

This file goes through the steps of Theorem 2 in the manuscript "Small Non-Negative Stencils for Anisotropic Diffusion". It includes the final step of the proof, not presented in the manuscript, which requires to solve a 21x21 linear system.

In[85]:= **ProvePositivity["n"]**

Proving the 3d non-negative matrix decomposition in case n

The matrix is
$$\begin{pmatrix} u^2 & uv & uw \\ uv & v^2 & vw \\ uw & vw & w^2 \end{pmatrix}$$

The comatrix is
$$\begin{pmatrix} -vw^2 + v^2 w^2 & uv vw - uv w^2 & -uw v^2 + uv vw \\ uv vw - uv w^2 & -uw^2 + u^2 w^2 & uv uw - u^2 vw \\ -uw v^2 + uv vw & uv uw - u^2 vw & -uv^2 + u^2 v^2 \end{pmatrix}$$

We assumed, up to a linear change of variables, that $u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $v = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $w = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

The six first elements of the stencil are
$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{pmatrix}$$

The basis of the space of 3x3 symmetric matrices is

$$\left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \right\}$$

The vector α (coefficients of the comatrix in the basis) is, in

terms of the coefficients of the comatrix
$$\begin{pmatrix} m[1, 1] + m[1, 2] + m[3, 1] \\ m[1, 2] + m[2, 2] - m[2, 3] \\ -m[2, 3] + m[3, 1] + m[3, 3] \\ -m[1, 2] \\ -m[3, 1] \\ m[2, 3] \end{pmatrix}$$

The vector α is, in terms of the coefficients of the matrix

$$\begin{pmatrix} -uw v^2 + uv vw + uw vw - vw^2 - uv w^2 + v^2 w^2 \\ -uv uw - uw^2 + u^2 vw + uv vw + u^2 w^2 - uv w^2 \\ -uv^2 - uv uw + u^2 v^2 - uw v^2 + u^2 vw + uv vw \\ -uw vw + uv w^2 \\ uv v^2 - uv vw \\ uv uw - u^2 vw \end{pmatrix}$$

The seventh and last element of the stencil is

$$\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$
, which defines the matrix
$$\begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$

The vector β (coefficients of the last matrix in the basis) is
$$\begin{pmatrix} -1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

The following linear forms (in the variables $uv, uw, vw, u^2, v^2, w^2$) take non-negative values by construction:

$$\begin{pmatrix} -uv - vw \\ uv - uw \\ uw \\ u^2 - 2uv \\ v^2 + 2vw \\ 2vw + w^2 \end{pmatrix}$$

The following 21 quadratic forms (in the variables $uv, uw, vw, u^2, v^2, w^2$) are obtained as products of the above non-negative linear forms:

$$\begin{pmatrix} uv^2 + 2uvvw + vw^2 \\ -uv^2 + uvuw - uvvw + uwvw \\ uv^2 - 2uvuw + uw^2 \\ -uvuw - uwvw \\ uvuw - uw^2 \\ uw^2 \\ -u^2uv + 2uv^2 - u^2vw + 2uvvw \\ u^2uv - 2uv^2 - u^2uw + 2uvuw \\ u^2uw - 2uvuw \\ u^2^2 - 4u^2uv + 4uv^2 \\ -uvv^2 - 2uvvw - v^2vw - 2vw^2 \\ uvv^2 - uvv^2 + 2uvvw - 2uvvw \\ uvv^2 + 2uvvw \\ u^2v^2 - 2uvv^2 + 2u^2vw - 4uvvw \\ v^2^2 + 4v^2vw + 4vw^2 \\ -2uvvw - 2vw^2 - uvw^2 - vw^2 \\ 2uvvw - 2uvvw + uvw^2 - uvw^2 \\ 2uvvw + uvw^2 \\ 2u^2vw - 4uvvw + u^2w^2 - 2uvw^2 \\ 2v^2vw + 4vw^2 + v^2w^2 + 2vw^2 \\ 4vw^2 + 4vw^2 + w^2^2 \end{pmatrix}$$

Our objective is to prove that the following 12 quadratic forms (in the variables $uv, uw, vw, u^2, v^2, w^2$) take non-negative values:

$$\begin{pmatrix} -uvvw + uvw^2 \\ -uvv^2 + uvvw - vw^2 + v^2w^2 \\ -uvuw - uw^2 + u^2vw + u^2w^2 \\ -uv^2 - uvuw + u^2v^2 - uvv^2 + u^2vw + uvvw - uvvw + uvw^2 \\ uvv^2 - uvvw \\ uvvw - vw^2 - uvw^2 + v^2w^2 \\ -uvuw - uw^2 + uvv^2 + u^2vw - uvvw + uvvw + u^2w^2 - uvw^2 \\ -uv^2 - uvuw + u^2v^2 + u^2vw \\ uvuw - u^2vw \\ uvuw - uvv^2 - u^2vw + uvvw + uvvw - vw^2 - uvw^2 + v^2w^2 \\ -uw^2 + uvvw + u^2w^2 - uvw^2 \\ -uv^2 + u^2v^2 - uvv^2 + uvvw \end{pmatrix}$$

A basis of the vector space of quadratic forms (in the variables $uv, uw, vw, u^2, v^2, w^2$):

Determinant of the first matrix is 1

The 12 quadratic forms of interest can be expressed as linear sums of the 21 first, with the following coefficients

$$\begin{pmatrix} 0 & 2 & 2 & 3 & 5 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 3 & 5 & 2 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 0 & 0 & 2 & 2 & 2 & 0 & 1 & 0 \\ 0 & 2 & 2 & 2 & 3 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 2 & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 & 4 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 4 & 1 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 2 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 2 & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 2 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 2 & 5 & 3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 5 & 2 & 2 & 2 & 0 & 1 & 1 & 1 & 0 & 2 & 2 & 1 & 0 & 0 & 2 & 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & 2 & 1 & 3 & 0 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 2 & 0 & 2 & 2 & 2 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

All these coefficients are non-negative. They take the values $\{0, 1, 2, 3, 4, 5\}$

In[86]= **ProvePositivity["p"]**

Proving the 3d non-negative matrix decomposition in case p

The matrix is $\begin{pmatrix} u^2 & uv & uw \\ uv & v^2 & vw \\ uw & vw & w^2 \end{pmatrix}$

The comatrix is $\begin{pmatrix} -vw^2 + v^2 w^2 & uv vw - uv w^2 & -uw v^2 + uv vw \\ uv vw - uv w^2 & -uw^2 + u^2 w^2 & uv uw - u^2 vw \\ -uw v^2 + uv vw & uv uw - u^2 vw & -uv^2 + u^2 v^2 \end{pmatrix}$

We assumed, up to a linear change of variables, that $u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $v = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $w = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

The six first elements of the stencil are $\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{pmatrix}$

The basis of the space of 3x3 symmetric matrices is

$$\left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \right\}$$

The vector α (coefficients of the comatrix in the basis) is, in

$$\text{terms of the coefficients of the comatrix} \begin{pmatrix} m[1, 1] + m[1, 2] + m[3, 1] \\ m[1, 2] + m[2, 2] + m[2, 3] \\ m[2, 3] + m[3, 1] + m[3, 3] \\ -m[1, 2] \\ -m[3, 1] \\ -m[2, 3] \end{pmatrix}$$

The vector α is, in terms of the coefficients of the matrix

$$\begin{pmatrix} -uw v^2 + uv vw + uv vw - vw^2 - uv w^2 + v^2 w^2 \\ uv uw - uw^2 - u^2 vw + uv vw + u^2 w^2 - uv w^2 \\ -uv^2 + uv uw + u^2 v^2 - uw v^2 - u^2 vw + uv vw \\ -uw vw + uv w^2 \\ uv v^2 - uv vw \\ -uv uw + u^2 vw \end{pmatrix}$$

The seventh and last element of the stencil is

$$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \text{ which defines the matrix } \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

The vector β (coefficients of the last matrix in the basis) is
$$\begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$$

The following linear forms (in the variables $uv, uw, vw, u^2, v^2, w^2$) take non-negative values by construction:

$$\begin{pmatrix} -uv + vw \\ uv - uw \\ uw \\ u^2 - 2uv \\ v^2 - 2vw \\ -2vw + w^2 \end{pmatrix}$$

The following 21 quadratic forms (in the variables $uv, uw, vw, u^2, v^2, w^2$) are obtained as products of the above non-negative linear forms:

$$\begin{pmatrix} uv^2 - 2uvvw + vw^2 \\ -uv^2 + uvuw + uvvw - uwvw \\ uv^2 - 2uvuw + uw^2 \\ -uvuw + uvvw \\ uvuw - uw^2 \\ uw^2 \\ -u^2uv + 2uv^2 + u^2vw - 2uvvw \\ u^2uv - 2uv^2 - u^2uw + 2uvuw \\ u^2uw - 2uvuw \\ u^2^2 - 4u^2uv + 4uv^2 \\ -uvv^2 + 2uvvw + v^2vw - 2vw^2 \\ uvv^2 - uvv^2 - 2uvvw + 2uvvw \\ uvv^2 - 2uvvw \\ u^2v^2 - 2uvv^2 - 2u^2vw + 4uvvw \\ v^2^2 - 4v^2vw + 4vw^2 \\ 2uvvw - 2vw^2 - uvw^2 + vw^2 \\ -2uvvw + 2uvvw + uvw^2 - uvw^2 \\ -2uvvw + uvw^2 \\ -2u^2vw + 4uvvw + u^2w^2 - 2uvw^2 \\ -2v^2vw + 4vw^2 + v^2w^2 - 2vw^2 \\ 4vw^2 - 4vw^2 + w^2^2 \end{pmatrix}$$

Our objective is to prove that the following 12 quadratic forms (in the variables $uv, uw, vw, u^2, v^2, w^2$) take non-negative values:

$$\begin{pmatrix} -uvv^2 + uvvw + uvvw - vw^2 - uvw^2 + v^2w^2 \\ uvuw - uw^2 - uvv^2 - u^2vw + uvvw + 2uvvw - vw^2 + u^2w^2 - 2uvw^2 + v^2w^2 \\ -uv^2 + uvuw + u^2v^2 - uvv^2 - u^2vw + uvvw \\ -uv^2 + 2uvuw - uw^2 + u^2v^2 - uvv^2 - 2u^2vw + uvvw + uvvw + u^2w^2 - uvw^2 \\ -uvvw + uvw^2 \\ uvuw - uw^2 - u^2vw + u^2w^2 \\ uvvw - vw^2 - uvw^2 + v^2w^2 \\ -uv^2 + uvuw + u^2v^2 - u^2vw \\ uvv^2 - uvvw - uvvw + uvw^2 \\ -uvuw + u^2vw \\ -uw^2 + uvvw + u^2w^2 - uvw^2 \\ -uvuw + uvv^2 + u^2vw - uvvw \end{pmatrix}$$

A basis of the vector space of quadratic forms (in the variables $uv, uw, vw, u^2, v^2, w^2$):

Determinant of the first matrix is -1

The 12 quadratic forms of interest can be expressed as linear sums of the 21 first, with the following coefficients

$$\begin{pmatrix} 3 & 5 & 2 & 4 & 3 & 1 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 0 & 0 & 2 & 1 & 1 & 0 & 1 & 0 \\ 3 & 5 & 2 & 5 & 5 & 2 & 1 & 1 & 1 & 0 & 2 & 2 & 1 & 0 & 0 & 2 & 2 & 2 & 1 & 1 & 0 \\ 0 & 3 & 2 & 1 & 3 & 1 & 1 & 1 & 1 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 2 & 2 & 5 & 2 & 2 & 2 & 2 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 1 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 2 & 2 & 5 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 0 & 0 \\ 3 & 4 & 1 & 5 & 3 & 2 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 2 & 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 2 & 3 & 2 & 1 & 1 & 1 & 0 & 0 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 2 & 3 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 3 & 5 & 2 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 & 3 & 2 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

All these coefficients are non-negative. They take the values {0, 1, 2, 3, 4, 5}

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In[83]= ConstructAlphaBeta[case_] := Block[{
  M = {{u2, uv, uw}, {uv, v2, vw}, {uw, vw, w2}},
  coM,
  u, v, w,
  VFirst, VLast,
  Vec3toVec6 = {#1^2, #2^2, #3^2, #1 * #2, #2 * #3, #3 * #1} &,
  Sym3toVec6 =
    {#[[1, 1]], #[[2, 2]], #[[3, 3]], #[[1, 2]], #[[2, 3]], #[[3, 1]]} &,
  syms, alpha, beta
},
Print["The matrix is ", M // MatrixForm];
coM = Det[M] * Inverse[M];
Print["The comatrix is ", coM // MatrixForm];
{u, v, w} = IdentityMatrix[3];
VFirst =
  If[case == "n", {u, v, w, u - v, u - w, v + w}, {u, v, w, u - v, u - w, v - w}];
Print["We assumed, up to a linear change of variables, that u=",
  u // MatrixForm, ", v=", v // MatrixForm, ", w=", w // MatrixForm];
Print["The six first elements of the stencil are ",
  Transpose[VFirst] // MatrixForm];
Print["The basis of the space of 3x3 symmetric matrices is ",
  (Transpose[{-#}].{#} // MatrixForm) &/@VFirst];

syms = Transpose[Vec3toVec6@@@VFirst];
alpha = Inverse[syms].Sym3toVec6[Table[m[i, j], {i, 1, 3}, {j, 1, 3}]];
Print[
  "The vector  $\alpha$  (coefficients of the comatrix in the basis) is, in terms
  of the coefficients of the comatrix ", alpha // MatrixForm];

alpha = Inverse[syms].Sym3toVec6[coM];
Print["The vector  $\alpha$  is, in terms of the coefficients of the matrix ",
  alpha // MatrixForm];

VLast = If[case == "n", u - v - w, u - v + w];
Print["The seventh and last element of the stencil is ",
  VLast // MatrixForm, ", which defines the matrix ",
  Transpose[{VLast}].{VLast} // MatrixForm];
beta = Inverse[syms].Vec3toVec6@@VLast;
Print["The vector  $\beta$  (coefficients of the last matrix in the basis) is",
  beta // MatrixForm];
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Return[{alpha, beta}];
];

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ProvePositivity[case_] := Block[{
  (*Monomials of the first and second degree*)
  Linear = {u2, v2, w2, uv, uw, vw},
  Quadratic,
  (*Linear quantities that are known to be non-negative*)
  NonNegativeLinear,
  (*Quadratic quantities that are known to be non-negative*)
  NonNegativeQuadratic,
  (*Objective*)
  ConjecturedNonNegativeQuadratic,
  alpha,
  beta,

  (*In matrix form*)
  NonNegativeQuadraticMatrix,
  ConjecturedNonNegativeQuadraticMatrix,
  Solution
},

Print["Proving the 3d non-negative matrix decomposition in case ", case];

{alpha, beta} = ConstructAlphaBeta[case];
NonNegativeLinear = If[case == "n",
  {-vw - uv, uv - uw, uw, u2 - 2 uv, v2 + 2 vw, w2 + 2 vw},
  {vw - uv, uv - uw, uw, u2 - 2 uv, v2 - 2 vw, w2 - 2 vw}];

Print[
  "The following linear forms (in the variables uv,uw,vw,u2,v2,w2) take
  non-negative values by construction:";
Print[ NonNegativeLinear // MatrixForm];

(*Non negative quadratic quantities that can be obtained
  by multiplying two given non-negative linear quantities*)
NonNegativeQuadratic = Table[
  Expand[NonNegativeLinear[[i]] * NonNegativeLinear[[j]]],
  {i, 1, Length[NonNegativeLinear]}, {j, 1, i}];
NonNegativeQuadratic = Join@@ NonNegativeQuadratic;

Print[
  "The following 21 quadratic forms (in the variables uv,uw,vw,u2,v2,w2)
  are obtained as products of the above
  non-negative linear forms:";
Print[ NonNegativeQuadratic // MatrixForm];

(*All quadratic quantities that are conjectured to be non-negative*)
ConjecturedNonNegativeQuadratic = {};
Do[
  If[beta[[i]] == 1, AppendTo[ConjecturedNonNegativeQuadratic, alpha[[i]]]];

Do[
  If[beta[[i]] * beta[[j]] == -1,
    AppendTo[ConjecturedNonNegativeQuadratic, alpha[[i]] + alpha[[j]]],
    {j, 1, i}];

```



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, {i, 1, Length[beta]}}];

Print[
  "Our objective is to prove that the following 12 quadratic forms (in the
    variables uv,uw,vw,u2,v2,w2) take non-negative values:"];
Print[ConjecturedNonNegativeQuadratic // MatrixForm];

(*All the Quadratic monomials*)
Quadratic = Table[Linear[[i]] Linear[[j]], {i, 1, Length[Linear]}, {j, 1, i}];
Quadratic = Join@@Quadratic;
Print["A basis of the vector space of
  quadratic forms (in the variables uv,uw,vw,u2,v2,w2):"];
Print[Quadratic // MatrixForm];

(*Now we want to express the ConjecturedNonNegativeQuadratic quantities,
as linear sums with non-negative coefficients of the
  given NonNegativeQuadraticQuantities. For that purpose,
we write all these in matrix form.*)

NonNegativeQuadraticMatrix = Table[
  Coefficient[NonNegativeQuadratic[[i]], Quadratic[[j]]],
  {i, 1, Length[NonNegativeQuadratic]},
  {j, 1, Length[Quadratic]}
];

Print["The matrix of the 21 first
  quadratic forms, known to take non-negative values:"];
Print[NonNegativeQuadraticMatrix // MatrixForm];

ConjecturedNonNegativeQuadraticMatrix = Table[
  Coefficient[ConjecturedNonNegativeQuadratic[[i]], Quadratic[[j]]],
  {i, 1, Length[ConjecturedNonNegativeQuadratic]},
  {j, 1, Length[Quadratic]}
];

Print["The matrix of the 12 quadratic forms of interest ",
  ConjecturedNonNegativeQuadraticMatrix // MatrixForm];

(*The conjecture holds if it is possible to express
the lines of the second matrix, as linear sums with non-
negative coefficients of those of the first matrix*)

Print["Dimensions of the first matrix are ",
  Dimensions[NonNegativeQuadraticMatrix]];
Print["Determinant of the first matrix is ",
  Det[NonNegativeQuadraticMatrix]];
Solution = ConjecturedNonNegativeQuadraticMatrix.
  Inverse[NonNegativeQuadraticMatrix];
Print["The 12 quadratic forms of interest can be expressed as linear
  sums of the 21 first, with the following coefficients "];
Print[Solution // MatrixForm];
Print["All these coefficients are non-negative. They take the values ",
  Union@@Solution];

(*Conjecture holds*)
];

```