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## Gravitational effects on domain walls with curvature corrections

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We derive the effective action for a domain wall with a small thickness in curved spacetime and show that, apart from the Nambu term, it includes a contribution proportional to the induced curvature. We then use this action to study the dynamics of a spherical thick bubble of false vacuum (de Sitter) surrounded by an infinite region of true vacuum (Schwarzschild).

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### I. INTRODUCTION

An intriguing implication of unified gauge theories is the possibility, within certain models, of the coexistence of two phases separated by a wall, which, at first approximation, can be seen as an infinitely thin bubble whose history is a timelike hypersurface [1]. Within this context, a number of articles have appeared in the literature studying the dynamics of bubbles with a surface layer described by the Nambu action of a domain wall. Such an action can be obtained from field theory, by considering, at first approximation, that the field is condensed along a three-dimensional timelike hypersurface, whose area gives the effective Nambu action. This last approach yields a description of the dynamics of a domain wall under the assumption that its dimensions are much greater than its thickness.

During the last few years, a number of authors studied the finite thickness corrections to the Nambu action. Gregory and Gregory *et al.* [2] have calculated the leading-order corrections to the equation of motion for a finite thickness domain wall, due to its extrinsic curvature and its self-gravity. Their conclusion was that the effective wall action must include, apart from the Nambu term, a contribution proportional to the induced curvature. On the other hand, Silveira and Maia [3], expanding the effective action in powers of the thickness, concluded that, in flat spacetime, there is a first-order correction term on the mean curvature and two second-order correction terms: one depending on the induced curvature and the other one depending on the Gaussian curvature. While we were writing our work, a paper of Larsen [4], regarding the finite thickness corrections to the Nambu action for a curved domain wall in Minkowski spacetime, came to our attention. Following the same ap-

proach as Silveira and Maia [3], Larsen [4] found that the correction term is proportional to the Ricci curvature of the induced metric. Hence, since the results in the literature seem to disagree, the first aim of the study we present in this paper is to obtain a consistent derivation of the effective action for a curved thick domain wall in curved spacetime. Our result agrees with that of Gregory [2], Larsen [4], and also Letelier [5], who studied the dynamics of test bubbles with curvature corrections in flat spacetime. The second aim of our work is to examine the effect of the curvature corrections on the bubble dynamics including gravitational effects. An analogous study was done by Blau *et al.* [6], who studied the dynamics of a spherically symmetric region of false vacuum separated by a Nambu domain wall from an infinite region of true vacuum.

This paper is organized as follows. In Sec. II we derive the effective action and the energy-momentum tensor for a domain wall with finite thickness corrections. In Sec. III we study the dynamics of such a domain wall separating two spherically symmetric static geometries. We then focus on the particular case of a spherical thick bubble of false vacuum (de Sitter) surrounded by an infinite region of true vacuum (Schwarzschild). We finally compare our results with those of Blau *et al.* [6].

Our system of units is such that  $\hbar=c=1$ .

### II. DOMAIN WALL WITH CURVATURE CORRECTIONS

In this section we will derive the effective action and effective energy-momentum tensor for a test domain wall moving in empty space, when curvature corrections are taken into account. We consider a real scalar field  $\phi$  with the action

$$A[\phi, g] = \int \sqrt{-g} \mathcal{L}(\phi, g) d^4x, \quad (2.1)$$

in a spacetime manifold  $\mathcal{M}$  with the signature  $(-+++)$ . The Lagrangian of  $\phi$  reads

$$\mathcal{L}(\phi, g) = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \lambda (\phi^2 - \eta^2)^2, \quad \lambda > 0. \quad (2.2)$$

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Taking the extremum of the action we obtain the field equation

$$(1/\sqrt{-g})\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi)-2\lambda\phi(\phi^2-\eta^2)=0. \quad (2.3)$$

A solution of the above field equation describes a membranelike structure provided there exists a timelike hypersurface  $\Sigma$ , such that the field  $\phi$  vanishes on  $\Sigma$ , while at a sufficiently large distance from it,  $\phi$  takes the value  $+\eta$  on one side and  $-\eta$  on the other one. Such a solution is called a domain wall and can be considered as a macroscopic membrane of a certain width, whose average history is given by  $\Sigma$ . In flat spacetime the explicit form of the solution of Eq. (2.3) has been obtained [7] in the particular case corresponding to a planar static domain wall. This particular solution, representing a flat membrane of width  $\delta$  lying along the plane  $z=0$ , reads

$$\phi_0(z)=\eta \tanh(z/\delta), \quad (2.4)$$

where

$$\delta=\eta^{-1}\lambda^{-1/2}. \quad (2.5)$$

The action corresponding to  $\phi_0$  is

$$A_0=-(\eta/\delta)^2 \int \cosh^{-4}(z/\delta)d^4x=-\mu \int dt dx dy, \quad (2.6)$$

where the mass per unit area  $\mu$  is given by

$$\mu=(2\eta^2/\delta) \int_0^\infty dz \cosh^{-4}z \approx 2\eta^2/\delta. \quad (2.7)$$

Here, we are interested in the effective action of a general solution of the field equation (2.3), describing membrane-like structures.

We find it convenient to introduce in a neighborhood of  $\Sigma$  Gaussian coordinates  $x^\alpha=(\tau^A, \rho)$ , where  $\tau^A$  ( $A=0,1,2$ ) are the parameters on  $\Sigma$  and  $\rho$  is the proper length along the geodesics orthogonal to  $\Sigma$ . In this system, the metric takes the form

$$\mathcal{G}_{\alpha\beta}=(\mathcal{G}_{AB}(\tau, \rho), \mathcal{G}_{\rho\rho}=1, \mathcal{G}_{A\rho}=0). \quad (2.8)$$

Let  $\partial_\alpha=(\partial_A, \partial_\rho)$  be the holonomic basis. We can then associate with each point on  $\Sigma$  four linearly independent vectors  $(e_A, N)$  defined as

$$e_A=(\partial_A)_0, \quad N=(\partial_\rho)_0. \quad (2.9)$$

The induced metric on  $\Sigma$  and its extrinsic curvature are, respectively,

$$\gamma_{AB}=e_A \cdot e_B, \quad (2.10)$$

$$K_{AB}=\nabla_A N \cdot e_B=-N \cdot \nabla_A e_B. \quad (2.11)$$

A second-order series expansion of the metric in the neighborhood of  $\Sigma$  gives

$$\mathcal{G}_{AB}=\gamma_{AB}+2\rho K_{AB}+\rho^2[K_A{}^C K_{BC}-(R_{A\rho B\rho})_0]+O(\rho^3), \quad (2.12)$$

where  $R_{\alpha\beta\gamma\delta}$  stands for the Riemann tensor. The inverse metric tensor  $\mathcal{G}^{\alpha\beta}$  is

$$\mathcal{G}^{AB}=\gamma^{AB}-2\rho K^{AB}+\rho^2[3K^{AC}K_C{}^B+(R^A{}_\rho B_\rho)_0]+O(\rho^3), \quad (2.13)$$

where  $\gamma^{AB}$  is the inverse of the induced metric  $\gamma_{AB}$ :

$$\mathcal{G}^{A\rho}=\mathcal{G}_{A\rho}=0, \quad (2.14)$$

and

$$\mathcal{G}^{\rho\rho}=\mathcal{G}_{\rho\rho}=1. \quad (2.15)$$

As can be easily verified,

$$\sqrt{-\mathcal{G}}=\sqrt{-\gamma}\{1+\rho K+\frac{1}{2}\rho^2[K^2-K^{AB}K_{AB}-(R_{\rho\rho})_0]+O(\rho^3)\}, \quad (2.16)$$

where  $\gamma=\det(\gamma_{AB})$ ,  $K=\text{Tr}(K^A{}_B)$ , and  $R_{\rho\rho}$  is the  $(\rho\rho)$  component of the Ricci tensor. Applying the Gauss-Codazzi equation

$$-2G_{\alpha\beta}N^\alpha N^\beta=({}^3R+K_{AB}K^{AB}-K^2), \quad (2.17)$$

where  ${}^3R$  denotes the induced Riemannian curvature on  $\Sigma$  and  $G_{\alpha\beta}$  stands for the Einstein tensor, we obtain that in an empty curved spacetime Eq. (2.16) reduces to

$$\sqrt{-\mathcal{G}}=\sqrt{-\gamma}[1+\rho K+\frac{1}{2}\rho^2({}^3R+O(\rho^3))]. \quad (2.18)$$

Let us now concentrate on solutions close to the planar static one. To do so, we suppose that the curvature radius of the membrane is large enough to assume that the solution  $\phi$  is near  $\phi_0(\rho)$ , where now  $\rho$  replaces  $z$ . Thus,

$$\phi=\phi_0(\rho)+\phi_1(x^\alpha), \quad (2.19)$$

where  $\phi_1$  denotes the perturbative term, which is such that  $\phi_1=0$  on  $\Sigma$  and  $\phi_1$  goes to zero far from the hypersurface  $\Sigma$ . Expanding the Lagrangian  $\mathcal{L}$  to second order in  $\phi_1$ , one gets

$$\mathcal{L}=L_0+L_1+L_2, \quad (2.20)$$

where

$$L_0=-\frac{1}{2}\partial_\rho\phi_0\partial_\rho\phi_0-\frac{1}{2}\lambda(\phi_0^2-\eta^2)^2, \quad (2.21)$$

$$L_1=-\partial_\rho\phi_0\partial_\rho\phi_1-2\lambda\phi_0(\phi_0^2-\eta^2)\phi_1, \quad (2.22)$$

$$L_2=-\frac{1}{2}\mathcal{G}^{\alpha\beta}\partial_\alpha\phi_1\partial_\beta\phi_1-\lambda(3\phi_0^2-\eta^2)\phi_1^2. \quad (2.23)$$

Note that the zero-order term  $L_0$  is identical to the Lagrangian for the planar static solution  $\phi_0$ . Using the field equation for the planar solution  $\phi_0$ , i.e.,

$$\partial_{\rho\rho}\phi_0(\rho)-2\lambda\phi_0(\phi_0^2-\eta^2)=0, \quad (2.24)$$

the first-order term  $L_1$  reduces to

$$L_1=-\partial_\rho(\phi_1\partial_\rho\phi_0). \quad (2.25)$$

Moreover, for the solution given by Eq. (2.19), the field equation (2.3) reads

$$(1/\sqrt{-\mathcal{G}})\partial_\rho\sqrt{-\mathcal{G}}\partial_\rho\phi_0+(1/\sqrt{-\mathcal{G}})\partial_\alpha(\sqrt{-\mathcal{G}}\mathcal{G}^{\alpha\beta}\partial_\beta\phi_1)-2\lambda[(3\phi_0^2-\eta^2)\phi_1+3\phi_0\phi_1^2+\phi_1^3]=0. \quad (2.26)$$

Replacing the above equation, to first order in  $\phi_1$ , in Eq. (2.23) we obtain

$$L_2 = -\left(\frac{1}{2}\sqrt{-\mathcal{G}}\right)[\phi_1\partial_\rho(\sqrt{-\mathcal{G}}\partial_\rho\phi_0) + \partial_\alpha(\phi_1\sqrt{-\mathcal{G}}\mathcal{G}^{\alpha\beta}\partial_\beta\phi_1)] . \quad (2.27)$$

We can now calculate the action

$$A = \int \sqrt{-\mathcal{G}}(L_0 + L_1 + L_2)d^4x , \quad (2.28)$$

where  $\sqrt{-\mathcal{G}}$  is given by Eq. (2.18). At this point we would like to remark that  $\phi$  is a test field, since Eq. (2.18) was derived using Einstein's equation in empty space. Replacing Eq. (2.4) in the expression (2.21) we get that

$$L_0 = -(\eta/\delta)^2 \cosh^{-4}(\rho/\delta) \quad (2.29)$$

and, therefore, the first term in Eq. (2.28) for the action reads

$$\int \sqrt{-\mathcal{G}}L_0d^4x = -\mu \int \sqrt{-\gamma}d^3\tau + \frac{1}{2} \int \sqrt{-\gamma}^{(3)}Rd^3\tau \int \rho^2L_0(\rho)d\rho , \quad (2.30)$$

where the mass per unit area  $\mu$  is given by Eq. (2.7). To get the second term in the action, we use Eq. (2.25) and perform an integration by parts. We get

$$\int \sqrt{-\mathcal{G}}L_1d^4x = \int \sqrt{-\gamma}K \left[ \int \phi_1\partial_\rho\phi_0d\rho \right] d^3\tau . \quad (2.31)$$

Finally, the last term in the action becomes

$$\int \sqrt{-\mathcal{G}}L_2d^4x = -\frac{1}{2} \int \sqrt{-\gamma}K \left[ \int \phi_1\partial_\rho\phi_0d\rho \right] d^3\tau , \quad (2.32)$$

where we have used Eq. (2.27). Combining the above results, the action reads

$$A = -\mu \int \sqrt{-\gamma}d^3\tau + \frac{1}{2} \int \sqrt{-\gamma}K \left[ \int \phi_1\partial_\rho\phi_0d\rho \right] d^3\tau + \frac{1}{2} \int \sqrt{-\gamma}^{(3)}Rd^3\tau \int \rho^2L_0(\rho)d\rho . \quad (2.33)$$

The above expression for the action can be further simplified, under the assumption of a domain wall having a small width  $\delta$ , arguing as follows: Let us first rewrite Eq. (2.33) as

$$A = -\mu \int \sqrt{-\gamma}d^3\tau + \frac{1}{2} \int \sqrt{-\gamma}K \left[ \int \phi_1\partial_\rho\phi_0d\rho \right] d^3\tau - \mu\alpha \int \sqrt{-\gamma}^{(3)}Rd^3\tau , \quad (2.34)$$

where  $\alpha$  is given from

$$\alpha\mu = -\frac{1}{2} \int \rho^2L_0d\rho = (\eta^2/2\delta^2) \int_{-\infty}^{+\infty} \rho^2 \cosh^{-4}(\rho/\delta)d\rho \approx \eta^2\delta/3$$

and, thus, from Eq. (2.7),

$$\alpha \approx \delta^2/6 . \quad (2.35)$$

We can always expand  $\phi_1$  in Taylor series as  $\phi_1 = \sum_i \tilde{\phi}_i \rho^i$ . The term  $i=0$  is absent ( $\tilde{\phi}_0=0$ ) because since the domain wall is placed at the origin  $\phi_1(\rho=0)=0$ . The general term contributes to the second integral of Eq. (2.34) as

$$\tilde{\phi}_i \int \rho^i \partial_\rho \phi_0 d\rho = \tilde{\phi}_i \eta \delta^i \int \left[ \frac{\rho}{\delta} \right]^i \cosh^{-2}(\rho/\delta) d(\rho/\delta) , \quad (2.36)$$

where the integral is of order 1, for  $i > 2$  (for  $i=1$  the integral vanishes by parity). Thus, the first term which contributes is for  $i=2$ , which is of higher order in  $\delta$  than the third integral in Eq. (2.34), so we neglect it. Thus the effective action simplifies to

$$A = -\mu \int \sqrt{-\gamma}d^3\tau - \mu\alpha \int \sqrt{-\gamma}^{(3)}Rd^3\tau , \quad (2.37)$$

where the first term is the usual Nambu action, while the second one represents the curvature correction due to the small thickness of the domain wall. To summarize, so far we have obtained the effective action for a domain wall with a small but nonzero thickness, embedded in an empty spacetime. At this point we would like to mention that the form given by Eq. (2.37) for the effective action with curvature corrections had already appeared in the literature. Our contribution was to do the analysis in a curved spacetime, give the explicit derivation of the expression for the effective action and, mainly, find the sign and the order of magnitude of  $\alpha$ .

Let us now proceed with the calculation of the effective energy-momentum tensor. Our objective is to study the motion of a bubble wall described by the action (2.37) in a curved spacetime. To do so, we will follow the same analysis as in Ref. [8]. We start with the variation for the effective action  $A$ , namely,

$$\delta A = -\frac{1}{2}\mu \int \sqrt{-\gamma}[\gamma^{AB} - 2\alpha^{(3)}G^{AB}]\delta\gamma_{AB}d^3\tau \quad (2.38)$$

(where  $^{(3)}G^{AB}$  stands for the induced Einstein tensor); since the term with the divergence dropped out once we have integrated over a closed surface. As was shown [8], for a purely geometrical variation of the three-dimensional world sheet of the domain wall, the variation of the induced metric reads

$$\delta_{\rho\gamma} \mu_{AB} = 2K_{AB} \delta\rho\tau . \quad (2.39)$$

To get the equation of motion for the test bubble, we take the extremum of the effective action  $A$  with respect to arbitrary variations in  $\rho$ . The resulting equation reads

$$S^{AB}K_{AB} = 0 , \quad (2.40)$$

where

$$S^{AB} = -\mu[\gamma^{AB} - 2\alpha^{(3)}G^{AB}] . \quad (2.41)$$

Because of the invariance of the action under infinitesimal coordinate transformations and under the assumption that the field satisfies the equation of motion,

one can show that the tensor  $T^{\alpha\beta}$ , defined as

$$T^{\alpha\beta} = \begin{pmatrix} T^{AB} & 0 \\ 0 & 0 \end{pmatrix}, \quad (2.42)$$

where

$$T^{AB} = S^{AB}\delta(\rho), \quad (2.43)$$

is the conserved energy-momentum tensor. Let us mention that the above expression for the energy-momentum tensor was also found by Letelier [5] in his study of the motion of a test bubble in flat spacetime. The quantity  $S^{AB}$  is formally obtained in the usual way, but using the induced metric and is conserved, i.e.,

$${}^{(3)}\nabla_A S^{AB} = 0, \quad (2.44)$$

where  ${}^{(3)}\nabla_A$  denotes the three-dimensional induced covariant derivative.

### III. DYNAMICS OF SPHERICAL DOMAIN WALLS SEPARATING TWO GEOMETRIES

In this section we will consider the gravitational effects of a spherically symmetric domain wall with a small, nonzero, thickness separating two spherically symmetric static geometries. We will then apply this formalism in the particular case where these geometries are the de Sitter and Schwarzschild metrics. This case has been studied earlier by Blau *et al.* [6] and could probably arise within the context of an inflationary scenario, in which one may have chaotic cosmological initial conditions. The difference between our study and the detailed analysis presented in Ref. [6] lies in the fact that we consider the action of the domain wall taking into account the effect of curvature corrections [see Eq. (2.37)]. Suppose that the timelike hypersurface  $\Sigma$  separates the spacetime  $\mathcal{M}$  into the manifolds  $\mathcal{M}^+$  and  $\mathcal{M}^-$ . As we have seen, the energy-momentum tensor  $T^{\alpha\beta}$  [see Eqs. (2.42), (2.43)] contains a  $\delta$  singularity without derivatives, and therefore  $\Sigma$  is a hypersurface of order one. In other words, as we go through  $\Sigma$ , the metric of the spacetime remains continuous, while the transverse derivatives are discontinuous. Thus we can apply the formalism presented by Israel in Ref. [9].

Let  $[A] = A_+ - A_-$  be the jump across the hypersurface  $\Sigma$ , and  $\bar{A} = \frac{1}{2}(A_+ + A_-)$  the average value of any discontinuous quantity  $A$ . With this notation, the junction equations are

$$[K_{AB}] - \gamma_{AB}[K] = -8\pi G S_{AB}, \quad (3.1)$$

$$S_{AB} \tilde{K}^{AB} = [t_{\alpha\beta} N^\alpha N^\beta], \quad (3.2)$$

$${}^{(3)}\nabla_B S_A^B = -[t_{\alpha\beta} e^\alpha_A N^\beta], \quad (3.3)$$

where  $t_{\alpha\beta}^\pm$  stand for the energy-momentum tensor of the continuous matter within the manifolds  $\mathcal{M}^\pm$ . In general, Eqs. (3.1)–(3.3) are not independent. Indeed, since the geometries  $\mathcal{M}^\pm$  have known metrics  $g_{\alpha\beta}^\pm$ , which are solutions of the Einstein equation associated with  $t_{\alpha\beta}^\pm$ , one can easily check that Eqs. (3.2) and (3.3) will be automatically satisfied once Eq. (3.1) is satisfied. Hence we have to veri-

fy Eq. (3.1) which is the Lanczos junction condition. Moreover, because of spherical symmetry, the off-diagonal components of the extrinsic curvature vanish, while the angular ones are related by

$$K_{\varphi\varphi} = \sin^2\vartheta K_{\vartheta\vartheta}. \quad (3.4)$$

So the dynamics of the domain wall will be completely determined by the  $(\tau\tau)$  and  $(\vartheta\vartheta)$  components of the junction condition (3.1). On the other hand, the spherical symmetry implies that the surface-energy tensor (2.41) cannot have  $(\tau\vartheta)$  and  $(\tau\varphi)$  components and takes a form which resembles that of a perfect fluid:

$$S^{AB} = (\sigma + p)u^A u^B + p\gamma^{AB}, \quad (3.5)$$

where  $\sigma, p, u = (1, 0, 0)$  denote the surface-energy density, surface pressure, and three-velocity of a point in the domain wall, respectively. From Eqs. (2.41) and (3.5) one gets

$$\sigma = \mu + 2\mu\alpha^{(3)}G^{AB}u_A u_B, \quad (3.6)$$

$$p = -\mu + (\mu\alpha/2)(2^{(3)}G^{AB}u_A u_B - {}^{(3)}R). \quad (3.7)$$

In the above expressions for  $\sigma$  and  $p$ , the first term is the usual Nambu term, while the second one is the result of the nonzero thickness of the domain wall.

Let the manifold  $\mathcal{M}^\pm$  have the spherically symmetric static metric

$$ds_\pm^2 = -f_\pm(r)dt_\pm^2 + f_\pm^{-1}(r)dr^2 + r^2 d\Omega^2, \quad (3.8)$$

where

$$d\Omega^2 = d\vartheta^2 + \sin^2\vartheta d\varphi^2. \quad (3.9)$$

The timelike hypersurface  $\Sigma$  generated by the spherical domain wall is parametrized by

$$\tau^A = (\tau, \vartheta, \varphi), \quad (3.10)$$

where  $\tau$  denotes the time parameter as measured by an observer moving with the domain wall. As a result of the matching condition on the induced geometry, the metric on the hypersurface  $\Sigma$  can be written as

$$ds_\Sigma^2 = -d\tau^2 + R^2(\tau)d\Omega^2, \quad (3.11)$$

with  $r = R(\tau)$ . In the coordinates  $(t, r, \vartheta, \varphi)$  the four components of the velocity  $u$  introduced in Eq. (3.5) are  $u_\pm^\alpha = (\dot{t}_\pm \dot{R}, 0, 0)$ , where the dot denotes derivation with respect to  $\tau$ . We introduce the notation

$$F_\pm \equiv \dot{t}_\pm f_\pm(R) = \epsilon_1^\pm [f_\pm(R) + \dot{R}^2]^{1/2}, \quad (3.12)$$

where  $\epsilon_1^\pm$  stands for the sign of  $\dot{t}_\pm$ , and the second equality means that the square of the velocity  $u_\pm^\alpha$  equals  $(-1)$ . The nonzero components of the extrinsic curvature are

$$K_{\tau r}^\pm = -\epsilon_2^\pm \dot{F}_\pm / \dot{R}, \quad (3.13)$$

$$K_{\vartheta\vartheta}^\pm = K_{\varphi\varphi}^\pm = -\epsilon_2^\pm F_\pm / R. \quad (3.14)$$

Note that the sign indeterminacy  $\epsilon_2^\pm$  depends on the expression for the normal  $N_\pm^\alpha$  to the hypersurface  $\Sigma$ , pointing from  $\mathcal{M}^-$  to  $\mathcal{M}^+$ . On the other hand, the nonzero components of the induced Ricci tensor are

$${}^{(3)}R_\tau = 2\ddot{R}/R, \quad (3.15)$$

$${}^{(3)}R_\vartheta = {}^{(3)}R_\varphi = (1/R^2) + (\dot{R}^2/R^2) + \ddot{R}/R. \quad (3.16)$$

The induced Ricci curvature scalar is

$${}^{(3)}R = 2[2\ddot{R}/R + \dot{R}^2/R^2 + 1/R^2], \quad (3.17)$$

and the nonzero components of the induced Einstein tensor read

$${}^{(3)}G_\tau = -[\dot{R}^2/R^2 + 1/R^2], \quad (3.18)$$

$${}^{(3)}G_\vartheta = {}^{(3)}G_\varphi = \ddot{R}/R. \quad (3.19)$$

We can now rewrite the expressions [see Eqs. (3.6) and (3.7)] for the surface-energy density  $\sigma$  and the surface pressure  $p$  as

$$\sigma = \mu[1 + 2\alpha(\dot{R}^2/R^2 + 1/R^2)], \quad (3.20)$$

$$p = -\mu[1 + 2\alpha\ddot{R}/R]. \quad (3.21)$$

Since both  $\mu$  and  $\alpha$  are positive, one can easily see that the effect of the curvature corrections is to increase  $\sigma$  with respect to its value in the case of a Nambu domain wall. On the other hand, the effect of the nonzero thickness of the domain wall on  $p$  depends on the sign of  $\ddot{R}$ . Stability conditions require  $p$  to be negative, and this is satisfied since  $\alpha$  is small [see Eq. (2.35)], provided we do not study very small values of  $R$ . As we have already mentioned earlier on, the dynamics of the domain wall will be completely determined once we solve the  $(\tau\tau)$  and  $(\vartheta\vartheta)$  components of the Lanczos junction condition (3.1). The  $(\vartheta\vartheta)$  component gives

$$-\epsilon_2^+ F_+ + \epsilon_2^- F_- = -4\pi\mu G [R + 2\alpha(1/R + \dot{R}^2/R)], \quad (3.22)$$

and the  $(\tau\tau)$  component becomes

$$-\epsilon_2^+ \dot{F}_+ + \epsilon_2^- \dot{F}_- = -4\pi\mu G [\dot{R} + 2\alpha\dot{R}(2\ddot{R}/R - 1/R^2 - \dot{R}^2/R^2)], \quad (3.23)$$

which is obviously the derivative of the  $(\vartheta\vartheta)$  component. Thus the only equation we have to solve is Eq. (3.22).

Let us apply the above analysis in the particular case of de Sitter and Schwarzschild geometries joined with a thick spherical domain wall. We consider a spherically symmetric region of false vacuum (de Sitter) which is separated from an infinite region of true vacuum (Schwarzschild) by a thick domain wall. The Schwarzschild geometry is determined by

$$f_+ = 1 - (2GM/r), \quad (3.24)$$

while the de Sitter one by

$$f_- = 1 - \chi^2 r^2, \quad (3.25)$$

where

$$\chi^2 = (8\pi/3)G\rho_0, \quad (3.26)$$

with  $\rho_0$  defined by the energy-momentum tensor of  $\mathcal{M}^-$ , i.e.,

$$t^{-\alpha\beta} = -\rho_0 g^{\alpha\beta}. \quad (3.27)$$

Doing some algebraic manipulations of Eq. (3.22) using Eqs. (3.12), (3.24), and (3.25), where we keep only the linear terms in  $\alpha$ , since  $\alpha$  is small [see Eq. (2.31)], we obtain an equation of the form

$$\mathcal{F}(R) = a(R)\dot{R}^2 + b(R)\dot{R}^4, \quad (3.28)$$

where

$$\begin{aligned} \mathcal{F}(R) = & (A - R\kappa)^2 + (8GM/R) \\ & - [4 + 4R\kappa(A - R\kappa)\epsilon - (16GM/R)\epsilon + 8\epsilon], \end{aligned} \quad (3.29a)$$

$$A(R) = (1/R\kappa)[(2GM/R) - R^2\chi^2], \quad (3.29b)$$

$$\kappa = 4\pi\mu G, \quad (3.29c)$$

$$\epsilon = (2\alpha/R^2), \quad (3.29d)$$

$$a(R) = 4 + 16\epsilon + 4R\kappa(A - R\kappa)\epsilon - (16GM/R)\epsilon, \quad (3.29e)$$

$$b(R) = 8\epsilon. \quad (3.29f)$$

As one can easily verify, Eq. (3.28) reduces to the equation of motion of the domain wall found by Blau *et al.* [6], once we set  $\alpha=0$ .

Let us first find the positions  $R$ , where the velocity of the spherical domain wall vanishes. These will be the solutions of the equation

$$\mathcal{F}(R) = 0. \quad (3.30)$$

To do so, we take  $\lambda \sim 1$ , the energy density of the false vacuum  $\rho_0$  to be of order  $\eta^4$ , the surface-energy density  $\mu$  to be of order  $\eta^3$  [see Eqs. (2.5) and (2.6)], and we choose for the Schwarzschild mass  $M$  arbitrarily the value of the grand unified theory (GUT) scale, which is lower than the critical mass  $M_{\text{crit}}$ , to be defined below. For a domain wall formed during the spontaneous symmetry breaking of GUT,  $\eta \approx 10^{14}$  GeV,  $\alpha \approx 10^{-28}$  GeV $^{-2}$  and Eq. (3.30) has two solutions [ $R_1 \approx 0.963 \times 10^{-15}$  GeV $^{-1}$  and  $R_2 \approx 3.125 \times 10^{-14}$  GeV $^{-1}$ ], almost identical to the case of a zero thickness ( $\alpha=0$ ) domain wall [ $R_1^* \approx 2.702 \times 10^{-15}$  GeV $^{-1}$  and  $R_2^* \approx 3.027 \times 10^{-14}$  GeV $^{-1}$ ].

At this point, we would like to remark that the equation governing the dynamics of the bubble is a second-order differential equation [see Eq. (3.23)]. As a consequence, one has to draw the two-dimensional-phase-space diagram of Eq. (3.28) to get a qualitative analysis of the domain wall dynamics. Here, we would like to remark that although Eq. (3.28) is a fourth-order equation, there are only two real solutions for  $\dot{R}$ . This can easily be verified solving Eq. (3.28) for  $\dot{R}^2$ , taking into account that  $\alpha$  is small and  $a(R), b(R)$  are positive. In Fig. 1 we show the phase-space diagram of that equation for a Nambu domain wall and a domain wall with curvature

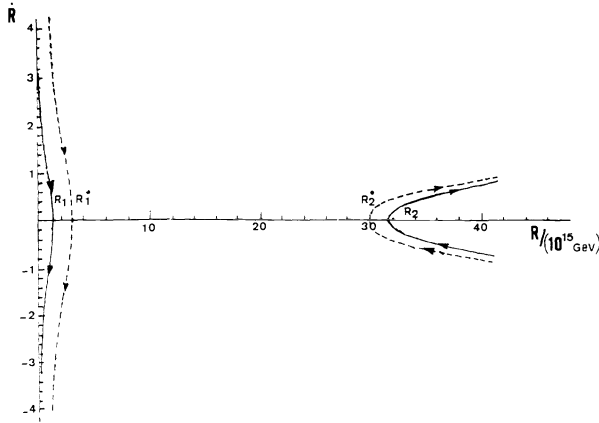


FIG. 1. Two-dimensional phase-space diagram of the equation describing the dynamics of a Nambu bubble [dashed line,  $\alpha=0$ ], and a bubble with curvature corrections [continuous line,  $\alpha \approx 10^{-28} \text{ GeV}^{-2}$ ] having a mass lower than a critical mass  $M_{\text{crit}}$ .

corrections, respectively. For  $M < M_{\text{crit}}$ , which is the case under consideration, we found *bounded*, as well as *bounce* solutions. Bounded solutions are the ones for which  $R$  starts at zero, grows to the maximum value  $R_1^*(R_1)$ , and then returns to zero. On the other hand,

bounce solutions are those for which  $R$  approaches infinity in the asymptotic past, falls to the minimum value given by  $R_2^*(R_2)$ , and then approaches infinity in the asymptotic future. This analysis holds for both a Nambu bubble, as well as a bubble with curvature corrections, as one can easily verify looking at Fig. 1. We thus agree with the analysis performed by Blau *et al.* [6] for a Nambu bubble, while we believe that our analysis is rather simpler. The critical mass  $M_{\text{crit}}$  is defined as the Schwarzschild mass, for which the phase-space diagram passes from the fixed points of the two-dimensional dynamical system describing the dynamics of the bubble. The numerical value of  $M_{\text{crit}}$ , for a Nambu domain wall ( $\alpha=0$ ), is found by demanding  $\mathcal{F}(R)=0$  and  $d\mathcal{F}(R)/dR=0$ , and it is  $M_{\text{crit}} \sim 10^{28} \text{ GeV}$ . The numerical value of the critical mass for a domain wall with curvature corrections is of the same order of magnitude. As one can see from Fig. 1, the effect of the curvature corrections on the bubble dynamics is to decrease  $R_1^*$  and increase  $R_2^*$ . This could be interpreted as a global dragging effect caused by the increase of the surface-energy density and the modification of the tension.

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