Weakly nonlinear second-order dynamical systems identification using a random parameters linear model

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WEAKLY NONLINEAR SECOND-ORDER DYNAMICAL SYSTEMS IDENTIFICATION USING A RANDOM PARAMETERS LINEAR MODEL

by

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ABSTRACT

The objective of this paper is to present an identification procedure which is based on the use of a stochastic linearization method with random coefficients. The model is then defined as a multidimensional linear second-order dynamical system with random coefficients. An optimization procedure is developed to identify the parameters of the probability law of the random coefficients. The identification procedure is described step by step. Finally, an example is presented and shows the interest of the method proposed.

1. INTRODUCTION

For linear multidimensional second-order dynamical systems (m.s-o.d.s.) with time independent coefficients (constant coefficients), modal identification procedures are known and well developed. In this paper, we are interested in the identification of weakly nonlinear m.s-o.d.s. with constant coefficients, using a linear model and a stationary random input. The main idea is to use an infinite family of linear models to represent the nonlinear dynamical system i.e., a linear m.s-o.d.s. with random coefficients. This means that the weakly nonlinear m.s-o.d.s. is identified by a linear m.s-o.d.s. with uncertainties. Consequently, such an identification yields a linear model whose operator-valued frequency response function is a random stochastic process indexed by the frequency. The eigenfrequencies and associated eigenmodes are then deduced from the linearized representation which is identified (consequently, the eigenfrequencies are random variables).

Using broad-band stationary random excitation and constant coefficients of the model yields the classical Stochastic Linearization Method with Constant Coefficients (SLMCC). The SLMCC was introduced by Caughey in 1963 [1] within the context of prediction methods. Many developments have been proposed in this area since this date and an excellent synopsis was made by Roberts and Spanos in 1990 [2]. An identification procedure based on SLMCC can be summarized as shown in Fig. 1 and will be refered in this paper as Method 1.

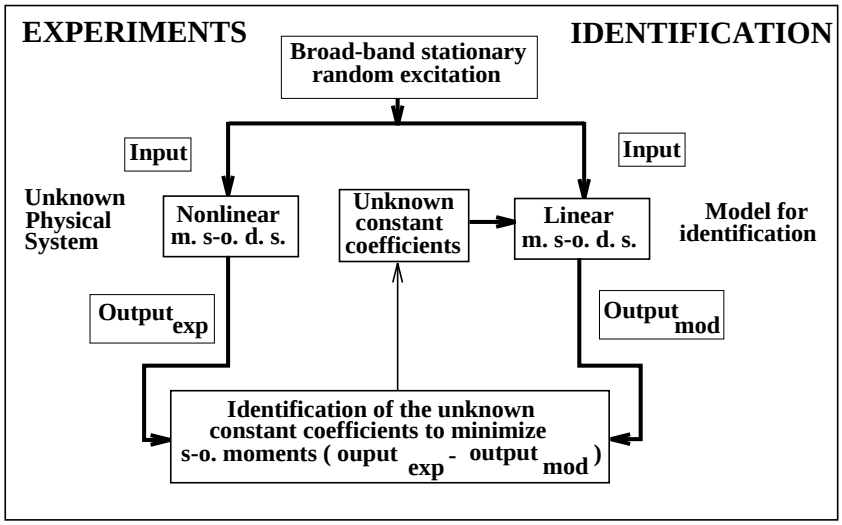

Fig. 1. Method 1 – Identification procedure based on a stochastic linearization method with constant coefficients (SLMCC)

In this field of identification procedures, it should be noted that difficulties arise due to the presence of the mass matrix which is unknown and which must be identified. Fillâtre [3] developed a method for identifying such an equivalent linear model in which mass, damping and stiffness matrices are constant and unknown. His approach is based on an extension of Kozin's works [4,5] and can be considered as a method based on the SLMCC. Generally speaking, it is known that the SLMCC yields a very good approximation of the second-order (s-o.) statistical moments of the stationary response of second-order dynamical systems. Consequently, an identification method based on such a procedure yields an equivalent linear dynamical system which can restitute the second-order moments. Unfortunately, in some cases, although the second-order moments are correctly estimated, the matrix-valued spectral density function (s.d.f.) of the response may be erroneous. This difficulty was first shown by Miles for a one-dimensional nonlinear dynamical system[6]. For such nonlinear dynamical systems, methods were proposed to calculate the power spectral density function of the stationary response without using Monte Carlo numerical simulation (Miles [6], Bouc [7,8] and Soize [9,10]). The matrix-valued spectral density function can generally not be calculated explicitly for multidimensional nonlinear dynamical systems, except for particular cases related to linear dynamical systems with random parametric excitations (see for instance Soize [11]). Recently, Bellizzi and Bouc [12] proposed an interesting method for multidimensional systems in the context of prediction methods.

The Stochastic Linearization Method with Random Coefficients (SLMRC) [9,10] is adapted to identification procedures and allows the identification to be improved with respect to the classical SLMCC. This fact was recently proved by Soize [13] for one-degree-of-freedom nonlinear second-order dynamical systems. This method, based on a linear dynamical model with random coefficients, has just been extended by Le Fur [14] for the identification of weakly nonlinear multidimensional second-order dynamical systems and the details of the method can be found in [15]. An identification procedure based on SLMRC can be summarized as shown in Fig. 2 and will be refered in this paper as Method 2. It should be noted that Method 2 uses Method 1. The purpose of this paper is to summarize a new approach developed by Soize and Le Fur [15] for identifying weakly nonlinear multidimensional second-order dynamical systems based on the identification of a linear model with random coefficients.

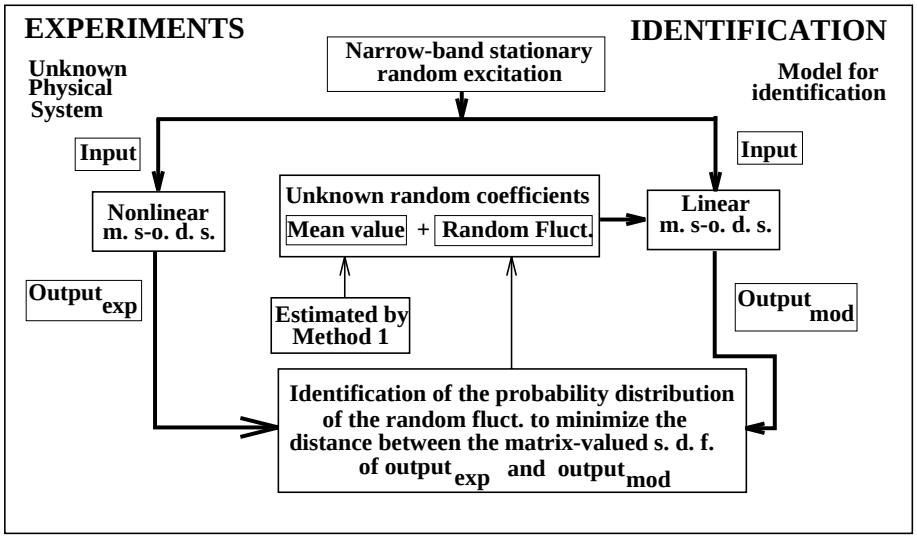

Fig. 2. Method 2 – Identification procedure based on a stochastic linearization method with random coefficients (SLMRC)

2. CONSTRUCTION OF THE MODEL WITH RANDOM COEFFICIENTS

We consider a weakly nonlinear dynamical system of dimension $n \geq 1$ subjected to an external random excitation. This dynamical system is written as the following stochastic differential equation

$$[M]\ddot{\boldsymbol{\mathcal{X}}}(t) + [C]\dot{\boldsymbol{\mathcal{X}}}(t) + [K]\boldsymbol{\mathcal{X}}(t) + \varepsilon \mathbf{f}\big(\boldsymbol{\mathcal{X}}(t)\big) = \mathbf{F}(t) \quad , \tag{1}$$

in which matrices $[M\,]$, $[C\,]$ and $[K\,]$ are positive definite; function **f** from $\mathbb{R}^n$ into $\mathbb{R}^n$ is odd, continuous and nonlinear; excitation force **F** is a Gaussian, second-order, centered, stationary, mean-square continuous stochastic process indexed by $\mathbb{R}$ with values in $\mathbb{R}^n$. The matrix-valued spectral density function of process **F** is written as

$$[S_{\mathbf{F}}(\omega)] = s(\omega)[B\,] \quad , \tag{2}$$

where $[B\,]$ is a positive matrix and s is a positive-valued function defined on $\mathbb{R}$ having the required properties such that process **F** is physically realizable and approaches an ideal normalized narrow-band noise.

Within the context of an identification problem, it is assumed that stochastic differential equation (1) has a unique stationary, second-order, centered stochastic solution $\boldsymbol{\mathcal{X}}$ having a matrix-valued spectral density function $[S_{\boldsymbol{\mathcal{X}}}]$. Furthermore, function $[S_{\boldsymbol{\mathcal{X}}}]$ is assumed to be square integrable on $\mathbb{R}$.

Applying the identification procedure developed by Fillâtre [3] (based on a stochastic linearization method with constant coefficients) yields the following linear stochastic differential equation on $\mathbb{R}^n$

$$[\underline{M}_c]\,\ddot{\mathbf{X}}(t) + [\underline{C}_c]\,\dot{\mathbf{X}}(t) + [\underline{K}_c]\,\mathbf{X}(t) = \mathbf{F}(t) \quad , \tag{3}$$

in which **F** is the stochastic process used in Eq. (1) and matrices $[\underline{M}_c]$, $[\underline{C}_c]$ and $[\underline{K}_c]$ result from the identification procedure and are positive definite.

We introduce the eigenmodes $\underline{\boldsymbol{\varphi}} \in \mathbb{R}^n$ and the associated eigenfrequencies $\underline{\omega}$ of the conservative problem associated with Eq. (3), which are the solutions of the generalized eigenvalue problem $[\underline{K}_c]\,\underline{\boldsymbol{\varphi}} = \underline{\omega}^2\,[\underline{M}_c]\,\underline{\boldsymbol{\varphi}}$. Let $[\underline{\Phi}]$ be the $(n \times n)$ real matrix of the eigenmodes such that $[\underline{\Phi}]_{jk} = \{\underline{\boldsymbol{\varphi}}_k\}_j$.

We introduce modal coordinates $\mathbf{Q}$ such that $\mathbf{X} = [\underline{\Phi}]\,\mathbf{Q}$. Substituting this change of coordinates in Eq. (3) yields

$$[\underline{M}_g]\,\ddot{\mathbf{Q}}(t) + [\underline{C}_g]\,\dot{\mathbf{Q}}(t) + [\underline{K}_g]\,\mathbf{Q}(t) = [\underline{\Phi}]^T\mathbf{F}(t) \quad , \tag{4}$$

where $[\underline{M}_g] = [\underline{\Phi}]^T\,[\underline{M}_c]\,[\underline{\Phi}]$, $[\underline{C}_g] = [\underline{\Phi}]^T\,[\underline{C}_c]\,[\underline{\Phi}]$ and $[\underline{K}_g] = [\underline{M}_g]\,[\underline{\Omega}^2] = [\underline{\Phi}]^T\,[\underline{K}_c]\,[\underline{\Phi}]$ are $(n\times n)$ real positive-definite matrices. Matrices $[\underline{M}_g]$, $[\underline{K}_g]$ and $[\underline{\Omega}^2]$ are diagonal and $[\underline{C}_g]$ is a dense matrix in the general case.

We associate with Eq. (4) the following stochastic differential equation with random coefficients

$$[\underline{M}_g]\,\ddot{\mathbf{Y}}(t) + [\underline{C}_g]\,\dot{\mathbf{Y}}(t) + [\underline{K}_g]\,\big([\,I\,] + [\Lambda]\big)\,\mathbf{Y}(t) = [\underline{\Phi}]^T\mathbf{F}(t) \quad , \tag{5}$$

where $[\Lambda]$ is a random variable with values in the $(n\times n)$ real diagonal matrices. We introduce the vector $\boldsymbol{\Lambda} = (\Lambda_1, \ldots, \Lambda_n)$ of its diagonal entries $\Lambda_i = [\Lambda]_{ii}$. It is assumed that $\{\Lambda_1, \ldots, \Lambda_n\}$ are independent real-valued random variables. The probability law $P_{\Lambda_i}(d\lambda)$ of the real-valued random variable Λ_i is defined by a probability density function $p_{\Lambda_i}(\lambda)$ on $\mathbb{R}$ with respect to $d\lambda$:

$$P_{\Lambda_i}(d\lambda) = p_{\Lambda_i}(\lambda)d\lambda \quad , \tag{6}$$

in which for all $\lambda \in \mathbb{R}$,

$$p_{\Lambda_i}(\lambda) = \alpha_{\Lambda_i}(1+\lambda)W_{\Lambda_i}(\lambda) \quad . \tag{7}$$

Real function $\lambda \mapsto W_{\Lambda_i}(\lambda)$ defined on $\mathbb{R}$ is such that

$$W_{\Lambda_i}(\lambda) = \mathbb{1}_{[\lambda_i^{(1)},+\infty[}(\lambda)(\lambda - \lambda_i^{(1)})e^{-\beta_{\Lambda_i}(\lambda-\lambda_i^{(1)})^2} \quad . \tag{8}$$

Equations (6)-(8) define a parametric family of probabilities where the unknown parameters α_{Λ_i} , β_{Λ_i} and $\lambda_i^{(1)}$ verify the conditions $\alpha_{\Lambda_i} > 0$, $\beta_{\Lambda_i} > 0$, $1 + \lambda_i^{(1)} > 0$. Since $P_{\Lambda_i}(d\lambda)$ is a probability, $P_{\Lambda_i}(\mathbb{R}) = 1$ and consequently, the three parameters α_{Λ_i}, β_{Λ_i} and $\lambda_i^{(1)}$ are dependent. Calculating α_{Λ_i} as a function of β_{Λ_i} and $\lambda_i^{(1)}$ yields

$$\alpha_{\Lambda_i} = \frac{2\beta_{\Lambda_i}}{1 + \lambda_i^{(1)} + \frac{1}{2}\sqrt{\frac{\pi}{\beta_{\Lambda_i}}}} \quad . \tag{9}$$

Because of the independence of random variables $\{\Lambda_1, \ldots, \Lambda_n\}$, the probability law of the $\mathbb{R}^n$-valued random variable $\boldsymbol{\Lambda}$ is written as

$$P_{\boldsymbol{\Lambda}} = \otimes_{i=1}^n P_{\Lambda_i} \quad . \tag{10}$$

It can be proved [14,15] that Eq. (5) has a unique second-order, centered, stationary solution $\mathbf{Y}$ which has a square integrable matrix-valued spectral density function given by the relation

$$[S_{\mathbf{Y}}(\omega)] = \int_{\mathbb{R}^n} [S_{\mathbf{Y}_\Lambda}(\omega;\boldsymbol{\lambda})]P_{\boldsymbol{\Lambda}}(d\lambda) \quad , \tag{11}$$

in which $P_{\boldsymbol{\Lambda}}$ is given by Eqs. (6)-(10) and matrix $[S_{\mathbf{Y}_\Lambda}(\omega;\boldsymbol{\lambda})]$ is such that

$$[S_{\mathbf{Y}_\Lambda}(\omega;\boldsymbol{\lambda})] = [H_\Lambda(\omega;\boldsymbol{\lambda})]\,[\Phi]^T\,[S_{\mathbf{F}}(\omega)]\,[\Phi]\,[H_\Lambda(\omega;\boldsymbol{\lambda})]^* \quad , \tag{12}$$

$$[H_\Lambda(\omega;\boldsymbol{\lambda})] = \big[-\omega^2\,[\underline{M}_g] + i\omega\,[\underline{C}_g] + [\underline{K}_g]([\,I\,] + [\,\lambda\,])\big]^{-1} \quad . \tag{13}$$

3. IDENTIFICATION PROCEDURE

The identification procedure consists in calculating the parameters of probability law P_{Λ} in order to minimize the "distance" between the matrix-valued spectral density function of the model responses and measured (experimental) responses.

Let $\boldsymbol{\mathcal{Q}}$ be the $\mathbb{R}^n$-valued stationary stochastic process such that $\boldsymbol{\mathcal{Q}} = [\underline{\Phi}]^{-1}\boldsymbol{\mathcal{X}}$ in which $\boldsymbol{\mathcal{X}}$ is the measured stationary stochastic process (experimental responses) and $[\underline{\Phi}]$ are the estimated eigenmodes introduced in Section 2 . We then deduce that for all real ω, the matrix-valued spectral density function $[S_{\boldsymbol{\mathcal{Q}}}(\omega)]$ of process $\boldsymbol{\mathcal{Q}}$ can be written as

$$[S_{\boldsymbol{\mathcal{Q}}}(\omega)] = [\underline{\Phi}]^{-1}[S_{\boldsymbol{\mathcal{X}}}(\omega)]\,[\underline{\Phi}]^{-T} \quad . \tag{14}$$

Let $\boldsymbol{\lambda}^{(1)}$ and $\mathbf{A}$ be the vectors in $\mathbb{R}^n$ such that $\boldsymbol{\lambda}^{(1)} = (\lambda_1^{(1)}, \ldots, \lambda_n^{(1)})$ and $\mathbf{A} = (A_1, \ldots, A_n)$ with

$$A_i = 1/\sqrt{\beta_{\Lambda_i}} \quad , \quad i \in \{1, \ldots, n\} \quad . \tag{15}$$

Let $\boldsymbol{\xi} = (\boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_n)$ be the vector in $\mathbb{R}^{2n}$ such that $\boldsymbol{\xi} = (\mathbf{A}, \boldsymbol{\lambda}^{(1)})$ and $\boldsymbol{\xi}_i = (A_i, \lambda_i^{(1)})$. Let $\mathcal{D}$ be the domain of $\boldsymbol{\xi}$ which is such that

$$\mathcal{D} = \left\{ (\mathbf{A}, \boldsymbol{\lambda}^{(1)}) \in \mathbb{R}^{2n} \quad | \quad A_i > 0\,,\, 1 + \lambda_i^{(1)} > 0\,,\, \forall\, i \in \{1, \ldots, n\} \right\} \quad . \tag{16}$$

In order to indicate the dependence of P_{Λ} and $[S_{\mathbf{Y}}]$ in $\boldsymbol{\xi}$, we rewrite these quantities as $P_{\Lambda}^{\boldsymbol{\xi}}$ and $[S_{\mathbf{Y}}^{\boldsymbol{\xi}}]$ respectively. Since the measured and the model matrix-valued spectral density functions are square integrable, the following cost function can be used,

$$H(\boldsymbol{\xi}) = \sum_{i=1}^{n} \| \, [S_{\boldsymbol{\mathcal{Q}}}]_{ii} - [S_{\mathbf{Y}}^{\boldsymbol{\xi}}]_{ii} \, \|^2 = \sum_{i=1}^{n} \int_{\mathbb{R}} \left([S_{\boldsymbol{\mathcal{Q}}}(\omega)]_{ii} - [S_{\mathbf{Y}}^{\boldsymbol{\xi}}(\omega)]_{ii} \right)^2 d\omega \quad . \tag{17}$$

The identification procedure is defined as the following optimization problem: find $\boldsymbol{\xi}_0$ in $\mathcal{D}$ such that

$$H(\boldsymbol{\xi}_0) = \min_{\boldsymbol{\xi} \in \mathcal{D}} H(\boldsymbol{\xi}) \quad . \tag{18}$$

It should be noted that diagonal terms $[S_{\mathbf{Y}}^{\boldsymbol{\xi}}]_{ii}$ depend on all the components of $\boldsymbol{\xi}$ due to the fact that matrix $[\underline{C}_g]$ used in the calculation of $[H_{\Lambda}(\omega; \boldsymbol{\lambda})]$ is not diagonal. In order to replace problem (18) by n independent optimization problems in $\mathbb{R}^2$, elements $[S_{\mathbf{Y}}^{\boldsymbol{\xi}}(\omega)]_{ii}$ are approximated by $[S_{\mathbf{Y}}^{\boldsymbol{\xi}_i}(\omega)]_{ii}$ obtained by neglecting the extra-diagonal part in matrix $[\underline{C}_g]$. It should be noted that this approximation (introduced only to simplify the optimization problem) is not used in the final calculation of matrix $[S_{\mathbf{Y}}^{\boldsymbol{\xi}}(\omega)]$ (see Section 4). Then, from Eqs. (2), (6)-(9) and (11)-(13), we deduce that for all i in $\{1, \ldots, n\}$

$$[S_{\mathbf{Y}}^{\boldsymbol{\xi}_i}(\omega)]_{ii} = \frac{2 e_{ii} s(\omega)}{1 + \lambda_i^{(1)} + A_i \sqrt{\pi}/2} \int_0^{+\infty} \frac{(1 + A_i x + \lambda_i^{(1)}) x e^{-x^2}\, dx}{\left((1 + A_i x + \lambda_i^{(1)}) [\underline{K}_g]_{ii} - \omega^2 [\underline{M}_g]_{ii} \right)^2 + \omega^2 [\underline{C}_g]_{ii}^2} \quad , \tag{19}$$

in which $e_{ii} = \{[\underline{\Phi}]^T\,[B]\,[\underline{\Phi}]\}_{ii}$. For each i in $\{1, \ldots, n\}$, we define the functional on $]0\,, +\infty[\times]-1\,, +\infty[\subset \mathbb{R}^2$ such that

$$J_i(\boldsymbol{\xi}_i) = \int_{\mathbb{R}} \left\{ [S_{\boldsymbol{\mathcal{Q}}}(\omega)]_{ii} - [S_{\mathbf{Y}}^{\boldsymbol{\xi}_i}(\omega)]_{ii} \right\}^2 d\omega \quad . \tag{20}$$

Consequently, the optimization problem on a subset of $\mathbb{R}^{2n}$ defined by Eq. (18), is replaced by the n following optimization problems on a subset of $\mathbb{R}^2$

$$J_i(\boldsymbol{\xi}_{i,0}) = \min_{\boldsymbol{\xi}_i \in]0,+\infty[\times]-1,+\infty[} J_i(\boldsymbol{\xi}_i) \quad , \quad i \in \{1,\ldots,n\} \quad . \tag{21}$$

It should be noted that each constraint optimization problem defined by Eq. (21) is not standard because J_i is not a convex function. Consequently the following method has been used:
- *Step1*: Determine a bounded subdomain $\mathcal{C}_i = [A_{2,i}, A_{1,i}] \times [\lambda_{1,i}^{(1)}, \lambda_{2,i}^{(1)}]$ included in unbounded domain $]0,+\infty[\times]-1,+\infty[$ such that $\mathcal{C}_i$ contains the solution (see [15]), in order to limit the space of the research for a solution.
- *Step 2*: Use a global optimization algorithm on $\mathcal{C}_i$ based on an adaptive random search [16] which allows a first approximation $\boldsymbol{\xi}_{i,a}$ of the solution of (21) to be constructed.
- *Step 3*: Finally, use a local optimization method on $\mathcal{C}_i$ based on the Gauss-Newton algorithm and initialized with $\boldsymbol{\xi}_{i,a}$, giving solution $\boldsymbol{\xi}_{i,0}$ of problem (21).

4. CALCULATION OF THE MODEL MATRIX-VALUED SPECTRAL DENSITY FUNCTION

Matrix-valued spectral density function $[S_{\mathbf{Y}}(\omega)]$ can be calculated for $\boldsymbol{\xi} = \boldsymbol{\xi}_0$ where $\boldsymbol{\xi}_0$ results from the identification procedure (see Section 3). Knowing $[S_{\mathbf{Y}}(\omega)]$ which is the identified model of measured matrix $[S_{\boldsymbol{\mathcal{Q}}}(\omega)]$ expressed in terms of modal coordinates, we deduce the identified model $[S_{\mathbf{Z}}(\omega)]$ of the measured matrix $[S_{\boldsymbol{\mathcal{X}}}(\omega)]$ relative to the physical coordinates. We have the relation $[S_{\mathbf{Z}}(\omega)] = [\underline{\Phi}]\,[S_{\mathbf{Y}}(\omega)]\,[\underline{\Phi}]^T$. Since matrix $[\underline{C}_g]$ is dense, the direct calculation of the model matrix-valued spectral density function defined by Eq. (11) requires calculating a n-uple integral on $\mathbb{R}^n$ for each ω. This calculation can only be carried out by numerical integration. Consequently, it cannot be made for large values of n (for instance when n is 10 or 20 (or more)). We therefore propose a construction of an approximation in ref. [15], which allows to calculate only n simple integrals on $\mathbb{R}$.

5. EXAMPLE

For this example, an "experimental data base" is constructed using a Monte Carlo numerical simulation in the time domain of the second-order nonlinear dynamical system defined by Eq. (1) with $n = 5$, where the nonlinear mapping $\mathbf{f}$ is defined by $\mathbf{f}\big(\boldsymbol{\mathcal{X}}(t)\big) = \big([K]_{11}\mathcal{X}_1(t)^3, \ldots, [K]_{55}\mathcal{X}_5(t)^3\big)$ with $\varepsilon = 1875$, where the frequency band of narrow-band process $\mathbf{F}$ is [14 Hz, 28 Hz] and where $[K]_{jj}$ are the diagonal terms of matrix $[K]$ appearing in Eq. (1). Matrices $[M]$, $[C]$ and $[K]$ were generated by the formulas $[M] = [S]^{-T}\,[\mathcal{M}]\,[S]^{-1}$, $[C] = [S]^{-T}\,[\mathcal{C}\,]\,[S]^{-1}$ and $[K] = [S]^{-T}\,[\mathcal{K}]\,[S]^{-1}$, where

$$[S] = \begin{bmatrix} 0.208513 & 0.333334 & 0.301512 & 0.447214 & 0.447214 \\ 0.208513 & -0.333334 & 0.301512 & -0.447214 & 0.447214 \\ 0.625543 & 0.577350 & 0.522233 & 0.447214 & 0.0 \\ 0.625543 & 0.577350 & -0.522233 & -0.447214 & 0.0 \\ 0.361158 & -0.333334 & -0.522233 & 0.447214 & 0.774597 \end{bmatrix} ,$$

and where $[\mathcal{M}]$ is the identity matrix (generalized masses equal to 1), $[\mathcal{K}] = [\mathcal{M}]\,[\Omega]^2$ in which $[\Omega]$ is the diagonal matrix whose diagonal is $2\pi \times [\,18.0\,,20.0\,,20.4\,,22.0\,,23.0\,]$ and finally, $[\,\mathcal{C}\,]$ is a diagonal matrix whose diagonal is $[\,4.5\,,5.0\,,5.13\,,5.5\,,5.8\,]$. Digital signal processing on the time-simulated sample paths of the stationary response was applied to estimate the "measured" matrix-valued spectral density function $[S_{\boldsymbol{\mathcal{X}}}(\omega)]$ for ω in the frequency band of analysis.
It should be noted that some eigenfrequencies Ω_j of the underlying linear dynamical system associated with the nonlinear dynamical system are close (20.0 Hz and 20.4 Hz). In presence of nonlinearities, this kind of situation is generally recognized as a difficult problem within the context of structural dynamic identification. The procedure presented in Section 3 is used to identify the parameters $\boldsymbol{\xi}_i$ of the model. Figure 3 is related to the comparisons between the matrix-valued spectral density functions

obtained by "experiments" and by identification of the model with constant coefficients. Fig. 3 shows the comparison between $[S_{\boldsymbol{\mathcal{X}}}]_{ii}$ and $[S_{\mathbf{X}}]_{ii}$ for i in $\{1,2,3,4,5\}$. These results correspond to those obtained by Fillâtre [3]. It should be noted that this first identification already yields a good identification (taking into account the intrinsic difficulties of the example considered), but as mentioned in Section 1, this kind of results can be improved using a more advanced model for identification (see below).

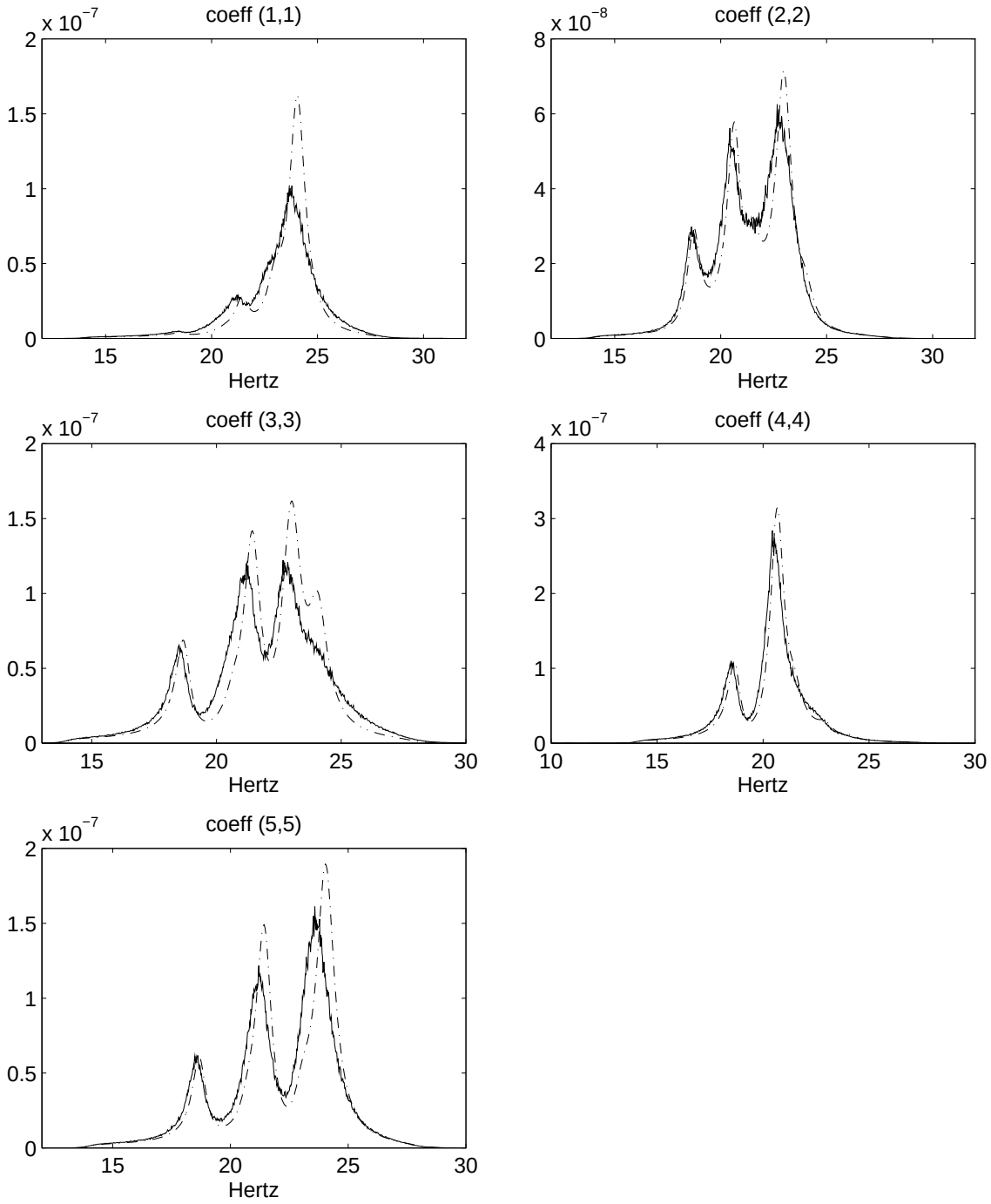

Fig. 3. Power spectral density functions $[S_{\boldsymbol{\mathcal{X}}}]_{ii}$ and $[S_{\mathbf{X}}]_{ii}$:
_____ experiments
- - - - - identification with stochastic linearization method with constant parameters

Figure 4 is related to the comparisons between the matrix-valued spectral density functions obtained by "experiments" and by identification of the model with random coefficients for which the procedure was described in Sections 2 to 4. Fig. 4 shows the comparison between $[S_{\boldsymbol{\mathcal{X}}}]_{ii}$ and $[S_{\mathbf{Z}}]_{ii}$ for i in $\{1, 2, 3, 4, 5\}$. It can be seen that the results obtained are much better than above. Complete results concerning this example are given in [15].

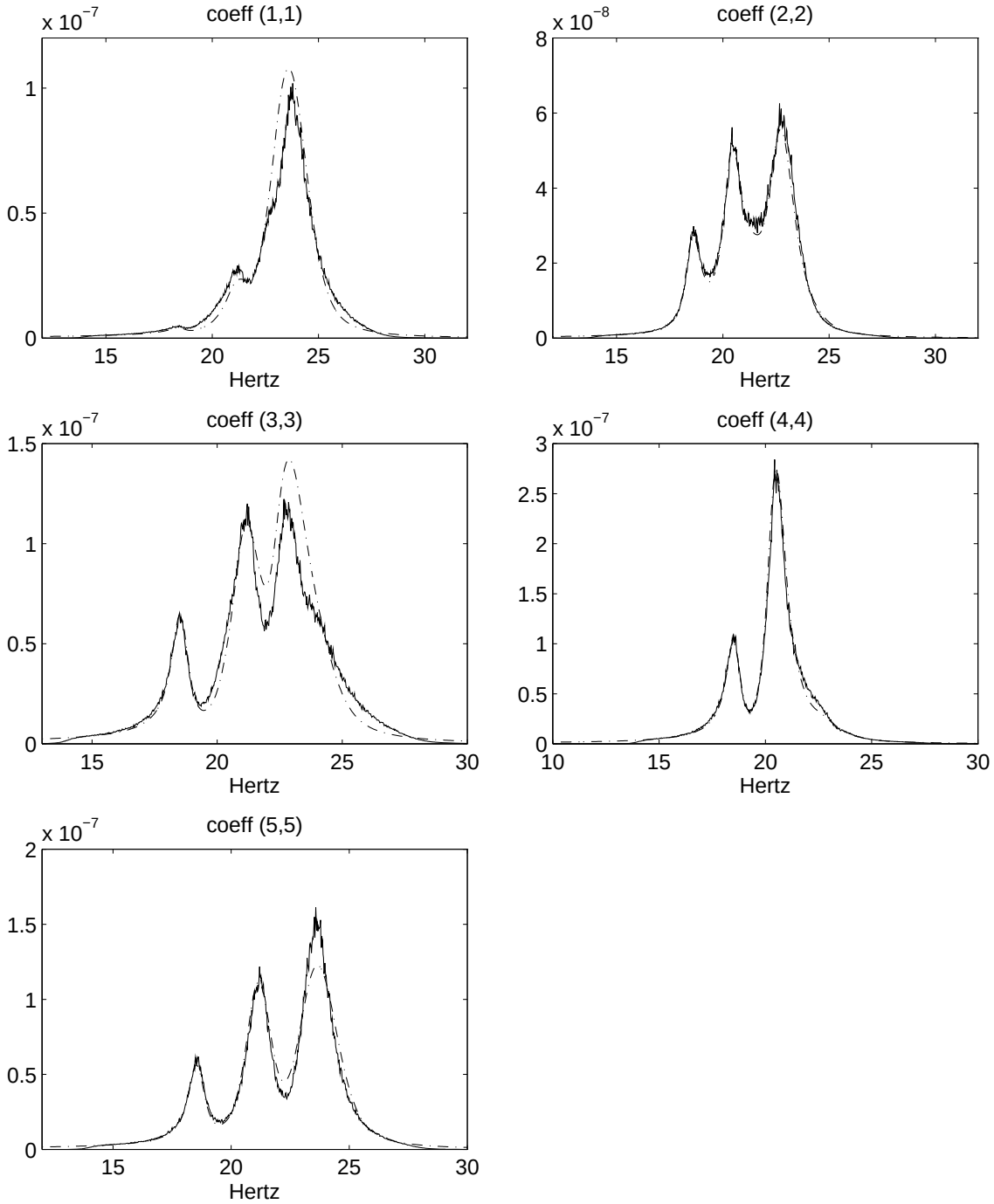

Fig. 4. Power spectral density functions $[S_{\boldsymbol{\mathcal{X}}}]_{ii}$ and $[S_{\mathbf{Z}}]_{ii}$:
_____ experiments
- - - - - identification with stochastic linearization method with random parameters

6. CONCLUSION

This work started from previous research on modal identification of weakly nonlinear multidimensional second-order dynamical systems (based on the use of the equivalent stochastic linearization with constant coefficients). In some cases, this method has difficulty identifying the matrix-valued spectral density function of the stationary responses. In the present work, we have used the previous work to identify the mean part of the model. In order to improve the identification of spectral quantities, we developed a method based on stochastic linearization with random coefficients. This new identification procedure seems to be very efficient and can be implemented easily. The results show that this method yields better results than the previous one. Nevertheless, this method could be improved by introducing some statistical dependence between the components of the random coefficients expressed in the modal coordinates (or possibly by introducing extra-diagonal terms) in oder to model energetic exchanges between eigenmodes due to the weak nonlinearities. It should be noted that in this last case, the optimization problem introduced in the method could not be split into several optimization problems with a smaller size. This being the case, the efficiency of such a procedure would have to be studied with great care.

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