# WELL-COMPOSED IMAGES AND RIGID TRANSFORMATIONS

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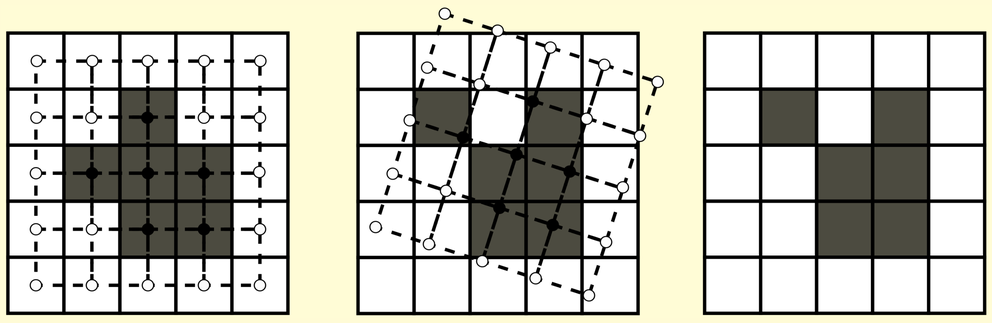
## Motivations

Image topology is generally not preserved during rigid transformations.



Original image with the associated grid | Rigid transformation applied on the grid | Transformed image

We study topological aspects of rigid transformations on 2D digital images, in particular considering the following questions:

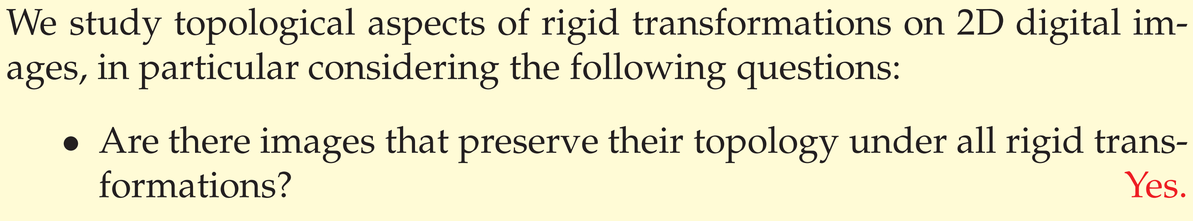
- Are there images that preserve their topology under all rigid transformations? Yes.
- Can we verify topological preservation for a given image? Yes, with a proposed algorithm in linear time.
- What condition for images to preserve their topology? Regularity.

## Regularity and topological invariance

A binary image $I$ is **well-composed** if in both foreground and background, any 8-connected component is also a 4-connected component.

Let $\frown_4$ denote the 4-adjacency relation. A binary image $I$ is **singular** if $\exists \mathbf{p} \in I, \forall \mathbf{q} \in I$: $(\mathbf{q} \frown_4 \mathbf{p}) \Longrightarrow (I(\mathbf{p}) \neq I(\mathbf{q}))$.

**Definition 1.** *A binary image $I$ is **regular** if $I$ is well-composed, non-singular and $\forall \boldsymbol{p}, \boldsymbol{q} \in I$ such that $I(\boldsymbol{p}) = I(\boldsymbol{q}) = 1$ (resp. 0):*
$(\boldsymbol{p} \frown_4 \boldsymbol{q}) \Longrightarrow (\exists \boxplus \subseteq I^{-1}(\{1\}(resp.\ \{0\})), \boldsymbol{p}, \boldsymbol{q} \in \boxplus)$.



Well-composed | Non well-composed | Singular | Non singular | Regular | Non regular

**Definition 2.** *A well-composed image $I$ is **topologically invariant** if all transformed image of $I$ is well-composed, and has an adjacency tree isomorphic to the one of $I$.*

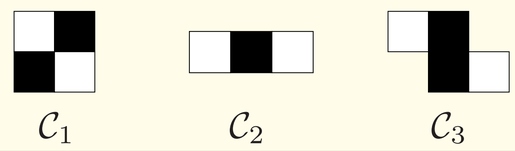
The following is our main result.

**Theorem 3.** ***An image is topologically invariant if it is regular.***

## Topological invariance verification and correction

We can verify in linear time whether a 2D binary well-composed image is topologically invariant using the introduced notion of regularity.

**Property 4.** *An image is regular iff it does not contain the configurations*

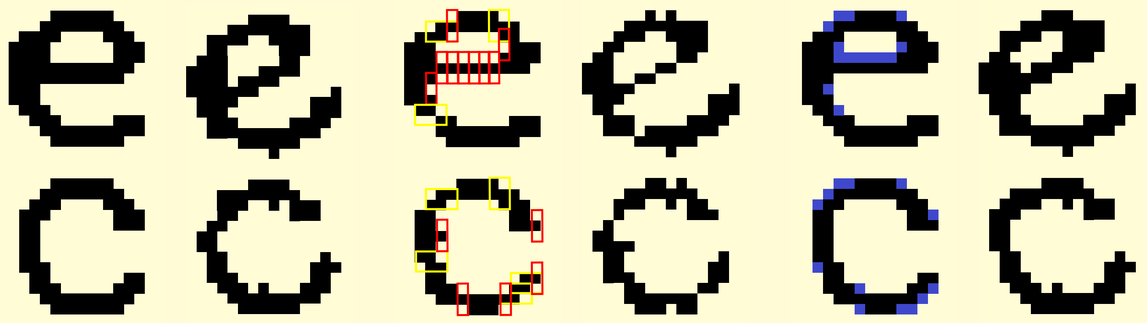


$\mathcal{C}_1$ $\mathcal{C}_2$ $\mathcal{C}_3$

**Algorithm 1.** *We verify the occurrences of the critical configurations $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$ in the given image.*

For preprocessing images in order to obtain regularity, we propose a strategy that consists of modifying progressively the image.

**Algorithm 2.** *We modify locally and iteratively the given image $I$ to eliminate the forbidden configurations $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$ in $I$, until stability.*



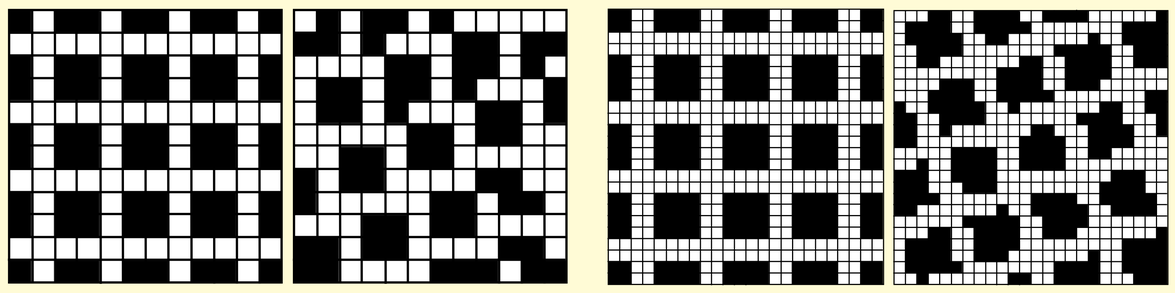
(a) Regular | Transformed of (a) | (b) Non regular | Transformed of (b) | (c) Corrected of (b) | Transformed of (c)

## Up-sampling regularisation

In presence of fine textures, the iterative approach for preprocessing images may not converge or even fail.

**Property 5.** *By doubling the resolution (i.e., associating a $2 \times 2$ pixel square to each initial pixel), a well-composed image becomes regular.*

**Algorithm 3.** *We propose an alternative up-sampling regularisation strategy in order to obtain a regular image.*



(a) Well-composed | (b) Transformed of (a) | (c) Up sampling of (a) | (d) Transformed of (c)

## Experiments

(a) Initial and sample | (b) Up-sampling regularisation | (e) Initial and sample | (f) Up-sampling regularisation | (i) Initial and sample | (j) Up-sampling regularisation

(c) Transformed of (a) topologically altered, by comparison to (a) | (d) Transformed of (b) topologically preserved, by comparison to (b) | (g) Transformed of (e) topologically altered, by comparison to (e) | (h) Transformed of (f) topologically preserved, by comparison to (f) | (k) Transformed of (i) topologically altered, by comparison to (i) | (l) Transformed of (j) topologically preserved, by comparison to (j)

Binary images | Grey images | Label images