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# Mixtures of Weibull tail-distributions

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## Abstract

We consider mixtures of Weibull tail-distributions. It is shown that such distributions are still Weibull tail-distributions and the associated Weibull tail-coefficient is exhibited.

Weibull tail-distributions are defined through their distribution function by:

$$F(x) = 1 - \exp \left\{ -x^{1/\theta} \ell(x) \right\}$$

where  $\ell$  is a slowly-varying function, see Table 1 for examples and [20] for a review. A sufficient condition for subexponentiality has been established in [19], Theorem 1.

There exist numerous estimators of  $\theta > 0$ . Berred [5] uses record values while most of estimators are based on the largest observations of the sample: Asimit *et al.* [1], Beirlant *et al.* [2, 3, 4], Broniatowski [6], Diebolt *et al.* [7, 8], Dierckx *et al.* [9], Gardes *et al.* [10, 11, 12, 13, 18], Girard [14], Goegebeur *et al.* [15, 16] and Mercadier *et al.* [17]. Our main result is a general result on Weibull tail-distributions which has not been shown yet in the extreme-value literature.

Let us consider the mixture distribution

$$F(x) = 1 - p \exp \left\{ -x^{1/\theta_1} \ell_1(x) \right\} + (p - 1) \exp \left\{ -x^{1/\theta_2} \ell_2(x) \right\} \quad (1)$$

where  $\ell_1, \ell_2$  are slowly-varying functions,  $p \in (0, 1)$  and  $\theta_2 > \theta_1 > 0$ .

**Theorem.** *Let  $F$  be defined following (1). Then,  $F$  is a Weibull tail-distribution with associated Weibull tail-coefficient  $\theta_2$ .*

**Proof.** Let us consider

$$\begin{aligned} 1 - F(x) &= p \exp \left\{ -x^{1/\theta_1} \ell_1(x) \right\} + (1 - p) \exp \left\{ -x^{1/\theta_2} \ell_2(x) \right\} \\ &= \exp \left\{ -x^{1/\theta_2} \ell_2(x) \right\} \left( 1 - p + p \exp \left\{ -x^{1/\theta_1} \ell_1(x) + x^{1/\theta_2} \ell_2(x) \right\} \right) \end{aligned}$$

$$\begin{aligned}
&= \exp \left\{ -x^{1/\theta_2} \ell_2(x) \right\} \left( 1 - p + p \exp \left\{ -x^{1/\theta_1} \ell_1(x) (1 - x^{1/\theta_2 - 1/\theta_1} \ell_2(x) / \ell_1(x)) \right\} \right) \\
&= \exp \left\{ -x^{1/\theta_2} \ell_2(x) \right\} \left( 1 - p + p \exp \left\{ -x^{1/\theta_1} \ell_1(x) (1 + o(1)) \right\} \right) \\
&= \exp \left\{ -x^{1/\theta_2} \ell_2(x) \right\} (1 - p + p(1 + o(1))) \\
&= \exp \left\{ -x^{1/\theta_2} \ell_2(x) \right\} (1 + o(1)),
\end{aligned}$$

and the conclusion follows.

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Distribution	$\theta$
Gaussian $\mathcal{N}(\mu, \sigma^2)$	$1/2$
Gamma $\Gamma(\alpha, \lambda)$	$1$
Weibull $\mathcal{W}(\alpha, \lambda)$	$1/\alpha$

Table 1: Weibull tail-distributions