amplitude of each resonance may be used as a damage indicator. Moreover, change of mode shapes depends on both the severity and the location of the damage and the spatial description of magnitude change with respect to each mode may vary from one to another due to the crack location. The main disadvantage of using mode shapes as damage assessment technique is the number of measurements at each of the points of the structures to perform and the duration of each measurement in order to estimate the detailed mode shape.

Gladwell and Morassi [46] investigated the effect of damage on the nodes in an axially vibrating thin rod. They demonstrated that nodes of the mode shapes move toward the damage: each node located to the right of the damage in the undamaged structure moves to the left, and each node on the left of it moves to the right. They concluded that the damage is located between nodes which move toward each other for every mode that has at least two nodes.

Then, Dilena and Morassi [47] defined the positive nodal displacement domain (PNDD) and negative nodal displacement domain (NNDD) that define the direction by which nodal points move. Using these domains allows the damage detection in the case of bending vibrations. One of the advantages to use the evolution of node shapes and the detection of the positive and negative nodal displacement domains is based on the fact that node positions are easier to measure than mode shapes because they only require the detection of modal component sign, rather than the measurements of amplitudes. They validated the damage assessment technique by undertaking experimental tests performed on cracked steel beams.

Damage detection based on the study of modal parameters (i.e. change in natural frequencies and mode shapes) conducted by Adams et al. [48], Cawley and Adams [49] and Yuen [50] for the vibration of bridges. Only the few lower modes are used to perform the detection of damage.

Natke and Cempel [51] used changes both in eigenfrequencies and mode shapes to detect damage in a cable-stayed steel bridge. Kullaa [52] indicated that changes both in the eigenfrequencies and mode shapes appear to be reliable indicators for the damage detection on the bridge Z24 in Switzerland [53]. They performed automatic identification of the modal parameters from the response data by combining control charts that corresponds to one of the primary techniques of statistical process control [54]. They observed that one of the limitations of some control charts was that the cumulative sum (CUSUM), and exponentially weighted moving average (EWMA) control charts [54] are sensitive to small shifts and may cause frequent false damage detection.

Law and Zhu [55] indicated that the deflection of mode shapes can be an effective indicator of damage in bridge structures. They observed that the deflection increases when the damage in the beam increases, the deflection also increases. However, they noted that the deflection of the damaged structure can be larger than the measured deflection under the light vehicle. They explained that a moving load (i.e. a vehicle in this case) can affect the opening of the damage so inducing evolutions of natural frequencies and mode shapes. So the breathing behavior of the crack due to moving load and the associated non-linear analysis is an important point to take under consideration for an efficient and robust detection of damage (see section 3. for more details).

MAC and other related assurance criteria

According to Doebling et al. [2, 3], West [56] presented what was possibly the first systematic use of mode shape information for the location of structural damage without the use of a prior FE model. The Modal Assurance Criterion (MAC) is used to determine the level of correlation between modes from the test of an undamaged Space Shuttle Orbiter body flap and the modes from the test of the flap after it has been exposed to acoustic loading. The MAC criteria that compares mode i and j has the form [57]

$$MAC\left(\Phi_i,\Phi_j\right)=\frac{\left|\sum_{k=1}^{n}\left(\Phi_j\right)_k\left(\Phi_i\right)_k^*\right|^2}{\left(\sum_{k=1}^{n}\left(\Phi_j\right)_k\left(\Phi_j\right)_k^*\right)\left(\sum_{k=1}^{n}\left(\Phi_i\right)_k\left(\Phi_i\right)_k^*\right)} \tag{10}$$

where $*$ denotes complex conjugate and $(\Phi)_k$ is an element of the mode-shape vector. The MAC value varies varies between 0 and 1. The MAC makes use of the orthogonality properties of the mode shapes to compare two modes. A value of one indicates that the mode shapes of the two sets of date are identical. If the modes are orthogonal and dissimilar, a value of zero is calculated. So, the low MAC value to some extent can be interpreted as a damage indicator in structures.

Srinivasan and Kot [58] noted that the changes in MAC values comparing the damaged and damaged modes shapes of a cylindrical shell are more sensitive than changes in resonant frequencies.

Although the MAC criteria can provide a good indication of the disparity between two sets of data and the detection of damage, it does not allow to show explicitly where the source of the damage in the structure lies. The Co-Ordinate Modal Assurance Criterion (COMAC) [59] has been proposed from the original MAC. The Co-Ordinate Modal Assurance Criterion (COMAC) identifies the co-ordinates at which two sets of mode shapes do not agree. The COMAC factor at a point i between two sets of the mode shape is given by

$$COMAC\,(i) = \frac{\left(\sum_{j=1}^{n} \mid (\Phi_A)_{ij}\,(\Phi_B^*)_{ij} \mid\right)^2}{\sum_{j=1}^{n} \mid ((\Phi_A)_{ij} \mid^2 \sum_{j=1}^{n} \mid ((\Phi_B)_{ij} \mid^2} \tag{11}$$

where n defines the number of correlated mode shapes. $(\Phi_A)_{ij}$ and $(\Phi_B)_{ij}$ denote the value of the j^{th} at a point i for the states A and B respectively.

Palacz and Krawczuk [60] showed that the damage location prediction is clearer when more than two mode shapes are used. However, they noted that a small number of measurements may induced worse damage detection by using the MAC criteria. Other related assurance criteria can be proposed: the frequency response assurance criterion (FRAC), coordinate orthogonality check (CORTHOG), frequency scaled modal assurance criterion (FMAC), partial modal assurance criterion (PMAC), Modal assurance criterion square Root (MACSR), scaled modal assurance criterion (SMAC), and modal assurance criterion using reciprocal modal vectors (MACRV). A review of the significant of each criteria may be found in [61]. For example, Kim et al. [62] investigated the use of the MAC and some of its forms for locating structural damage. By using the COMAC and the PMAC in conjunction, they demonstrated that the damage location can be clearly isolated.

One of the main disadvantage in using mode shapes and the original modal assurance criterion (MAC) or the other related assurance criteria as damage indicators is the capability to estimate a detailed mode shape; measurements at a lot of points are needed and the duration of measurements may considerably increase if the change of mode shapes obtained from successive tests is used as a damage indicator.

Performing experimental tests on a clamped board structure, Parloo et al. [63] used the modes shapes and the modal assurance criterion (MAC) and co-ordinate modal assurance criterion (COMAC) for damage detection. Experiments performed on the I-40 highway bridge in New Mexico allow the detection of damage. However they indicated that these criteria are not able to detect all the proposed damage scenarios. They indicated that only the most severe damage of the bridge was identified and that the presence of measurement noise strongly affected the damage detection.

Mode shapes curvature

Mode-shape derivatives such as curvature are widely used as an alternative to damage identification from mode shape changes to obtain spatial information about vibration changes.

Pandey et al. [64] demonstrated that the absolute change in mode shape curvature can be an efficient indicator of damage. They proposed to estimate the mode shape curvatures using the central difference approximation

$$\Phi''_{q,i} = \frac{\Phi_{q-1,i} - 2\Phi_{q,i} + \Phi_{q+1,i}}{h^2} \tag{12}$$

where h is the distance between the measurement co-ordinates. $\Phi_{q,i}$ define the modal displacement for the i^{th} mode shape at the measurement co-ordinate q. To detect and quantify damage, it is supposed that the local increase in the curvature $\Phi''_{q,i}$ is observed due to the local reduction in stiffness and so the presence of damage. Then the Mode Shape Curvature (MSC) defined by Pandey et al. [64] is given by mode shape curvature (MSC)

$$MSC_q = \sum_i |\left(\Phi_{q,i}^{damaged}\right)'' - \left(\Phi_{q,i}^{undamaged}\right)''| \tag{13}$$

Ho and Ewins [65] proposed other related criteria based on the mode shapes curvatures as damage indicators: Mode Shape Amplitude Comparison (MSAC), Flexibility Index (FI), Mode Shape Slope (MSS), and Mode Shape Curvature Square (MSCS) that are defined as follows

$$MSAC_q = \sum_i |\Phi_{q,i}^{damaged} - \Phi_{q,i}^{undamaged}| \tag{14}$$

$$FI_q = \sum_i |\left(\Phi_{q,i}^{damaged}\right)^2 - \left(\Phi_{q,i}^{undamaged}\right)^2| \tag{15}$$

$$MSS_q = \sum_i |\left(\left(\Phi_{q,i}^{damaged}\right)'\right)^2 - \left(\left(\Phi_{q,i}^{undamaged}\right)'\right)^2| \tag{16}$$

$$MSCS_q = \sum_i |\left(\left(\Phi_{q,i}^{damaged}\right)''\right)^2 - \left(\left(\Phi_{q,i}^{undamaged}\right)''\right)^2| \tag{17}$$

Ho and Ewins [65] indicated that the previous indicators and absolute changes in mode-shape curvature serve as good indicators for damage detection. However, false damage detections can be observed at mode shape nodal points or at the boundaries. Then, one of the main disadvantage in using mode shape curvature as damage indicators is the quality of the measurements and uncertainties at the boundaries conditions that may drastically affect the methods based on mode shapes and their derivatives.

Maeck and De Roeck [53, 66] investigated the effects of different damage scenarios for the prestressed concrete bridge Z24 in Switzerland, tested in the framework of the Brite Euram project SIMCES [53]. They used the mode shape curvatures in a direct stiffness calculation technique: an indicator for damage detection was established by examining the changes in the dynamic stiffness, given by changes in the modal bending moment over the modal curvature. They indicated that an increase of the relative curvatures in the damage zone and a deviation from the symmetrical character of the mode shape of the undamaged bridge appear when the damage is present. Moreover, they shown that modal curvatures are very sensitive to damage in the bridge.

Abdel Wahab and De Roeck [67] used the application of the change in modal curvatures to detect damage in simply supported beams and the concrete bridge Z24. The structures contain damage at different locations. They proposed the Curvature Damage Factor

$$CDF = \frac{1}{N}\sum_{i=1}^{N} \mid \left(\Phi_i^{damaged}\right)'' - \left(\Phi_i^{undamaged}\right)'' \mid \tag{18}$$

where N is the number of modes to be considered. By using measured data on a prestressed concrete bridge Z24, they indicated that the modal curvatures of the lower modes are more accurate than those of the higher ones for damage detection. Even if the detection of multiple damages can be difficult with the classical mode shape curvature from the results of only one mode, the authors proved that the Curvature Damage Factor (CDF) allows a clear identification of these damage locations. They also mentioned that irregularities in the measured mode shapes or uncertainties need to be carefully examined in order to avoid worse diagnostic. They concluded that future techniques that can improve the quality of the measured mode shapes are highly recommended.

The modes shapes curvatures were applied by Parloo et al. [63] for different damages on the I-40 highway bridge in New Mexico. The authors demonstrated that only the most severe damage is identified due to the uncertainties, measurement noise and ambient conditions. They concluded that the mode shape curvature can not be used in practical cases for the detection of small damages or in early state.

Dutta and Talukdar [68] investigated changes in natural frequencies, modes shapes and curvature mode shapes between the damaged and intact structures for continuous bridges containing damaged parts at different locations. They used the Curvature Damage Factor (CDF) previously defined by Abdel Wahab and De Roeck [67]. They demonstrated that a better localization of damage is obtained by considering curvature of the mode shapes instead of the mode shapes that are less sensitive to damage. They also noted that adequate numbers of modes are needed when multiple cracks are present and the choice of the modes for damage identification is a very important fact for proper and efficient evaluation of multiple damage locations.

Modal strain energy

A damage detection method based both on the changes in strain mode shape and change in resonant frequency was proposed by Dong et al. [69]. The difference between the damaged and undamaged structures was calculated by considering the index $\Delta\Phi_i$ that is given by

$$\Delta\Phi_i = \left(\frac{\omega_i^{undamaged}}{\omega_i^{damaged}}\right)^2 \Phi_i^{damaged} - \Phi_i^{undamaged} \tag{19}$$

where $\omega_i^{undamaged}$ and $\omega_i^{damaged}$ are the pulsations of the i^{th} mode for the undamaged and damaged structures. $\Phi_i^{undamaged}$ and $\Phi_i^{damaged}$ are the i^{th} strain mode shape of the undamaged and damaged structures, respectively. It was demonstrated that the index $\Delta\Phi_i$ is more sensitive to the damage severity than the similar index calculated by using the displacement eigenparameter.

Another damage detection method considers the decrease in modal strain energy between two structural degrees of freedom. This technique was proposed by Stubbs et al.

$[70, 71]$. For a Bernouilli-Euler beam, the damage index β_{ij} is given by

$$\beta_j = \frac{\displaystyle\sum_{i=1}^{n} \mu_{ij}^{damaged}}{\displaystyle\sum_{i=1}^{n} \mu_{ij}^{undamaged}} \tag{20}$$

with

$$\mu_{ij}^{damaged} = \frac{\displaystyle\int_a^b \left(\left(\Phi_i^{damaged}(x)\right)''\right)^2 dx + \int_0^L \left(\left(\Phi_i^{damaged}(x)\right)''\right)^2 dx}{\displaystyle\int_0^L \left(\left(\Phi_i^{damaged}(x)\right)''\right)^2 dx} \tag{21}$$

$$\mu_{ij}^{undamaged} = \frac{\displaystyle\int_a^b \left(\left(\Phi_i^{undamaged}(x)\right)''\right)^2 dx + \int_0^L \left(\left(\Phi_i^{undamaged}(x)\right)''\right)^2 dx}{\displaystyle\int_0^L \left(\left(\Phi_i^{undamaged}(x)\right)''\right)^2 dx} \tag{22}$$

$\mu_{ij}^{undamaged}$ and $\mu_{ij}^{damaged}$ are the fractional strain energies for the i^{th} mode of the undamaged and damaged structures, respectively. L corresponds to the length of the beam. a and b define the two endpoints of the element j of the beam where the damage is estimated. Increasing of the damage at element j of the beam increases the value of index β_{ij}.

The modes strain energy was performed by Parloo et al. [63] for the identification of various damages on the I-40 highway bridge in New Mexico. It was shown that the method can not allow an efficient and robust detection of small damage due to measurement noise.

Alvandi and Cremona [72] used the strain energy method for damage detection not only in beam but also in civil engineering structures with experimental data. They showed that the the strain energy method presents a best stability regarding the noisy signals. Even if the strain energy method appears to be more efficient than three other methods tested by the authors (the changes in flexibility, see section 2.1.; change in mode shape curvature, see section 2.1.; and change in flexibility curvature, see section 2.1.) in the complex and simultaneous damage cases, they indicated that the detection of two damages in the structures can be more difficult and a specific procedure needs to be applied. The same conclusions were reported by the authors if the damage is located near the supports or joints.

Changes in dynamic flexibility

The dynamic flexibility matrix can be used as a damage detection method in the static behavior of the structure [73].

The dynamic flexibility matrix $\mathbf{G}$ is defined as the inverse of the static stiffness matrix

$$\mathbf{u} = \mathbf{G}\mathbf{f} \tag{23}$$

where $\mathbf{f}$ is the applied static force and $\mathbf{u}$ corresponds to the resulting structural displacement.

By only keeping the first few modes of the structure, the expression of the flexibility matrix can be approximating by

$$\mathbf{G} = \Phi\Omega^{-1}\Phi^T = \sum_{i=1}^{n} \frac{1}{\omega_i^2} \Phi_i \Phi_i^T \tag{24}$$

where ω_i is the i^{th} resonant frequency of the structure. Ω is the diagonal matrix of rigidity given by

$$\Omega = diag\left(\omega_1, \omega_2, \cdots, \omega_n\right) \tag{25}$$

Φ_i defines the i^{th} mode shape and Φ is the mode shapes matrix given by

$$\Phi = [\Phi_1 \quad \Phi_2 \cdots \Phi_n] \tag{26}$$

Each column of the flexibility matrix represents the displacement pattern of the structure associated with an unit force applied at the associated degree of freedom. It is noted that small changes in the lower order modes can induce highly evolutions of the dynamic flexibility matrix due to the inverse relation to the square of the resonant frequencies ω_i.

Doebling et al. [2] reviewed five different approaches to detect damage by comparing the dynamic flexibility matrices of the damaged and undamaged structures: comparison of flexibility changes, unity check method, stiffness error matrix method, effects of residual flexibility and changes in measured stiffness matrix.

The changes in the flexibility matrices before and after damage in structures can be obtained by considering the variation matrix

$$\Delta\mathbf{G} = \mathbf{G} - \mathbf{G}_{undamaged} \tag{27}$$

where $\mathbf{G}$ and $\mathbf{G}_{undamaged}$ are respectively the flexibility matrices of the damaged and undamaged structures. It is evident that the column of the flexibility variation matrix $\Delta\mathbf{G}$ where the maximum variation is observed corresponds to the damage location.

Lin [74] proposed the use of the unity check method to locate damage in structures. This criterion was previously proposed by the author for location of modeling errors using modal test data [75]. The error matrix is given by

$$\mathbf{E} = \mathbf{G}\mathbf{K}_{undamaged} - \mathbf{I} \tag{28}$$

As shown in the previous equation, the method is based on the pseudo-inverse relationship between the dynamic flexibility matrix $\mathbf{G}$ of the damaged structure and the structural stiffness matrix $\mathbf{K}_{undamaged}$ of the undamaged structure: the product of a stiffness matrix and a flexibility matrix produces an unity matrix at any stage of damage. So, if no damage is present, the error matrix $\mathbf{E}$ returns a zero matrix. The plots of the stiffness error matrix $\mathbf{E}$ indicate the damage location that is related to the highest peak.

The stiffness error matrix method is based on the definition of the following matrix

$$\mathbf{E} = \mathbf{K}_{undamaged}\left(\mathbf{G} - \mathbf{G}_{undamaged}\right)\mathbf{K}_{undamaged} \tag{29}$$

that is a function of the flexibility change in the damaged structure $\Delta\mathbf{G} = \mathbf{G} - \mathbf{G}_{undamaged}$ and the stiffness matrix $\mathbf{K}_{undamaged}$ of the undamaged structure. Gysin [76] demonstrated that the stiffness error matrix is very dependent of the number of modes retained to form the flexibility matrix. Park et al. [77] proposed to extend the error stiffness matrix by the weighted error matrix which magnifies the amount of stiffness error only at certain nodal points related to the damaged element (by applying a division by the variance in natural frequency). It was demonstrated that this weighted error matrix is more sensitive to damage allowing the detection of structural damage at early state.

The effects of residual flexibility are defined by

$$\mathbf{G} = \Phi\Omega^{-1}\Phi^T + \mathbf{G}_{residual} \tag{30}$$

where $\mathbf{G}_{residual}$ defines the contribution of the flexibility matrix from modes no retained in the approximated flexibility matrix $\mathbf{G}$. As explained by Doebling et al. [2], the purpose is to be able to obtain a good estimation of the static flexibility matrix allowing more accurate detection and identification of damage in structures.

Aktan et al. [78] proposed to apply changes in dynamic flexibility as an indicator for damage detection in bridge. Even if the influence of the truncation previously observed by Gysin [76] was observed by the authors, they concluded that evolution of the dynamic flexibility is suitable for damage detection.

Mayes [79] used the measured dynamic flexibility for damage detection of the I-40 bridge over the Rio Grande.

Park et al. [80] illustrated the use of the flexibility matrix for damage detection in ten-story building, a bridge and an engine structure. They demonstrated that the damage can be correctly predicted and located.

Topole [81] proposed to investigate the sensitivity and robustness of the dynamic flexibility matrix. Considering various damage scenarios, including multiple damages and damage at joints, he concluded that this damage assessment technique works well for locating and the location and quantifying a simple damage but provides erroneous results for multiples damages.

Parloo et al. [63] used the change in flexibility method on a clamped board experiments and the I-40 highway bridge in New Mexico for damage detection.

Alvandi and Cremona [72] used the change in flexibility to detect and identify damaged elements on a simple supported beam. However the authors used experimental data of various civil engineering structures (for example the Interstate 40 highway bridge in Albuquerque, Z24 Bridge in Switzerland, a High Speed Railway Bridge in France and Saint Marcel Bridge in Canada) and indicated that changes in dynamic flexibility shows less efficiency in the case of more complex and/or simultaneous damages, when damages are closed to joints or in the case of low quality of experimental data.

Dynamic flexibility curvature method

Zhang and Aktan [82] proposed to combine the mode shape curvature technique (see section 2.1.) and the change in dynamic flexibility matrix (see section 2.1.). They used the change in curvature obtained by considering the flexibility instead of the mode shapes as described for the mode shape curvature method.

The Dynamic Flexibility Curvature Change is defined by

$$\mathbf{DFCC} = \sum_{i=1}^{n} \mid \left(\Phi\Omega^{-1}\Phi^{T}\right)'' - \left(\Phi_{undamaged}\Omega_{undamaged}^{-1}\Phi_{undamaged}^{T}\right)'' \mid \qquad (31)$$

where Ω and $\Omega_{undamaged}$ are the diagonal matrix of rigidity for the damaged and undamaged structures, respectively. Φ and $\Phi_{undamaged}$ correspond to the mode shapes matrix for the damaged and undamaged systems. n defines the number of modes shapes. So a localized increase curvature change in $\mathbf{DFCC}$ indicates a loss of stiffness at the same location indicating the presence of damage.

Alvandi and Cremona [72] investigated the use of the dynamic flexibility curvature method for damage detection in beam structures with one or two damages and noisy measurements. They concluded that if the presence of damage is easily detectable in the case of one damage, it can be more difficult for multiple damages.

Sensitivity-based approach

The sensibility-based approach uses the mode shapes of the damaged and undamaged structures and the natural frequencies of the undamaged modes. The damage assessment technique localizes the damage by means of the mode shape sensitivities to changes in stiffness between adjacent structural degree of freedom or/and changes in mass in structural degree of freedom [83, 84].

The sensitivity of the i^{th} degree of freedom for the j^{th} mode shape in stiffness between the p^{th} and q^{th} degrees of freedom is defined by

$$\frac{\partial \Phi_{ij}}{\partial k_{pq}} = (\Phi_{pj} - \Phi_{qj}) \sum_{r=1, r\neq j}^{n} \frac{1}{\lambda_r - \lambda_j} \frac{\Phi_{pr}\Phi_{qr}}{a_r} \Phi_{ir} \tag{32}$$

where λ_r are the poles of the system and a_r the modal scaling factors.

The sensibility of the i^{th} degree of freedom for the j^{th} mode shape to local mass at the k^{th} degree of freedom is estimated by

$$\frac{\partial \Phi_{ij}}{\partial m_k} = -\lambda_j \frac{\Phi_{kj}^2}{a_j} \Phi_{ij} + \Phi_{kj} \sum_{r=1, r\neq j}^{n} \frac{\lambda_j^2}{\lambda_r - \lambda_j} \frac{\Phi_{kr}\Phi_{ir}}{a_r} \tag{33}$$

n defines the number of modes retained for the approximation of the sensibility indicators. Generally, even if a limited number of modes is used for the calculation of these sensibility factors, a good approximation can be obtained in comparison with the exact expression that corresponds to the case in which all mode shapes of the structure are taken into account. Moreover, it is admitted that calculating mass sensitivities is numerically more stable that calculating stiffness sensitivities due to the presence of noise measurement.

Parloo et al. [63] compared the mode shape sensitivities with various damage indicators such as the modal flexibility change method, the mode shape curvature change and the strain energy method. Comparisons were performed by conducting damage detection experiments on a clamped board as well as on the data from the I-40 highway bridge in New Mexico. The authors clearly indicated that the sensitivity-based approach is the most efficient damage assessment technique. Even if the damage is very small, this technique allows a correct identification of the damage. Moreover, it was observed that even if the survey can be done in only one part of the clamped board structure, the sensitivity-based technique is still able to identify the location of damage. However, the method is less efficient in the case of the real civil engineering structure due to the fact that the sensibility-based approach is strongly affected by the presence of measurement noise.

Changes in antiresonances

Structural modification due to the presence of damage can drastically change the phenomenon of antiresonances and the relationship between the resonance and antiresonance behavior. So physical interpretation of the phenomenon of antiresonances and the significance of antiresonances in experimental structural analysis can be used for the detection and location of structural damage in complex structure. The antiresonance frequencies can be interpreted as the resonance frequencies of the system fixed at the excitation points in the excitation directions. For mored details, we refer the interested reader to the book of Ewins [57].

First if all, it is well known that the resonances and antiresonances alternate continuously only for the Frequency Response Function (see section 2.2. for more details) of the driving point where the response co-ordinate and the excitation co-ordinate are identical. Increasing the distance between the excitation co-ordinate and the response co-ordinate, the number of antiresonance ranges decreases [85].

Bamnios et al. [86] proposed to study both analytically and experimentally the influence of damage on the mechanical impedance of beams under various boundary conditions. Experimental tests were performed on plexiglas beams damaged for different locations and for different damage severities. They demonstrated that the driving-point impedance changes due to the damage in case of flexural vibrations. For a cantilever beam, they indicated that the first antiresonance moves towards the first resonance as the driving point approaches the damage. Then, the antiresonance tends to coincide with the first resonance in the vicinity of the damage. Finally, the first antiresonances does not move after crossing the damage whereas the first antiresonances moves toward the first resonance in the case of undamaged structure. They also indicated that changes in antiresonances strongly follow definite trends depending upon the damage location. Then they showed that there is a jump in the slope of the curve of the changes in the first antiresonances in the vicinity of the damage. Increasing the damage severity increases substantially the jump phenomenon allowing an efficient prediction of the damage localization. The results were validated by considering both cantilever beam and beam clamped at both ends.

Douka et al. [87] studied changes in antiresonances in double-cracked beams. As previously noted in the paper of Bamnios et al. [86], they concluded that a shift in the antiresonances of the damage structure occurs depending on the severity and location of the damage. Using changes in antiresonances and resonances allows the identification of the crack size and location of a double-cracked beam. Due to the presence of two damages, two jump in the slope of the curve of the changes in the first resonances were observed by the authors; each slope was detected in the vicinity of each damage. However, they indicated that the proposed method based on changes in antiresonances appears not to be efficient for small damage due to the fact that small irregularities in the slope of the antiresonance curve cannot be reliably estimated.

Showing the studies of Bamnious et al. [86] and Douka et al. [87], Dharmaraju and Sinha [88] proposed to comment the methodology of the use of antiresonances by conducting experiments on a freefree beam with open cracks. They clearly demonstrated that the identification of the crack location due to the change in antiresonance can be difficult. Then, they finally concluded that a more robust identification based on the previous methodology has to be developed for practical applications.

Dilena and Morassi [39, 89] proposed to extend the previous results to avoid the non-uniqueness of the damage location in symmetrical beams by an appropriate use of resonances and antiresonances for damage identification in symmetrical beams. The study was conducted on a freefree uniform beam either under axial or bending vibration. They demonstrated that an appropriate use of the changes in ratio between the first resonance and the first antiresonance uniquely determines the damage localization. However, they indicated that the methodology may be more difficult to be extend in the case of complex structures due to the noise on antiresonances.

2.2. Changes in Frequency Response function

One of the limitations of the indexes based on changes in frequencies or modes shapes, evolution of modal shape curvature, and mode strain energy is the only use of the resonant frequencies and mode shapes for the vibrational description of the mechanical structures. So, extension of the damage detection based on the use of the Frequency Response Function must be investigated.

Theoretical description of FRF

The equation of motion for a complex structure is often described by the following equation written as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}(t) \tag{34}$$

where **x** is the vector of nodal degrees of freedom of the structure. t defines the time instant. $\mathbf{M}$, **K** and $\mathbf{C}$ are the mass, stiffness and damping matrices. $\mathbf{f}(t)$ is the external force vector, and dot represents the derivative with respect to the time.

For the case of harmonic excitation, the force vector can be defined as

$$\mathbf{f}(t) = \mathbf{F}e^{i\omega t} \tag{35}$$

where ω is the forcing frequency, and **F** defines the force amplitude vector. Therefore, the response vector may be assumed as

$$\mathbf{x}(t) = \mathbf{X}e^{i\omega t} \tag{36}$$

and equation 34 may be rewritten as

$$\left(-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}\right)\mathbf{X} = \mathbf{F} \tag{37}$$

Consequently, the relation between the response $\mathbf{X}\left(\omega\right)$ and the excitation $\mathbf{F}\left(\omega\right)$ at each frequency ω is given by

$$\mathbf{X}\left(\omega\right) = \mathbf{H}\left(\omega\right)\mathbf{F}\left(\omega\right) \tag{38}$$

where $\mathbf{H}\left(\omega\right)$ defines the receptance matrix of the system or the Frequency Response Function matrix that is given by

$$\mathbf{H}\left(\omega\right) = \left(-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}\right)^{-1} \tag{39}$$

The relation between the response at the i^{th} co-ordinate with a single excitation applied at the j^{th} coordinate defines the individual Frequency Response Function $\mathbf{H}_{ij}\left(\omega\right)$ that is given by

$$\mathbf{H}_{ij}\left(\omega\right) = \frac{\mathbf{X}_i}{\mathbf{F}_j} \tag{40}$$

with $\mathbf{F}_n = 0$ for $n = 1, \ldots, m$ and $n \neq j$ (m is the total number of degree-of-freedom). It may be noted that Operational Deflection Shape (ODS) that describes the normalized structure shape at each frequency ω is given by the column vector of the receptance matrix $\mathbf{H}_j\left(\omega\right)$.

Extension of the MAC criteria for the Frequency Response Function

As previously explained, the stiffness and the damping of a structure are affected by the damage. So, changes in the receptance matrix of the damaged structure $\mathbf{H}_{ij}^{damaged}(\omega)$ may be used as a damaged indicator.

As an extension of the MAC criteria in the frequency domain, Heylen and Lammens [90] proposed the Frequency Response Assurance Criterion

$$FRAC_{ij}(\omega) = \frac{|\mathbf{H}_{ij}^{damaged}(\omega)\left(\mathbf{H}_{ij}^{undamaged}(\omega)\right)^{*}|^{2}}{\mathbf{H}_{ij}^{undamaged}(\omega)\left(\mathbf{H}_{ij}^{undamaged}(\omega)\right)^{*}\mathbf{H}_{ij}^{damaged}(\omega)\left(\mathbf{H}_{ij}^{damaged}(\omega)\right)^{*}} \tag{41}$$

where $*$ defines the complex conjugate operator. In this case, the excitation is applied at the j^{th} co-ordinate, and the response function at the i^{th} co-ordinate. The values of $FRAC_{ij}$ varies between zero to unity. If the $FRAC_{ij}$ value is equal to unity, no damage is found. Increasing the damage decreases the value of $FRAC_{ij}$.

Zang et al. [91,92] proposed the first Global Shape Correlation function (GSC) to detect damage in structure. The GCS indicator is defined by

$$GCS(\omega) = \frac{|\mathbf{H}_{undamaged}^{*}(\omega)\,\mathbf{H}_{damaged}(\omega)|^{2}}{\left(\mathbf{H}_{undamaged}^{*}(\omega)\,\mathbf{H}_{undamaged}(\omega)\right)\left(\mathbf{H}_{damaged}^{*}(\omega)\,\mathbf{H}_{damaged}(\omega)\right)} \tag{42}$$

where $\mathbf{H}_{undamaged}$ and $\mathbf{H}_{damaged}$ are the column of the Frequency Response Function data measured at frequency ω for the undamaged and damaged structures. This function exists for all frequencies and the sum is over all locations. The $GCS(\omega)$ returns a real value between zero to unity, indicating no match at all or a complete match (i.e. no damage). So if the value $GCS(\omega)$ is different from unity, damage is detected.

Then Zang et al. [91, 92] proposed the second Global Amplitude Correlation function (GAC) based on response amplitudes. The GAC indicator is defined by

$$GAC(\omega) = \frac{2|\mathbf{H}_{undamaged}^{*}(\omega)\,\mathbf{H}_{damaged}(\omega)|}{\mathbf{H}_{undamaged}^{*}(\omega)\,\mathbf{H}_{undamaged}(\omega) + \mathbf{H}_{damaged}^{*}(\omega)\,\mathbf{H}_{damaged}(\omega)} \tag{43}$$

They also proposed the averaged integration of first Global Shape Correlation function (AIGSC) and the second Global Amplitude Correlation function (AIGAC)

$$AIGSC(\omega) = \frac{1}{N}\sum_{i=1}^{N} GSC(\omega_i) \tag{44}$$

$$AIGAC(\omega) = \frac{1}{N}\sum_{i=1}^{N} GAC(\omega_i) \tag{45}$$

where N is the number of frequencies chosen in the frequency range of interest. The AIGSC and AIGAC indicators are real constant between zero to unity to indicate total damage or undamaged structure. The authors investigated these various indicators to a bookshelf structure with various case of damage, including location and level for single or multiple presence of damage. They concluded that all the correlation criteria are able to detect the damaged structures.

Zang and Imregun [93] proposed to use the sensitivity of the global correlation functions with respect to the damaged selected parameters for damage identification.

Pascual et al. [94, 95] proposed to quantify the correlation between two operational deflection shapes and to detect the presence of damage by using the Frequency Domain Assurance Criterion (FDAC)

$$FDAC_j\left(\omega_1, \omega_2\right) = \frac{\left|\sum_{i=1}^{n} \mathbf{H}_{ij}^{damaged}\left(\omega_2\right)\left(\mathbf{H}_{ij}^{undamaged}\left(\omega_1\right)\right)^*\right|^2}{\sum_{i=1}^{n} \mathbf{H}_{ij}^{undamaged}\left(\omega_1\right)\left(\mathbf{H}_{ij}^{undamaged}\left(\omega_1\right)\right)^* \sum_{i=1}^{n} \mathbf{H}_{ij}^{damaged}\left(\omega_2\right)\left(\mathbf{H}_{ij}^{damaged}\left(\omega_2\right)\right)^*} \tag{46}$$

where n is the total number of co-ordinates. The measurement of correlation is done between the operational deflection shapes of the damaged structure for each frequency ω_2 and the operational deflection shapes of the undamaged structure for each frequency ω_1.

Generally, a simplified form of the Frequency Domain Assurance Criterion (FDAC) is proposed based on the measure of correlation between the operational deflection shapes of the damaged and undamaged structures at the same frequency ω with only one applied force [96]. This simplified form that is referred to as the Response Vector Assurance Criterion (RVAC) is given by

$$RDAC\left(\omega\right) = \frac{\left|\sum_{i=1}^{n} \mathbf{H}_{i}^{damaged}\left(\omega\right)\left(\mathbf{H}_{i}^{undamaged}\left(\omega\right)\right)^*\right|^2}{\sum_{i=1}^{n} \mathbf{H}_{i}^{undamaged}\left(\omega\right)\left(\mathbf{H}_{i}^{undamaged}\left(\omega\right)\right)^* \sum_{i=1}^{n} \mathbf{H}_{i}^{damaged}\left(\omega\right)\left(\mathbf{H}_{i}^{damaged}\left(\omega\right)\right)^*} \tag{47}$$

Sampaio et al. [96,97] proposed to use an adaptation of the Response Vector Assurance Criterion, the Detection and Relative damage Quantification indicator

$$DRQ\left(\omega\right) = \frac{\sum_{\omega} RVAC\left(\omega\right)}{N} \tag{48}$$

where N defines the number of frequencies. This indicator can be considered as an arithmetic average of the $RVAC$ along the frequency range of consideration. If the DRQ factor is equal to unity no damage is detected. The larger the damage is, the smaller the degree of correlation between the operational deflection shapes of the undamaged and damaged structures. The value of DRQ returns a constant number between zero to unity.

Moreover, Sampaio et al. [96] proposed to measure the degree of correlation between the second spatial derivative of the operational deflection shapes of the damaged and undamaged structures to locate damage. The relation is given by

$$RDAC''\left(\omega\right) = \frac{\left|\sum_{i=1}^{n} \left(\mathbf{H}_{i}^{damaged}\left(\omega\right)\right)''\left(\left(\mathbf{H}_{i}^{undamaged}\left(\omega\right)\right)''\right)^*\right|^2}{\sum_{i=1}^{n} \left(\mathbf{H}_{i}^{undamaged}\left(\omega\right)\right)''\left(\left(\mathbf{H}_{i}^{undamaged}\left(\omega\right)\right)''\right)^* \sum_{i=1}^{n} \left(\mathbf{H}_{i}^{damaged}\left(\omega\right)\right)''\left(\left(\mathbf{H}_{i}^{damaged}\left(\omega\right)\right)''\right)^*} \tag{49}$$

Finally, the authors [96, 97] proposed to use an adaptation of the Response Vector Assurance Criterion, the Detection and Relative damage Quantification indicator

$$DRQ^{''}(\omega) = \frac{\sum_{\omega} RVAC^{''}(\omega)}{N} \tag{50}$$

They validated the use of the previous versions of the DRQ and $DRQ^{''}$ on a free-free beam in transverse vibration. They indicated that the DRQ indicator is able to detect small damage and to distinguish different severity of damage. They also concluded that the versions of the $DRQ^{''}$ indicator does not permit to enhance the damage assessment in comparison with the classical DRQ indicator.

Frequency response curvature method

This damage assessment technique is based on the extension of the mode shapes curvature for a given frequency range. The frequency response curvature can be estimated by using a central difference approximation and is defined by

$$\left(\mathbf{H}_{ij}(\omega)\right)^{''} = \frac{\mathbf{H}_{i+1,j}(\omega) - 2\mathbf{H}_{i,j}(\omega) + \mathbf{H}_{i-1,j}(\omega)}{h} \tag{51}$$

where $\mathbf{H}_{ij}$ is the receptance or individual Frequency Response Function measured at the i^{th} location with a single excitation applied at the j^{th} coordinate for a given frequency ω. h is the distance between the measurement co-ordinates. One of the main advantage of this method is the simplicity of use due to the fact that the receptances are known and the calculation of the frequency response curvature approximation an be easily done.

The difference between the frequency response curvature of the damaged and undamaged structures for a given range of frequency $\omega = [\omega_{initial}; \omega_{final}]$ at the i^{th} location with an applied force at the j^{th} point is given by

$$\Delta\left(\mathbf{H}_{ij}(\omega)\right)^{''} = \sum_{\omega=\omega_{initial}}^{\omega_{final}} \left| \left(\mathbf{H}_{ij}^{damaged}(\omega)\right)^{''} - \left(\mathbf{H}_{ij}^{undamaged}(\omega)\right)^{''} \right| \tag{52}$$

Palacz et al. [60] proposed the use of the frequency curvature technique for damage detection in a cantilever beam. They clearly demonstrated that increasing the number of measurements drastically increases the robustness of the damage location. Moreover, measurement errors can result in disturbances in proper assessment of the damage location if a small number of measurement points is considered. They also indicated that errors in the frequency curvature do not strongly affect the identification of the damage parameters.

2.3. Coupling responses measurements

Basic theory

Considering that the damage induces only a decrease of the stiffness at the damage location, the equation of motion for a damaged structure can be written as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \tilde{\mathbf{K}}\mathbf{x} = \mathbf{f}(t) \tag{53}$$

where **x** is the vector of nodal degrees of freedom of the damage system. $\mathbf{M}$ and $\mathbf{C}$ are the mass and damping matrices. $\tilde{\mathbf{K}}$ is the global stiffness matrix that contains the stiffness reduction $\mathbf{K}_{crack}$ at the crack location. $\mathbf{f}(t)$ is the external force vector, and dot represents the derivative with respect to the time.

As previously shown in Section 2.2., if the harmonic excitation force vector is assumed on the form

$$\mathbf{f}(t) = \mathbf{F}e^{i\omega t} \tag{54}$$

where ω is the forcing frequency and $\mathbf{F}$ defines the force amplitude vector, the harmonic response vector can be determined by using the following form

$$\mathbf{x}(t) = \mathbf{X}e^{i\omega t} \tag{55}$$

and the previous equation of motion 53 can be rewritten as

$$\left(-\omega^2\mathbf{M} + i\omega\mathbf{C} + \tilde{\mathbf{K}}\right)\mathbf{X} = \mathbf{F} \tag{56}$$

Due to the fact that the damage introduces a stiffness matrix $\mathbf{K}_{crack}$ at the damage location, the equation of motion of the damaged system can be rewritten as

$$\left(-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}\right)\begin{bmatrix} \mathbf{X}^c \\ \mathbf{X}^{uc} \end{bmatrix} = \mathbf{F} - \mathbf{F_c} = \begin{bmatrix} \mathbf{F}^c \\ \mathbf{F}^{uc} \end{bmatrix} - \begin{bmatrix} \mathbf{F_c}^c \\ \mathbf{0} \end{bmatrix} \tag{57}$$

where $\mathbf{K}$ defines the stiffness matrix of the undamaged structure. The subscripts c and uc represent the cracked and uncracked elements (i.e. damaged and undamaged structures), respectively. $\mathbf{F}$ contains the external force vector, and $\mathbf{F_c}$ represents the force vector only due to the contribution of the crack. It may be noted that the vector $\mathbf{F_c}$ contains non-zero terms only at the crack nodal degrees of freedom due to the local reduction of stiffness into the global stiffness matrix of undamaged structure. The expression of $\mathbf{F_c}^c$ is given by

$$\mathbf{F_c}^c = \mathbf{K}_{crack}\mathbf{X}^c \tag{58}$$

Then, assuming modal viscous damping and using the normal mode substitution

$$\mathbf{x} = \tilde{\Phi}\mathbf{q} = \sum_{r=1}^{n} q_r\tilde{\Phi}_r \tag{59}$$

the equation set 56 is diagonalized as follows:

$$\forall r \qquad \left(\tilde{\omega_r^2} - \omega^2 + 2i\zeta_r\tilde{\omega_r}\omega\right)q_r = f_r \tag{60}$$

q_r defines the modal participation of the r^{th} mode. $\tilde{\Phi}_r$ corresponds to the associated eigenvector. n is the number of retained modes for the normal mode substitution. f_r is the generalized modal force given by

$$f_r = \frac{\tilde{\Phi}_r^T\mathbf{F}}{m_r} \tag{61}$$

and $\tilde{\omega_r}$ is the r^{th} mode undamped natural frequency of the cracked beam

$$\tilde{\omega_r} = \sqrt{\frac{\tilde{k_r}}{m_r}} \tag{62}$$

with

$$m_r = \tilde{\Phi}_r^T\mathbf{M}\tilde{\Phi}_r \tag{63}$$

$$\tilde{k_r} = \tilde{\Phi}_r^T \tilde{\mathbf{K}} \tilde{\Phi}_r \tag{64}$$

ζ_r is the r^{th} mode damping ratio

$$\zeta_r = \frac{c_r}{2 m_r \omega_r} \tag{65}$$

with $c_r = \alpha m_r + \beta k_r$ due to the orthogonality property of $\mathbf{M}$ and $\tilde{\mathbf{K}}$; here $\omega_r = \sqrt{\frac{k_r}{m_r}}$ is the r^{th} undamped natural frequency of the undamaged structure. k_r is defined as follows

$$k_r = \Phi_r^T \mathbf{K} \Phi_r \tag{66}$$

where $\mathbf{K}$ defines the stiffness matrix of the undamaged system; and Φ_r is the eigenvectors of the undamaged structure.

Consequently, the relationship between the output vector $\mathbf{X}(\omega)$ and the input vector $\mathbf{F}(\omega)$ is given by

$$\mathbf{X}(\omega) = \mathbf{H}(\omega)\mathbf{F}(\omega) = \sum_{r=1}^{n} \frac{\tilde{\Phi}_r \tilde{\Phi}_r^T}{m_r \left(\tilde{\omega_r^2} - \omega^2 + 2i\zeta_r \tilde{\omega_r} \omega\right)} \mathbf{F}(\omega) \tag{67}$$

where $\mathbf{H}(\omega)$ defines the Frequency Response Function matrix previously defined in section 2.2.. $\mathbf{H}(\omega)$ is the linear combination of each mode.

Thereby, the Frequency Response Function $\mathbf{H}_{kl}(\omega)$ (that corresponds to an excitation force only applied at the l^{th} degree of freedom with the response located at the k^{th} degree of freedom) is given by

$$\mathbf{H}_{kl}(\omega) = \sum_{r=1}^{n} \frac{\tilde{\Phi}_{lr} \tilde{\Phi}_{kr}}{m_r \left(\tilde{\omega_r^2} - \omega^2 + 2i\zeta_r \tilde{\omega_r} \omega\right)} \tag{68}$$

It clearly appears that the amplitude of the resonance peaks are affected by the damage severity and location due to the fact that $\tilde{\omega_r}$ and $\tilde{\Phi}_r$ are functions of the properties of damage. Then, the equation of motion defined in equation 68 can be expressed by rearranging the equation of motion and extracting the force vector due to the contribution of the damage

$$\mathbf{X}(\omega) = \sum_{r=1}^{n} \frac{\tilde{\Phi}_r \tilde{\Phi}_r^T}{m_r \left(\omega_r^2 - \omega^2 + 2i\zeta_r \omega_r \omega\right)} \mathbf{F}(\omega) - \sum_{r=1}^{n} \frac{\tilde{\Phi}_r \tilde{\Phi}_r^T}{m_r \left(\omega_r^2 - \omega^2 + 2i\zeta_r \omega_r \omega\right)} \mathbf{F_c}(\mathbf{x}, \omega) \tag{69}$$

where $\omega_r = \sqrt{\frac{k_r}{m_r}}$ is the r^{th} undamped natural frequency of the undamaged structure.

So, considering this last equation 69, the possible coupling of response measurements (i.e. two lateral vibrations, lateral/axial or lateral/torsional vibrations for example) due to the damage can be clearly explained. The first term of equation 69 corresponds to the effect of the external force $\mathbf{f}(t) = \mathbf{F}e^{i\omega t}$ where ω is the forcing frequency. The second term of equation 69 corresponds to the effect of damage in coupling vibrations. For a damaged structure, the term $\mathbf{F_c}(\mathbf{x}, \omega)$ introduces an excitation force due to the damage. So the response of an excitation is observed not only in the direction of the excitation but also in other directions. It may be observed that the detection of damage based on coupling measurements is possible if the measurements of the Frequency Response Function are done not only in the direction of the external excitation force $\mathbf{f}(t)$ but also in another direction, and if the excitation of the external force $\mathbf{f}(t)$ and the excitation due to the damage $\mathbf{F_c}$ are not in the same direction [40]. Moreover, the coupling is strongly dependent on the position and severity of the damage (i.e. the intensity of the excitation force due to damage $\mathbf{F_c}(\mathbf{x}, \omega)$). One of the main advantage of the coupling responses measurements is that the detection is not sensitive to uncertainties and can be easily applied for the detection of small cracks.

Applications for damage detection in practical cases

The identification of damage in beams by coupled response measurements was proposed by Gounaris et al. [98]. The methodology for the determination of the severity and the location of a transverse crack requires an external excitation to dynamically excite the beam and two response measurements at a point of the beam. By measuring the displacement in one direction when exciting in another direction allows the determination of the presence of damage. The authors indicated that the main advantage of this method is the possibility to detect damage for small cracks and that more than one coupling measurements are analytically obtained and may be experimentally observed allowing an efficient and robust identification of the severity and localization of damage.

Liu et al. [99] illustrated both analytically and experimentally the damage detection in hollow section structure in freefree boundary conditions through coupled response measurements. They proposed to investigate the coupling measurements of the lateral and axial responses due to the presence of damage: for the uncracked beam, lateral or axial force only excite the corresponding bending or axial modes. When a damage is added in the structure, they observed the coupling behavior of axial and lateral displacements (i.e. the axial and bending modes). They demonstrated that the coupling of lateral and axial vibrations is clearly observable in experimental Frequency Response Function: the presence of damage introduces an extra peak in the bending direction close to the undamaged axial natural frequency. Moreover, they proposed to validate the coupling measurement method with the modal assurance criterion (MAC, see section 2.1. for more details) for physically interpreting the appearance of new peaks. They concluded that the coupling measurements of axial and lateral modes is a very good indicator of the presence of damage.

Chasalevris and Papadopoulos [100] investigated the coupled bending vibrations of a stationary shaft with two cracks. They considered the case of bending vibrations caused by a vertical excitation. They noted that the coupling measurement is observable (i.e. response in the horizontal plane) when the crack orientations are not symmetrical to the vertical plane. Experimental tests in a clamped-free shaft were performed to prove that the coupling phenomenon is more intense for deeper damage. They also observed that the coupling becomes stronger or weaker depending on the relative angular position of the cracks and the damages severity.

Lee et al. [101] also proposed to use the coupling measurements in Frequency Response Function for the identification of damage parameters. Numerical examples for cantilever beam and simply-supported beam.

Extension of coupling measurements for rotating machinery

For rotating machinery, all the previous criteria and methodologies based on linear measurements can be useful for damage detection even if these approaches have to be carried out during static condition of rotor and are an "‘off-line process"’ and so time consuming for practical engineering applications in rotors. However extension of the detection of open cracks in rotating machinery based on coupling measurements presented in section 2.3. were also proposed by some researchers.

Papadopoulos and Dimaragonas [102] studied the coupling of longitudinal and bending vibrations of a rotating shaft, due to an open transverse crack. Then, they proposed to investigate the significant influence of the bending vibration on the torsional vibration spectrum [103]. They also demonstrated that both the evolution of eigenfrequencies and the coupling of the vibration modes are verified analytically and experimentally and can be efficiency used for the detection of damage. They indicated that the severity of damage clearly affects the decrease of frequencies and the coupling measurements of modes [104]. They concluded phenomena of coupling between bending and torsion, bending and tension, and the general vibration coupling [105] can be very useful for rotor crack identification in service, which is of importance to turbo-machinery.

Ostachowicz and Krawczuk [106] demonstrated that an open crack can be detected by using coupling of torsional and bending vibration in a rotating shaft.

Collins et al. [107] used the coupling mechanism in lateral and longitudinal directions of a cracked rotor. They introduced an impulse axial excitation to the cracked shaft and observed the responses in the longitudinal direction for diagnosis of damage.

Gounaris and Papadopoulos [102] proposed to identify the damage parameters in rotating shafts bu using the response coupled crack. The basis of the method is the measurement coupled response measurements. They introduced a model that considers the gyroscopic effect and the axial vibration with a single harmonic excitation in one direction (bending), while measuring in an other (axial). Considering different rotational speeds and excitations, they observed that the phenomena of coupled measurements is an easy and efficient way to detect the existence of damage in rotating shafts.

3. Non-linear analysis

Even if in the case of simple structures, the damage position and severity can be determined from changes by using various linear analysis previously presented in this review (changes in natural frequencies, modes shapes, MAC criteria, Frequency Response Functions, antiresonances, coupling measurements,...), some researchers have illustrated the fact that the presence of damage can induce more complicated behavior.

For example Gudmundson [109] noted during experimental tests on a cantilever beam that decrease in natural frequencies is not always observed due to the closing of the crack. So the linear analysis appears to be insufficient to describe the non-linear behavior of the crack, the so called "‘breathing phenomena"’ corresponding to the fact that the crack alternately opens and closes during experimental tests.

3.1. Non-linear behavior and equation of damage structures

The equation of motion for a complete system with a damage can be defined as follows

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{Q} + \mathbf{W} + \mathbf{W}_{damage} \tag{70}$$

where overdots indicate differentiation with respect to time. $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices. In the case of rotating machinery, the damping matrix includes not only the external damping but also the gyroscopic contribution. $\mathbf{Q}$ and $\mathbf{W}$ define the vector of gravity force and external force. $\mathbf{W}_{damage}$ is the vector contribution due to the damage in the structure. This vector contains the non-linear expressions due to the breathing mechanism associated with the crack. In a damage structure, at the crack location, the part of the cross section which is cracked is not capable of supporting tensile. This fact induces that the crack may open and close according to the stresses developed in the cracked surface. When the crack is in a compression zone, the crack remains closed and the damage structure has the same stiffness as the intact structure. If the crack is in a tensile zone, the crack is open, resulting in reduced structural stiffness. It may be said that the two previous situations are generally completed by considering the transition situation where the crack is partially closed or opened and the extent of crack opening is determined by the proportion of the crack face which is subject to tensile axial stresses. So, during the transition from close to open crack or vice versa, the structure is more or less stiff due to the variation of the stiffness at the crack location.

To a first approximation, a damage structure may be treated by considering the resulting equations of motion linear with time dependent coefficients. However, in general cases for rotating machinery or civil engineering structures, the opening-closing effect of the crack may be a function of the vibration amplitudes of the system and thus a full non-linear treatment is needed: the equations become totally nonlinear because deflection give rise to local axial stretching and this effect the position of the neutral axis of any specified cross section of structure.

Generally speaking, equation 70 are rewritten as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} + \left(\mathbf{K} + f\left(t\right)\mathbf{K}_{damage}\right)\mathbf{x} = \mathbf{Q} + \mathbf{W} \tag{71}$$

where $\mathbf{K}_{damage}$ is the stiffness matrix due to the crack. $f\left(t\right)$ is defined as the function describing the breathing mechanism and varies between zero to one.

Modeling the breathing mechanism is one of the first difficulty when dealing with the non-linear effect of damage in dynamic responses of structures, the non-linear opening and closing of a crack being generally dependent on the physical system under study.

For example, in the case of rotating machinery, a simple model, called the "'hinge model"', of an opening and closing crack that assumes to change from its closed to open state abruptly as the shaft rotates was proposed by Gasch [110, 111]. When the crack is assumed to open and close during the revolution of the shaft, some researchers [112–118] proposed models in which the opening and closing of the crack was described by a cosine function. For more details on different non-linear breathing mechanisms for engineering applications, we refer the interested reader to the following references [119–123].

For civil engineering applications, the breathing of the crack can depend on the deflection of the structure. For example, Law and Zhu [55] proposed to open and close the crack zone when a load (that represents a vehicle) is moving along the beam.

3.2. Changes in frequencies and dynamic response due to the breathing mechanism

First of all, we have to compare the modal properties and changes in resonant frequencies obtained by considering a breathing crack and an open crack. The open crack corresponds to the case where $f\left(t\right) = 1$.

The first studies that investigated the closing and breathing effects of cracks on the dynamical characteristics were proposed by Carlson [124] and Gudmunston [109]. Considering an edge-cracked cantilever beam, they observed differences between the modal properties and dynamic response of damaged structures with open and closing cracks.

Later, the vibrations with the crack opening and closing has been proposed by Zastrau [125]. Changes in the spectrum of a simply-supported beam with multiple cracks and the bilinear nature of a beam with a breathing crack was also investigated.

Ostachowicz and Krawczuk [126] studied changes in the dynamic characteristics of a structure with the assumption of an open and closed crack leads.

Qian et al. [127] observed that a closing crack can induce modifications of the amplitudes responses of damaged structures. They indicated that the differences of displacement response between the damaged and undamaged structures are reduced due to the crack closure. They also performed numerical and experimental tests on a cantilever beam with an edge-crack that show good agreement. Finally they proposed to identify the modal parameters by means of an identification technique in the time domain to consider the closing effect of the crack.

Collins et al. [128] studied the non-linear vibration behavior by using direct numerical integration for forced vibrations of a beam with breathing crack.

Chondros et al. [129] investigated changes in vibration frequencies for a fatigue-breathing crack. As previously explained by Gudmunston [109], a smaller drop in frequencies for a fatigue-breathing crack than an open-crack model predicts is observed.

Luzzato [130] evaluated the evolution of the first three eigenfrequencies of a simply supported beam by considering an open crack and a breathing crack. He also found that the relationships between eigenfrequencies and damage severity for an open and breathing cracks are different. Decreases for the breathing crack appears to be less significant.

A numerical study of the vibration of edge-cracked beam on elastic foundation for different damage severity and location with both an opening crack and a breathing crack was performed by Hsu [131]. The author also investigated the effects of axial loading and foundation stiffness. Using an excitation force at at the end of the beam, it appears that the responses with opening crack and closing crack are very different. A typical non-linear behavior of the beam with a breathing crack is obtained. A deformation of the mode shape is also observed when the vibration amplitudes increase.

Bikri et al. [132] studied the non-linear free vibrations of a clampedclamped beam with an edge crack. They indicated that the non-linear frequencies increase with increasing vibration amplitudes. They also obtained the classical result that the natural frequency shift is smaller for the non-linear breathing crack than for the linear open crack. Moreover, they observed that the non-linear mode shapes with a breathing crack are very different from the mode shapes of an open crack. Increasing the damages severity increase the the deformation of the normalized first mode shapes. They concluded that a robust identification of damage parameters only based on changes in frequencies can be very difficult due to the non-linear effects of the breathing behavior of the crack.

Law and Zhu [55] studied the effect of the breathing crack on the dynamic behavior of the concrete bridge. Using a damage model for reinforced concrete structures proposed by Abdel Wahab et al. [133], the crack zone is open or close when the vehicular loads is moving along the bridge. Even if the dynamic response appears to be a good indicator of damage in bridge, the authors indicated that significant changes in the dynamic responses are observed between the open crack and the breathing crack. So, a robust damage detection is possible, however, a complete identification of the damage severity is clearly dependent of the non-linear breathing behavior that has been chosen.

3.3. Higher-Order Frequency Response Functions or Order Function

As previously seen, the opening and closing of the crack can drastically change the modal properties of the damaged structures inducing worse damage identification. So a more complex non-linear analysis as to be taken into account based on the presence of the non-linear components in the vibrational responses of damaged structures.

To be able to identify the effects of a breathing crack on the non-linear responses of damaged structures, the use of the Volterra series and Harmonic Balance Method are discussed in the two following sections. Firstly, the Higher-Order Frequency Response Functions based on the Volterra series and the motion of the order functions are discussed. Then, the different Harmonic balance decompositions based on a simple or multiple excitations are treated. In addition, non-exhaustive example for civil engineering systems and rotating machinery are given to illustrate the feasibility, advantages and limitations of the different indicators based on these non-linear formulations.

Basic theory of the Volterra series and higher-order Frequency Response Functions

Due to the fact that the damaged structures exhibit non-linear behavior if the crack is breathing, an extension of the Frequency Responses Functions used for linear systems (presented in section 2.2.) can be considered for harmonic excitations by investigating the higher-order Frequency Response Functions with the Volterra series.

Volterra [134] proposed to extend the conventional input-output form that describes a linear system

$$y(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau \tag{72}$$

by the Volterra series in the case of non-linear system. This extended non-linear form of $y(t)$ can be written as

$$y(t) = y_1(t) + y_2(t) + \cdots + y_n(t) \tag{73}$$

where

$$y_1(t) = \int_{-\infty}^{+\infty} h_1(\tau_1)\, x(t-\tau_1)\, d\tau_1 \tag{74}$$

$$y_2(t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_2(\tau_1,\tau_2)\, x(t-\tau_1)\, x(t-\tau_2)\, d\tau_1 d\tau_2 \tag{75}$$

$$y_n(t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} h_n(\tau_1,\tau_2,\cdots,\tau_n)\, x(t-\tau_1)\, x(t-\tau_2)\cdots x(t-\tau_n)\, d\tau_1 d\tau_2\cdots d\tau_n \tag{76}$$

The kernels h_n can be considered to be symmetric without loss of generality [135] (i.e. $h_2(\tau_1,\tau_2) = h_2(\tau_2,\tau_1)$ for example).

Then, the extension of the Frequency Response Functions can be defined for each corresponding Volterra kernels h_i (for $i = 1,\cdots,n$). The High-Order Frequency Response Functions (HOFRFs) or Volterra kernel transforms $H_n(\omega_1,\cdots,\omega_n)$ are defined by using Multi-dimensional Fourier Transforms (MFTs)

$$H_n(\omega_1,\omega_2,\cdots,\omega_n) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} h_n(\tau_1,\tau_2,\cdots,\tau_n)\, e^{i(\omega_1\tau_1+\omega_2\tau_2+\cdots+\omega_n\tau_n)} d\tau_1 d\tau_2\cdots d\tau_n \tag{77}$$

It is noted that the visualization of the High-Order Frequency Response Functions (HOFRFs) can be difficult and generally expressions for only one periodic excitation composed of a single harmonic $x(t) = e^{i\omega t}$ are considered for damage detection. In this case, the non-linear response up to the n^{th} order is given by

$$y_n(t) = H_1(\omega)e^{i\omega t} + H_2(\omega,\omega)e^{i2\omega t} + \cdots + H_n(\omega,\omega,\cdots,\omega)e^{in\omega t} \tag{78}$$

One of the most important point to note is that only components of the output at multiples of the excitation frequency ω are observed. So the effect of damage and the breathing crack that characterize the non-linear behavior of the damaged system can be detected by following evolution of the High-Order Frequency Response Functions (HOFRFs). For the reader comprehension, it is remembered that no damage results in a linear structural system and only the first FRF H_1 with the associated linear response $y_1(t) = H_1(\omega)e^{i\omega t}$ is observed.

To be more general and to allow a damage detection for mechanical structure with multi-excitations, expression of the non-linear response $y(t)$ for a two-frequency harmonic input $x(t) = e^{i\omega_1 t} + e^{i\omega_2 t}$ is given up to the second order by [136]

$$\begin{aligned} y_n(t) = \; & H_1(\omega_1)e^{i\omega_1 t} + H_1(\omega_2)e^{i\omega_2 t} \\ & + H_2(\omega_1,\omega_1)e^{i2\omega_1 t} + H_2(\omega_2,\omega_2)e^{i2\omega_2 t} + 2H_2(\omega_1,\omega_2)e^{i\omega_1 + i\omega_2 t} + \cdots \end{aligned} \tag{79}$$

It can be noted that not only the multiples of the excitation frequencies ω_1 or ω_2 are observed, but also the component at the sum frequency for the excitation $\omega_1 + \omega_2$ is obtained for the second order Frequency Response Function $H_2(\omega_1,\omega_2)$. It is not difficult to understand that for a multi-excitation with n frequencies $(\omega_1,\omega_2,\cdots,\omega_n)$ the non-linear response is described by adding all the Higher-Order Frequency Response Functions with the all the components at the "'sum"' frequency $\omega_1 + \omega_2 + \cdots + \omega_n$.

So if the breathing behavior can be expressed by a non-linear function $f(\omega_d)$ (where ω_d is the fundamental frequency of the opening and closing phenomena) and the damaged structure is excited by the multi-frequencies $(\omega_1,\omega_2,\cdots,\omega_n)$ (with $\omega_d \neq \omega_i$ for $i = 1,2,\cdots,n$). All components of the "'sum frequency"' including ω_d (for example $m_i\omega_i + m_d\omega_d$ with $i = 1,2,\cdots,n$; $m_i = 1,2,\cdots,\infty$ and $m_d = 1,2,\cdots,\infty$ for the second order Frequency Response Functions) can be used for indicating the presence of damage in the structure. It clearly appears that all the Higher-Order Frequency Response Functions can be used for an efficient and robust detection of damage based on the non-linear responses and combinations between the fundamental frequency of the breathing crack and the excitation frequencies.

Basic theory of the Harmonic Balance Method and order n

In this section, the use of the Harmonic Balance Method for a simple excitation is developed to illustrate and to enhance the previous conclusions done by the Volterra series.

As previously explained in section 3.1., the breathing behavior of the crack induces the time-dependent coefficient of equation 71. So the system of the damaged structure can be rewritten in a "'non-linear"' form as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{Q} + \mathbf{W} + \mathbf{f}_{NL}(\mathbf{x},\omega,t) \tag{80}$$

with

$$\mathbf{f}_{NL}(\mathbf{x},\omega,t) = \mathbf{f}(t)\,\mathbf{K}_{damage}\mathbf{x} \tag{81}$$

The expression $\mathbf{f}_{NL}(\mathbf{x},\omega,t)$ represents the non-linear breathing behavior of the crack.

The general idea of the harmonic balance method is to represent the periodic solution of the non-linear system by its frequency content.

Firstly, we consider the case of an external periodic excitation force of frequency ω that is also the fundamental frequency of the breathing behavior of the crack. So the non-linear dynamical response of the damaged structure is represented as truncated Fourier series with m harmonics:

$$\mathbf{x}(t) = \mathbf{B}_0 + \sum_{k=1}^{m} \left(\mathbf{B}_k \cos(k\omega t) + \mathbf{A}_k \sin(k\omega t)\right) \tag{82}$$

where ω defines the fundamental frequency of the breathing behavior. $\mathbf{B}_0$, $\mathbf{A}_k$ and $\mathbf{B}_k$ (with $k = 1, \cdots, m$) define the unknown coefficients of the finite Fourier series.

Considering that the non-linear force $\mathbf{f}_{NL}$, the gravity force $\mathbf{Q}$ and the external excitation force $\mathbf{W}$ can be represented as truncated Fourier series of fundamental frequency ω, we have

$$\mathbf{f}_{NL}(\mathbf{x},\omega,t) = \mathbf{C}_0^f + \sum_{k=1}^{m} \left(\mathbf{C}_k^f \cos(k\omega t) + \mathbf{S}_k^f \sin(k\omega t)\right) \tag{83}$$

$$\mathbf{Q}(\mathbf{x},\omega,t) = \mathbf{C}_0^Q \tag{84}$$

$$\mathbf{W}(\mathbf{x},\omega,t) = \mathbf{C}_1^W \cos(\omega t) + \mathbf{S}_1^W \sin(k\omega t) \tag{85}$$

By substituting expressions 82, 83, 84 and 85 into 80, the analytical expressions of the various harmonic components of the non-linear response of the damaged structures can be easily obtained.

The constant terms $\mathbf{B}_0$ of the non-linear responses are given by

$$\mathbf{K}\mathbf{B}_0 = \mathbf{C}_0^Q + \mathbf{C}_0^f \tag{86}$$

The first harmonic components $\mathbf{A}_1$ and $\mathbf{B}_1$ are determined by considering the relations

$$\begin{bmatrix} \mathbf{K} - \omega^2\mathbf{M} & -\omega\mathbf{D} \\ \omega\mathbf{D} & \mathbf{K} - \omega^2\mathbf{M} \end{bmatrix} \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{B}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{S}_1^W + \mathbf{S}_1^f \\ \mathbf{C}_1^W + \mathbf{C}_1^f \end{bmatrix} \tag{87}$$

Finally, the k^{th} Fourier coefficients $\mathbf{A}_k$ and $\mathbf{B}_k$ for $2 \leq k \leq m$ are given by

$$\begin{bmatrix} \mathbf{K} - (k\omega)^2\mathbf{M} & -k\omega\mathbf{D} \\ k\omega\mathbf{D} & \mathbf{K} - (k\omega)^2\mathbf{M} \end{bmatrix} \begin{bmatrix} \mathbf{A}_k \\ \mathbf{B}_k \end{bmatrix} = \begin{bmatrix} \mathbf{S}_k^f \\ \mathbf{C}_k^f \end{bmatrix} \tag{88}$$

For the reader comprehension, it is noted that the non-linear expression $\mathbf{f}_{NL}(\mathbf{x},\omega,t)$ is a unknown function in the frequency domain. The determination of the Fourier coefficients is generally obtained by using an alternate frequency-time procedure [137, 138].

Showing equations 88, it clearly appears that the non-linear breathing behavior induces the appearance of the Fourier components of the n^{th} order with $n \geq 2$, whereas the influence of the external excitation force influences only the first order of the non-linear responses. This results can be compared with the appearance of the High-Order Frequency Response Functions presented in the previous section.

Extension of the Harmonic Balance Method for multiple excitation

Now, the general case in which the structural system is excited by several incommensurable frequencies $\omega_1, \omega_2, \ldots, \omega_n$ is discussed. The multiple excitations are contained in the external excitation forces $\mathbf{W}(\omega_1, \omega_2, \ldots, \omega_n)$.

For the sake of simplicity in the following developments, we supposed that at least one of the previous frequencies corresponds to the fundamental frequency of the non-linear breathing behavior $\mathbf{f}_{NL}(\mathbf{x}, \omega_j, t)$.

So, the non-linear response contains the frequency components of any linear combinations of the incommensurable frequency components

$$k_1\omega_1 + k_2\omega_2 + \cdots + k_j\omega_j + \cdots + k_n\omega_n \quad with \quad k_j = -m, -m+1, \ldots, 1, 0, 1, \ldots, m-1, m \tag{89}$$

where m defines the order for each fundamental frequency.

Thus the approximation of the truncated Fourier series for the non-linear response can be written in the following form

$$\mathbf{x}(t) = \sum_{k_1=-m}^{m} \sum_{k_2=-m}^{m} \cdots \sum_{k_n=-m}^{m} \left(\mathbf{A}_{k_1,k_2,\ldots,k_n} cos\left(\sum_{j=1}^{n} k_j\omega_j t \right) + \mathbf{B}_{k_1,k_2,\ldots,k_n} sin\left(\sum_{j=1}^{n} k_j\omega_j t \right) \right) \tag{90}$$

$\mathbf{A}_{k_1,k_2,\ldots,k_n}$ and $\mathbf{B}_{k_1,k_2,\ldots,k_n}$ define the unknown Fourier coefficients of any linear combinations of the incommensurable frequency components $\omega_1, \omega_2, \ldots, \omega_n$ that have been previously defined in equation 89.

Then the non-linear expression that contains the presence of damage with the breathing mechanism $\mathbf{f}_{NL}(\mathbf{x}, \omega, t)$ can be rewritten in the frequency domain

$$\mathbf{f}_{NL}(t) = \sum_{k_1=-m}^{m} \sum_{k_2=-m}^{m} \cdots \sum_{k_n=-m}^{m} \left(\mathbf{C}^{f}_{k_1,k_2,\ldots,k_n} cos\left(\sum_{j=1}^{n} k_j\omega_j t \right) + \mathbf{S}^{f}_{k_1,k_2,\ldots,k_n} sin\left(\sum_{j=1}^{n} k_j\omega_j t \right) \right) \tag{91}$$

Considering that all harmonics at negative combination frequencies can be replaced by harmonic terms at positive combination frequencies due to the following trigonometric relation

$$cos\left(\sum_{j=1}^{n} k_j\omega_j t \right) = cos\left(\sum_{j=1}^{n} -k_j\omega_j t \right) \tag{92}$$

$$sin\left(\sum_{j=1}^{n} k_j\omega_j t \right) = -sin\left(\sum_{j=1}^{n} -k_j\omega_j t \right) \tag{93}$$

it may be concluded that only terms at positive combination frequencies (i.e. $\sum_{j=1}^{n} k_j\omega_j t \geq 0$) can be retained in the non-linear response and non-linear expression force. Finally the total number of harmonic terms N_h is equal to $N_h = N_c + N_s$ where N_c and N_s correspond to the number of cosine and sine Fourier components.

So, the non-linear equation of motion 80 can be rewritten in the form of a linear algebraic matrix equation system for unknown vector of harmonic coefficients with only terms at positive combination frequencies

$$\mathbf{H} \cdot \mathbf{a} = \mathbf{F}_{NL} + \mathbf{F}_{ext} \tag{94}$$

where $\mathbf{a}$ defines the vector of harmonic coefficients

$$\mathbf{a} = \left[\left[\mathbf{a}_{1,cos}^T \mathbf{a}_{1,sin}^T\right], \left[\mathbf{a}_{2,cos}^T \mathbf{a}_{2,sin}^T\right], \ldots, \left[\mathbf{a}_{m,cos}^T \mathbf{a}_{m,sin}^T\right]\right]^T \tag{95}$$

The superscript ^{T} denotes the vector or matrix transposition. Considering the previous relation, the j^{th} contributions $\mathbf{a}_{j,cos}$ and $\mathbf{a}_{j,sin}$ contain the new Fourier decomposition of cosine and sine terms corresponding to the positive combination frequencies. They are defined by

$$\mathbf{a}_{j,cos} = \left[\mathbf{a}_{j,cos}^1, \mathbf{a}_{j,cos}^2, \ldots, \mathbf{a}_{j,cos}^{N_c}\right]^T \tag{96}$$

$$\mathbf{a}_{j,sin} = \left[\mathbf{a}_{j,sin}^1, \mathbf{a}_{j,sin}^2, \ldots, \mathbf{a}_{j,sin}^{N_s}\right]^T \tag{97}$$

For the reader comprehension, the relation between the non-linear response $\mathbf{x}(t)$ is given by

$$\mathbf{x}(t) = \mathbf{Y} \cdot \mathbf{a} \tag{98}$$

and the matrix $\mathbf{Y}$ is defined by

$$\mathbf{Y} = \begin{bmatrix} \mathbf{T} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{T} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{T} \end{bmatrix} \tag{99}$$

with

$$\mathbf{T} = \left[\mathbf{T}^{cos} \quad \mathbf{T}^{sin}\right] \tag{100}$$

By introducing $\tau_j = \omega_j t$, the sub-matrices $\mathbf{T}^{cos}$ and $\mathbf{T}^{sin}$ are composed from elements of cosine and sine terms:

$$\mathbf{T}^{cos} = \left[cos\left(\sum_{j=1}^{n} k_j^1 \tau_j\right), cos\left(\sum_{j=1}^{n} k_j^2 \tau_j\right), \ldots, cos\left(\sum_{j=1}^{n} k_j^m \tau_j\right)\right] \tag{101}$$

$$\mathbf{T}^{sin} = \left[sin\left(\sum_{j=1}^{n} k_j^1 \tau_j\right), sin\left(\sum_{j=1}^{n} k_j^2 \tau_j\right), \ldots, sin\left(\sum_{j=1}^{n} k_j^m \tau_j\right)\right] \tag{102}$$

where k_j^i (for $i = 1, \ldots, n$) are integers.

In equation 94, the expressions of the matrices and vectors $\mathbf{H}$, $\mathbf{F}_{NL}$ and $\mathbf{F}_{ext}$ are given by

$$\mathbf{H} = \left\langle \mathbf{Y}, \sum_{j=1}^{n} \omega_j \left(\sum_{i=1}^{n} \omega_i \mathbf{M} \frac{\partial^2 \mathbf{Y}}{\partial \tau_j \partial \tau_i} + \mathbf{D} \frac{\partial \mathbf{Y}}{\partial \tau_j}\right) + \mathbf{KY} \right\rangle \tag{103}$$

$$\mathbf{F}_{NL} = \langle \mathbf{Y}, \mathbf{f}_{NL}(\mathbf{x}, t) \rangle \tag{104}$$

$$\mathbf{F}_{ext} = \langle \mathbf{Y}, \mathbf{Q} + \mathbf{W} \rangle \tag{105}$$

where $\langle \cdot, \cdot \rangle$ is defined by

$$\langle \mathbf{P}, \mathbf{R} \rangle = \int_0^{2\pi} \int_0^{2\pi} \cdots \int_0^{2\pi} \mathbf{P}^T \mathbf{R} d\tau_1 d\tau_2 \ldots d\tau_n \tag{106}$$

In conclusion, the linear combinations of the incommensurable frequency components $\omega_1, \omega_2, \ldots, \omega_n$ are defined by the vector $\mathbf{a}$ (see equation 95) that verifies the linear multi-harmonic equations given in equation 94. Du to the presence of damage and the non-linear breathing behavior that is assumed to be approximated by equations 91 and 104, not only harmonic components of a specific frequency $k_j\omega_j$ (with $k_j = 1, \ldots, m$, $\omega_j = \omega_1, \ldots, \omega_n$), but also the combination of harmonic components of multi-frequencies $\sum_{j=1, k_j \in [0,\ldots,m]}^{n} k_j\omega_j$ can appear. Moreover, new peaks of linear combinations of frequency $\omega_i \pm \omega_j$ (with $i = 1 \ldots n$, $j = 1 \ldots n$, and $i \neq j$) can appear due to the coupling measurements of lateral, axial or torsional vibrations that are not present if the structure is undamaged.

3.4. Examples of the identification of damage based on the non-linear responses and harmonic components

In the two previous section the basic theory of the effect of a non-linear breathing crack has been developed. In the following sections, illustrative examples and applications considering the non-linear responses, appearance of multiples harmonics based on the Higher-Order Frequency Response Functions and Order Functions for damage detection in civil engineering (non-rotating systems) and rotating machinery are presented.

Appearance of multiples harmonics for non-rotating structures

Ruotolo et al. [139] used the Higher Order Frequency Response Function to characterize the non-linear behavior of the cracked beam. By performing a simulation of a cantilever steel beam with a closing crack, they clearly showed that the evolution of the non-linear behavior can be very useful for structural damage assessment in structures. The higher order function response functions are extremely sensitive to the damage location and severity. They indicated that the second, third and fourth orders change with increasing the damage severity whereas the first order does not exhibit dependence on the damage severity.

Peng et al. [140] used the concept of nonlinear output frequency response functions (NOFRFs) based on the Volterra series. The higher-order transform function (HOTF) of a cracked cantilevered beam was estimated by considering different experimental studies with small and large damages. The authors observed that the the nonlinear output frequency response functions are a quite sensitive indicator of the presence of damage.

The non-linear behavior of a beam with a breathing crack was also simulated by Friswell and Penny [5]. Considering the dynamic behavior of first mode of vibration, they demonstrated numerically that the the frequency spectrum of the response contains the integer Fourier harmonics which are multiple of the exciting frequency due to the breathing crack.

Pugno et al. [141] studied the non-linear vibration of a cantilever beam with a harmonic force and different damage location and severity. They noted that the presence of multiple breathing cracks in a beam induces the non-linear dynamic behavior of the damage structure. The super-harmonics (i.e. harmonic components that are multiple of the forcing frequency) appear due to the presence of damage. The amplitudes of each harmonic are drastically dependent on the number, location and severity of damage. They also concluded that the use of the appearance and evolution of harmonics can be an efficient assessment technique for damage detection.

Kisa and Brandon [142] proposed a numerical study with finite element modeling of the cracked beam. They indicated that the closure of crack drastically affects the evolution of the non-linear behavior of the damage structure.

Saavendra and Cuitino [143] simulated the non-linear behavior of a structure with an opening and closing of the crack that is assumed to vary with the excitation force. Based on experimental and theoretical tests on a free-free supported beam with one crack and a U-frame, they observed that the breathing effect drastically change the frequency spectrum of the steady-state vibration by introducing new peaks at integer harmonics of the forcing frequency. They also indicated that the second and fourth harmonics are the most important components due to the breathing crack.

Sinha and Friswell [144] simulated the experimental tests of Saavendra and Cuitino [143] in order to investigate the non-linear vibration behavior of the freefree beam with the breathing crack. They also observed the appearance of a response at twice the forcing frequency and concluded that the presence of the $2\times$ components can be used for damage detection in beams.

The appearance of the second harmonic of the excitation frequency was observed by Loutridis et al. [145]. Moreover, they indicated that the breathing of the crack is present due to the instantaneous frequency oscillations that vary between frequencies corresponding to open and closed states of the crack. Then, the magnitudes of the second harmonic component $2\times$ increase with increasing the damage severity.

Bovsunovsky and Surace [146] proposed to detect damage by using both the appearance of sub- and super-harmonic resonances. They demonstrated that the evolutions of sub and super-harmonic are very sensitive to the damage parameters and can be used at very early stages. They also proposed to take into account the energy dissipated in a crack by means of the relationship between energy and the nominal stress intensity factor range. Then, the nonlinear effects that depends not only on the damage parameters but also on the level of damping, are investigated. It is shown that the non-linear effect are less important if the level of damping increases.

Combination of multi-harmonics frequencies for non-rotating structures

Luzzato [130] studied the dynamic behavior of both open and breathing cracks in a flexural vibrating beam. The author proposed to apply two distinct excitation frequencies. The first excitation frequency f_1 is chosen to be very close to the first eigenfrequency of the simply supported beam. The second excitation frequency f_2 is chosen to be far from any eigenfrequency of the beam and as any harmonic of the first eigenfrequency. Considering the global Frequency Response Function of the non-linear responses, not only the multiple harmonics of the first excitation frequencies are obtained but also magnitudes obtained at modulation frequencies $f_2 - f_1$ and $f_2 + f_1$ are observed. The author indicated that the magnitudes of modulation frequencies are less important that the magnitudes of the harmonics of the first excitation frequency. Moreover, it clearly appears that the multiples harmonics and combination frequencies can be used for the detection of the early presence of a crack in structures.

Appearances of $\frac{1}{n}$ sub-critical speed for rotating structures

In rotating machinery, one of the first study on the non-linear components contribution was performed by Gasch [110, 111]. He demonstrated that a slight decrease in the natural frequencies and the $2\times$ harmonic components of the system in the frequency domain are key indicators for the detection of transverse cracks in a rotating shaft. The author also found that the new resonance can appear when the rotational speeds of the rotor reach $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ of the resonant frequencies of the rotor. He also suggested that the non-linear behavior of the damage rotor and the opening and closing of the crack during its rotation are due mainly to the shaft's self-weight.

Mayes and Davies [147] analyzed the response of a multi-rotor-bearing system containing a transverse crack in a rotor both experimentally and theoretically. They also reported that the $\frac{1}{2}$ sub-critical speeds appears due to the presence of damage. Moreover, they suggested that the vibrational amplitude at $\frac{1}{2}$ sub-critical speeds reaches a maximum when the crack is in phase with the shaft's imbalance, whereas it is minimized in the case of specific angular separations between the crack and the imbalance. They also concluded that even if the appearances of the $\frac{1}{2}$ sub-critical speeds can be used for damage detection, they can be rendered ineffective in the case of specific configurations.

Henry and Okah [148] observed the response of a cracked rotor at half of the sub-critical speed and other sub-critical speed. They explained that both the linear asymmetric and cracked shafts, in combination with gravity, resonate with a twice-per-revolution vibration at one-half of the resonant speed. So, only considering changes in the non-linear responses at one-half of the resonant speed of the cracked rotor is not sufficient for an efficient crack detection. Then, they indicated that only the cracked shafts exhibit resonant peaks at other sub-critical speeds. For the cracked rotor under study, the authors observed new peaks in the non-linear responses only at odd fractions of the critical speeds. They concluded that crack detection can be performed if the evolution of the non-linear responses is conducted not only at one-half but also one-third of resonant speeds.

Muszynska et al. [149] demonstrated experimentally that damage induces the presence subharmonic torsional resonance when the rotating speed of the rotor system is at $\frac{1}{8}$, $\frac{1}{5}$, $\frac{1}{4}$ and $\frac{1}{2}$ of the first torsional critical speed.

An extensive study of the effect of crack on sub-critical speeds was performed by Zhu et al. [150]. They indicated that resonances appear when the rotational speeds of the shaft reach $\frac{1}{2}$ and $\frac{1}{3}$ of the critical speeds of the rotor system. With the increase of the crack depth, the $\frac{1}{2}$ and $\frac{1}{3}$ sub-critical resonant peaks increase. They mentioned that the amplitudes of the sub-critical speeds can be suppressed with any increase of damping.

Sinou and Lees [138] illustrated that the evolutions of the maximum vertical and horizontal amplitudes at the first critical speed are not only drastically affected by the crack parameters but also very different due to the breathing behavior of the crack. The authors concluded that an increase in the amplitude of the response, as well as a decrease in the rotor speed at which the response is maximum around half of the first critical speed, are two important characteristics of the presence of damage in rotating machinery.

Then, Sinou [151] indicated that the vibration amplitudes in the $\frac{1}{n}$ sub-critical resonances (with $n \geq 2$) depend not only on the rotor damping, unbalance, position and depth of the crack, but also on the combinations of the unbalance and the crack parameters. The sensibility of the magnitudes of $\frac{1}{2}$ and $\frac{1}{3}$ sub-critical resonances with respect to the unbalance angle and the unbalance-crack interactions are different in the vertical and horizontal directions. However, the author suggested that the magnitude of the sub-critical resonances peaks do not greatly change if the crack effect is predominant.

Appearances of harmonic components for rotating structures

Liao and Gasch [152] proposed to detect crack by considering not only the non-linear response and appearances of new sub-critical resonances, but also the evolution of each harmonic components for different orders. Moreover, they indicated that observing the magnitudes amplitudes at the resonant frequencies, one-half and one-third of the critical resonant frequencies is not sufficient for early crack detection. They explained that the first harmonic component can be masked by the signal of the unbalance responses, and that the second harmonic can result from an unequal stiffness of the rotor. The evolution of the third harmonic component at one-third of the resonance peak was considered by the authors to be the most efficient signal for crack detection. However, the author observed experimentally that the third harmonic component can be very low and they suggested to compare the current state of vibrations of the damaged rotor with an initial reference state of the undamaged rotor to avoid worse diagnostic during crack identification. Experimental results indicated that both decreasing of the natural frequencies of the non-rotating shaft and the second and third harmonic components at one-half and one-third resonance peaks for the rotating shaft can be observed even if noise measurements due to ball bearing are present.

Schmied and Kramer [114] observed that the detection of crack based on the first and second harmonic components can be affected by the position of the unbalance. For example, the first harmonic can decrease if the unbalance is opposite the crack. However, if subharmonic components are dominated by the weight vibration, the evolutions of the second and third harmonic at one-half and one-third resonant speeds clearly indicate the presence of crack in rotors.

Darpe et al. [153] illustrated the fact that that the measured signals of damage rotor clearly indicate predominant fifth, third and second harmonics while passing through the respective sub-critical resonances. The second harmonic component at one-half of the first critical speed was very strong, both in the vertical and horizontal directions. Comparing the breathing and switching crack models, the authors concluded that similar results are obtained at one-half of the first resonant speed.

The effects of damage on the harmonic components have been extensively studied in the paper [151]. It was demonstrated that the third harmonic component increases near the rotational speeds at $\frac{1}{3}$, $\frac{1}{2}$ and 1 of the critical speeds. Increasing the damage severity induces a decrease in the critical speeds of the rotor system due to the reduction in system stiffness. Moreover, the vibration amplitudes of the second, third and fourth harmonic components increase with increasing the crack depth. It was also suggested that the interaction between the crack and the unbalance may mask the presence of the crack: the second and third orders super-harmonic frequency components and the resonant amplitudes in the $\frac{1}{2}$ and $\frac{1}{3}$ sub-critical resonances may disappear for specific cases, depending on the relative angle between unbalance and crack vectors. Finally, the author suggested that the variation of non-linear harmonic components and the emerging of new resonance - antiresonance peaks of the cracked rotor at second, third and fourth harmonic component can provide useful information on the presence of crack and may be used on an on-line crack monitoring rotor system for different damage parameters even if the crack is very small.

Cheng et al. [154] studied numerically the non-linear behavior of a rotor system with considerations of the effects of the crack depth, the crack location, the locations of the disc, and the shaft's rotational speed. They found that the amplitudes of the second, third and fourth harmonic components of the rotor are clearly present due to the damage for different configurations of the rotor system. However, they noted that these amplitudes reduce dramatically when the crack depth is small and that the crack location is another main factor that affects the non-linear behavior of the rotor system and changes in the harmonic components.

Adewusi and Al-Bedoor [155,156] investigated an experimental study on the non-linear dynamic response of an overhung rotor with a propagating transverse crack. They observed that the appearance of second harmonic component is the first signature of the presence of damage. It was demonstrated that the third harmonic components are excited just before fracture, and so can not be used as an efficient and robust factor for the detection of damage in early stages. They showed that the first harmonic component may increase or decrease depending on the location of the crack and the direction of vibration measurement while the second harmonic components always increases during crack propagation. Finally, the authors suggested to follow changes in amplitudes of the first and second harmonic components at a constant running speed to detect a propagating crack. This procedure allows to distinguish the presence of damage from imbalance and misalignment that also introduce harmonic components in the non-linear responses of the cracked rotor.

Appearances of the loops phenomena for rotating structures

As explained by Sinou and Lees [138], the presence of damage in a rotating shaft can be done by using the evolution of the orbit over time at around one-half of the critical speeds. The signature of the presence of damage based on orbital patterns indicates the change in amplitude and phase at half any resonance speed, and is also a characteristic for signals containing two vibration components with the same direction of precession. The authors explained that the orbit changes firstly from a simple loop to a double loop when the rotating speed of the cracked rotor is passing through half of the first critical speed. Then, one distortion in the orbit appears and the shape of the orbit changes finally to a simple loop containing a small inside loop. They also indicated that the shape of the orbit changes from a simple loop to a triple loop when the rotating speed passes through one-third of the first critical

Changes in loops and orbital patterns were studied by Henry and Okah [148]. They indicated the shaft executes two, three and five loops per shaft revolution at the $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{5}$ sub-critical speeds, respectively. However, they indicated that appearances of the two-loops at one-half of the resonant speed can be generated by the gravity in combination with the linear asymmetric of an uncracked rotor. To avoid worse diagnostic, not only responses and orbital patterns at the half sub-critical speed but also at other sub-critical speed must be considered.

Schmied and Kramer [114] indicated that the orbits at one-half and one-third of the critical resonant speeds define double and triple loops due to the presence of the second and third harmonic components. They also observed that these orbits do not depend on the position of the unbalance, whereas the orbits at the resonant speeds change with the unbalance position.

Adewusi and Al-Bedoor [156] observed experimentally the inside loops when a cracked rotor is passing through one-half of the first critical speed. They indicated that the orbital patterns are similar to the two-loop orbits that have been reported by Bently and Bosmans [157].

Darpe [153] studied both numerically and experimentally the evolution of orbital patterns during passage through sub-critical resonances. The authors explained that the inner loop that is present at one-half of the first resonant speed due to dominant second harmonic component. When the rotor is passing through the $\frac{1}{2}$ sub-critical speed, the inner loop changes its orientation by almost 180 degrees.

However, in a recent study [151], it was illustrated that the double or triple loops only appears for a deep crack in practical cases. Moreover, the author indicated that even if the damage severity is important, the inside loop or triple loop can be masked due to the predominance of the first harmonic component in the non-linear response due to the fact that the inside loop is the signature of the second harmonic component.

Appearances of multi-harmonics frequencies for rotating structures

Iwasubo et al. [158] proposed to detect crack in rotating shaft by adding a periodic external force or an impact force. The detection is based on the appearances of the combination harmonics in the non-linear responses with the frequencies of the rotor rotation and the external periodic force or the free vibration due to the impact force. If an external force ω_1 is adding, the non-linear time responses include the combination harmonics $\omega_1 \pm n\omega$ due to the interaction between the rotation of the cracked rotor ω and the harmonic force. To demonstrate the efficient of the proposed methodology based on the appearances of the combination harmonics, the authors compared the non-linear responses of an uncracked and cracked rotors for an experimental rotor system. They observed that harmonics with frequencies ω, 2ω, 3ω and $\omega_1 - \omega$ occur for the uncracked rotor. They explained that the second harmonic component is introduced by the asymmetric of the shaft rotor, and that the 3ω and $\omega_1 - \omega$ are due to the non-linearities of the rotor system. However, they observed that the amplitudes associated with the combination harmonics are very small. Then, by adding a crack in rotor, the authors observed not only strong increases of the 2ω, 3ω and $\omega_1 - \omega$, but also the appearances of the combination harmonics $\omega_1 + \omega$ and $\omega_1 \pm 2\omega$. Increasing the crack depth increases the contribution of the combination harmonics in the non-linear responses. The authors obtained the same conclusions by using an impact force. Finally they suggested that the periodic external force should be selected in the neighborhood of the main or one of the combination resonance such as for example $\omega_0 - \omega < \omega_1 < \omega_0 + \omega$ (where ω_0, ω_1 and ω define the natural frequency of the rotor system, the frequency if the external force and the rotational speed of the cracked rotor). Finally the authors indicated that the detection should be more robust and efficient if the rotational speed is very small, allowing the reduction of the influence of the unbalance force.

Darpe et al. [159] used impulse axial excitation to a rotating cracked shaft to detect damage. It was demonstrated that the presence of damage induces not only the coupling mechanism in lateral and longitudinal vibrations but also the combination harmonics due to interaction of rotational frequency and its harmonics with the constant excitation frequency and its harmonics. Then the same results of coupling measurements and appearance of combination harmonics have been observed for slant crack in rotor: coupled longitudinal, bending and torsional vibrations for crack rotor and slant crack rotor have been studied by Darpe et al. [160, 161]. In [160], Darpe et al. illustrated that the interaction of the torsional excitation frequency with the rotational frequency and its harmonics leads to the appearance of sum and difference frequencies around the bending natural frequency. However, they observed that the severity of damage does not change the amplitudes of the combinations harmonics. Then, the same results were proposed for the coupling of longitudinal and torsional vibrations by applying an axial impulse excitation. The authors demonstrated that adding a torsional harmonic excitation or/and a periodic axial impulse excitation to damage rotor can be an efficient tool for damage detection based on the response of the cracked rotor in torsional, lateral and axial directions and the appearance of interaction and combined torsional, lateral and axial frequencies and harmonics. In [161], Darpe demonstrated that not only the appearance of the components harmonics of the rotating speed can be used for damage detection but also the interaction of the torsional excitation frequency ω_t with the rotational frequency ω and its harmonics ca be consider. The appearance of sum frequencies $\omega_t + n\omega$ and difference frequencies $\omega_t - n\omega$ (with n integer) around the torsional excitation frequency is observed due to the presence of crack. So the torsional excitation frequency induces coupling between the torsional and longitudinal vibrations. However the author demonstrated that the amplitude of the sum and difference frequencies $\omega_t + n\omega$ and $\omega_t - n\omega$ are not drastically affected by increasing the crack depth.

Ishida and Inoue [162] proposed to extent the study of Iwasubo et al. [158] in the case of the non-linear vibration of a cracked rotor with external harmonic force. In their study, they investigated both experimentally and numerically the combination harmonics by exciting the shaft for different frequencies. First of all, they obtained the results of Iwatsubo et al. with the crack detection based on the combination harmonics $\omega_1 \pm n\omega$ due to the interaction between the rotation of the cracked rotor ω and the harmonic force frequency ω_1. Then, they observed that the appearances of resonances that satisfy the relationships $m\omega_1 \pm n\omega$ (with n and m integers) are useful for detection of cracks not only in a horizontal rotor system, but also in vertical rotor system (i.e. with the influence or not of the gravity on the non-linear responses of the cracked shaft). It was indicated that the amplitudes of additional peaks at $m\omega_1 \pm n\omega$ increases as the crack grows.

4. Transient signals and Wavelet Transform

4.1. Basic theory

Due to the limitation of the conventional Fourier analysis that is suitable for steady state vibration signals, but provides a poor representation of signals well localized in time, time-scale signal processing tools have to be used for damage detection.

In civil engineering, Newland [163–165] was one of the first researcher that proposed to apply the wavelet approach to analyze the vibration of structures. When a non-linear signal is judged non-stationary, the characteristics of transient responses and changes in the properties of non-stationary signal of damaged structures can be precisely described and analyzed by means of the Continuous Wavelet Transform (CWT). The signal is decomposed into wavelets, small oscillations that are highly localized in time, whereas the Fourier transform decomposes the signal into infinite length sines and cosines losing all time-localization information.

The wavelet analysis transforms a signal into wavelets that are well localized both in frequency and time. The continuous wavelet transform (CWT) of a function $f(t)$ is a wavelet transform defined by

$$W(a,b) = \int_{-\infty}^{+\infty} f(t)\, \psi_{a,b}^{*}(t)\, dt \tag{107}$$

where

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \tag{108}$$

are the daughter wavelets (i.e. the dilated and shifted versions of the "'mother"' wavelet ψ that is continuous in both time and frequency). a is the scale parameter, b is the time parameter. The asterisk $\psi_{a,b}^{*}$ indicates the complex conjugate of $\psi_{a,b}$. The following admissibility condition has to be satisfied $0 < C_\psi < +\infty$ where C_ψ defines the admissibility constant

$$C_\psi = \int_{-\infty}^{+\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega \tag{109}$$

and $\hat{\psi}$ is the Fourier transform of ψ

$$\hat{\psi} = \int_{-\infty}^{+\infty} f(t)\, e^{i\omega t} dt \tag{110}$$

For a time signal $f(t)$ represented by N sampled data points (with uniform time step δt), the Continuous Wavelet Transform of equation 107 is a convolution of the data sequence $f(n')$ (with $n' = 1, \ldots, N$) with a scaled and normalized wavelet. It can be represented as follows:

$$C(a,n) = \sum_{n'=0}^{N-1} f(n') \sqrt{\frac{\delta t}{a}} \Psi_0^* \left(\frac{(n'-n)\,\delta t}{a} \right) \tag{111}$$

where n defines the localized time index and δt is the sampling interval.

The specification of the mother wavelet is one of the main important point due to the fact that the mother wavelet has to be fully specified and serves as the source function from which which scaled and translated basis functions are constructed. Generally speaking, the notion of "'wavelet function"' is used to refer to either orthogonal or nonorthogonal wavelets whereas the notion of "'wavelet basis"' refers only to an orthogonal set of functions. The one-dimensional wavelet transform previously defined in this section can be extended to two-dimensional case for plate structures damage localization [166] or any dimensions dimensions [167].

As previously explained, the selection of an appropriate type of a wavelet is essential for an effective damage detection based on wavelet analysis. Some of the most commonly used CWT wavelet [145, 168–178] are the Morlet, the Paul, the Gabor, the Derivative of a Gaussian (DOG), the Symlet, the Coiflet and bi-orthogonal wavelets. For example, the Morlet wavelet is a Gaussian-windowed complex sinusoid that quite well localized in both time and frequency space. The Morlet wavelet is defined as following in the time and frequency domains respectively

$$\psi_0(\eta) = \pi^{-\frac{1}{4}} e^{im\eta} e^{-\frac{\eta^2}{2}} \tag{112}$$

$$\hat{\psi}_0(s\omega) = \pi^{-\frac{1}{4}} H(\omega)\, e^{\frac{(s\omega-\omega_0)^2}{2}} \tag{113}$$

where m is the wavenumber and η is non-dimensional time parameter. $H(\omega)$ is the Heaviside step function with $H(\omega) = 1$ if $\omega > 0$ and $H(\omega) = 0$ otherwise. ω_0 defines the frequency.

The Paul wavelet is given in the time and frequency domains by

$$\psi_0(\eta) = \frac{2^m i^m m!}{\sqrt{\pi(2m)!}} (1 - i\eta)^{-(m+1)} \tag{114}$$

$$\hat{\psi}_0(s\omega) = \frac{2^m!}{\sqrt{m(2m-1)!}} H(\omega)\,(s\omega)^m\, e^{-s\omega} \tag{115}$$

The Paul wavelet localizes most efficiently in the time domain whereas the Morlet offers the best frequency localization.

The Derivative of Gaussian (DOG) wavelet is given in the time and frequency domains by

$$\psi_0(\eta) = \frac{(-1)^{m+1}}{\sqrt{\Gamma\left(m + \frac{1}{2}\right)}} \frac{d^m}{d\eta^m} e^{\frac{-\eta^2}{2}} \tag{116}$$

$$\hat{\psi}_0\left(s\omega\right) = \frac{-i^m + 1}{\sqrt{\Gamma\left(m + \frac{1}{2}\right)}}\left(s\omega\right)^m e^{\frac{-(s\omega)^2}{2}} \tag{117}$$

The GaussDeriv wavelet's adjustable parameter is the derivative order m. If $m = 2$ the wavelet is named the Marr or Mexican Hat wavelet.

For the interested reader, Kijewski and Kareem [179] presented a review of the use of wavelet transforms for system identification in civil engineering.

Staszewski [180] and Kim and Melhem [181] proposed a global review of the damage detection in structures by wavelet analysis. They explained that damage detection via the wavelet-based methods can be divided into three categories: the first category considers the variation of wavelet coefficients and is used to detect the presence and severity of damage. The second class of methods allows the localization of damage by considering the local perturbation and irregularity of wavelet coefficients in a space domain near the location of damage. The last category is based on the reflections of wave caused by local damage involves to estimate both the position and severity of damage.

4.2. Application for non-rotating structures

Surace and Ruotolo [182] proposed to use the wavelet to detect the presence of a single crack in a cantilever beam. They clearly demonstrated that changes in the wavelet coefficients can be used for the damage detection: the non-linear behavior corresponding to the breathing of the crack (i.e. opening and closing of the crack) was identified via the wavelet analysis.

Wang et al. [183, 184] investigated the localization of damage in a simply supported beam or a cantilever beam with a transverse open crack by considering the wavelet theory. They observed that the presence of damage induces an abrupt and local evolution of the wavelet coefficients near the location of damage. The authors explained that one of the advantages of the wavelet-based technique is the possibility to detect and locate damage without an accurate knowledge of the material properties of undamaged structures.

The same results were obtained by Lu and Hsu [185, 186]. Using the Marr or Mexican hat wavelet, they analyzed the vibration signal of the damage structure and an abrupt change in the wavelet coefficients is observed close to the damage location.

Loutridis et al. [187] investigated the feasibility of using the wavelet transform to double-cracked cantilever beams. They indicated that the wavelet transform allows the capture of local perturbations due to the damage in the structural response. The authors proposed to validate the numerical results of the wavelet analysis by performing experiments on a plexiglas rectangular cross-section beam. They concluded that even if measuring errors and noise corrupt the response data and the wavelet coefficients, the sudden changes in the spatial response at the damage location can be used to identify the doubled cracks in beam. However, they showed that the damage severity may be difficult to identify in practical case due to measurement errors and uncertainties. Finally, they also indicated that the application and limitation of the wavelet transform for the detection of damage need to be strongly investigated for more complex structures.

Chang and Chen [188] proposed to estimate the positions the positions and depths of multi-cracks in beam structures by using spatial wavelet based method. The authors used the mode shapes analyzed by wavelet transformation in order to identify the positions of damages. The authors proposed the use of the Gabor wavelet that was previously consider for damage detection by Kim and Kim [189] and Quek et al. [190]. The authors explained that the Gabor wavelet functions are used due to the fact that the magnitude of Gabor wavelet function can be used for the location of damage. The local perturbations in the mode shapes due to the presence of damage are found in the fine scale wavelets. The number of damages and the positions of damages are determined from the plot of wavelet coefficients. The authors demonstrated that a sudden change in the distributions of the wavelet coefficients allows the damage localization. The proposed method appears to be efficient even if the damage is small. However, the main limitation of the method is observed if the damage is near the boundaries.

Li et al. [191] proposed a damage detection method based on the Gabor wavelet. Numerical and experimental tests are conducted on a plexiglas rectangular cross-section cantilever beam. The authors noted that not only the location of the damage but also the orientation of an inclined crack can be identify. By considering the relations between the flexural waves, reflections and transmission ratios of the inclining crack, an efficient identification of the crack depth and of the slanting orientation of the inclining crack can be provided.

Zhu and Law [192] studied the dynamic behavior of a bridge beam structure under a moving load and proposed the detection of damage based on wavelet transform. The sudden changes in the spatial variation of the responses and the evolutions of the coefficients of the wavelet transform are used to locate and estimate the damage severity. The authors indicated that multiple damages can be easily identified in terms of location and severity even if noisy measurements are present. Moreover, different parameters of moving load (i.e. speed and magnitude) are tested and confirmed that the proposed methodology seems to be robust in regard to external conditions.

Ovanesova and Suarez [177] noted that damage can be performed in structures through wavelet transform without the knowledge of the original undamaged structure. They pointed out that the most appropriate wavelet needs to be selected for an optimal identification of damage. They indicated that actual limitations such as elimination of the effects of the boundary conditions or unavoidable uncertainties associated with experimental procedures need to be extensively addressed in the future to establish a rigorous methodology for damage detection based on wavelet transform.

Damage location in plate structures based on the formulation of the two-dimensional continuous wavelet transform was investigated by Rucka and Wilde [166]. Considering an experimental steel plate with four fixed said that one of the main advantage of wavelet analysis is the possibility to identify small damage and the defect position without knowledge of neither the structure characteristics nor its analytical models.

4.3. Application for non-rotating structures

Some researchers [155, 193–195] proposed to introduce time-scale signal processing tools based on wavelet transform for the detection of damage in rotating machinery due to the limitation of the conventional Fourier analysis that is suitable for steady state vibration signals, but provides a poor representation of transient signals due to a strong increase of the shaft's rotating speed.

Sekhar and Prabhu [196] studied the effects of different factors such as crack depth, unbalance eccentricity and acceleration influencing transient vibrations. They observed that the measurement of transient response when the rotor is passing through the critical speed or one-half of the critical speed can be used for the non destructive detection of cracks in rotors.

The discrete wavelet transform were used by Adewusi and Al-Bedoor [155] to detect experimentally a propagating transverse crack for an overhang rotor during transient vibration signals. The authors indicated that the crack reduces the critical speed of the rotor system. Changes in amplitudes of the first and second vibration harmonic components are easily detected and appear to be an important feature that distinguishes a propagating crack from imbalance and misalignment.

Zou and Chen [195] used the wavelet transform and the WignerVille distribution that is yet another tool for analyze non-stationary non-linear responses of the cracked rotor. Comparing the two methods, they concluded that the wavelet transform is more sensitive to the stiffness variation. They also indicated that the crack can be easily identify even if the unbalance increases significantly.

5. Conclusion

This review aims to provide a comprehensive review of the state of the art in fault diagnosis techniques based on linear and non-linear vibrational measurements with particular regard to academics applications, civil engineering and rotating machinery. It can be observed that the area of crack detection is continuously evolving.

Firstly, basic methods based on linear condition monitoring techniques have been extensively developed by considering not only the changes in frequencies and modes shapes, but also the motion of anti-resonances, the changes nodes position, the changes in dynamic flexibility or mode shapes curvature, the appearances of resonant peaks due to vibration coupling or different changes in the measurements of the Frequency Responses Functions, the definition of the Modal Assurance Criterion and other related assurance criteria, etc...

Secondly, the emergence of the common use of robust developments based on the non-linear responses of mechanical structures due to the presence of crack allows a more efficient identification of crack in complex mechanical engineering structures. An important observation is the recognition of the appearances of the non-linear harmonic components due to the damage and the combination of harmonics if the mechanical system is subjected to multi-excitations.

Finally, the developments of tools to analyze the non-stationary responses of mechanical systems such as the wavelet transform allows the detection of crack for transient signals.

It clearly appears that one of the most important limitations of an efficient identification of the damage parameters is due to the effect of various controlled or uncontrolled factors such as the ambient conditions (evolution of the temperature for example), the evolutions of boundary conditions, the variations in the mass of the structure (due to the location of accelerometers in academic structures, position of vehicle or human in bridges structures or values of the unbalance mass in rotating structures for example), the variations in the stiffness in real engineering structures (more particularly in civil engineering applications and rotating systems). All these variations can introduce uncertainties in both the linear and non-linear vibrational measurements.

For a practical point of view, using condition monitoring techniques based on linear approaches can be considered to be the first step for a rapid estimation of the presence of damage if the damage is not very small. Even if a lot of robust and new tools based on linear measurements have been developed, using frequency shifts to detect damage appears to be more practical in engineering applications. Then, different techniques can be used in combination with changes in modal parameters to identify the damage parameters.

If the emergence of a non-linear behavior for mechanical structures is observed or if the undamaged mechanical systems is non-linear by nature, alternative indicators based on non-linear analysis can be more effective for a robust detection and identification of the presence of damage.

Even if new techniques and extensive studies have been undertaken in the past decade for both the detection and identification of simple or multi damages in structures, there are still many unanswered questions and future research must be attempted to increase the reliability and safety of complex engineering structures in civil applications and rotating machinery. One of the most crucial next step can be to be able to propose more practical and commonly implemented techniques only based on classical linear and non-linear vibration measurements by keeping in mind that the number of measured points is limited in practical case of civil applications or rotating machinery.

Given an exhaustive list of topics of interest for future developments is not possible. However, some non-exhaustive interesting further studies can be considered:

- the development and comparison of crack models by considering the effect of more or less complex breathing mechanisms or/and the effect of the propagating crack if necessary,
- the effects of changes in damping and thermal effects for the damage identification, due to the linear or non-linear vibration in structures and the crack propagation,
- the extension of criteria that have been developed for the linear theory, for the non-linear detection of damage based on the harmonic components or the Higher Order Frequency Response Functions,
- the development of linear and non-linear condition monitoring techniques by adding theories that take into account uncertainties due to the ambient conditions or the evolutions of boundary conditions,
- the improvement of correlations between experimental and numerical analysis for linear and non-linear assessment techniques with the inclusions of modeling errors and uncertainties,
- the developments of optimization procedures, genetic algorithms or artificial neural network (that have not been developed in this review even if they have been extensively used of damage identification in the past decade).

Bibliography

[1] Rytter, A., 1993. *Vibration Based Inspection of Civil Engineering Structures*. PhD Thesis, Aalborg University, Denmark.

[2] Doebling, S., Farrar, C., Prime, M., and Shevits, D., 1996. "Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: A literature review". *Los Alamos National Laboratory, USA,* **1**, p. 1–136.

[3] Doebling, S., Farrar, C., and Prime, M., 1998. "A summary review of vibration-based damage identification methods". *Shock and Vibration Digest,* **30**, p. 91-105.

[4] Sohn, H., Farrar, C., Hemez, F., Shunk, D., Stinemates, D., and Nadler, B., 2003. *A Review of Structural Health Monitoring Literature: 19962001*. Los Alamos National Laboratory, USA.

[5] Friswell, M., and Penny, J., 1992. "A simple nonlinear modal of a cracked beam". *Proceeding of the 10th International Modal Analysis Conference, San Diego, Californie,* **1**, p. 516–521.

[6] Salawu, O., 1997. "Detection of structural damage through changes in frequency: a review". *Engineering Structures,* **19(9)**, p. 718–723.

[7] Wauer, J., 1990. "Dynamics of cracked rotors: Literature survey". *Applied Mechanics Review,* **43**, p. 13–17.

[8] Dimarogonas, A., 1996. "Vibration of cracked structures: a state of the art review". *Engineering Fracture Mechanics,* **55**, p. 831–857.

[9] Sabnavis, G., Kirk, R., and Kasarda, M., 2004. "Cracked shaft detection and diagnostics: a literature review". *Shock and Vibration Digest,* **36**, p. 287-296.

[10] Montalvo, D., Maia, N., and Ribeiro, A., 2006. "A review of vibration-based structural health monitoring with special emphasis on composite materials". *The Shock and Vibration Digest,* **38(4)**, p. 1–30.

[11] Lifshitz, J., and A.Rotem, 1969. "Determination of reinforcement unbonding of composites by a vibration technique". *Journal of Composite Materials,* **3**, p. 412-423.

[12] Hearn, G., and Testa, R., 1991. "Modal analysis for damage detection in structures". *Journal of Structural Engineering,* **117**, p. 3042-3062.

[13] Hasan, W., 1995. "Crack detection from the variation of the eigenfrequencies of a beam on elastic foundation". *Engineering Fracture Mechanics,* **52(3)**, p. 409–421.

[14] Davini, C., Morassi, A., and Rovere, N., 1995. "Modal analysis of notched bars: tests and comments on the sensitivity of an identification technique". *Journal of Sound and Vibration,* **179(3)**, p. 402–416.

[15] Dado, M., 1997. "A comprehensive crack identification algorithm for beams under different end conditions". *Applied Acoustics,* **51(4)**, p. 381–398.

[16] Boltezar, M., Strancar, B., and A.Kuhelj, 1998. "Identification of transverse crack location in flexural vibrations of free-free beams". *Journal of Sound and Vibration,* **211(5)**, p. 618–623.

[17] Shifrin, E., and Ruotolo, R., 1999. "Natural frequencies of a beam with an arbitrary number of cracks". *Journal of Sound and Vibration,* **222(3)**, p. 409–423.

[18] Fernandez-Saez, J., Rubio, L., and Navarro, C., 1999. "Approximate calculation of the fundamental frequency for bending vibrations of cracked beams". *Journal of Sound and Vibration,* **225(2)**, p. 345–352.

[19] Chaudhari, T., and Maiti, S., 2000. "A study of vibration of geometrically segmented beams with and without crack". *International Journal of Solids and Structures,* **37**, p. 761–779.

[20] Morassi, A., 2001. "Identification of a crack in a rod based on changes in a pair of natural frequencies". *Journal of Sound and Vibration,* **242(4)**, p. 577–596.

[21] Zheng, D., and Kessissoglou, N., 2004. "Free vibration analysis of a cracked beam by finite element method". *Journal of Sound and Vibration,* **273**, p. 457-475.

[22] Chondros, T., 2005. "Variational formulation of a rod under torsional vibration for crack identification". *Theoretical and Applied Fracture Mechanics,* **44**, p. 95–104.

[23] Nahvi, H., and Jabbari, M., 2005. "Crack detection in beams using experimental modal data and finite element model". *International Journal of Mechanical Sciences,* **47**, p. 1477-1497.

[24] Loya, J., Rubio, L., and Fernandez-Saez, J., 2006. "Natural frequencies for bending vibrations of Timoshenko cracked beams". *Journal of Sound and Vibration,* **290**, p. 640–653.

[25] Narkis, Y., 1994. "Identification of crack location in vibrating simply supported beams". *Journal of Sound and Vibration,* **172(4)**, p. 549–558.

[26] Narkis, Y., and Elmalah, E., 1996. "Crack identification in a cantilever beam under uncertain conditions". *International Journal of Mechanical Sciences,* **38(5)**, p. 499–507.

[27] Lee, Y., and Chung, M., 2000. "A study on crack detection using eigenfrequency test data". *Computers and Structures,* **77**, p. 327–342.

[28] Sinou, J.-J., and Lees, A. W., 2005. "Influence of cracks in rotating shafts". *Journal of Sound and Vibration,* **285(4-5)**, p. 1015–1037.

[29] Sinou, J.-J., 2007. "Numerical investigations of a robust identification of crack location and size in beams using only changes in ratio pulsations of the cracked beams". *Structural Engineering Mechanics,* **25(6)**, p. 691–716.

[30] Messina, A., Jones, I., and Williams, E., 1992. "Damage detection and localization using natural frequency changes". *In Proceedings of the 1st Conference on Identification, Cambridge, England, UK,* **1**, p. 67-76.

[31] Messina, A., Williams, E., and Contursi, T., 1998. "Structural damage detection by a sensitivity and statistical-based method". *Journal of Sound and Vibration,* **216(5)**, p. 791-808.

[32] Koh, B., and Dyke, S., 2007. "Structural health monitoring for flexible bridge structures using correlation and sensitivity of modal data". *Computers and Structures,* **85**, p. 117-130.

[33] P. G. Nikolakopoulos, D. E. K., and Papadopoulos, C. A., 1997. "Crack identification in frame structures". *Computers and Structures,* **64(14)**, p. 389–406.

[34] Yang, X., Swamidas, A., and Seshadri, R., 2001 "Crack identification in vibrating beams using the energy method". *Journal of Sound and Vibration* **244(2)**, p.339–357.

[35] Dong, G., Chen, J., and Zou, J., 2004. "Parameter identification of a rotor with an open crack". *European Journal of Mechanics A/Solids,* **23(2004)**, p. 325–333.

[36] Swamidas, A., Yang, X., and Seshadri, R., 2004. "Identification of cracking in beam structures using Timoshenko and Euler formulations". *Journal of Engineering Mechanics,* **130(11)**, p. 1297–1308.

[37] Lele, S., and Maiti, S., 2002. "Modeling of transverse vibration of short beams for crack detection and measurement of crack extension". *Journal of Sound and Vibration,* **257 (3)**, p. 559–583.

[38] Li, B., Chen, X., Ma, J., and He, Z., 2005. "Detection of crack location and size in structures using wavelet finite element methods". *Journal of Sound and Vibration,* **285**, p. 767-782.

[39] Dilena, M., and Morassi, A., 2004. "The use of antiresonances for crack detection in beams". *Journal of Sound and Vibration,* **276**, p. 195-214.

[40] Sinou, J.-J., 2007. "Damage assessment based on the frequencies' ratio surfaces intersection method for the identification of the crack depth, location and orientation". *Structural Durability and Health Monitoring,* **3(3)**, p. 133–164.

[41] Modena, C., Sonda, D., and D.Zonta, 1999. "Damage localization in reinforced concrete structures by using damping measurements". *Proceedings of the International Conference on Damage Assessment of Structures, Dublin, Ireland,* **1**, p. 132–141.

[42] Bovsunovsky, A., 2004. "The mechanisms of energy dissipation in the non-propagating fatigue cracks in metallic materials". *Engineering Fracture Mechanics,* **71(16-17)**, p. 2271-2281.

[43] Kyriazoglou, C., Page, B. L., and Guild, F., 2004. "Vibration damping for crack detection in composite laminates". *Composites Part A: Applied Science and Manufacturing,* **35(7-8)**, p. 945–953.

[44] Panteliou, S., Chondros, T., Chondros, V., Argyrakis, V., and Dimarogonas, A., 2001. "Damping factor as an indicator of crack severity". *Journal of Sound and Vibration,* **241(2)**, p. 235–245.

[45] Leonard, F., Lanteigne, J., Lalonde, S., and Turcotte, Y., 2001. "Free-vibration behaviour of cracked cantilever beam and crack detection". *Mechanical Systems and Signal Processing,* **15(3)**, p. 529–548.

[46] Gladwell, G. M. L., and Morassi, A., 1999. "Estimating damage in a rod from changes in node positions". *Inverse Problems in Engineering,* **7**, p. 215–233.

[47] Dilena, M., and Morassi, A., 2002. "Identification of crack location in vibrating beams from changes in node positions". *Journal of Sound and Vibration,* **255(5)**, p. 915–930.

[48] Adams, R., Cawley, P., Pye, C., and Stone, B., 1978. "A vibration technique for nondestructively assessing the integrity of structures". *Journal of Mechanical Engineering Science,* **20**, p. 93–100.

[49] Cawley, P., and Adams, R., 1979. "The location of defects in structures from measurements of natural frequency". *Journal of Strain Analysis,* **14 (2)**, p. 49-57.

[50] Yuen, M., 1985. "A numerical study of the eigenparameters of a damaged cantilever". *Journal of Sound and Vibration,* **103**, p. 301-310.

[51] Natke, H., and Cempel, C., 1997. "Model-aided diagnosis based on symptoms". *Proceedings of DAMAS 97, Sheffield, United-Kingdom,* **1**, p. 363-375.

[52] Kullaa, J., 2003. "Damage detection of the z24 bridge using control charts". *Mechanical Systems and Signal Processing,* **17(1)**, p. 163–170.

[53] Maeck, J., and De Roeck, G., 2003. "Damage assessment using vibration analysis on the z24-bridge". *Mechanical Systems and Signal Processing,* **17(1)**, p. 133–142.

[54] Montgomery, D., 1995. *Introduction to statistical quality control*. New York, John Wiley and Sons, 3rd edn.

[55] Law, S., and Zhu, X., 2004. "Dynamic behavior of damaged concrete bridge structures under moving vehicular loads". *Engineering Structures,* **26**, p. 1279-1293.

[56] West, W., 1984. "Illustration of the use of modal assurance criterion to detect structural changes in an orbiter test specimen". *Proceedings of the Air Force Conference on Aircraft Structural Integrity,* **1**, p. 1–6.

[57] Ewins, D., 1985. *Modal Testing: Theory and Practice.* New York: Wiley.

[58] Srinivasan, M., and Kot, C., 1992. "Effects of damage on the modal parameters of a cylinder shell". *Proceedings of the 10th International Modal Analysis Conference, Kissimmee, Florida, USA,* **1**, p. 529–535.

[59] Lieven, N., and Ewins, D., 1988. "Spatial correlation of mode shapes: The coordinate modal assurance criterion (comac)". *Proceedings of the 6th International Modal Analysis Conference, Kissimmee, Florida, USA,* **1**, p. 690-695.

[60] Palacz, M., and Krawczuk, M., 2002. "Vibration parameters for damage detection in structures". *Journal of Sound and Vibration,* **249(5)**, p. 999–1010.

[61] Allemang, R., 2003. "The modal assurance criterion twenty years of use and abuse". *Sound and Vibration*, p. 14–21.

[62] Kim, J., Jeon, H., and C.W.Lee, 1992. "Application of the modal assurance criteria for detecting and locating structural faults". *Proceedings of the 10th International Modal Analysis Conference, San Diego, California, USA,* **1**, p. 536-540.

[63] Parloo, E., Guillaume, P., and Overmeire, M. V., 2003. "Damage assessment using mode shape sensitivities". *Mechanical Systems and Signal Processing,* **17(3)**, p. 499–518.

[64] Pandey, A., Biswas, M., and Samman, M., 1991. "Damage detection from changes in curvature mode shapes". *Journal of Sound and Vibration,* **145**, p. 321–332.

[65] Ho, Y., and Ewins, D., 2000. "On the structural damage identification with mode shapes". *In Proceedings of the European COST F3 Conference on System Identification and Structural Health Monitoring Madrid, Spain,* **1**, p. 677–686.

[66] Maeck, J., and De Roeck, G., 1999. "Damage detection on a prestressed concrete bridge and rc beams using dynamic system identification". *Proceedings DAMAS 99, Trans TechPublications, Dublin, Ireland,* **1**, p. 320-327.

[67] Abdel Wahab, M., and De Roeck, G., 1999. "Damage detection in bridges using modal curvatures: application to a real damage scenario". *Journal of Sound and Vibration,* **226(2)**, p. 217–235.

[68] Dutta, A., and Talukdar, S., 2004. "Damage detection in bridges using accurate modal parameters". *Finite Elements in Analysis and Design,* **40**, p. 287-304.

[69] Dong, C., Zhang, P., and Huang, W. F. T., 1994. "The sensitivity study of the modal parameters of a cracked beam". *Proceedings of 12th International Modal Analysis Conference,* **1**, p. 98–104.

[70] Stubbs, N., Kim, J., and Topole, K., 1992. "An efficient and robust algorithm for damage localization in offshore platforms". *In Proceedings ASCE Tenth Structures Congress,* **1**, p. 543–546.

[71] Stubbs, N., Kim, J., and Farrar, C., 1995. "Field verification of a nondestructive damage localization and severity estimation algorithm". *In Proceedings of the 13th International Modal Analysis Conference, Nashville, Tennessee, USA,* **1**, p. 210–218.

[72] Alvandi, A., and Cremona, C., 2006. "Assessment of vibration-based damage identification techniques". *Journal of Sound and Vibration,* **292**, p. 179-202.

[73] Pandey, A., and Biswas, M., 1994. "Damage detection in structures using changes in flexibility". *Journal of Sound and Vibration,* **169 (1)**, p. 3-17.

[74] Lin, C., 1994. "Unity check method for structural damage detection". *Proceeding of 35th AIAA/ASME/ASCE/AHS/ASC structures, structural dynamics and materials conference*, **1**, p. 347–354.

[75] Lin, C., 1990. "Location of modeling errors using modal test data". *AIAA Journal*, **28**, p. 1650-1654.

[76] Gysin, H., 1986. "Critical application of an error matrix method for location of finite element modeling inaccuraties". *Proceedings of the 4th International Modal Analysis Conference*, **1**, p. 1339–1351.

[77] Park, Y., Park, H., and Lee, S., 1988. "Weighted error matrix application to detect stiffness damage characteristic measurement". *International Journal of Analytical and Experimental Modal Analysis*, **3(3)**, p. 101–107.

[78] Aktan, A., Lee, K., Chuntavan, C., and Aksel, T., 1994. "Modal testing for structural identification and condition assessment of constructed facilities". *Proceedings of the 12th International Modal Analysis Conference*, **1**, p. 462–468.

[79] Mayes, R., 1995. "An experimental algorithm for detecting damage applied to the i-40 bridge over the Rio Grande". *Proceedings of the 13th International Modal Analysis Conference*, **1**, p. 219–225.

[80] Park, K., Reich, G., and Alvin, K. F., 1998. "Structural damage detection using localized flexibilities". *Journal of Intelligent Material Systems and Structures*, **9**, p. 911–919.

[81] Topole, K., 1995. "Damage evaluation via flexibility formulation". *Smart Systems for Bridges, Structures, and Highways, Proceedings of SPIE*, **3**, p. 145-154.

[82] Zhang, Z., and Aktan, A., 1995. "The damage indices for constructed facilities". *Proceedings of the 13th International Modal Analysis Conference*, **1**, p. 1520-1529.

[83] Vanhonacker, P., 1980. "Differential and difference sensitivities of natural frequencies and mode shapes of mechanical structures". *AIAA Journal*, **18**, p. 1511-1514.

[84] Heylen, W., Lammens, S., and Sas, P., 1995. *Modal Analysis Theory and Testing*. Katholieke Universiteit, Leuven.

[85] Wahl, F., Schmidt, G., and Forrai, L., 1996. "On the significance of antiresonance frequencies in experimental structural analysis". *Journal of Sound and Vibration*, **219 (3)**, p. 379-394.

[86] Bamnios, Y., Douka, E., and Trochidisi, A., 2002. "Crack identification in beam structures using mechanical impedance". *Journal of Sound and Vibration*, **256 (2)**, p. 287-297.

[87] Douka, E., Bamnios, G., and Trochidis, A., 2004. "A method for determining the location and depth of cracks in double-cracked beams". *Applied Acoustics*, **65**, p. 997–1008.

[88] Dharmaraju, N., and Sinha, J. K., 2005. "Some comments on use of antiresonance for crack identification in beams". *Journal of Sound and Vibration*, **286**, p. 669-671 .

[89] Dilena, M., and Morassi, A., 2006. "Damage detection in discrete vibrating systems". *Journal of Sound and Vibration,* **289**, p. 830-850.

[90] Heylen, W., and Lammens, S., 1996. "FRAC : a consistent way of comparing frequency response functions". *Identification in Engineering systems, Swansea,* **1**, p. 48–57.

[91] Zang, C., and Imregun, M., 2001. "Structural damage detection using artificial neural networks and measured frf data reduced via principal component projection". *Journal of Sound and Vibration,* **242(5)**, p. 813–827.

[92] Zang, C., Friswell, M., and Imregun, M., 2003. "Structural health monitoring and damage assessment using measured frfs from multiple sensors, part i: The indicator of correlation criteria". *In Proceedings of the 5th International Conference on Damage Assessment of Structures, Southampton, UK,* **1**, p. 131–140.

[93] Zang, C., and Imregun, M., 2003. "Structural health monitoring and localization using frf based model umdating approach". *In Proceedings of the 5th International Conference on Damage Assessment of Structures, Southampton, UK,* **1**, p. 191–199.

[94] Pascual, R., Golinval, J., and Razeto, M., 1997. "A frequency domain correlation technique for model correlation and updating". *In Proceedings of the 15th International Modal Analysis Conference, Orlando, Florida, USA,* **1**, p. 587-592.

[95] Pascual, R., Golinval, J., and Razeto, M., 1999. "On-line damage assessment using operating deflection shapes". *In Proceedings of the 17th International Modal Analysis Conference, Kissimmee, USA*, **1**, p. 238–243.

[96] Sampaio, R., Maia, N., and Silva, J., 2003. "The frequency domain assurance criterion as a tool for damage identification". *In Proceedings of the 5th International Conference on Damage Assessment of Structures, Southampton, England, UK,* **1**, p. 69–76.

[97] Maia, N., Silva, J., Almas, E., and Sampaio, R., 2003. "Damage detection in structures: from mode shape to frequency response function methods". *Mechanical Systems and Signal Processing,* **17**, p. 489-498.

[98] Gounaris, G. D., Papadopoulos, C. A., and Dimarogonas, A. D., 1996. "Crack identification in beams by coupled response measurements". *Computers and Structures,* **58(2)**, p. 299–305.

[99] Liu, D., Gurgenci, H., and Veidt, M., 2003. "Crack detection in hollow section structures through coupled response measurements". *Journal of Sound and Vibration,* **261**, p. 17–29.

[100] Chasalevris, A., and Papadopoulos, C., 2008. "Coupled horizontal and vertical bending vibrations of a stationary shaft with two cracks". *Journal of Sound and Vibration,* **309**, p. 507-528.

[101] Lee, J., Kim, J., Yun, C., Yi, J., and Shim, J., 2002. "Health-monitoring method for bridges under ordinary traffic loadings". *Journal of Sound and Vibration,* **257(2)**, p. 247–264.

[102] Papadopoulos, C., and Dimarogonas, A., 1987. "Coupled longitudinal and bending vibrations of a rotating shaft with an open crack". *Journal of Sound and Vibration,* **117**, p. 81-93.

[103] Papadopoulos, C., and Dimarogonas, A., 1987. "Coupling of bending and torsional vibration of a cracked Timoshenko shaft". *Archive of Applied Mechanics,* **57(4)**, p. 257–266.

[104] Papadopoulos, C., and Dimarogonas, A., 1988. "Coupled longitudinal and bending vibrations of a cracked shaft". *Journal of vibration, acoustics, stress, and reliability in design,* **110**, p. 1-8.

[105] Papadopoulos, C., and Dimarogonas, A., 1992. "Coupled vibrations of cracked shafts". *Journal of Vibration and Acoustic,* **114**, p. 461-467.

[106] Ostachowicz, W., and Krawczuk, M., 1992. "Coupled torsional and bending vibrations of a rotor with an open crack". *Archives of Applied Mechanics,* **62**, p. 191-201.

[107] Collins, K., Plaut, R., and Wauer, J., 1991. "Detection of cracks in rotating Timoshenko shafts using axial impulses". *Journal of Vibration and Acoustics,* **113**, p. 74-78.

[108] Gounaris, G. D., and Papadopoulos, C. A., 2002. "Crack identification in rotating shafts by coupled response measurements". *Engineering Fracture Mechanics,* **69**, p. 339-352.

[109] Gudmundson, P., 1983. "The dynamic behaviour of slender structures with cross-sectional cracks". *J. Mech. Phys. Solids,* **31**, p. 329–345.

[110] Gasch, R., 1976. "Dynamic behaviour of a simple rotor with a cross sectional crack". *International Conference on Vibrations in Rotating Machinery, Paper C178/76, IMechE,* **1**, p. 123–128.

[111] Gasch, R., 1993. "A survey of the dynamic behaviour of a simple rotating shaft with a transverse crack". *Journal of Sound and Vibration,* **160(2)**, p. 313–332.

[112] Davies, W. G. R., and Mayes, I. W., 1984. "The vibrational behaviour of a multi-shaft, multi-bearing system in the presence of a propagating transverse crack". *Transactions of the ASME Journal of Vibration, Acoustics, Stress, and Reliability in Design,* **106**, p. 146–153.

[113] Mayes, I. W., and Davies, W. G. R., 1984. "Analysis of the response of a multi-rotor-bearing system containing a transverse crack in a rotor". *Transactions of the ASME Journal of Vibration, Acoustics, Stress, and Reliability in Design,* **106**, p. 139–145.

[114] Schmied, J., and Kramer, E., 1984. "Vibrational behaviour of a rotor with a cross-sectional crack". *IMechE Conference on Vibrations in Rotating Machinery, C279/84,* **1**, p. 15-19.

[115] Papadopoulos, C., and Dimarogonas, A., 1988. "Stability of cracked rotors in the coupled vibration mode". *Journal of Vibration Acoustics Stress and Reliability in Design-Transactions of the ASME,* **110(3)**, p. 356-359.

[116] Jun, O., Eun, H., Earmme, Y., and Lee, C.-W., 1992. “Modelling and vibration analysis of a simple rotor with a breathing crack”. *Journal of Sound and Vibration*, **155(2)**, p. 273–290.

[117] Pu, Y., Chen, J., and Zou, J., 2002. “Quasi-periodic vibration of cracked rotor on flexible bearings”. *Journal of Sound and Vibration*, **251(5)**, p. 875-890.

[118] Sinou, J.-J., and Lees, A. W., 2005. “Influence of cracks in rotating shafts”. *Journal of Sound and Vibration*, **285(4-5)**, p. 1015–1037.

[119] Papadopoulos, C., and Dimarogonas, A., 2008. “The strain energy release approach for modeling cracks in rotors: A state of the art review”. *Mechanical Systems and Signal Processing*, **22**, p. 763-789.

[120] Keiner, H., and Gadala, M., 2002. “Comparison of different modelling techniques to simulate the vibration of a cracked rotor”. *Journal of Sound and Vibration*, **254(5)**, p. 1012–1024.

[121] Friswell, M., and Penny, J., 2002. “Crack modeling for structural health monitoring”. *Structural Health Monitoring*, **1**, p. 139-148.

[122] Sinha, J., Friswell, M., and Edwards, S., 2002. “Simplified models for the location of cracks in beam structures using measured vibration data”. *Journal of Sound and Vibration*, **251**, p. 13-38.

[123] Bachschmid, N., Pennacchi, P., and Tanzi, E., 2008. “Some remarks on breathing mechanism, on non-linear effects and on slant and helicoidal cracks”. *Mechanical Systems and Signal Processing*, **22**, p. 879-904.

[124] Carlson, R., 1974. “An experimental study of the parametric excitation of a tensioned sheet with a crack like opening”. *Experimental Mechanics*, **14**, p. 452–458.

[125] Zastrau, B., 1985. “Vibrations of cracked structures”. *Archives of Mechanics*, **138(2)**, p. 731–743.

[126] Ostachowicz, W. M., and Krawczuk, M., 1990. “Vibration analysis of a cracked beam”. *Computers and Structures*, **36(2)**, p. 245–250.

[127] Qian, G., Gu, S., and Jiang, J., 1990. “The dynamic behaviour and crack detection of a beam with a crack”. *Journal of Sound and Vibration*, **138(2)**, p. 233–243.

[128] Collins, K., Plaut, P., and Wauer, P., 1992. “Free and forced longitudinal vibrations of a cantilevered bar with a crack”. *Journal of Vibration and Acoustics*, **114**, p. 171–177.

[129] Chondros, T., Dimarogonas, A., and Yao, J., 2001. “Vibration of a beam with a breathing crack”. *Journal of Sound and Vibration*, **239(1)**, p. 57–67.

[130] Luzzato, E., 2003. “Approximate computation of non-linear effects in a vibrating cracked beam”. *Journal of Sound and Vibration*, **265**, p. 745-763.

[131] Hsu, M., 2005. “Vibration analysis of edge-cracked beam on elastic foundation with axial loading using the differential quadrature method”. *Comput. Methods Appl. Mech. Engrg*, **194**, p. 1-17.

[132] Bikri, K. E., Benamar, R., and Bennouna, M., 2006. “Geometrically non-linear free vibrations of clampedclamped beams with an edge crack”. *Computers and Structures*, **84**, p. 485-502.

[133] Abdel Wahab, M., De Roeck, G., and Peeters, B., 1999. “Parameterization of damage in reinforced concrete structures using model updating”. *Journal of Sound and Vibration*, **228(4)**, p. 717-730.

[134] Volterra, V., 1959. *Theory of Functionals and Integral Equations*. Dover Publications, New York.

[135] Schetzen, M., 1980. *The Volterra and Wiener Theories of Non-Linear Svstems*. Wiley, New York.

[136] Kerschen, G., Worden, K., Vakakis, A., and Golinval, J.-C., 2006. “Past, present and future of nonlinear system identification in structural dynamics”. *Mechanical Systems and Signal Processing*, **20**, p. 505-592.

[137] Cameron, T., and Griffin, J., 1989. “An alternating frequency/time domain method for calculating the steady-state response of nonlinear dynamic systems”. *Journal of Applied Mechanics*, **56**, p. 149–154.

[138] Sinou, J.-J., and Lees, A. W., 2007. “A non-linear study of a cracked rotor”. *European Journal of Mechanics A/Solids*, **26(1)**, p. 152–170.

[139] Ruotolo, R., Surace, C., Crespo, P., and Storer, D., 1997. “Harmonic analysis of the vibrations of a cantilevered beam with a closing crack”. *Computers and Structures*, **61(6)**, p. 1057–1074.

[140] Peng, Z., Lang, Z., and Billings, S., 2007. “Crack detection using nonlinear output frequency response functions”. *Journal of Sound and Vibration*, **301**, p. 777-788.

[141] Pugno, N., Surace, C., and Ruotolo, R., 2000. “Evaluation of the non-linear dynamic response to harmonic excitation of a beam with several breathing cracks”. *Journal of Sound and Vibration*, **235(5)**, p. 749–762.

[142] Kisa, M., and Brandon, J., 2000. “The effects of closure of cracks on the dynamics of a cracked cantilever beam”. *Journal of Sound and Vibration*, **238(1)**, p. 1–18.

[143] Saavendra, P., and Cuitino, L., 2001. “Crack detection and vibration behaviour of cracked beams”. *Computers and Structures*, **79**, p. 1451-1459.

[144] Sinha, J., and Friswell, M., 2002. “Simulation of the dynamic response of a cracked beam”. *Computers and Structures*, **80**, p. 1473-1476.

[145] Loutridis, S., Douka, E., and Hadjileontiadis, L., 2005. “Forced vibration behaviour and crack detection of cracked beams using instantaneous frequency”. *NDT and E International*, **38**, p. 411-419.

[146] Bovsunovsky, A., and Surace, C., 2005. “Considerations regarding superharmonic vibrations of a cracked beam and the variation in damping caused by the presence of the crack”. *Journal of Sound and Vibration*, **288**, p. 865-886.

[147] Mayes, I., and Davies, W., 1976. "The vibrational behaviour of a rotating system containing a transverse crack". *IMechE Conference on Vibrations in Rotating Machinery, C/168/76*, **1**, p. 53–64.

[148] Henry, T., and Okah, B., 1976. "Vibration in cracked shafts". *IMechE Conference on Vibrations in Rotating Machinery, C162/76*, **1**, p. 15-19.

[149] Muszynska, M., Goldman, P., and Bently, D., 1992. "Torsional-lateral vibration cross-coupled responses due to shaft anisotropy: a new tool in shaft crack detection". *Vibrations in Rotating Machinery, Institution of Mechanical Engineers Conference Publications, London*, **1**, p. 257-262.

[150] Zhu, C., Robb, D., and Ewins, D., 2003. "The dynamics of a cracked rotor with an active magnetic bearing". *Journal of Sound and Vibration*, **265**, p. 469-487.

[151] Sinou, J.-J., 2008. "Detection of cracks in rotor based on the 2x and 3x superharmonic frequency components and the crackunbalance interactions". *Communications in Nonlinear Science and Numerical Simulation*, **13(9)**, p. 2024–2040.

[152] Liao, M., and Gasch, R., 1992. "Crack detection in rotating shafts - an experimental study". *International Conference on Vibrations in Rotating Machinery, Paper C432/106, IMechE*, **1**, p. 289–295.

[153] Darpe, A., Gupta, K., and Chawla, A., 2004. "Transient response and breathing behaviour of a cracked Jeffcott rotor". *Journal of Sound and Vibration*, **272**, p. 207-243.

[154] Chen, C., Dai, L., and Fu, Y., 2007. "Nonlinear response and dynamic stability of a cracked rotor". *Communications in Nonlinear Science and Numerical Simulation*, **12(6)**, p. 1023–1037.

[155] Adewusi, S., and Al-Bedoor, B., 2001. "Wavelet analysis of vibration signals of an overhang rotor with a propagating transverse crack". *Journal of Sound and Vibration*, **246(5)**, p. 777–793.

[156] Adewusi, S., and Al-Bedoor, B., 2002. "Experimental study on the vibration of an overhung rotor with a propagating transverse crack". *Shock and Vibration*, **9**, p. 91-104.

[157] Bently, D., and Bosmans, R., 1989. "Case study of shaft crack failure". *Orbit*, **10(4)**, p. 8–12.

[158] Iwasubo, T., Arii, S., and Oks, A., 1992. "Detection of a transverse crack in rotor shaft by adding external force". *International Conference on Vibrations in Rotating Machinery, Paper C432/093, IMechE*, **1**, p. 275–282.

[159] Darpe, A., Gupta, K., and Chawla, A., 2002. "Analysis of the response of a cracked Jeffcott rotor to axial excitation". *Journal of Sound and Vibration*, **249**, p. 429-445.

[160] Darpe, A., Gupta, K., and Chawla, A., 2004. "Coupled bending, longitudinal and torsional vibrations of a cracked rotor". *Journal of Sound and Vibration*, **269**, p. 33-60.

[161] Darpe, A., 2007. “Coupled vibrations of a rotor with slant crack”. *Journal of Sound and Vibration,* **305**, p. 172-193.

[162] Ishida, Y., and Inoue, T., 2006. “Detection of a rotor crack using a harmonic excitation and nonlinear vibration analysis”. *Journal of Vibration and Acoustics,* **128**, p. 741–749.

[163] Newland, D., 1993. *An introduction to random vibrations, spectral and wavelet analysis*. 3rd ed. Harlow: Longman Scientific and Technical.

[164] Newland, D., 1994. “Wavelet analysis of vibration. part 1: theory”. *Journal of Vibration and Acoustic,* **116**, p. 409-416.

[165] Newland, D., 1994. “Wavelet analysis of vibration. part 2: wavelet maps”. *Journal of Vibration and Acoustic,* **116**, p. 417-424.

[166] Rucka, M., and Wilde, K., 2006. “Application of continuous wavelet transform in vibration based damage detection method for beams and plates”. *Journal of Sound and Vibration,* **297**, p. 536-550.

[167] Mallat, S., 1989. “A theory for multiresolution signal decomposition: the wavelet representation”. *IEEE Transaction on Pattern Analysis and Machine Intelligence,* **11**, p. 674-693.

[168] Hong, J., Kim, Y., Lee, H., and Lee, Y., 2002. “Damage detection using the Lipschitz exponent estimated by the wavelet transform: applications to vibration modes of a beam”. *International Journal of Solids and Structures,* **39**, p. 1803-1816.

[169] Douka, E., Loutridis, S., and Trochidis, A., 2003. “Crack identification in beams using wavelet analysis”. *International Journal of Solids and Structures,* **40**, p. 3557-3569.

[170] Rucka, M., and Wilde, K., 2006. “Crack identification using wavelets on experimental static deflection profiles”. *Engineering Structures,* **28**, p. 279-288.

[171] Wang, Q., and Deng, X., 1996. “Damage detection with spatial wavelets”. *International Journal of Solid and Structures,* **36**, p. 3443-3468.

[172] Chang, C., and Chen, L., 2003. “Vibration damage detection of a Timoshenko beam by spatial wavelet based approach”. *Applied Acoustics,* **64**, p. 1217-1240.

[173] Gentile, A., and Messina, A., 2003. “On the continuous wavelet transforms applied to discrete vibrational data for detecting open cracks in damaged beams”. *International Journal of Solids and Structures,* **40**, p. 295-315.

[174] Naldi, G., and Venini, P., 1997. “Wavelet analysis of structures: statics, dynamics and damage identification”. *Meccanica,* **32**, p. 223-230.

[175] Messina, A., 2004. “Detecting damage in beams through digital differentiator filters and continuous wavelet transforms”. *Journal of Sound and Vibration,* **272**, p. 385-412.

[176] Bechou, L., Angrisiani, L., Ousten, Y., Dallet, D., Levi, H., Daponte, P., and Danto, Y., 1999. “Localization of defects in die-attach assembly by continuous wavelet transform using scanning acoustic microscopy”. *Microelectronics Reliability,* **39**, p. 1095–1101.

[177] Ovanesova, A., and Suarez, L., 2004. “Applications of wavelet transforms to damage detection in frame structures”. *Engineering Structures,* **26**, p. 39-49.

[178] Douka, E., Loutridis, S., and Trochidis, A., 2004. “Crack identification in plates using wavelet analysis”. *Journal of Sound and Vibration,* **270**, p. 279-295.

[179] Kijewski, T., and Kareem, A., 2003. “Wavelet transform for system identification in civil engineering”. *Computer-Aided Civil and Infrastructure Engineering,* **18 (5)**, p. 339-355.

[180] Staszewski, W., 1998. “Structural and mechanical damage detection using wavelets”. *The Shock and Vibration Digest,* **30 (6)**, p. 457-472.

[181] Kim, H., and Melhem, H., 2004. “Damage detection of structures by wavelet analysis”. *Engineering Structures,* **26**, p. 347-362.

[182] Surace, C., and Ruotolo, R., 1994. “Crack detection of a beam using the wavelet transform”. *Proceedings the 12th International Modal Analysis Conference, Honolulu, Hawaii,* **1**, p. 1141-1147.

[183] Liew, K., and Wang, Q., 1998. “Application of wavelet theory for crack identification in structures”. *Journal of Engineering Mechanics ASCE,* **124(2)**, p. 152-157.

[184] Wang, Q., Wang, D., and Su, X., 1999. “Crack detection of structure for plane problem with spatial wavelets”. *Acta Mech Sin,* **15(1)**, p. 39-51.

[185] Lu, C., and Hsu, Y., 1999. “Application of wavelet transform to structural damage detection”. *Proceedings of the 17th International Modal Analysis Conference, Kissimmee, Floride,* **1**, p. 908–914.

[186] Lu, C., and Hsu, Y., 2002. “Vibration analysis of an inhomogeneous string for damage detection by wavelet transform”. *International Journal of Mechanical Sciences,* **44**, p. 745-754.

[187] Loutridis, S., Douka, E., and Trochidis, A., 2004. “Crack identification in double-cracked beams using wavelet analysis”. *Journal of Sound and Vibration,* **277**, p. 1025-1039.

[188] Chang, C., and Chen, L., 2005. “Detection of the location and size of cracks in the multiple cracked beam by spatial wavelet based approach”. *Mechanical Systems and Signal Processing,* **19**, p. 139-155.

[189] Kim, Y., and Kim, E., 2000. “A new damage deflection method based on a wavelet transform”. *Proceedings of the International Modal Analysis Conference,* **1**, p. 1207-1212.

[190] Quek, S., Wang, Q., Zhang, L., and Ong, K., 2001. “Practical issues in the detection of damage in beams using wavelets smart materials and structures”. *Institute of Physics Publishing,* **10**, p. 1009-1017.

[191] Li, Z., Xia, S., Wang, J., and Su, X., 2006. "Damage detection of cracked beams based on wavelet transform". *International Journal of Impact Engineering,* **32**, p. 1190-1200.

[192] Zhu, X., and Law, S., 2006. "Wavelet-based crack identification of bridge beam from operational deflection time history". *International Journal of Solids and Structures,* **43**, p. 2299-2317.

[193] Sekhar, A. S., 2003. "Crack detection and monitoring in a rotor supported on fluid film bearings: Start-up vs run-down". *Mechanical Systems and Signal Processing,* **17(4)**, p. 897-901.

[194] Sekhar, A., 2003. "Crack detection through wavelet transform for a run-up rotor". *Journal of Sound and Vibration,* **259**, p. 461-472 .

[195] Zou, J., and Chen, J., 2004. "A comparative study on timefrequency feature of cracked rotor by wignerville distribution and wavelet transform". *Journal of Sound and Vibration,* **276**, p. 1-11.

[196] Sekhar, A., and Prabhu, B., 1998. "Condition monitoring of cracked rotors through transient response". *Mechanism and Machine Theory,* **33(8)**, p. 1167–1175.