A Hybrid Positioning Method Based on Hypothesis Testing

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Abstract—We consider positioning in the scenario where only two reliable range estimates, and few less reliable power observations are available. Such situations are difficult to handle with numerical maximum likelihood methods which require a very accurate initialization to avoid being stuck into local maxima. We propose to first estimate the support region of the two peaks of the likelihood function using a set membership method, and then decide between the two regions using a rule based on the less reliable observations. Monte Carlo simulations show that the performance of the proposed method in terms of outlier rate and root mean squared error approaches that of maximum likelihood when only few additional power observations are available.

Index Terms—Location estimation, decision theory, estimation theory, time of arrival, received signal strength, set membership methods, interval analysis.

I. INTRODUCTION

WE consider the scenario where only two reliable range estimates and few less reliable observations are available. This situation occurs when only few links can provide time-of-arrival range estimates, but where a number of power observations can be achieved from elsewhere. In such scenarios, range-based positioning generally offers high accuracy, especially if considering ultra wideband observations. Power measurements in comparison, generally lead to inaccurate estimated ranges due to the log-normal relation between power and distance. The hybrid position estimator should thus fuse heterogeneous observations of very different accuracies. However, as reported in [1], this fusion is non-trivial for some hybrid positioning algorithm. Indeed, the introduction of additional, but less informative, power observations when a few accurate range observations are already available, may in fact lower the positioning accuracy. Thus, to take full advantage of the power information, suitable hybrid positioning algorithms are needed.

For the particular problem at hand, when considering only two reliable range estimates, the likelihood function can be dominated by two narrow peaks at the exact same height, leading to an ambiguous maximum likelihood (ML) estimator. This ambiguity is addressed in analysis of flip ambiguity problems, see e.g. in [2]. Introducing extra power observations largely changes the heights, but not the widths, of these peaks. Albeit the ML estimation is in this case unambiguous, the required global numerical optimization of a likelihood

function with multiple narrow peaks renders the ML too computationally demanding for many practical applications. The use of a local optimizer can be considered, but this optimization requires a good initialization to avoid to be trapped in a local maximum.

An alternative approach is to solve the problem in two steps: First estimate the support regions of the peaks of the likelihood function, then calculate the final position estimate. The problem of estimating the regions can be solved among others by set-membership methods [3], [4]. In a set-membership method, each observation defines a subset in space of possible position estimates. The support region is then be computed by intersecting these subsets by using, e.g. RSIVIA algorithm [5]. In a heterogeneous positioning problem, as considered here, the high accuracy observations lead to small subsets, whereas the less informative observations lead to larger subsets, which include the smaller subsets. Thus, the support regions estimated by intersection of the small subsets alone do not shrink further by intersecting with the larger subsets. Consequently, in the problem at hand, the introduction of low accuracy information, such as the power observations, neither improves nor degrades the performance of the algorithm. In the case where only two range estimates are available, the set membership methods return one or two disjoint subsets corresponding to the supports of the peaks of the likelihood function. If the algorithm returns two subsets, a positioning ambiguity arises.

In the present contribution, we propose a method to obtain the final position estimate with the use of the two support regions returned by a set membership approach. We formulate the selection of the two disjoint subsets as a standard hypothesis test based on the less informative power observations. The final position estimate is afterward obtained as the centroid of the chosen subset. Simulations show that the performance of the proposed method are close to that of the ML estimator.

II. POSITION ESTIMATION BASED ON A DECISION CRITERION

A. Description of Scenario

The considered scenario is illustrated in Fig. 1. The position B of a blind node is estimated from the two noisy range observations r_1, r_2 provided by the range nodes at known positions R_1 and R_2,

$$r_i = \|R_i - B\| + \delta_i, \qquad i = 1, 2, \qquad (1)$$

where δ_i is the error in the range estimate. Given a probability model for δ_i, it is possible to determine a confidence interval for the range estimate r_i, which as shown in Fig. 1, yields a confidence region shaped as an annulus with center R_i. The ambiguity problem occurs when the intersection of two
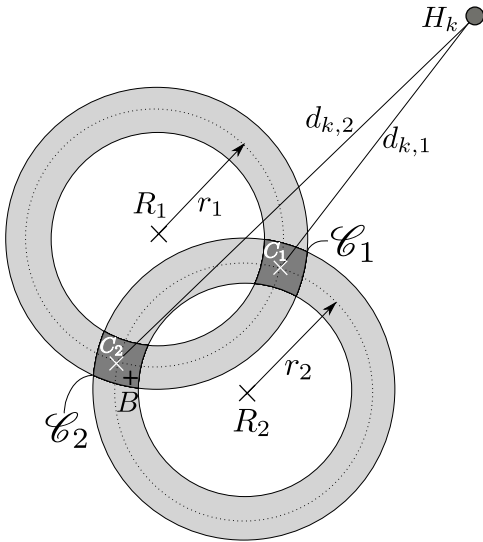
Fig. 1: The blind node at position B receives two range estimates r_1 and r_2 from ranging nodes at position R_1 and R_2 respectively. Errors in range estimates are modeled by two confidence annuli centered at R_1 and R_2, respectively. The intersection of annuli are the two shaded regions $\mathscr{C}_1$ and $\mathscr{C}_2$, with their centroids C_1 and C_2 respectively. In addition, the blind node achieves a log power observation from each helping node at position H_k. The distance from H_k to C_i is denoted by $d_{k,i}$.

confidence annuli splits into two disconnected subsets $\mathscr{C}_1$ and $\mathscr{C}_2$ with centroids C_1 and C_2, respectively. The two subsets and their centroids can be obtained using, e.g the algorithm proposed in [5].

To provide an unambiguous position estimate, additional information is required. We assume that additional observations $\{P_k\}$ of the log power are available from the helping nodes at positions $\{H_k\}$. We model the log power observation P_k as a distance dependent mean $\mu(d_k)$ distorted by an additive error term X_k, i.e.,

$$P_k = \mu(d_k) + X_k, \tag{2}$$

with $d_k = \|B - H_k\|$. We assume the joint probability density function (pdf) of $X_1, \ldots, X_K$ to be known. We gather the log power observations into a vector $\mathbf{P} = [P_1, \ldots, P_K]$. Practically, the log power information can be obtained from the received signal strength indicators.

B. Proposed Decision Rule

We approximate the conditional pdf for the power observation conditioned by the position of the blind node as:

$$f_{\mathbf{P}|B,r_1,r_2}(\mathbf{p}) \approx \begin{cases} f_{\mathbf{P}|\mathscr{C}_1,r_1,r_2}(\mathbf{p}), & B \in \mathscr{C}_1 \\ f_{\mathbf{P}|\mathscr{C}_2,r_1,r_2}(\mathbf{p}), & B \in \mathscr{C}_2 \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

with $\mathbf{p} \in \mathbb{R}^K$. In (3) we neglect the probability of the event $B \notin \mathscr{C}_1 \cup \mathscr{C}_2$. This approximation is valid by appropriately choosing the regions $\mathscr{C}_1$ or $\mathscr{C}_2$. Since B is unknown, $f_{\mathbf{P}|B,r_1,r_2}(\mathbf{p})$ cannot be computed. However, if accurate range estimates are available,

TABLE I: Parameter Settings

Parameter	Value	
$\mathcal{S}$	$[-20, 20] \times [-20, 20]$	m^2
$\mathcal{L}$	$[-80, 80] \times [-80, 80]$	m^2
σ_δ	0.9	m
σ_X	4	dB
n_p	3	
P_0	-40	dB

then B can be approximated by C_1 if $B \in \mathscr{C}_1$ or by C_2 if $B \in \mathscr{C}_2$, and thus:

$$d_k \approx \begin{cases} d_{k,1} = \|C_1 - H_k\|, & B \in \mathscr{C}_1 \\ d_{k,2} = \|C_2 - H_k\|, & B \in \mathscr{C}_2. \end{cases} \tag{4}$$

With the above approximations, the solution of the ambiguity problem can be phrased as a classical decision problem where λ is the likelihood ratio. The decision threshold γ can be defined to account with a priori information or costs [6]:

$$\lambda = \frac{f_{\mathbf{P}|\mathscr{C}_1,r_1,r_2}(\mathbf{p})}{f_{\mathbf{P}|\mathscr{C}_2,r_1,r_2}(\mathbf{p})} \underset{\mathscr{C}_1}{\overset{\mathscr{C}_2}{\gtrless}} \gamma. \tag{5}$$

The ML decision rule is obtained for $\gamma = 1$.

C. Special Case: Uncorrelated Gaussian Log Power Errors

In the special case where $X_1, \ldots, X_K$ are independent Gaussian random variables with zero mean and variances $\sigma_1^2, \ldots, \sigma_K^2$, yields for B in $\mathscr{C}_i$:

$$f_{\mathbf{P}|\mathscr{C}_i,r_1,r_2}(\mathbf{p}) = \prod_{k=1}^{K} \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{(p_k - \mu_{k,i})^2}{2\sigma_k^2}\right), i = 1, 2, \tag{6}$$

with $\mu_{k,i} = \mu(d_{k,i})$. Hence, the log likelihood ratio $\Lambda = \ln \lambda$ reads:

$$\Lambda = \sum_{k=1}^{K} \left[\frac{(p_k - \mu_{k,2})^2}{2\sigma_k^2} - \frac{(p_k - \mu_{k,1})^2}{2\sigma_k^2} \right] \tag{7}$$

We obtain the ML decision rule upon insertion of (7) into (5) with $\gamma = 1$:

$$\sum_{k=1}^{K} \frac{1}{2\sigma_k^2} \left[\mu_{k,2}^2 - \mu_{k,1}^2 \right] \underset{\mathscr{C}_2}{\overset{\mathscr{C}_1}{\gtrless}} \sum_{k=1}^{K} \frac{1}{\sigma_k} P_k(\mu_{k,1} - \mu_{k,2}). \tag{8}$$

It can be observed that for fixed centroids C_1 and C_2, the left hand terms are constants, while the right hand terms are a Gaussian random variable. Thus, the computation of error probability is well-known [6].

III. NUMERICAL ANALYSIS OF THE PROPOSED METHOD

In this section the performance of the proposed method is compared to ML approaches via Monte Carlo simulations of the scenario described in Subsection III-A. We consider a true ML estimator relying on global optimization and an ML approximation (ML-WLS) in which a local optimizer is initialized with a weighted least squares solution [7], both introduced in Subsection III-B.
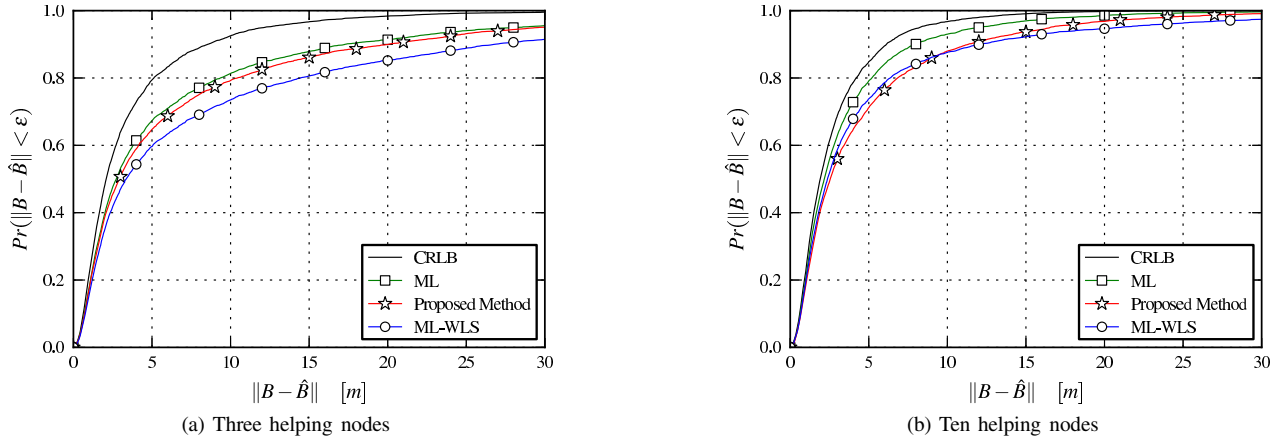
(a) Three helping nodes

(b) Ten helping nodes

Fig. 2: CDF comparison of the absolute positioning error between the Cramer Rao Lower Bound (CRLB) [10], the ML estimator (ML), the iterative ML estimator initialized with a weighted least squares solution (ML-WLS) and the proposed method with: 2(a) three helping nodes nodes, and 2(b) ten helping nodes.

A. Simulations Scenario

The performance of the proposed solution is assessed via Monte Carlo simulations. We consider the special case described in Subsection II-C for the setup given in Fig. 1 with the parameters settings as in Table I. We draw the positions B, R_1, R_2 independently and uniformly on the area $\mathcal{S}$. The positions $\{H_k\}$ of the helping nodes are independently drawn according to an uniform distribution on the larger area $\mathcal{L}$. The range errors δ_1 and δ_2 assumed to be independent zero mean Gaussian random variables with variance $\sigma_{\delta_i}^2 = \sigma_\delta^2$. The mean of the received log power is modeled according to the standard path loss model $\mu(d_k)$:

$$\mu(d_k) = P_0 - 10 n_p \log_{10}(d_k), \qquad (9)$$

where P_0 is the power received at 1 meter and n_p is the path loss exponent. The variance σ_k^2 is chosen equal to σ_X^2 for all k. Values for P_0, n_p, σ_δ^2 and σ_X^2 are chosen according to the measurements reported in [8].

B. Maximum Likelihood Estimation

The ML estimator for the hybrid positioning problem reads $B \in \arg\max_z \Lambda_{\text{Hybrid}}(z)$, where $\Lambda_{\text{Hybrid}}(z)$ denotes the log likelihood function for B based on $\{r_i\}$ and $\mathbf{P}$. One approach is to find the maxima of the local extrema of the log likelihood function, obtained by equating the gradient to zero. For independent range estimates and power measurements, the gradient of the log likelihood function reads:

$$\nabla \Lambda_{\text{Hybrid}}(z) = \nabla \Lambda_{\text{Power}}(z) + \nabla \Lambda_{\text{Range}}(z), \qquad (10)$$

where Λ_{Power} and Λ_{Range} are the log likelihood functions of the power measurements and of the range estimates, respectively. The two gradients read [7], [9],

$$\nabla \Lambda_{\text{Power}}(z) = \sum_{k=1}^{K} \frac{1}{s^2} \frac{M_k - s^2 - \ln\|z - H_k\|}{d_k^2} (z - H_k),$$

$$\nabla \Lambda_{\text{Range}}(z) = \sum_{i=1}^{2} \frac{1}{\sigma_\tau^2} \frac{r_i - \|z - R_i\|}{r_i} (z - R_i), \qquad (11)$$

with the definitions

$$s = -\frac{\sigma_X \ln 10}{10 n_p}, \quad M_k = \frac{(P_0 - P_k)\ln 10}{10 n_p} + \ln d_0. \qquad (12)$$

Due to the non-linear relation (11), finding the roots of (10) requires global numerical optimization, which is not feasible for most applications. However, an approximate solution can be obtained by initializing a numerical local optimizer with an initial guess, e.g. a weighted least squares (ML-WLS) approach [7].

C. Comparison of Performance

We compare the performances of the three algorithms in term of cumulative density functions (CDF), outlier rates and root mean square errors (RMSE). From the empirical CDFs shown in Fig. 2 it appears that for a low number of helping nodes, the performance of the proposed method outperforms ML-WLS and is close to that of ML. For high number of helping nodes, the proposed method and ML-WLS has similar performances, except in a large errors regime, where the proposed method prevails. These large errors are observed to be less frequent when the number of helping nodes is high. To inspect this difference, we consider the occurrence of outliers. We define an outlier as follows: if $B \in \mathscr{C}_i$, the estimate of B is called an outlier if it lies in the complement of $\mathscr{C}_i$. Note that for the proposed method, an outlier is equivalent to a decision error in (8). On Fig. 3 we observe that the outlier rate decreases with the number of helping nodes increases. Not surprisingly, the ML estimator yields the lowest outlier rate of the three methods. It also appears that the proposed method consistently outperforms the ML-WLS in terms of outlier rate. This is most significant when the number of helping nodes is less than four. These differences of performance are also reflected in the RMSEs reported in Fig. 4. In particular, for four or less helping nodes we observe that proposed method is close to the ML curve, compared to ML-WLS curve.

The above observations suggest that the RMSE for the proposed method approach can be attributed to two types of errors: large errors outliers due to decision error in (8), and small errors resulting from the approximation in (4). The small
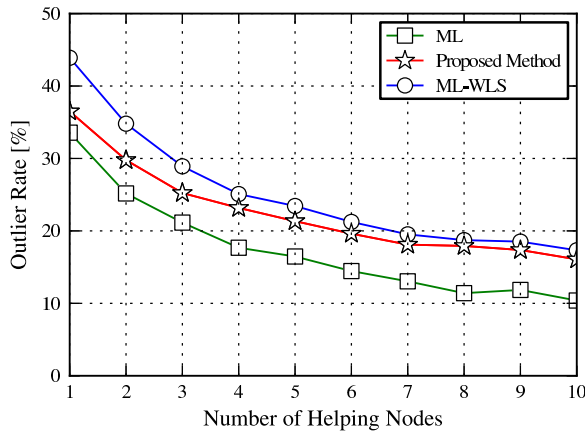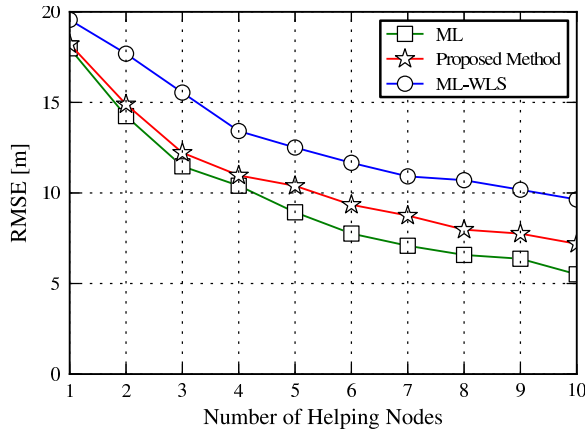
Fig. 3: Outlier rate vs. number of helping nodes.

Fig. 4: RMSE vs. number of helping nodes.

In such a situation, the ML estimator necessitates numerical global optimization of an objective function with local maxima located at narrow peaks. The proposed method relies on an approximate ML decision rule with the hypothesis corresponding to the blind node residing in the support regions of each of the peaks of the likelihood function. The decision rule is formed using the less informative power observations. Finally, the position estimate is computed as the centroid of the selected peak's support regions. Monte Carlo simulations show that the performance of the proposed method in terms of outlier rate and root mean squared error, in a realistic scenario, approaches that of ML. This is in particular the case when only few additional power observations are available, i.e. when the errors due to outliers dominate. Furthermore, the proposed method outperforms an alternative procedure where a least squares approach provides initialization for numerical optimization of the likelihood function. Further improvement of the accuracy of the proposed method could be achieved by refining the estimate of the local maxima of the likelihood function.

errors occur since the centroids C_1 or C_2 are used to estimate the position of the blind node. Thus, we conjecture that the effect of these small errors can be reduced by improving this approximation, considering additional knowledge of the probability model for the range error δ_i. This information could be included directly as a weighting function in the computation of centroids. Alternatively, the proposed method could be used to provide an initial guess for a numerical optimization of the likelihood function. We further conjecture that the outlier rate, which is equivalent to the rate of false decision in (8), could be also reduced by improving the approximation (4).

IV. CONCLUSIONS

The proposed method yields a position estimate for situations where only two reliable range estimates are available along with a number of less informative observables, e.g. information on the received log power from other nodes.

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