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Adaptive trajectory following control of a fixed-wing UAV in presence of crosswind

A. Brezoescu, T. Espinoza, P. Castillo and R. Lozano

Abstract—An adaptive backstepping approach to obtain directional control of a fixed-wing UAV in presence of unknown crosswind is developed in this paper. The dynamics of the cross track error with respect to a desired trajectory is derived from the lateral airplane equations of motion. Adaptation laws are proposed to estimate the parameters of the unknown disturbances and are employed in closed-loop system. The stability analysis is proved using Lyapunov theory. In addition, several simulations taking into account unknown wind gusts are performed to analyze the behavior and the robustness of the control scheme. A test platform has been developed in order to validate the proposed control law.

I. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) represent an area of great interest in the automatic control community. The absence of the pilot renders them best suited to solve dangerous situations. However, it requires significant attention in the flight control design since the vehicle may experience large parameter variations and external disturbances. The largest use of the UAVs is within military applications but they are also used in a growing number of civil applications such as firefighting, digital mapping or monitoring. To increase the usefulness of UAVs, the capability of the autonomous controller to track a reference path is essential. Moreover, the robustness with respect to environmental disturbances must be considered. For example, small UAVs are significantly sensitive to wind since its magnitude may be comparable to the UAVs speed.

A wide range of control approaches for trajectory tracking purpose could be found in literature for both underwater and aerial vehicles. The problem of trajectory tracking for an underwater vehicle was formulated as a gain scheduling control problem in [1] while Repoulias and Papadopoulos [2] employed a method based on feedback linearization, backstepping and nonlinear damping design tools. In gain scheduling control the system dynamics are considered slowly varying [3][4] and that reduces the flight capabilities of an airplane. Trajectory linearization control (TLC) was used in [5] to avoid the use of gain scheduling and to enable operation across the full flight-envelope for a 6DoF fixed-wing aircraft model. Nelson et al. proposed in [6] a method based on the vector field approach for the case where the time dimension of the reference trajectory is removed. The

algorithm was validated through simulations and real flight tests of a fixed-wing miniature air vehicle.

When accurate knowledge of the vehicle dynamics is not available, adaptive control design can be employed in order to estimate the unknown parameters. Many of the results in adaptive control are derived from Lyapunov stability theory [7]-[10]. Several flight control algorithms which combines adaptation with other control tools, such as backstepping, neural networks or sliding mode control, can be found in the literature. For instance in [11], flight control laws for two different control objectives were designed employing backstepping technique : maneuvering purpose and automatic control for the flight path angle. Also, two schemes based on adaptive backstepping and nonlinear observer design were proposed for estimating model errors. Likewise, a Lyapunov-based adaptive backstepping approach with online estimation of the uncertain aerodynamic forces and moments was used in [12] to design a flight-path controller for a nonlinear high-fidelity F-16 model. It was shown that trajectory control can still be accomplished with these uncertainties while good tracking performance is maintained. On the other hand, in [13] the authors introduced the design of an adaptive backstepping controller for longitudinal flight-path control when the aerodynamic coefficients are not known exactly. The system followed references in velocity and flight path angle and showed good performance in simulations.

Even if there are many adaptive approaches to flight control design, only few have been developed to realize airplane directional control in presence of unknown wind gusts. The goal of this work is to stabilize an airplane under crosswind and to realize the convergence to zero of the cross track error with respect to a desired trajectory. Moreover, the adaptive controller must be robust, by construction, with respect to external and unknown disturbances. We focus mainly in the lateral dynamic of the plane, for this, an analysis of this dynamic is presented in section II. Likewise, in this section we introduce the cross track error and the dynamic velocity of the plane with respect to the desired path. An adaptive control strategy is developed and presented in section III in order to follow the trajectory in presence of wind. Besides, the stability properties of the controller are discussed at the end of this section. The validation of the proposed control scheme is done in simulations and the main results are depicted in graphs in section IV. Additionally, a prototype of the airplane was developed to validate the control algorithm, the main characteristics of this prototype are described in section V. And finally in section VI, the conclusion and future work are discussed.

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II. AIRCRAFT MODEL DESCRIPTION

The dynamic characteristics of an airplane strongly depend on many parameters such as altitude, speed, configuration or environmental disturbances. As a result, its complete dynamic is nonlinear, coupled and complex for control purposes. In this study we focus mainly in the trajectory following problem of an airplane flying in level flight in presence of crosswind. This kind of flight occurs when lift equals weight, thrust equals drag and the airplane flies at constant velocity, and this implies that the roll and pitch angle are small. In addition, level flight implies zero flight path angle, γ , and positive angle of attack, α , see [14]. Therefore, using the relationship of the latter to pitch angle θ

$$\alpha = \theta - \gamma$$

it can be concluded that $\theta = \alpha$. Consequently, the airplane velocity, the angle of attack and, implicitly, the pitch angle vary slowly compared to the other parameters, and their time derivatives can be neglected in the flight dynamic.

In real conditions an airplane is generally exposed to crosswind. We define a crosswind like a wind which occurs perpendicular to the vehicle but parallel to the ground. If a plane is experiencing a crosswind, it will be pushed over or yawed away from the wind.

In order to derive the equations of motion two reference frames are used : the Earth-fixed frame, denoted by F_E , and the body-fixed frame represented by F_B . F_E and F_B have two dimensions since only the lateral dynamics are considered in this study. The origin of F_B coincides with the vehicle's center of mass and the direction of its axes is according to Fig. 1. F_E is employed as an inertial frame since the rotation of the Earth is neglected. Its origin is denoted by O_E while $O_E x_E$ is chosen northwards and $O_E y_E$ points east.

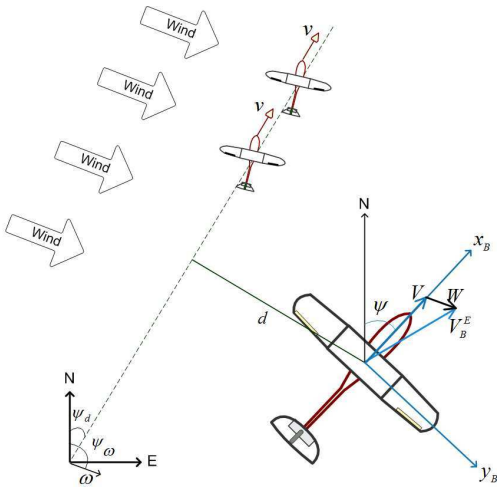


Fig. 1. Tracking formulation problem

The classical relation of the aircraft velocity relative to the Earth is

$$\bar{V} = V + W \quad (1)$$

where $V = [u \ v]^T$ denotes the velocity of the aircraft relative to the local atmosphere and W represents the wind relative to F_E . Besides, we only consider in this study a lateral wind having North, W_N , and East velocity components, W_E .

Denote $\bar{V}_B = [u^E \ v^E]^T$ as the velocity of the aircraft relative to the Earth in the directions of the body frame axes. Thus, from (1) it follows

$$\begin{bmatrix} u^E \\ v^E \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix} + B_B \begin{bmatrix} W_N \\ W_E \end{bmatrix}$$

where B_B defines the complete transformation from F_E to F_B assuming constant pitch angle and it is given by

$$B_B = \begin{pmatrix} c_\theta c_\psi & c_\theta s_\psi \\ s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi \end{pmatrix}$$

where s_θ and c_θ denote $\sin(\theta)$ and $\cos(\theta)$, respectively.

Then, the differential equations for the coordinates of the flight path in F_E are

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = B_B^T \bar{V}_B$$

or

$$\begin{aligned} \dot{x} &= u^E c_\theta c_\psi + v^E s_\phi s_\theta c_\psi - v^E c_\phi s_\psi \\ \dot{y} &= u^E c_\theta s_\psi + v^E s_\phi s_\theta s_\psi + v^E c_\phi c_\psi \end{aligned}$$

with

$$\begin{aligned} u^E &= u + W_N c_\theta c_\psi + W_E c_\theta s_\psi \\ v^E &= v + W_N s_\phi s_\theta c_\psi - W_N c_\phi s_\psi + W_E s_\phi s_\theta s_\psi \\ &\quad + W_E c_\phi c_\psi \end{aligned}$$

where x and y represent the inertial position in the x -axis (North) and in the y -axis (East).

Remember that the pitch and roll angles are small so that $\sin(\theta, \phi) \approx 0$ and $\cos(\theta, \phi) \approx 1$. Moreover, considering a symmetrical airplane with a rigid spinning rotor placed in the front of its body, it can then be considered, without loss of generality, V acting only in the x -axis, see Fig. 1. Hence, the following expression can be stated

$$\begin{aligned} v &\ll 1 \\ u &\approx V \end{aligned}$$

and consequently

$$\begin{aligned} \dot{x} &= V \cos \psi + \omega \cos \psi_\omega \\ \dot{y} &= V \sin \psi + \omega \sin \psi_\omega \end{aligned}$$

where $\omega \cos \psi_\omega = W_N$, $\omega \sin \psi_\omega = W_E$, ω is the wind velocity and ψ_ω describes the wind direction.

Notice that the above equations are relatively proportional to the variation of the yaw angle, and it can be controlled

using the rudder deflection of the airplane. The differential equations describing this dynamics are

$$\begin{aligned}\dot{\psi} &\approx r \\ \dot{r} &\approx c\tau_{\psi}\end{aligned}$$

where r stands for yaw rate, τ_{ψ} represents the yawing moment and c is a constant related to the aircraft moment of inertia.

An analysis of the nonlinear model is presented for two different flight conditions (with and without wind). Notice from Fig. 2 the behavior of the plane when it flies in stable or moving atmosphere. The desired trajectory is plotted in thick dashed line while the solid path describes the real airplane trajectory. The crosswind has North and East velocity components of $W_N = -3$ m/s and $W_E = 5$ m/s, respectively. The airplane velocity relative to the surrounding air mass is 20 m/s.

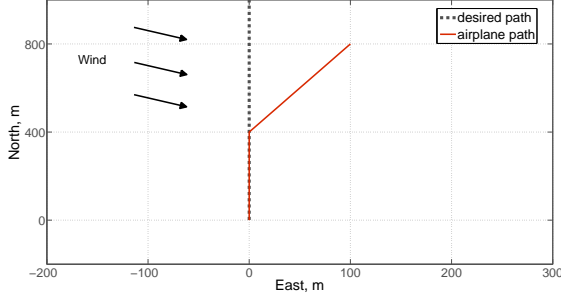


Fig. 2. Earth-Relative Aircraft Location. First, the plane flies in stable atmosphere and it is capable to follow the desired path. When the atmosphere moves relative to the Earth, the airplane diverge from the path.

III. CONTROL DESIGN

The main control objective is to obtain directional control in order to follow a desired trajectory even in presence of unknown crosswind. To simplify the analysis, let assume that the desired trajectory is aligned with the North axis of the reference frame, then, the desired path angle, ψ_d , is equal to zero. Therefore, the amount of the trajectory deviation will depend on the velocity of the airplane and wind and also on the angle of the wind in relation to the airplane. In addition we consider, for control design, that the wind velocity changes slowly such that it can be considered quasi-constant. However, it will be proved in simulations that the closed-loop system remains stable even with no constant wind.

Thus, without loss of generality, the airplane dynamics for trajectory following purpose can be defined as

$$\dot{d} \equiv \dot{y} = V \sin \psi + k_{\omega} \quad (2)$$

$$\dot{\psi} = r \quad (3)$$

$$\dot{r} = c\tau_{\psi} \quad (4)$$

where $k_{\omega} = \omega \sin(\psi_{\omega})$ is considered quasi-constant and due to the wind perturbation, and d is the cross track error from the desired trajectory.

To stabilize the system resulted in (2), (3) and (4), the control law will be constructed using the Adaptive Backstepping approach. Then, we define the following error variable

$$e_1 = d - d_{min} \quad (5)$$

where d_{min} is the minimum constant distance from the desired trajectory. Thus,

$$\dot{e}_1 = V \sin \psi + k_{\omega} \quad (6)$$

A. Convergence of e_1 to zero

Propose the following positive function

$$V_{L_1} = \frac{1}{2} e_1^2$$

thus

$$\dot{V}_{L_1} = e_1 (V \sin \psi + k_{\omega})$$

To stabilize e_1 we introduce ψ^v as a virtual control in the following form

$$V \sin \psi^v = -c_1 e_1 - \hat{k}_{\omega_1}$$

where \hat{k}_{ω_1} is the estimate of k_{ω} and $c_1 > 0$ is a constant. Evaluating \dot{V}_{L_1} when $\psi \rightarrow \psi^v$ it follows that

$$\dot{V}_{L_1}|_{\psi=\psi^v} = -c_1 e_1^2 + e_1 \tilde{k}_{\omega_1}$$

where $\tilde{k}_{\omega_1} = k_{\omega} - \hat{k}_{\omega_1}$. Notice from the above equation that if $\hat{k}_{\omega_1} \rightarrow k_{\omega}$ then $\dot{V}_{L_1} \leq 0$. Thus, rewriting V_{L_1} , it yields

$$V_{L_1} = \frac{1}{2} \left(e_1^2 + \frac{1}{\gamma_1} \tilde{k}_{\omega_1}^2 \right)$$

where $\gamma_1 > 0$ is a constant adaptation gain. Then

$$\dot{V}_{L_1}|_{\psi=\psi^v} = -c_1 e_1^2 + \left(e_1 - \frac{\dot{\hat{k}}_{\omega_1}}{\gamma_1} \right) \tilde{k}_{\omega_1}$$

Choosing the update law as

$$\dot{\hat{k}}_{\omega_1} = \gamma_1 e_1 \quad (7)$$

It follows that

$$\dot{V}_{L_1}|_{\psi=\psi^v} = -c_1 e_1^2$$

B. Convergence of ψ to ψ^v

Define the error

$$e_2 = V \sin \psi - V \sin \psi^v = V \sin \psi + c_1 e_1 + \hat{k}_{\omega_1} \quad (8)$$

and rewrite (6) in terms of e_1 and e_2

$$\dot{e}_1 = e_2 - c_1 e_1 + \tilde{k}_{\omega_1} \quad (9)$$

This implies that

$$\dot{e}_2 = Vr \cos \psi + (\gamma_1 - c_1^2) e_1 + c_1 e_2 + c_1 \tilde{k}_{\omega_1} \quad (10)$$

Notice that $\cos \psi = \sqrt{1 - (\sin \psi)^2}$. From (8)

$$\sin \psi = \frac{e_2 - c_1 e_1 - \hat{k}_{\omega_1}}{V}$$

and assuming that $-\frac{\pi}{2} < \psi < \frac{\pi}{2}$ it follows that (10) becomes

$$\dot{e}_2 = rR + (\gamma_1 - c_1^2) e_1 + c_1 e_2 + c_1 \tilde{k}_{\omega_1} \quad (11)$$

with $R = \sqrt{V^2 - (e_2 - c_1 e_1 - \hat{k}_{\omega_1})^2}$.

Introduce the following positive function

$$\begin{aligned} V_{L_2} &= V_{L_1} + \frac{1}{2}e_2^2 \\ &= \frac{1}{2} \left(e_1^2 + \frac{1}{\gamma_1} \tilde{k}_{\omega_1}^2 + e_2^2 \right) \end{aligned}$$

From (7), (9) and (11) the derivative reads

$$\dot{V}_{L_2} = -c_1 e_1^2 + e_2 \left[c_1 e_2 + e_1 (\gamma_1 + 1 - c_1^2) + c_1 \tilde{k}_{\omega_1} + rR \right]$$

By selecting the virtual control as

$$r^v R = -e_2(c_1 + c_2) - e_1(\gamma_1 + 1 - c_1^2) - c_1(\hat{k}_{\omega_2} + \hat{k}_{\omega_1})$$

\dot{V}_{L_2} becomes when $r \rightarrow r^v$

$$\dot{V}_{L_2}|_{r=r^v} = -c_1 e_1^2 - c_2 e_2^2 + c_1 e_2 \tilde{k}_{\omega_2}$$

where $\tilde{k}_{\omega_2} = k_{\omega} - \hat{k}_{\omega_2}$, \hat{k}_{ω_2} represents a new estimate for k_{ω} and c_2 denotes a positive constant gain. Notice that if we had employed the existing estimate \hat{k}_{ω_1} , we would have had no design freedom left to cancel the unknown parameter from \dot{V}_{L_2} . Additionally, \hat{k}_{ω_2} could be seen as a factor correction for \hat{k}_{ω_1} .

Notice from the above equation that if $\hat{k}_{\omega_2} \rightarrow k_{\omega}$ then $\dot{V}_{L_2} \leq 0$. Thus, rewriting V_{L_2} , it yields

$$V_{L_2} = V_{L_1} + \frac{1}{2} \left(e_2^2 + \frac{1}{\gamma_2} \tilde{k}_{\omega_2}^2 \right)$$

with $\gamma_2 > 0$ and constant. Hence \dot{V}_{L_2} becomes

$$\dot{V}_{L_2}|_{r=r^v} = c_1 e_1^2 - c_2 e_2^2 + \tilde{k}_{\omega_2} \left(c_1 e_2 - \frac{\dot{\hat{k}}_{\omega_2}}{\gamma_2} \right)$$

Proposing the update law

$$\dot{\hat{k}}_{\omega_2} = \gamma_2 c_1 e_2$$

then, it follows

$$\dot{V}_{L_2}|_{r=r^v} = c_1 e_1^2 - c_2 e_2^2$$

C. Convergence of r to r^v

Let us define the third error variable

$$\begin{aligned} e_3 &= rR - r^v R \\ &= rR + L_2 e_2 + L_1 e_1 + c_1(\hat{k}_{\omega_2} - \hat{k}_{\omega_1}) \end{aligned} \quad (12)$$

where $L_1 = 1 - c_1^2 + \gamma_1$, $L_2 = c_1 + c_2$. Rewriting the error system representation, we obtain

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} -c_1 & 1 \\ -1 & -c_2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} \tilde{k}_{\omega_1} \\ e_3 + c_1 \tilde{k}_{\omega_2} \end{bmatrix}$$

thus, the derivative of e_3 yields

$$\begin{aligned} \dot{e}_3 &= c\tau_{\psi} R - \frac{r(e_2 - c_1 e_1 - \hat{k}_{\omega_1})(e_3 - L_2 e_2 - L_1 e_1)}{R} \\ &\quad - \frac{r(e_2 - c_1 e_1 - \hat{k}_{\omega_1})(c_1 \hat{k}_{\omega_1} - c_1 \hat{k}_{\omega_2})}{R} \\ &\quad + L_2 e_3 + L_3 e_2 + L_4 e_1 + c_1 L_2 \tilde{k}_{\omega_2} + L_1 \tilde{k}_{\omega_1} \end{aligned}$$

with $L_3 = -c_1 c_2 - c_1^2 - c_2^2 + 1 + \gamma_1 + c_1^2 \gamma_2$ and $L_4 = -2c_1 - c_2 + c_1^3 - 2c_1 \gamma_1$.

Finally, introduce the following Lyapunov function

$$V_L = \frac{1}{2} \left(e_1^2 + \frac{1}{\gamma_1} \tilde{k}_{\omega_1}^2 + e_2^2 + \frac{1}{\gamma_2} \tilde{k}_{\omega_2}^2 + e_3^2 \right)$$

then

$$\dot{V}_L = -c_1 e_1^2 - c_2 e_2^2 + e_3 (\dot{e}_3 + e_2) \quad (13)$$

Propose the control input as

$$\begin{aligned} c\tau_{\psi} &= - \frac{e_3(L_2 + c_3) + e_2(L_3 + 1 - r^2) + e_1(L_4 + c_1 r^2)}{R} \\ &\quad - \frac{\hat{k}_{\omega_3}(L_1 + c_1 L_2) - \hat{k}_{\omega_2} c_1 L_2 - \hat{k}_{\omega_1}(L_1 - r^2)}{R} \end{aligned}$$

where $\tilde{k}_{\omega_3} = k_{\omega} - \hat{k}_{\omega_3}$ and c_3 is a positive constant gain. Notice that the unknown term k_{ω} appears again in \dot{V}_L , thus we propose a correction factor in order to realize the convergence of the states.

Introducing the above into (13), we have

$$\dot{V}_L = -c_1 e_1^2 - c_2 e_2^2 - c_3 e_3^2 + e_3 (L_1 + c_1 L_2) \tilde{k}_{\omega_3}$$

Observe that $\dot{V}_L \leq 0$ if $\hat{k}_{\omega_3} \rightarrow k_{\omega}$. Therefore augmenting V_L , it yields

$$V_L = \frac{1}{2} \left(e_1^2 + \frac{1}{\gamma_1} \tilde{k}_{\omega_1}^2 + e_2^2 + \frac{1}{\gamma_2} \tilde{k}_{\omega_2}^2 + e_3^2 + \frac{1}{\gamma_3} \tilde{k}_{\omega_3}^2 \right)$$

and

$$\dot{V}_L = -c_1 e_1^2 - c_2 e_2^2 - c_3 e_3^2 + \tilde{k}_{\omega_3} \left[e_3 (L_1 + c_1 L_2) - \frac{\dot{\hat{k}}_{\omega_3}}{\gamma_3} \right]$$

Choosing

$$\dot{\hat{k}}_{\omega_3} = \gamma_3 (L_1 + c_1 L_2) e_3$$

\dot{V}_L becomes

$$\dot{V}_L = -c_1 e_1^2 - c_2 e_2^2 - c_3 e_3^2 \quad (14)$$

The error representation of the closed-loop adaptive system is summarized below

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} -c_1 & 1 & 0 \\ -1 & -c_2 & 1 \\ 0 & -1 & -c_3 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + \begin{bmatrix} \tilde{k}_{\omega_1} \\ c_1 \tilde{k}_{\omega_2} \\ L_5 \tilde{k}_{\omega_3} \end{bmatrix}$$

$$\begin{bmatrix} \dot{\hat{k}}_{\omega_1} \\ \dot{\hat{k}}_{\omega_2} \\ \dot{\hat{k}}_{\omega_3} \end{bmatrix} = \begin{bmatrix} \gamma_1 & 0 & 0 \\ 0 & c_1 \gamma_2 & 0 \\ 0 & 0 & L_5 \gamma_3 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \quad (15)$$

where $L_5 = c_1 c_2 + \gamma_1 + 1$.

Rewriting the control input $c\tau_{\psi}$ in terms of d, ψ, r we have

$$\begin{aligned} c\tau_{\psi} &= \tan \psi (r^2 - L_6) - L_7 r \\ &\quad - \frac{L_8 d + L_9 \hat{k}_{\omega_1} + L_{10} \hat{k}_{\omega_2} + L_{11} \hat{k}_{\omega_3}}{V \cos \psi} \end{aligned} \quad (16)$$

with the updated parameters

$$\begin{aligned}\dot{\hat{k}}_{\omega_1} &= \gamma_1 d \\ \dot{\hat{k}}_{\omega_2} &= \gamma_2 c_1 \left(V \sin \psi + c_1 d + \hat{k}_{\omega_1} \right) \\ \dot{\hat{k}}_{\omega_3} &= \gamma_3 L_{11} V \left[r \cos \psi + L_2 \sin \psi \right] + \\ &\quad + \gamma_3 L_{11} \left[d L_{11} + c_1 \hat{k}_{\omega_2} + c_2 \hat{k}_{\omega_1} \right]\end{aligned}$$

where

$$\begin{aligned}L_6 &= 1 + L_2 c_3 + L_2^2 + L_3 \\ L_7 &= L_2 + c_3 \\ L_8 &= L_7 (L_1 + c_1 L_2) + c_1 (L_3 + 1) + L_4 \\ L_9 &= 1 - c_1 L_7 + L_3 - L_1 + L_2 L_7 \\ L_{10} &= c_1 L_7 - c_1 L_2 \\ L_{11} &= L_1 + c_1 L_2\end{aligned}$$

Notice from (14) that $\dot{V}_L \leq 0$ and it estabes the global stability of the equilibrium $(e_i, \tilde{k}_{\omega_i}) = (0, 0)$. From the LaSalle-Yoshizawa theorem, we have that e_i and $\tilde{k}_{\omega_i}; i = 1, 2, 3$; are bounded and go to zero as $t \rightarrow \infty$. From (5) it follows that $d \rightarrow d_{min}$. (8) implies that \hat{k}_{ω_1} is also bounded and

$$\lim_{t \rightarrow \infty} \psi = \arcsin \left(-\frac{\hat{k}_{\omega_1}}{V} \right) \quad (17)$$

Observe that from (12) r is bounded and $r \rightarrow 0$. On the other hand, from (16) it follows that $c\tau_{\psi}$ is bounded.

LaSalle's invariance principle assures that the state $(e_i, \tilde{k}_{\omega_i})$ converges to the largest invariant set M contained in $\{(e_1, e_2, e_3, \tilde{k}_{\omega_1}, \tilde{k}_{\omega_2}, \tilde{k}_{\omega_3}) \in \mathbb{R}^6 | \dot{V}_L = 0\}$. On this invariant set, we have $e_i \equiv 0$ and $\dot{e}_i \equiv 0$. From (15) it yields $\dot{\tilde{k}}_{\omega_i} = 0$ and $\tilde{k}_{\omega_i} = 0$. Thus, the largest invariant set M is

$$\begin{aligned}M &= \{(e_i, \tilde{k}_{\omega_i}) \in \mathbb{R}^6 | e_i = 0, \tilde{k}_{\omega_i} = 0\} \\ &= \{(d, \psi, r, \hat{k}_{\omega_1}, \hat{k}_{\omega_2}, \hat{k}_{\omega_3}) \in \mathbb{R}^6 | (d, \psi, r, \hat{k}_{\omega_1}, \hat{k}_{\omega_2}, \hat{k}_{\omega_3}) \\ &= (0, \arcsin(-\frac{\hat{k}_{\omega_1}}{V}), 0, k_{\omega}, k_{\omega}, k_{\omega})\}\end{aligned}$$

The manifold M is the single point $d = 0$, $\psi = \arcsin(-\frac{\hat{k}_{\omega_1}}{V})$, $r = 0$, $\hat{k}_{\omega_i} = k_{\omega}$ for $i = 1, 2$ and 3 , which is globally asymptotically stable.

IV. SIMULATION RESULTS

The proposed control strategy was validated in closed-loop system in simulations with various wind conditions. For simplicity we consider that the desired trajectory is aligned with the North axis of the inertial frame which makes the desired path angle $\psi_d = 0^\circ$. In addition, the airplane is flying with a constant speed equal to 20 m/s and the crosswind has a direction West-East perpendicular to the desired path. For a smoother convergence we have used the following parameters in simulations : $c_1 = c_3 = 1.5$; $c_2 = 1.3$; $\gamma_1 = 1$; $\gamma_2 = 1.1$; $\gamma_3 = 1.4$.

A. Case constant wind

Several simulations were performed to validate the controller and representative results are presented. The first simulations were carried out with a constant wind velocity of 7 m/s. The initial conditions are : $d = 2$ m/s, $\psi = -10^\circ$ and $r = 0$ rad/s. For comparative control purpose, a standard nonlinear backstepping algorithm was developed to control the system (2)-(4) and it is given by

$$c\tau_{\psi_b} = -3r + \tan \psi (r^2 - 5) - \frac{3d + 5k_{\omega}}{V \cos \psi} \quad (18)$$

In Fig. 3 we show the time evolution of the aircraft deviation from the desired trajectory for constant wind when employing the controllers (16) and (18). The wind parameter, denoted by k_{ω} , is not known and therefore considered zero in the simulation. Notice from this figure that the controller developed in (16) is able to provide cross track error regulation due to the adaptation laws presented in (15). For this case, the closed-loop adaptive system shows good response even in presence of unknown disturbance.

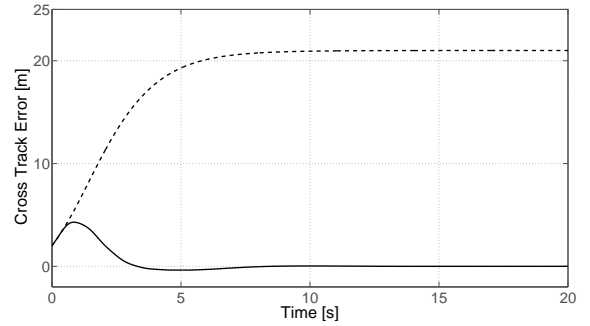


Fig. 3. Position Error for unknown wind. Solid line represents the proposed controller (16) whilst dashed line the standard backstepping control algorithm (18).

Fig. 4 reveals the fact that to maintain alignment with the desired trajectory during a crosswind flight requires the controller to fly the airplane at a sideslip angle. Indeed, when the position error converges to zero, the yaw angle is stabilized around a constant value and the airplane keeps moving toward North. Notice that the yaw angle is nonzero unless the atmosphere is at rest.

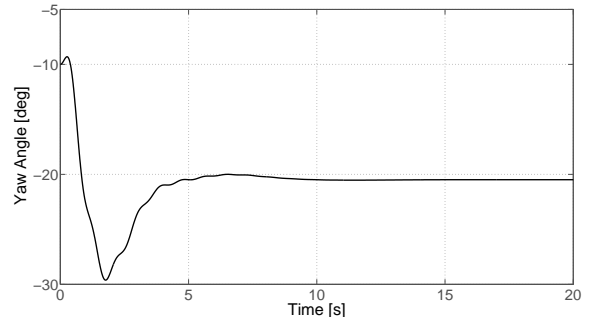


Fig. 4. Yaw angle for $k_{\omega} = 7$ m/s

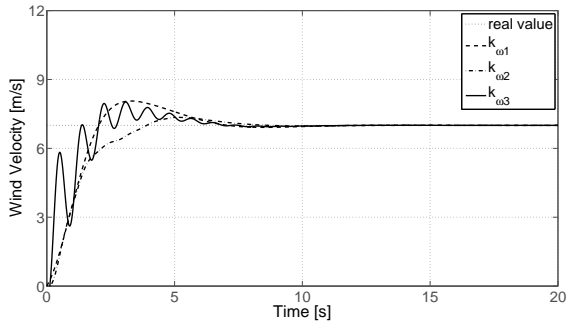


Fig. 5. Parameter estimation for $k_{\omega} = 7$ m/s

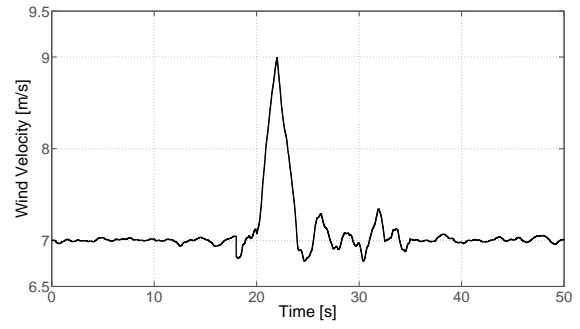


Fig. 8. Variable wind gust

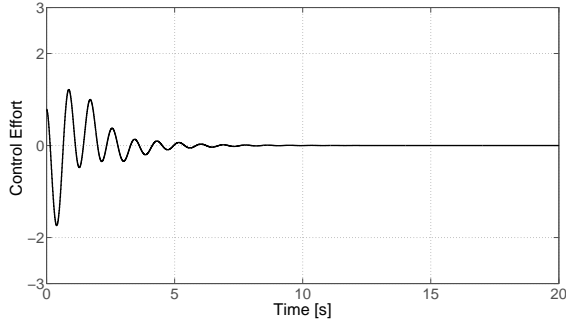


Fig. 6. Control input for $k_{\omega} = 7$ m/s

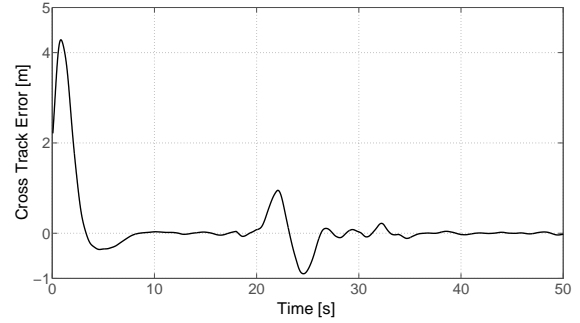


Fig. 9. Position error for variable wind

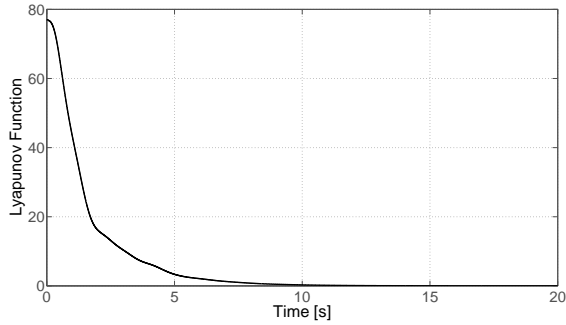


Fig. 7. Lyapunov function for $k_{\omega} = 7$ m/s

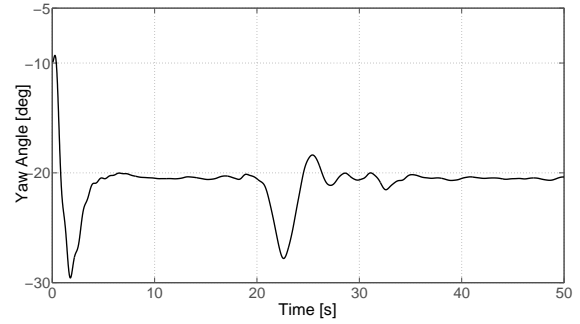


Fig. 10. Yaw angle for variable wind

On the other hand, the proposed adaptation scheme guarantees the convergence of the unknown parameter estimates towards its true constant value, see Fig. 5. The Lyapunov function, plotted in Fig. 7, is semi-positive definite and continually decreasing which proves the stability properties of the system. Indeed, in Fig. 6 we illustrate the control input response.

B. Case variable wind gust

In order to demonstrate the robustness of the proposed control algorithm, some variations in the wind parameters are added. For this purpose, we assume that the wind varies in magnitude as shown in Fig. 8. Notice that, at time 20s, a sudden increase of 2 m/s is presented in speed of the wind. The initial conditions are the same as those for constant wind.

The main results are displayed in figures 9 - 12. The wind deviates the airplane from the reference trajectory toward the wind direction but the controller (16) is able to recover the aircraft and to converge the position error to zero.

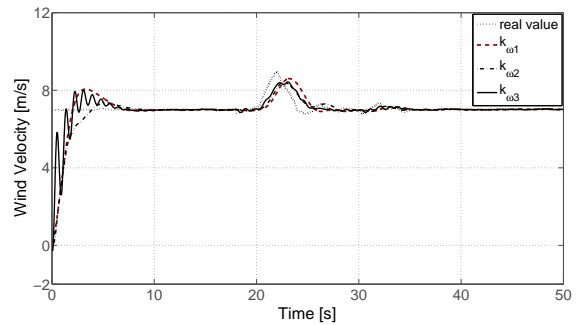


Fig. 11. Parameter estimation for variable wind

The adaptation laws show relatively small convergence time and the estimated wind velocities are in agreement with the real values. When aligned with the reference trajectory, the airplane is flown at a sideslip angle to maintain directional control.

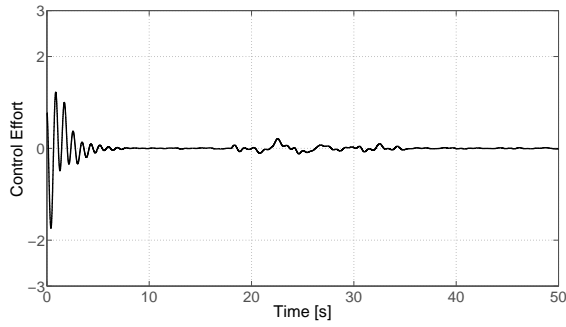


Fig. 12. Control input for variable wind

V. AIRPLANE CONFIGURATION

In this section we introduce an overview of the onboard control system developed in order to carry out flight tests.

The airplane used is the Multiplex TwinStar II whose technical characteristics are given in Table I. Its configuration is based on the classic aerodynamic layout and it is made of molded Elapor foam. Two brushless motors were mounted on the airfoil-shaped wings to power the airplane. A couple of ailerons, an elevator and a rudder are used as control surfaces and servo motors are attached to them as control surface actuators.

TABLE I
PARAMETERS OF THE AIRPLANE.

Parameter	Value
Wingspan	1420 mm
Fuselage length	1085 mm
Wing area	43 dm^2
Weight approx.	1500 g
Wing loading	35 g / dm^2
RC functions	Aileron, elevator, rudder, throttle

The central processing unit, represented by the RabbitCore RCM4300 Microprocessor, collects the measurements of the IMU (Inertial Measurement Unit employed to estimate the airplane attitude and angular rates), of the airspeed sensor and of the GPS system, to compute the control law. The control responses are sent to the servo signal generator/receiver unit and also to the two electric speed controllers to activate the brushless motors. Indeed, a modem is added to send and receive data from a base station. The developed prototype is presented in Fig. 13.

VI. CONCLUSIONS AND FUTURE WORK

An adaptive control algorithm based on the backstepping approach has been proposed in this paper. The control strategy was focused on reducing the position deviation of the airplane with respect to a desired path in the lateral dynamics in presence of unknown wind. The control scheme was derived considering adaptation laws to estimate the unknown wind parameters.

The closed-loop system was evaluated in several simulations and the main results, showing the good performance, were introduced by some graphs. An embedded control

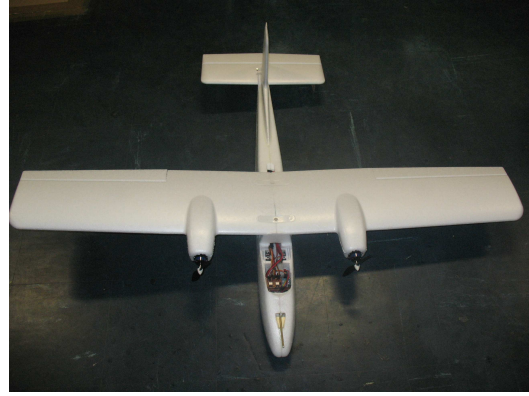


Fig. 13. The airplane prototype

system was developed in order to validate the control strategy in flight tests.

Future work will include real time implementation of the flight controller using the developed hardware platform.

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