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# Nonlinear aggregation of conditional extreme-value index estimators

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## Abstract

When an extreme phenomena depends on a covariate, one has to deal with Conditional Extreme Value Analysis. In this paper, we propose a nonlinear aggregation technique to estimate the conditional extreme-value index.

Extreme value analysis (EVA) is a branch of statistics dealing with the extreme deviations from the mean of probability distributions. It seeks to assess the probability of events that are more extreme than any observed prior. Extreme value analysis is widely used in many disciplines, ranging from structural engineering, finance, earth sciences, traffic prediction, geological engineering, etc. For example, EVA might be used in the field of hydrology to estimate the value an unusually large flooding event, such as the 100-year flood. Similarly, for the design of a breakwater, a coastal engineer would seek to estimate the 50-year wave and design the structure accordingly.

Let  $X$  be a random variable and  $F(x) = P(X \leq x)$  its distribution function. We can define a new random variable,  $Y_n$ , as the maximum of  $n$  copies of the random variable  $X$ :  $Y_n = \max\{X_1, X_2, \dots, X_n\}$ .  $Y_n$  is the  $n$ -sample maximum of the random variable  $X$ . If the events generating the realizations of  $X$  are independent, the cumulative distribution of  $Y_n$  may be expressed as  $[F(y)]^n$ . Upon definition of a renormalized variable  $S_n = (Y_n - b_n)/a_n$ , the extreme value theorem establishes that, if

$$\lim_{n \rightarrow \infty} P(S_n < s) = \lim_{n \rightarrow \infty} F^n(a_n s + b_n) = H(s) \quad (1)$$

where  $a_n > 0$  and  $b_n$  are normalization constants, then the function  $H(s)$  in Eq. (1) must be one of the three following types:

- EV1 or Gumbel:  $H(s) = \exp(-\exp(-s))$
- EV2 or Fréchet:  $H(s) = \exp(-s^{-\alpha})$
- EV3 or Weibull:  $H(s) = \exp(-|s|^\alpha)$

The three asymptotic types, EV1-EV3, can be thought of as special cases of a single Generalized Extreme Value distribution (GEV) :

$$H_{GEV}(s) = \exp \left\{ - \left( 1 + \gamma \frac{s - \mu}{\sigma} \right)_+^{-1/\gamma} \right\} \quad (2)$$

where  $(\cdot)_+ = \max(\cdot, 0)$ ,  $\mu$  is the location parameter,  $\sigma > 0$  is the scale parameter, and  $\gamma$  is a shape parameter. The limit  $\gamma = 0$  corresponds to the EV1 distribution,  $\gamma > 0$  to the EV2 distribution (with  $\alpha = 1/\gamma$ ) and  $\gamma < 0$  to the EV3 distribution (with  $\alpha = -1/\gamma$ ).

When the studied phenomena depends on a covariate, one has to deal with Conditional Extreme Value Analysis (CEVA). This branch of statistics has become very active these past ten years, the main contributions to this domain are listed below:

- Theoretical issues: [11, 15, 9, 14, 16, 13, 4, 12, 7, 18, 10, 3, 1, 26, 25, 22]
- Quantile regression: [19, 8, 23]
- Application to finance: [5, 2, 17, 6, 20, 24]

Our main result is the following:

**Theorem** *Let  $\hat{\gamma}_1(x), \dots, \hat{\gamma}_K(x)$  be  $K$  consistent estimators of the conditional extreme-value index  $\gamma(x)$ . Let  $w_1, \dots, w_K$  be  $K$  weights summing to one and  $\psi$  a continuous bijective function with continuous inverse  $\psi^{-1}$ . Then,*

$$\hat{\gamma}(x) := \psi^{-1} \left( \sum_{i=1}^K w_i \psi(\hat{\gamma}_i(x)) \right)$$

*is a consistent estimator of the conditional extreme-value index  $\gamma(x)$ .*

**Proof.** The continuity property of  $\psi$  implies that  $\psi(\hat{\gamma}_i(x))$  converges to  $\psi(\gamma(x))$  in probability. Besides, since  $\sum_{i=1}^K w_i = 1$ , it follows that  $\sum_{i=1}^K w_i \psi(\hat{\gamma}_i(x))$  converges to  $\psi(\gamma(x))$  in probability as well and the result follows.

Let us highlight that the positivity of the weights is not required. Moreover, this result is valid whatever the sign of  $\gamma(x)$ . Finally, it extends the linear aggregation procedure of [21] to the nonlinear case where the function  $\psi$  is not the identity.

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