

PROPOSITION OF A NEW SEMANTICS FOR STANDARDIZED TOLERANCING

PRINCIPLE OF ITS VERIFICATION ON COORDINATE MEASURING MACHINE

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Abstract

The semantics of standardized Dimensioning and Tolerancing (D&T), that is to say its meaning by relationship to actual surfaces, on one hand renders difficult the synthesis of a functional D&T and on other hand can not be correctly inspected, in particular with the Coordinate Measuring Machines (CMM).

We suggest a **new semantics for the tolerancing** based on the fitting of an **envelope surface** on the actual surface.

We define the envelope surface, in a unique and general way, as being the theoretical surface in contact with the actual surface on the outside of the material and so that the volume between them is minimal. In others terms, we suggest a unique fitting criterion enabling to define the envelope surface and which we call the **criterion of minimal volume**. We show the interesting properties of this criterion notably from the point of view of its functional interpretation.

We present then a method, for the **software of CMMs**, enabling the calculation of the envelope surface following the suggested criterion. The performances of this method and the obtained numeric result are compared to the two principals fitting criterions used, the least squares criterion and the minimal form defect criterion (again called minimax or Tschebyscheff criterion).

Introduction

The semantics of tolerancing standards is inspired of traditional technologies of verification: Instruments of measurement (calipers, micrometers), elements of metrology (surface plates) and of control (gauges). However it is not very well formalized. The most significant example is the semantics associated to the linear size tolerance: Inspired of the measurement by micrometers, it is based on the ambiguous notion of *local size*.

This origin of tolerancing standards renders very difficult the synthesis of a functional D&T. Moreover, the standardized semantics cannot be respected by the new technology of verification that is the CMM.

The object of this paper is to suggest a new semantics for the standardized tolerancing, based on the fitting of a theoretical surface on the actual surface. So that the theoretical surface is representative of the actual surface, we define a general fitting criterion for any types of surface and show how it can be calculated by the software of a CMM.

Principle of standardized tolerancing

The principle of standardized tolerancing is to tolerancing individually each simple surfaces (plane, cylinder, cone) composing the surface of the part.

For every simple surface, the standards discriminates three types of defects: A form defect, a size defect and a position defect (in this paper, the term position means the terms *location* and *orientation* used by the standards).

The object of the D&T is to tolerancing these three defects. The standards provides two graphic means: The *size tolerancing*, represented by a tolerated dimension, and the *geometric tolerancing*, represented by a *tolerance frame*.

The form and the position are toleranced with the geometric tolerancing. The linear size (diameter of a shaft) or angular size (angle between two planes), is toleranced with the size tolerancing.

It is necessary to note that these three defects constitute a model of reality, because nothing doesn't allow discriminate them naturally on the actual surface. It means that it is necessary to give them a sense with regard to the actual surface, which the norm calls the *interpretation* and which we prefer to call the semantics.

Standardized Semantics of the tolerancing

Presentation

For the linear sizes, the size tolerance limit the *actual local sizes (measurements between two opposite points) of the feature* [1] (Fig. 1.a). The feature nominates a cylindrical surface or a pair of flat parallel surfaces [1].

For the angular sizes, the meaning of the size tolerancing is very ambiguous. One notices that for the conical surfaces, the standards [2] doesn't tolerance the angle but only a form defect.

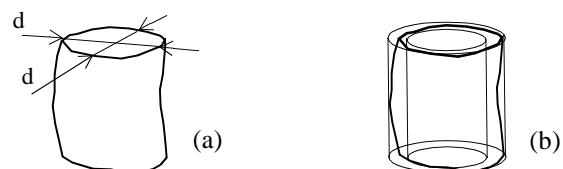


Fig. 1 : Size and Form

The form defect is the maximal distance between the actual surface and a theoretical surface in contact with it on the outside of the material. According to the standards, the theoretical surface has to be chosen of manner to minimize this distance. The theoretical surface is therefore fitted to the actual surface following the

criterion of minimal form defect, again called minimax criterion or Tschebyscheff criterion. This is the first fitting criterion used by the standards.

The standards, in its presentation of form defect, considers an other theoretical surface, «parallel» to the previous (coaxial cylinder), in contact with the actual surface on the inside of the material (Fig. 1.b).

Moreover, the standards offers a means to limit the form defect of the surface, through the size tolerance, thanks to the *envelope requirement*. This requirement specifies, that in addition of conditions on the local sizes, the actual surface has to respect a limit theoretical surface of which the size corresponds to the maximum material size authorized by the size tolerance.

If the position is toleranced, the tolerance is expressed by a *tolerance zone*. The «actual axis» must lie within this (Fig. 2.a).

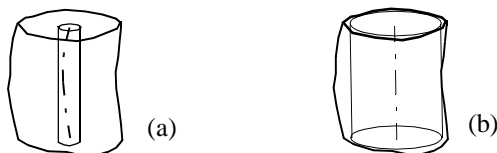


Fig. 2 : Toleranced position or datum position

Remark: The notion of «actual axis» is not definite in the standards.

If the position serves of datum, the datum axis that has to be considered is the axis of the envelope cylinder. In the case of a reaming, the envelope cylinder is the cylinder of maximal diameter incoming in the reaming (Fig 2.b.). This is the second criterion used by the standards. It is operable only for the surfaces characterized by an alone linear size (Cylinder and sphere).

Implication on the functional D&T

The difficulty felt by the drawer during the tolerancing of a part, is bound in the fact that the three tolerances have some interplays between them: The position tolerance limit indirectly the form defect of the surface and the form tolerance limit the size tolerance, which can not be superior to it (Fig. 1.b). However it is impossible to determine the accumulated effect of these interplays because the three defects are not relative to the same theoretical surface.

Remark: One notes here that the *principle of independency*, presented by the standards as the basic principle for the interpretation of tolerances, has a very limited scope!

Moreover, the interpretation of a linear size tolerance doesn't correspond to any real functional condition. Sometimes, it can be said that it is useful when the form is not functional, taking for example the condition of strength. In fact, for this condition, the variation of the form is more influential that the variation of the size as it is underlined by Professor Srinivasan [3].

Implication on the verification

The interpretation in terms of local sizes of a linear

size tolerance presents several problems for the verification:

- The notion of local size is very difficult to define simply. Until now, it isn't define by the standards. Some recent efforts provided definitions; but they are complicated and haven't got any functional justification.
- The local size cannot be measured when there is no material (Example of a shaft with a flat laid).
- The local size is incompatible with the techniques of verification on CMM.
- Finally, in a more theoretical view of view, it is impossible to warrant that none actual local size is out of the tolerance, being given that there is an infinity of it.

The verification of a position tolerance following the standardized interpretation necessitates the «construction» of the *actual axis*. In practices one prefers verify the position defect of the axis of a measuring rod adjusted in the reaming, as it is describes in the french standards relative to the verification, from NF E 10-105 to E 10-108 [4]. Note that the measuring rod materializes the envelope of the surface.

In conclusion, the standardized semantics is not functional and leads to some difficulties in metrology.

Proposition of a new semantics

Presentation

We suggest that the form, the size and the position are relative to the theoretical envelope surface fitted to the actual surface. In the continued of this paper, we give an unique and general definition of the theoretical envelope surface.

In the case of a shaft, the envelope is the theoretical cylinder of minimal diameter, outside to the material:

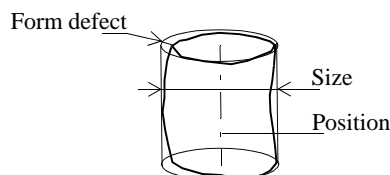


Fig. 3 : New semantics

The semantics that we suggest is the following:

- The size tolerance limit the envelope's size of the actual surface: The envelope's size has to be in the tolerance.
- Note that this semantics is also applicable to angular sizes like the angle of a cone because the definition of the envelope, that we suggest in the continued, is general.
- The form tolerance limit the maximal deviation of the actual surface with regard to the envelope.
- The position tolerance applies to the geometric feature which characterizes the position of the envelope: Axis of the envelope cylinder, centre of the envelope sphere, axis and apex of the envelope cone. We name these features, the positional features. In the case of the plane, the positional feature is the plane itself. Consequently, following the suggested semantics, a position tolerance applied to a flat surface, concerns the envelope plane of

this (*tangent plane*).

- Finally, if the surface serves of datum, this is also the positional feature of the envelope which is considered. This last point corresponds to the standards.

Remark: The suggested semantics can be used with other fitting criterions enabling the definition of a theoretical fitted surface; the least squares criterion for example. However, the one than we propose to adopt seems to be the most functional.

Implication on the functional D&T

The three tolerances, the size, the form and the position, are independent between them while attributing to the same theoretical surface:

The position tolerance doesn't limit the form defect of the surface.

The form tolerance doesn't limit the envelope's size.

The size tolerance concerns directly the envelope of the surface allowing to the drawer to control the maximum and minimum clearances in the assemblies.

Remark: If the envelope, which we want to tolerance the size, must apply just on a restraint length of the surface (example of a long shaft), the standards [6] provides a means to limit the zone in which we want that applies the tolerance:

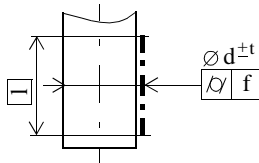


Fig. 4 : Tolerancing on a restraint zone

This drawing means, that in any premises of the shaft, the size tolerance and the form tolerance has to be true on the envelope of length l .

Implication on the verification

With the traditional technologies, these new interpretations don't lay more difficulties that the standardized interpretations:

For the position tolerances, the current practices are more in conformity with the suggested interpretation that with the one of the standards (except for the plane).

For the form tolerances, the standardized methods enable the verification of form defect following the suggested interpretation.

For the linear size tolerances, one could use gauges. The use of the micrometers doesn't enable to inspect the envelope.

The CMMs agrees particularly to this new semantics. However the criterion of the least squares, used by the software of these machines, leads to a theoretical surface passing through the sampled points. Some software shift this last surface toward the most outside point of the material. But in the two cases the obtained surface is not the envelope surface.

We present in the next part of this paper, an algorithm for the software of CMMs enabling the calculation of the envelope surface.

General definition of the envelope

Suggested definition

The envelope of an actual simple surface is the theoretical surface, outside to the material, which minimizes the volume between it and the actual surface.

In others terms, the envelope is the theoretical surface fitted to the actual surface following the criterion of Minimal Volume. We present lower its mathematical form.

Remark: To our knowledge, this criterion is original and has never been proposed before.

Comparison with the standardized fitting criterions

The standards [7] describes, for every type of surface, how has to be fitted the theoretical surface to the actual surface when this has to serve of datum. Alone the cylindrical surface has got a precisely definition.

We show here that the suggested criterion generalizes any fitting given by the standards and formalizes them further.

- For the cylindrical surfaces, the standards specifies that we have to consider, for a shaft, the *axis of minimal cylinder circumscribes*; in others terms, the axis of the fitted cylinder, outside to the material, of minimal diameter. The criterion of minimal volume is strictly equivalent to the criterion of minimal/maximal size definite by the standards. The demonstration is simple: Call v_1 the interior volume of the shaft and v_2 the interior volume of the circumscribes cylinder of diameter d and of height corresponding to the one of the shaft. The volume between the two surfaces is equals to the difference of these two volumes: $v = v_2 - v_1$.

v_1 being a constant as the height, to minimize this volume, it is necessary to minimize the diameter d of the theoretical cylinder.

- For the spherical surfaces, the standards speaks of *centre of the sphere envelope* [7]. The criterion of minimal volume permits to obtain the sphere of minimal diameter (or maximal). This is therefore compliant to the standards.

- For the conical surfaces, the criterion is quite ambiguous: *Axis of the cone, the best fitted on the actual surface* [7]. The criterion of minimal Volume enables to define in a unique way the conical envelope.

- Finally, for the plane surfaces, the criterion is also quite ambiguous: *Plane tangent to the actual surface. If the plane can have several orientations, the orientation of reference is the position average which allotted the deviations* (NF E 04-554 [7]). In the previous edition of this standards, the criterion was the one of minimal form defect: *Tangent plane on the outside of the material which minimizes the maximal deviation from the actual surface.*

In the current cases of flatness defects, the different fitting criterions lead to surfaces different little. However, we show on the following figure that, in some case of particular defects, the criterion of minimal volume allows to obtain a position of the plane more representative of the

actual surface than the criterion of minimal form defect.

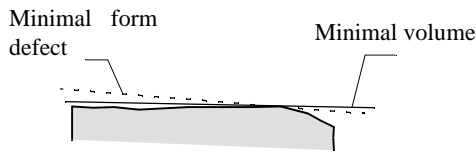


Fig. 5 : Fitting of a plane

Mathematical expression of the Minimal Volume criterion

Consider an initial theoretical surface outside to the material and sufficiently close from the actual surface.

e , the deviation between a point P of the actual surface and the theoretical surface.

ds , the elementary surface surrounding the point P . ds generates a elementary volume $dv = e ds$.

The included volume between the two surfaces is then: $V = \int_S eds$

The CMMs allow to know actual surfaces in a finished number of points $\{P_i\}$. To every point P_i we can associate an elementary surface Δs_i , surrounding it.

The volume is then: $V \approx \sum_S e_i \Delta s_i$

The suggested fitting criterion consists in minimizing this volume with the constraint that the theoretical surface stays on the outside of the material; that is to say, the deviations e_i stays positive: $e_i \geq 0$

It is possible, by an algorithm of «carving», to computing, for every sampled point P_i , the elementary surface Δs_i surrounding it. However it is more simple to impose a regular distribution of the sampled points on the actual surface in a such way that they have all the same elementary surface. Δs_i becomes then a constant that it is possible to exit of the sign sum.

The obtained criterion is the minimisation of the sum of the deviations, which are forced to stay positive:

$$\begin{aligned} &\text{Minimise } \sum e_i \\ &\text{under the constraints } e_i \geq 0 \end{aligned}$$

The deviations e_i are functions of the variation of size of the initial theoretical surface and also of its displacement with regard to the actual surface.

To solve this optimization problem, we use the «model of small displacements», developed by Professor Bourdet and Clement [10], which enables to linearize the expression of the deviation e_i in function of the displacement and size parameters.

Remark: To our knowledge, the obtained criterion, that would be named criterion of the minimal sum of the deviations, has never been suggested before. It is presumably not equivalent in the criterion of the minimal sum of the absolute values of the deviations which leads to a surface passing through the sampled points and for which Professor Shunmugam [8] suggests a method of calculation.

Mathematical model for the fitting of a theoretical surface on a sampled surface

The mathematical model described here is due to Professors Bourdet and Clement [9][10]. It is named the «model of small displacements».

The sampled points are expressed in a datum frame (frame of the CMM).

We define an initial theoretical surface with some sampled points (for the plane we choose three points among the set).

For every sampled point P_i , we define the normal to the theoretical surface (n_i), oriented toward the outside of the material, and the theoretical point M_i belonging to the surface.

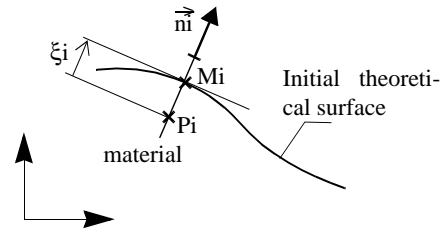


Fig. 6 : Initial position of the theoretical surface

The initial deviation between the theoretical surface and the point P_i is noted ξ_i .

To fit the theoretical surface to the actual surface, represented by the sampled points $\{P_i\}$, following a given criterion, we have to displace the initial theoretical surface and to vary its size.

The initial situation of the theoretical surface is very close of its optimal situation; that allows to characterize its displacement (translation and rotation) by a screw operator of which the components in a datum frame are:

$$\{D\}_O = \begin{cases} \vec{\Omega} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \\ \vec{D}_O = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \end{cases}$$

$\vec{\Omega}$ is the vector of rotation and \vec{D}_O represents the displacement of the origin point due to the displacement of the theoretical surface.

This model of the surface displacement enables to calculate the displacement of the point M_i by the relation:

$$\vec{D}_{M_i} = \vec{D}_O + \vec{\Omega} \wedge \vec{OM}_i$$

With the additional hypothesis that the displacement of M_i is small with regard to the curvature radius of the surface in M_i , we can write that the new deviation between the sampled point P_i and the theoretical surface is: $e_i = \xi_i + \vec{D}_{M_i} \cdot \vec{n}_i$

It is necessary to complete this model in order to take into account of the small size variation of the initial theoretical surface.

The size variation, noted r , is represented locally at the point P_i by the «shift» of the theoretical surface of a

small algebraic quantity equals to: $\rho_i \cdot r$

In the case of the fitting of a cylinder, if r represents the variation of its radius, then ρ_i is constant for every points and is worth 1.

The equation of the deviation becomes then:

$$e_i = \xi_i + \overrightarrow{D_{Mi}} \cdot \overrightarrow{n_i} + \rho_i \cdot r$$

In noting the coordinates of n_i and of M_i in the following way,

$$\overrightarrow{n_i} = \begin{pmatrix} a_i \\ b_i \\ c_i \end{pmatrix} \quad \overrightarrow{OM_i} = \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}$$

The development of e_i deal the linear following expression:

$$e_i = \xi_i + (a_i \cdot u + b_i \cdot v + c_i \cdot w + Li \cdot \alpha + Mi \cdot \beta + Ni \cdot \gamma) + \rho_i \cdot r$$

Where Li , Mi and Ni are the coordinates of the vector: $\overrightarrow{OP_i} \wedge \overrightarrow{n_i}$

Remark: In reason of the equiprojection theorem, the quantity $\overrightarrow{D_{Mi}} \cdot \overrightarrow{n_i}$ is equals also to the quantity $\overrightarrow{D_{Pi}} \cdot \overrightarrow{n_i}$

This property is advantageous because it allows to avoid the calculation of the coordinates of theoretical point M_i , in taking directly those of sampled point P_i to calculate the deviation e_i .

The authors used first this model for the fitting following the least squares criterion. The minimal value of the squared deviations sum is obtained when the partial derivatives of this sum nullifies. The solution is obtained by the resolution of a linear equations system.

Then, they used it for the criterion of minimal form defect and the one of maximal/minimal diameter for the inscribed/circumscribed cylinders.

In the two last cases, the problem to solve is a Linear Program.

Calculation of the envelope surface

Starting from the previous expression of e_i , the objective function, corresponding to the criterion of minimal volume, is:

$$\sum e_i = u \sum a_i + v \sum b_i + w \sum c_i + \alpha \sum Li + \beta \sum Mi + \gamma \sum Ni + r \sum \rho_i + \sum \xi_i$$

The problem consists in minimizing this linear function under the constraints system formed with linear inequalities $\{e_i \geq 0\}$. We obtain a linear program to solve.

We developed a resolution algorithm for linear programs. This algorithm is based on the simplex method. It was developed for the needs of a more important work which one will find the ideas in [11].

Numeric comparisons of fitting criterions

The resolution algorithm, that we developed, enabled us to fit a plane or a cylinder following the four criterions hereunder:

- Minimal form defect: Minimize $e_{i_{Max}}$. We have a linear program with $2N$ inequalities (N is the number of sampled points). The fitted surface is outside to the material.

- Least square: We have a system of Cramer of order three for the plane and five for the cylinder. This system is transformed in linear program (null objective function) in order to be resolved by the same algorithm that the other criterions. The surface is shifted toward the point the most outside of the material.

- Optimal diameter: Minimize r with $\{e_i \geq 0\}$. We obtain a linear program with N inequalities. The fitted cylinder is outside to the material.

- Minimal deviations sum (suggested criterion): Minimize $\sum e_i$ with $\{e_i \geq 0\}$. We have a linear program with N inequalities. The fitted surface is outside to the material.

We could verify the exactness of the numeric result given by the resolution algorithm in using some fictional surfaces of size envelope and of form defect knew in advance.

From the different tests that we conducted on a dozen of actual surfaces (8 to 16 points by surface having sizes inferior to 50mm), it stands out that:

- The criterion of minimal form defect is very long to compute (more iterations on more constraints: One second on a PC DX2-66 for a cylinder in 16 points). The obtaining of a minimal form defect is often done in the detriment of a smaller diameter for the reamings, or larger for the shafts, that the envelope diameter.

- The criterion of the least squares is very fast to compute. The obtained fitting is quite close from the criterion of minimal form defect, but the diameter of the «shifted» cylinder is smaller, for a reaming, or larger, for a shaft, that the one obtained by this last criterion. The deviation by relationship to the envelope diameter is therefore larger.

- The criterion of the minimal deviations sum (suggested criterion) is faster that the one of minimal form defect (Some tenths of second for the cylinder in 16 points). The obtained diameter corresponds to the envelope diameter.

- The criterion of Optimal diameter is lightly faster that the suggested criterion. But it leads to a form defect too large (larger than any others criterions). This results of the fact that it only takes into account of the highest points.

In conclusion, the suggested criterion leads to the numeric results which are the nearest of reality (envelope diameter, form defect) with a time of calculation completely acceptable.

Conclusion

The standardized semantics is based on the traditional practices of verification.

The consequence of this origin is that it is very difficult to formalize this semantics. One notes moreover the complexity of the definitions suggested in this goal, in particular for the notion of local size.

The standardized semantics is not adapted to the functional tolerancing.

Moreover, the verification on CMM following this semantics is difficult.

We suggested in this paper a new semantics for the tolerancing, based on the fitting of a theoretical envelope on the actual surface.

It is perfectly adapted to the verification on Coordinate Measuring Machine, especially if one uses the criterion of «minimal volume» definite in this paper in the place of the least squares criterion.

Finally, the suggested semantics simplifies cleanly the analysis and the synthesis of the tolerancing, without any modification of the D&T syntax, that is to say its graphic representation.

Simpler and more coherent, it constitutes one rigorous basis for the development of methods of functional D&T.

References

- [1] E04-561 «Principe de tolérancement de base»; Norme expérimentale AFNOR, Oct. 1991; Integral reproduction of the standards ISO8015:1985.
- [2] NF E 04-557 «Cotation et tolérancement - Cône»; Norme AFNOR, Déc. 1991. Corresponding to the standards ISO 3040:1990.
- [3] V. Srinivasan, «Recent Effort in Mathematization of ASME/ANSI Y14.5M»; Proceedings of 3rd CIRP Seminars on Computer Aided Tolerancing, pp223-232, April 1993.
- [4] NF E 10-105 «Méthodes de mesurage dimensionnel - Sixième partie : Etablissement des références spécifiées»; Norme expérimentale AFNOR, Déc. 1990.
- [5] E 10-108 «Méthodes de mesurage dimensionnel - Neuvième partie : Ecart de localisation»; Norme AFNOR, Déc. 1992.
- [6] NF E 04-552 «Tolérancement Géométrique. Généralités, définitions, symboles, indications sur les dessins»; Norme AFNOR, Nov. 1983. Corresponding to the standards ISO 1101:1983
- [7] NF E 04-554 «Cotation et tolérancement - Références et systèmes de référence pour tolérances géométriques»; Norme AFNOR, Déc 1988. Corresponding to the standards ISO 5459:1981.
- [8] M.S. Shunmugam, «New approach for evaluating form errors of engineering surfaces»; Journal of Computer-aided-design, Vol. 19, No 7, pp368-374, Sept. 1987.
- [9] P. Bourdet, «Contribution à la mesure tridimensionnelle: Modèle d'identification géométrique des surfaces; Métrologie fonctionnelle des pièces mécaniques; Correction géométrique des machines à mesurer tridimensionnelles»; Thèse de l'université de Nancy I (France), 1987.
- [10] P. Bourdet, A. Clement, «A study of optimal-criterion identification based on the small-displacement screw model»; Annals of the CIRP, Vol. 37/1/1988, pp 503-506.