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Nonuniform cratering of the Moon and a

revised crater chronology of the inner solar system. Mathieu Le Feuvre<sup>\*</sup> and Mark A. Wieczorek

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#### 1 ABSTRACT

We model the cratering of the Moon and terrestrial planets from the present knowledge of the orbital and size distribution of asteroids and comets in the inner solar system, in order to refine the crater chronology method. Impact occurrences, locations, velocities and incidence angles are calculated semi-5 analytically, and scaling laws are used to convert impactor sizes into crater sizes. Our approach is generalizable to other moons or planets. The lunar cratering rate varies with both latitude and longitude: with respect to the 8 global average, it is about 25 % lower at  $(\pm 65^{\circ}N, 90^{\circ}E)$  and larger by the 9 same amount at the apex of motion  $(0^{\circ}N, 90^{\circ}W)$  for the present Earth-Moon 10 separation. The measured size-frequency distributions of lunar craters are rec-11 onciled with the observed population of near-Earth objects under the assump-12 tion that craters smaller than a few kilometers in diameter form in a porous 13 megaregolith. Varying depths of this megaregolith between the mare and high-14 lands is a plausible partial explanation for differences in previously reported 15 measured size-frequency distributions. We give a revised analytical relation-16 ship between the number of craters and the age of a lunar surface. For the 17 inner planets, expected size-frequency crater distributions are calculated that 18 account for differences in impact conditions, and the age of a few key geologic 19 units is given. We estimate the Orientale and Caloris basins to be 3.73 Ga old, 20 and the surface of Venus to be 240 Ma old. The terrestrial cratering record 21 is consistent with the revised chronology and a constant impact rate over the 22 last 400 Ma. Better knowledge of the orbital dynamics, crater scaling laws and 23 megaregoltih properties are needed to confidently assess the net uncertainty 24 of the model ages that result from the combination of numerous steps, from 25 the observation of asteroids to the formation of craters. Our model may be 26

inaccurate for periods prior to 3.5 Ga because of a different impactor population, or for craters smaller than a few kilometers on Mars and Mercury, due
to the presence of subsurface ice and to the abundance of large secondaries,
respectively. Standard parameter values allow for the first time to naturally
reproduce both the size distribution and absolute number of lunar craters up
to 3.5 Ga ago, and give self-consistent estimates of the planetary cratering
rates relative to the Moon.

<sup>8</sup> Keywords: cratering; Moon; terrestrial planets; crater chronology; impact

9 processes

#### 1 1 Introduction

The counting of impact craters offers a simple method to estimate the ages of 2 geologic units on planetary surfaces when in situ rock samples are lacking. The 3 crater chronology method is based on the simple idea that old surfaces have accumulated more impact craters than more recent ones (Baldwin, 1949). The relationship between geologic age and number of lunar craters, based on the ra-6 diometric dating of existing lunar rock samples, is found to be approximately 7 linear from the present to about 3 Ga ago, and approximately exponential beyond that time (Neukum et al., 2001a; Stöffler and Ryder, 2001, and references therein). For statistical robustness, craters are generally counted within 10 a number of consecutive diameter ranges, allowing one to recognize certain 11 biases, such as erosion, resurfacing or crater saturation processes. Measure-12 ments over various geologic units has led to the postulate that the relative 13 shape of the size-frequency distributions (SFD) of lunar impact craters was 14 similar for all surfaces, but unfortunately, the exact shape of this production 15 function in the 2–20 km kilometer range is still debated after decades of study. 16 The total predicted size-frequency distribution for any given time is obtained 17 by multiplying the production function, assumed independent of age, by a 18 time-variable constant. The age of a geologic unit is estimated by finding the 19 best fit between the standardized and measured distributions. 20

The approach taken in this work models directly crater distributions from the current knowledge of the impactor population. This allows us to infer properties of the impact history of a planetary body that the sole observation of craters could not reveal. In particular, whereas the crater chronology method assumes that craters accumulate uniformly on the surface of the planetary

body, this method allows to quantify possible spatial variations in the impact
rate. Moreover, in the absence of dated samples with known geological context
from Mercury, Venus and Mars, the generalization of the crater chronology to
these planets requires an estimate of the relative cratering rates with respect
to the Moon: in the same period of time, the number of craters of a given
size that form on two planets differ according to both the different impact
probabilities of the planet crossing objects and the impact conditions (e.g.,
impact velocity, surface gravity).

Using this bottom-up approach, Neukum et al. (2001b) proposed age estimates of mercurian geologic features based on a Mercury/Moon cratering rate ra-10 tio estimated from telescopic observations. Similarly, Hartmann and Neukum 11 (2001) adapted the lunar cratering chronology to Mars, using the Mars/Moon 12 cratering rate ratio of Ivanov (2001). The venusian surface, which contains a 13 small number of craters that appear to be randomly distributed, has been the 14 subject of several attempts of dating. The most recent estimates can be found 15 in Korycansky and Zahnle (2005) and McKinnon et al. (1997), and are both 16 based on the population of Venus crossers as estimated by Shoemaker et al. 17 (1990). In this study, we use improved estimates of the orbital characteristics 18 and size-frequency distribution of the impactor population. 19

The early lunar cratering record indicates that impactors were hundred of times more numerous than today, possibly due to the massive injection in the inner system of main-belt and/or Kuiper-belt objects about 700 Ma after the Moon formed, an event known as the Late Heavy Bombardment (LHB) (see Gomes et al., 2005; Tera et al., 1974; Hartmann et al., 2000). More than 3 Ga ago, the impactor population appears to have reached a relative state of equilibrium, being replenished both in size and orbit by, respectively, collisions

inside the main asteroid belt and the ejection by resonances with the giant
planets (Bottke et al., 2002). Under the assumption of a steady state distribution of impactors, the distribution of craters on ~ 3 Ga old surfaces should
be consistent with the present astronomically inferred cratering rates.

The lunar cratering record, in the form of the standardized production func-5 tions, agrees reasonably well with telescopic observations of planet-crossing ob-6 jects down to a few kilometers in diameter (Stuart, 2003; Werner et al., 2002). Smaller craters appear to be not numerous enough to have been formed by an impacting population similar to the present one. But, in Ivanov and Hartmann (2007), it was suggested that the presence of a porous lunar megaregolith may 10 reduce the predicted size of small craters, accounting for this observation. In 11 Strom et al. (2005), the distinction was made between pre- and post-LHB lu-12 nar crater distributions. Older distributions, depleted in small craters, would 13 reflect the SFD of Main Belt asteroids - massively provided by the LHB -14 rather than the SFD of Near-Earth asteroids. Marchi et al. (2009) revised the 15 crater chronology method by deriving a new production function from impact 16 modeling that differs significantly from those based on measurements. In this 17 study, we attempt to fully reconcile the measured lunar production functions 18 with the telescopic observations of planet-crossing objects. 19

Modeling the impact bombardment also allows us to quantify spatial cratering asymmetries that are not accounted for in the traditional crater chronology method. The presence of such asymmetries would bias the ages based on crater densities according to the location of the geologic unit. The Moon is subject to nonuniform cratering, since it is not massive enough to gravitationally homogenize encounter trajectories. Latitudinal asymmetries are produced by the anisotropy of encounter inclinations (Le Feuvre and Wieczorek, 2008), whereas

longitudinal asymmetries result from synchronous rotation since the relative
encounter velocity, hence impact rate, are maximized at the apex of motion
(e.g., Morota et al., 2005). In addition, the Earth may focus low inclination
and velocity projectiles onto the nearside of the satellite or act as a shield at
small separation distances.

Several studies have attempted to give estimates of the lunar cratering asym-6 metries. Wiesel (1971) used a simplified asteroid population, and Bandermann and Singer (1973) used analytical formulations based on strongly simplifying assumptions in order to calculate impact locations on a planet. These formulations did not allow to investigate any latitudinal effects. Wood (1973) 10 numerically integrated the trajectories of ecliptic projectiles; Pinet (1985) nu-11 merically studied the asymmetries caused by geocentric projectiles that were 12 potentially present early in the lunar history. Horedt and Neukum (1984), 13 Shoemaker and Wolfe (1982), Zahnle et al. (1998) and Zahnle et al. (2001) all 14 proposed analytical formulations for the apex/antapex effect, but the range of 15 predicted amplitudes is very large (the first, in particular, claimed that this ef-16 fect is negligible for the Moon). Moreover, these four studies based their results 17 on isotropic encounter inclinations. Gallant et al. (2009) numerically modeled 18 projectile trajectories in the Sun-Earth-Moon system, and reported a signif-19 icant apex/antapex effect. In our study, we derive a semi-analytic approach 20 for calculating cratering rates of synchronously locked satellites, and apply 21 it to the Moon. This method yields results that are nearly identical to full 22 numerical simulations, is computationally very rapid, and easily generalizable 23 to other satellites. 24

The "asteroid to crater" modeling requires several steps to create synthetic crater distributions. First, one needs to know the impactor population as

function of orbital elements, size and time. Second, impact probabilities a 1 are calculated over precession and revolution cycles of both the projectile 2 and target, based on geometrical considerations (Öpik, 1951; Wetherill, 1967; Greenberg, 1982; Bottke and Greenberg, 1993). As these impact probabilities predict typically that a given planetary crosser will collide with a given planet over timescales of about 10 Ga, whereas the typical lifetime of these 6 objects is thought to be a thousand times lower (Michel et al., 2005) as a result of ejection from the solar system or collision with another body, the 8 calculated bombardment must be seen as the product of a steady-state pop-9 ulation, where vacant orbital niches are continuously reoccupied (Ivanov and 10 Hartmann, 2007). Third, scaling laws derived from laboratory experiments 11 and dimensional analyses are used to calculate the final crater size produced 12 for given a impact condition (e.g., impactor size, velocity and cohesion of 13 the target material), for which several different scaling laws have been pro-14 posed (Schmidt and Housen, 1987; Holsapple and Schmidt, 1987; Gault, 1974; 15 Holsapple, 1993; Holsapple and Housen, 2007). Subsequent gravitational mod-16 ification of the transient cavity that gives rise to the final crater size have been 17 deduced from crater morphological studies (Pike, 1980; Croft, 1985). 18

The rest of this paper is organized as follows. In the following section, we 19 describe the employed orbital and size distribution of the impactor population. 20 In section three, the necessary equations used in generating synthetic cratering 21 rates are summarized; the full derivations are given in the appendix. In section 22 four, we compare our synthetic lunar crater distribution with observations, we 23 describe the lunar spatial asymmetries, and give the predicted planet/Moon 24 cratering ratios. We also provide simple analytic equations that reproduce 25 the predicted cratering asymmetries on the Moon, as well as the latitudinal 26

<sup>1</sup> variations predicted for the terrestrial planets in Le Feuvre and Wieczorek

<sup>2</sup> (2008). Section five is dedicated to revising the crater chronology method.

 $_{\scriptscriptstyle 3}$   $\,$  New age estimates are proposed for geologic units on the Moon, Earth, Venus

<sup>4</sup> and Mercury.

#### **5 2** Impactor population

Let us denote a, e, i and d respectively the semi-major axis, eccentricity,
inclination and diameter of those objects whose orbits can intersect the inner
planets. The entire population can be written

$$n(>d, a, e, i) = \bar{n}(>1) \times o(a, e, i) \times s(>d) , \qquad (1)$$

where n can be expressed as the product of  $\bar{n}(d > 1)$ , the total number of 9 objects with a diameter greater than 1 km; o(a, e, i), the relative number of ob-10 jects with a given set of orbital elements, normalized so that  $\int o(a, e, i) da de di =$ 11 1; and s(>d), the normalized cumulative number of objects larger than a given 12 diameter, such that s(> 1 km) = 1. This formulation assumes that no corre-13 lations exist between the size of the object and its orbit, which is consistent 14 with the observations of Stuart and Binzel (2004) for diameters ranging from 15  $\sim 10$  m to  $\sim 10$  km. 16

### 17 2.1 Orbital distribution

The orbital distribution of near Earth-objects (NEOs) is taken from the model of Bottke et al. (2002), which provides a debiased estimate of the orbital distribution of bolides that can potentially encounter the Earth-Moon system.

This model assumes that the NEO population is in steady-state, continuously replenished by the influx coming from source regions associated with
the main asteroid belt or the transneptunian disk. The model was determined
through extensive numerical integrations of test particles from these sources,
and calibrated with the real population observed by the Spacewatch survey.
The relative number of objects is discretized in orbital cells that spans 0.25
AU in semi-major axis, 0.1 in eccentricity, and 5° in inclination.

In order to model the martian impact flux, we have amended this model by 8 including the known asteroids that cross the orbit of Mars, but which are not part of the NEO population. These are taken from the database provided by E. 10 Bowell (Lowell Observatory), that gives the orbit and absolute magnitude H11 of telescopic discoveries. We consider the population of H < 15 objects to be 12 the best compromise between a sufficient number of objects (in order to avoid 13 sparseness in the orbital space), and completeness. Their distribution as a 14 function of perihelion is very similar in shape to brighter (hence larger) H < 1315 objects (see Le Feuvre and Wieczorek, 2008), the latter being large enough 16 to ensure they do not suffer observational bias in the martian neighborhood. 17 The H < 15 population is therefore considered as complete, and is scaled to 18 match the modeled NEO population. From the relationship between absolute 19 magnitude and geometric albedo of Bowell et al. (1989), and a mean albedo 20 of 0.13 (Stuart and Binzel, 2004), H < 15 corresponds to d > 4 km. 21

22 2.2 Size distribution

The size distribution of impactors is taken from a compilation of various estimates of the size-dependent impact rate on Earth, as shown in figure 1. This

compilation gathers atmospheric recordings of meteoroids and impact proba-1 bilities calculated with Opik equations from debiased telescopic observations. 2 Estimates from telescopic observations are those of Rabinowitz et al. (2000), Morbidelli et al. (2002), Harris, A. W. (2002) and Stuart and Binzel (2004). Estimates based on atmospheric recordings include Halliday et al. (1996). ReVelle (2001) and Brown et al. (2002). Concerning the LINEAR survey, esti-6 mates of Harris, A. W. (2002) have been scaled at large sizes to the estimates of Stuart and Binzel (2004). In contrast to Morbidelli et al. (2002) and Stuart 8 and Binzel (2004), Rabinowitz et al. (2000) did not use a debiased albedo 9 distribution, but rather a constant mean geometric albedo of 0.1 in order to 10 convert their telescopic observations from magnitude to diameter at all sizes. 11 Though small objects are expected to possess a larger albedo as they are gener-12 ally younger than large objects, we did not attempt to correct for this effect, 13 since estimates of Rabinowitz et al. (2000) show a general agreement with 14 other studies. In atmospheric flash estimates, masses have been deduced from 15 kinetic energy, the latter being estimated from luminous energy. We convert 16 kinetic energies of Brown et al. (2002) into diameters using a mean density 17 of 2700 kg m<sup>-3</sup> and a mean impact velocity of 20 km s<sup>-1</sup> (Stuart and Binzel, 18 2004). In order to increase the range of sizes in our compilation, and to reduce 19 the statistical uncertainties associated with the larger objects, we have also 20 included the size-frequency distribution of Mars-crossing objects with sizes 21 greater than 4 km, and scaled these to the terrestrial impact rates of Stuart 22 and Binzel (2004). 23

Various assumptions have led to all these estimates. Among them, the assumed impact velocity and bolide density are only of moderate influence. As an example, varying the density from 2700 to 2000 kg m<sup>-3</sup>, or the mean impact

velocity from 20 to 17 km s<sup>-1</sup>, changes the estimates of Brown et al. (2002)
only by about 10 %. Of major influence are the luminous efficiency used to
obtain kinetic energy from flashes (see Ortiz et al., 2006), and the debiasing
process in the case of telescopic observations, which are difficult to assign a
statistical uncertainty to. Consequently, we simply fit a 10th-order polynomial
to the entire dataset, assuming each data is error free, and that the average
combination of all estimates gives a good picture of the impactor population.

8

18

### [Table 1 about here.]

We express the resulting analytic size-frequency distribution as the product 9 of two terms: the normalized size distribution  $\log s(>d) = \sum_{i=0}^{10} s_i (\log d_{\rm km})^i$ , 10 whose coefficients are listed in table 1, and the Earth's impact rate for objects 11 larger than 1 km,  $\bar{\phi}_e(d > 1) = 3.1 \times 10^{-6} \text{ Ga}^{-1} \text{ km}^{-2}$ . An overbar is appended 12 to the Earth's impact rate symbol, denoting that this quantity is spatially av-13 eraged over the planet's surface. The size-frequency distribution of impactors 14 is here assumed to be the same for all bodies in the inner solar system. The 15 relative impact rates for these bodies with respect to Earth are calculated in 16 Section 3 using the orbital distribution of the planet crossing objects. 17

[Fig. 1 about here.]

From asteroids to craters

Here we describe how is calculated the number of craters that form on the Moon and planets, per unit area and unit time, as a function of the crater diameter and location on the surface. We first need to calculate the encounter conditions generated by the impactor population, then the corresponding im-

- <sup>1</sup> pact rate and conditions (impact velocity and incidence angle), in order to
- <sup>2</sup> finally obtain a cratering rate by the use of impact crater scaling laws. For the
- <sup>3</sup> reader's convenience, derivations are given in the appendix.

#### 4 3.1 Encounter probability

Following Öpik (1951), the assumptions under which an encounter is considered to occur can be summarized as follows:

An encounter between the target (Moon or planet) and impactor occurs at the geometrical point of crossing of the two orbits (the mutual node).
The geometry of encounter is given by the relative velocity vector U at this point, which is expressed here in a frame where the X-axis points towards the central body (planet or Sun), (XY) defines the target's orbital plane, and the Z-axis points upward.

The relative encounter velocity does not account for the acceleration generated by the mass of the target.

The impactor, as seen by the target, is treated as if it were approaching from
an infinite distance, under only the gravitational influence of the target.
The encounter trajectory is therefore hyperbolic in the reference frame of
the target.

For a given U, there are an infinite number of hyperbolic trajectories that
 can actually strike the target, that are distributed uniformly on a circle
 perpendicular to U with a surface equal to the target's gravitational cross
 section.

These approximations hold as long as the radius of the target's Hill sphere
is large enough with respect to the target size. We performed three-body
numerical simulations that show that a factor of ten between the Hill sphere
and target radii suffices to ensure the validity of the above approximations.
For the terrestrial planets, this condition is largely verified. For the Moon, it
corresponds to a minimum Earth-Moon separation of ~ 17 Earth's radii.

In the case of planets in nearly circular orbits, the encounter geometry **U** and probability P (providing the two orbits intersect) are simply given by the well known Öpik equations (Öpik, 1951). In order to account for the eccentricity 9 of the target (which is important for Mercury and Mars, but not for the 10 Earth), we use the improved formulation of Greenberg (1982) and Bottke and 11 Greenberg (1993). The probability is largest for low inclination encounters, 12 and for encounters occurring near the projectile's pericenter and apocenter. 13 Singularities of the encounter probability are avoided following Dones et al. 14 (1999). For a given orbital geometry, the encounter probability is proportional 15 to the gravitational cross section, whose radius is

$$\tau = R\sqrt{1+\Gamma} , \qquad (2)$$

<sup>17</sup> where R is the target's radius and

$$\Gamma = 2\frac{GM}{RU^2} \tag{3}$$

<sup>18</sup> is the Safronov parameter, with G the gravitational constant and M the tar-<sup>19</sup> get's mass.

Note that the calculated impact probabilities are long-term averages over precession cycles of both the projectile and target (i.e., the longitude of node and argument of pericenter can take any value between 0 and  $2\pi$ ). We account

for secular variations of the planetary orbital elements using the probability
distributions given as a function of time in Laskar (2008). Our results are
only sensitive to secular variations for Mars, and in the following, two values
are quoted for this planet, that correspond to the present day value and to a
long-term average (1 Ga and 4 Ga averages yield nearly identical results).

<sup>6</sup> By calculating the encounter probability P and velocity **U** associated with a <sup>7</sup> given orbital element set (a, e, i), by weighting this probability with the relative <sup>8</sup> number of objects o(a, e, i), and by summing over the entire planet-crossing <sup>9</sup> population for each terrestrial planet, we build the probability distribution of <sup>10</sup> the encounter conditions,  $p(\mathbf{U})$ , using bins of 1 km s<sup>-1</sup> for each component of <sup>11</sup> the encounter velocity. Analytically, we have

$$p(\mathbf{U}) = \frac{p'(\mathbf{U})}{\int_{\mathbf{U}} p'(\mathbf{U}) \, \mathrm{d}\mathbf{U}} \,, \tag{4}$$

12 with

$$p'(\mathbf{U}) = \int_{\mathcal{D}} P(\mathcal{D}) \ o(a, e, i) \ \delta(\mathbf{U}(\mathcal{D}) - \mathbf{U}') \ \mathrm{d}\mathcal{D} , \qquad (5)$$

<sup>13</sup> where the integration is performed over the 8-dimensional domain <sup>14</sup>  $\mathcal{D} = (U'_X, U'_Y, U'_Z, a, e, i, w_0, \Delta \Omega)$ , with  $w_0$  the target's argument of perihelion, <sup>15</sup>  $\Delta \Omega$  the difference between the target and projectile's longitudes of the ascend-<sup>16</sup> ing node (see Greenberg, 1982), and  $\delta$  the Kronecker function which equals 1 <sup>17</sup> when  $\mathbf{U} = \mathbf{U}'$  and 0 otherwise. The impact rate relative to the Earth is

$$r = \frac{\int_{\mathbf{U}} p'(\mathbf{U}) \, \mathrm{d}\mathbf{U}}{\int_{\mathbf{U}} p'_e(\mathbf{U}) \, \mathrm{d}\mathbf{U}} \,. \tag{6}$$

where  $p'_e$  is calculated from equation 5 with the Earth being the target.

The lunar case requires a specific treatment, which is detailed in appendix A.2.
For simplicity and without altering the results, it is assumed that the lunar
orbit is circular about the Earth and possess a zero inclination with respect

to the ecliptic. We first calculate encounters probabilities P' and velocities
V with the entire Earth-Moon system, whose expression is the same as for
the Earth, except that the gravitational cross section radius is replaced in the
Öpik equation by what we call here the lunar orbit cross section, defined as
(Zahnle et al., 1998)

$$\tau' = a_m \sqrt{1 + 2\frac{v_m^2}{V^2}} \; ,$$

(7)

where  $a_m$  and  $v_m$  are respectively the lunar semi-major axis and velocity. On the cross sectional disk, the distance from the center is denoted by the impact parameter b. Only when  $b \leq \tau'$  is it possible for a hyperbolic orbit to impact the Moon (note that the gravitational cross section of the Moon itself is accounted for later in the calculation, see appendix A.2). The probability distribution of 10 encounter conditions  $p(\mathbf{V})$  is first calculated according to this new definition 11 of the encounter probability. Then, the relative lunar encounter velocity U 12 and probability  $P_m$  are derived analytically for each hyperbolic orbit crossing 13 the Earth-Moon system (equations A.19 - A.18, A.24 - A.25, A.30 - A.33). 14 The probability distribution of lunar encounter conditions,  $p(\mathbf{U})$ , is then de-15 termined by integrating numerically  $P_m$  for all possible hyperbolic orbits of 16 each encounter V, and for all encounter velocities. Similarly, the impact prob-17 ability with Earth  $P_e$  is determined for each hyperbola, and the Moon/Earth 18 impact ratio r is calculated over all possible encounters. Mathematically,  $p(\mathbf{U})$ 10 and r are given by equations (A.34-A.36). 20

<sup>21</sup> Note that there is a dependence of the lunar impact rate on the Earth-Moon <sup>22</sup> separation distance,  $a_m$ . This distance has evolved outward with time, and we <sup>23</sup> test various separation distances in the simulations. A major difference be-<sup>24</sup> tween our approach and previous investigations (Shoemaker and Wolfe, 1982; <sup>25</sup> Zahnle et al., 1998, 2001) is that the argument of pericenter of the hyperbolic

<sup>1</sup> orbits is not assumed to precess uniformly within the Earth-Moon system,

<sup>2</sup> but is explicitly given by the encounter geometry. Our formulation allows to

<sup>3</sup> calculate explicitly lateral asymmetries in the lunar cratering rate.

4 3.2 Impact rate

5 Let us express the cumulative impact flux, that is the number of objects with

 $_{6}$  diameters greater than d that hit the planet per unit time and area, as

$$\phi(>d,\lambda,\varphi) = \bar{\phi}(>d) \times \Delta\phi(\lambda,\varphi) , \qquad (8)$$

<sup>7</sup> where  $\lambda$  and  $\varphi$  are respectively the latitude and longitude,  $\bar{\phi}(>d)$  is the <sup>8</sup> spatially-averaged impact rate for projectiles larger than d, and  $\Delta \phi(\lambda, \varphi)$  is the <sup>9</sup> relative impact rate as a function of position, normalized to the global average. <sup>10</sup> Using our normalized impactor SFD, the average impact rate expresses as

$$\bar{\phi}(>d) = \bar{\phi}(>1) \ s(>d) \ ,$$
 (9)

and  $\bar{\phi}(d > 1)$  is obtained from the impact ratio between the target and Earth, r, and the terrestrial impact rate  $\bar{\phi}_e(d > 1)$  as

$$\bar{\phi}(>1) = r \; \bar{\phi}_e(>1) \; .$$
 (10)

<sup>13</sup> The net spatial asymmetry  $\Delta \phi(\lambda, \varphi)$  is found by integrating the spatial as-<sup>14</sup> symmetries  $\delta \phi(\lambda, \varphi, \mathbf{U})$  associated with each encounter geometry:

$$\Delta\phi(\lambda,\varphi) = \int_{\mathbf{U}} \delta\phi(\lambda,\varphi,\mathbf{U}) \ p(\mathbf{U}) \ \mathrm{d}\mathbf{U} \ , \tag{11}$$

where  $\delta \phi$  is given in appendix A.3 as a function of the Safronov parameter  $\Gamma$ and obliquity of the target (equations A.47 – A.49). The impact flux is homogeneous for  $\Gamma = \infty$ , that is, for encounter velocities negligible with respect to

the target's surface gravitational potential (equation 3). On the other hand, for  $\Gamma = 0$ , encounter trajectories are straight lines, and the impact flux is a simple geometrical projection of the spatially uniform encounter flux (on a plane perpendicular to the radiant) onto the target's spherical surface. The associated impact velocities and incidence angles are calculated from equations (A.51) and (A.54). The impact velocity u is only dependent on U and  $\Gamma$ , while the incidence angle  $\theta$  further depends on position.

55

#### <sup>8</sup> 3.3 Cratering rate

To obtain cratering rates from impact rates, we need to convert the impactor 9 diameters into crater diameters. For this purpose, we use equations that have 10 been derived in the framework of  $\pi$ -scaling dimensional analysis (Holsapple 11 and Schmidt, 1987; Holsapple, 1993), where the transient crater is given as 12 a function of the projectile diameter, impact velocity, surface gravity and 13 projectile/target density ratio (see appendix A.4). It is assumed that only the 14 vertical component of the impact velocity, whose value is obtained from the 15 impact angle, contributes to the crater size (Pierazzo et al., 1997), though 16 other relations could be easily incorporated into this analysis. The scaling 17 equation and parameters are taken from the summary of Holsapple and Housen 18 (2007) for the case of porous and non-porous scaling. It will be seen that both 19 formation regimes are necessary to reconcile the impactor and crater SFDs. 20 We only consider craters that form in the gravity regime, where the tensile 21 strength of rock is negligible, that is, craters larger than a few hundred meters 22 in competent rock, and larger than a few meters in consolidated soils. An 23 increase of the transient crater diameter by wall slumping and rim formation is 24

accounted for as given in Melosh (1989, 253 pp.). Finally, large craters collapse due to gravity, becoming complex craters, and the relationship between simple and complex crater diameters is taken from Holsapple (1993). Putting this altogether, we obtain the relation  $d(D, u, \theta)$  that gives the impactor diameter d as a function of the crater diameter D, impact velocity u and incidence angle  $\theta$  (appendix A.4, equations (A.63 – A.59)). The impactor diameter drequired to create a crater of size D is ultimately a function of  $\lambda$ ,  $\phi$  and U (see equations A.51 and A.54).

The cratering rate, that is the number of craters larger than D that form at
(λ, φ) per unit time and area, is

$$C(>D,\lambda,\varphi) = \int_{\mathbf{U}} \phi(>d,\lambda,\phi) \ p(\mathbf{U}) \ \mathrm{d}\mathbf{U} , \qquad (12)$$

11 where

$$d = d(D, \lambda, \phi, \mathbf{U}) . \tag{13}$$

<sup>12</sup> For convenience, we separate the cratering rate into

 $C(>D,\lambda,\varphi) = \bar{C}(>D) \times \Delta C(>D,\lambda,\varphi) .$ (14)

where  $\bar{C}(>D)$  is the spatially averaged rate and  $\Delta C(>D, \lambda, \varphi)$  is the relative spatial variation. Note that  $\Delta C$  depends on D, though in practice, this dependence is moderate (see next section).

#### 1 4 Results

#### 2 4.1 Crater size-frequency distributions

We first present our synthetic size-frequency distribution of lunar craters. Following the terminology of Marchi et al. (2009), we refer to this as a model production function. We compare our model production function with the two standard measured production functions of Neukum (Neukum, 1983, 186 pp.; Neukum and Ivanov, 1994) and Hartmann (Basaltic Volcanism Study Project, 1981; Hartmann, 1999). We note that the two are in good agreement over the crater diameter range from 300 m to 100 km, but differ between 2 and 20 km, with a maximum discrepancy of a factor 3 at 5 km.

Using the traditional non-porous scaling relations and a standard target den-11 sity of 2800 kg m<sup>-3</sup>, we calculate that  $2.88 \times 10^{-11}$  craters larger than 1 km 12 would be created each year on the lunar surface by the present impactor pop-13 ulation. Using the time-dependence established by Neukum (1983, 186 pp.) 14 that predicts a quasi-constant impact flux over the last  $\sim 3$  Ga, the Hartmann 15 and Neukum production functions return respective values of  $7.0 \times 10^{-13}$  and 16  $8.2 \times 10^{-13}$ , which are about forty times lower, implying that the present flux 17 must be considerably larger than the time averaged value. 18

However, by using the porous scaling law instead, in order to account for the presence of megaregolith on the lunar surface, our calculated spatially averaged lunar cratering rate is

$$\bar{C}_m(D>1) = 7.5 \times 10^{-13} \text{ yr}^{-1} \text{ km}^{-2}$$
, (15)

<sup>22</sup> a value in excellent agreement with the two measured production functions

<sup>1</sup> under the assumption of a constant impact flux over the last  $\sim 3$  Ga.

Let us now reconcile the entire shape of the measured production functions 2 with the observed impactor population. As shown in figure 2, the two mea-3 sured distributions are very well fitted by using the porous regime for small craters (D < 2 km), and the non-porous regime for larger craters (D > 205 km). We model a simple smooth transition between the two regimes by con-6 sidering that the impactor size d required to create a crater of diameter D is a linear combination of the sizes required from the porous and non-porous scaling relations, the influence of each regime depending on the depth of material 9 excavated by the crater. The depth of excavation  $z_T$  is about 1/10 of the tran-10 sient crater diameter  $D_T$ , and does not seem to depend on target properties 11 (Melosh and Ivanov, 1999). The impactor size is averaged over the depth of 12 excavation: 13

$$d = \frac{1}{z_T} \int_0^{z_T} d_z(z) \, \mathrm{d}z \,, \tag{16}$$

where  $d_z$  is the impactor size required by the material at depth z, given by the porous regime at the surface, by the non-porous regime at depths larger than a given "megaregolith thickness" T, and by a linear combination between z = 0 and z = T:

$$d_z(z) = \frac{1}{T} \left( (T-z) \ d_p + z \ d_{np} \right) \text{ for } z \le T ,$$

$$d_z(z) = d_{np} \qquad \text{ for } z \ge T ,$$
(17)

<sup>18</sup> with  $d_p$  and  $d_{np}$  the impactor diameters respectively required from the porous <sup>19</sup> and non-porous regimes. In the calculation of  $d_{np}$ , the target density is set to <sup>20</sup> 2800 kg m<sup>-3</sup> (solid rocks), whereas we assume in calculating  $d_p$  that the density <sup>21</sup> of the porous material is 2500 kg m<sup>-3</sup>, based on Bondarenko and Shkuratov <sup>22</sup> (1999, abstract no. 1196) who inferred an upper regolith density comprised

<sup>1</sup> between 2300 and 2600 kg m<sup>-3</sup> from correlations between the surface regolith <sup>2</sup> thickness and the Soderblom's crater parameter (Soderblom and Lebofsky, <sup>3</sup> 1972). We note that given the simplicity of our crater-scaling procedure in the <sup>4</sup> transition zone, the correspondance between T and the actual megaregolith <sup>5</sup> thickness should not be expected to be exact.

As shown in figure 2, our model reproduces both the Hartmann and Neukum production functions within the 100 m - 300 km diameter range, for respective values of T equal to 250 and 700 m, respectively. For these diameter ranges, 8 the maximum discrepancy between our model and the Neukum production 9 function is only 30 % at 200 m, and always less than 20 % for craters larger 10 than 500 m. Below 100 m, we note that our model is in reasonable agreement 11 with the Neukum production function, and we leave the implications for the 12 contribution of secondary craters to further investigations. The model pro-13 duction function proposed by Marchi et al. (2009) is also shown in figure 2. A 14 detailed comparison with this latter study is given in the discussion section. 15

#### 16

### [Fig. 2 about here.]

The use of porous scaling was first suggested by Ivanov (2006) (see also Ivanov, 17 2008; Ivanov and Hartmann, 2007), and is a natural consequence of a highly 18 fractured megaregolith on airless bodies. Also natural is to expect that the 19 thickness of the megaregolith will depend upon both age and local geology. 20 We note that the need for a transition regime falls within the diameter range where the measured production functions differ the most (excluding very large 22 craters). We suspect that this is partially a result of the Hartmann produc-23 tion function being based on crater counts performed solely over mare units, 24 whereas the Neukum production function also includes older highlands ter-

<sup>1</sup> rains (Neukum et al., 2001a).

Estimated megaregolith thicknesses are roughly consistent with seismic models of the lunar crust (e.g., Warren and Trice, 1977; Lognonné et al., 2003) 3 that generally predict reduced seismic velocities for the upper km, which is 4 attributed to an increased porosity and fractures. Furthermore, seismic data at the Apollo 17 landing site, overlaid by mare basalt, indicates that the upper 250 / 400 m show a very low P-wave velocity with respect to the deeper basalt (Kovach and Watkins, 1973; Cooper et al., 1974), the lower estimate being in 8 agreement with our calculated megaregolith depth of 250 m for the Hartmann 9 production function. Finally, Thompson et al. (1979) show by analysis of radar 10 and infrared data (which are dependent on the amount of near surface rocks) 11 that craters overlying highlands show different signatures for craters greater 12 and less than 12 km, and that mare craters down to 4 km in diameter possess 13 a similar signature to that of highlands craters greater than 12 km. They at-14 tribute this difference to the presence of a pulverized megaregolith layer that 15 is thicker in the older highlands than the younger mare. 16

By the use of a porous regime dictated by the properties of a megaregolith, our model production function reproduces the measured crater distributions in shape and in the absolute number of craters formed over the past 3 Ga, under the assumption of a constant impact flux. We caution that our simple formulation of the porous / non-porous transition does not account for the temporal evolution of the megaregolith and that the inferred megaregolith thicknesses are only qualitative estimates.

The present Earth-Moon distance has been used in the above calculation of the lunar cratering rate, and temporal variations in the lunar semimajor axis

could, in principle, modify the Earth/Moon impact ratio and encounter velocity distribution with the Moon. Nevertheless, it is found in our simulations
that, for a lunar semi-major axis as low as 20 Earth radii, the average lunar
cratering rate is changed by less than 3%. This implies that both the shielding
and gravitational focusing of projectiles by the Earth are of very moderate
effects, especially since the Moon is believed to have spent the vast majority
of its history beyond 40 Earth radii.

8

### [Table 2 about here.]

Our globally averaged planetary cratering rates  $\bar{C}(>D)$  are fitted by 10th-9 order polynomials for the Moon and inner planets. The coefficients (with units 10 of  $yr^{-1}$  km<sup>2</sup>) are listed in table 2. Since the megaregolith thickness is not 11 necessarily the same on each planet, and may depend on the age and geology 12 of the surface, coefficients are given for the two scaling regimes  $(T = \infty$  and 13 T = 0 km) for diameters between 0.1 and 1000 km (except for the Earth and 14 Venus, where only non-porous scaling is given). A linear transition simpler 15 than ours can be used by defining two threshold diameters,  $D_p$  and  $D_{np}$ , such 16 that the porous and non-porous regime applies alone respectively below  $D_p$ 17 and above  $D_{np}$ . The cratering rate in the transition regime is then calculated 18 as  $C(>D) = C_p(>D_p) + \frac{C_{np}(>D_{np}) - C_p(>D_p)}{D_{np} - D_p}(D - D_p)$ , where  $C_p$  and  $C_{np}$  are 19 given in table 2 in the porous and non-porous columns, respectively. Note that 20 the martian cratering rate is sensitive to the eccentricity of the planet, since 21 the number of potential impactors increases dramatically as this planets gets 22 closer to the Main Asteroid Belt (Le Feuvre and Wieczorek, 2008; Ivanov, 23 2001). In addition to calculating the present day martian cratering rate, we 24 also used the probability distribution of the martian eccentricity provided by 25 Laskar (2008) to calculate an average over the past 1 Ga (note that this value 26

<sup>1</sup> is nearly insensitive for longer averages).

<sup>2</sup> Planetary size-frequency distributions are generally expressed with respect to

the lunar one. This is done by defining the size-dependent quantity  $R_c$ , which

<sup>4</sup> is the cratering ratio with respect to the Moon,

$$R_c(>D) = \frac{\bar{C}(>D)}{\bar{C}_m(>D)}$$
.

(18)

For illustrative purpose,  $R_c$  is shown for the inner planets in figure 3 by assuming that craters with diameters less than 10 km form in a porous soil on both the planet and Moon, while craters with greater sizes form in solid rocks (except for the Earth and Venus where only the non-porous regime is used). Note that  $R_c$  can be easily calculated from equation 18 and table 2 for a different (and more realistic) transition between porous and non-porous regimes.

[Fig. 3 about here.]

12

11

### [Fig. 4 about here.]

The mean impact velocity on the Moon is calculated to be  $\bar{u}_m = 19.7$  km 13  $s^{-1}$ . The full probability distribution of impact velocities for each planet is 14 shown in figure 4. The quantities  $\bar{\phi}/\bar{\phi}_m$ ,  $\bar{u}/\bar{u}_m$  and  $g/g_m$ , that give the relative 15 impact flux, impact velocity and surface gravity with respect to the Moon, 16 are given in table 3 for the inner planets. Mars experiences a high impact rate 17 with respect to the Moon (about 3) due to its proximity to the main asteroid 18 belt. In comparison, the martian cratering ratio is reduced (between about 19 0.5 and 2.5) because the impact velocity on Mars is significantly lower than 20 on the Moon, requiring larger (and hence less numerous) impactors to create 21 a crater of a given size. Mercury exhibits also a high impact rate, and the 22 impact velocity is about twice as large as on the Moon, resulting in a high 23

value of the cratering ratio  $R_c$ , comprised between 2 and 4. The impact rate is larger on the Earth and Venus than on the Moon, as these planets possess a higher gravitational attraction. Their higher surface gravities compensate the differences in impact velocities with the Moon, and  $R_c$  is comprised between 5 0.5 and 1.5 for the Earth and between 1 and 2 for Venus.

6

14

[Table 3 about here.]

50

#### 7 4.2 Spatial variations

<sup>8</sup> The relative spatial cratering variations on the Moon,  $\Delta C(>D, \lambda, \varphi)$ , are <sup>9</sup> shown in figure 5 for the present Earth-Moon distance of about 60 Earth <sup>10</sup> radii, and for crater diameters larger than 1 km. The cratering rate varies <sup>11</sup> from approximately -20% to +25% with respect to the global average. It is <sup>12</sup> minimized at about ( $\pm 65^{\circ}N, 90^{\circ}E$ ), whereas the maximum, which is a factor <sup>13</sup> 1.5 higher, is located at the apex of motion ( $0^{\circ}N, 90^{\circ}W$ ).

### [Fig. 5 about here.]

Two effects conjugate to give such a distribution. First, a latitudinal effect, 15 detailed in Le Feuvre and Wieczorek (2008), comes from the higher propor-16 tion of low inclination asteroids associated with the higher probabilities of low 17 inclination encounters. The pole/equator ratio is 0.80. Second, a longitudinal 18 apex/antapex effect comes from the synchronous rotation of the Moon and 19 the higher relative encounter velocities at the apex. The lunar orbital velocity 20 is added to the projectile velocity for impacts at the apex, whereas it is sub-21 tracted at the antapex. The apex/antapex ratio is 1.37. We note that there is 22 a negligible nearside/farside effect: the nearside experiences the formation of 23

about 0.1% more craters that the farside. The Earth does indeed concentrate
very low inclination (and moderate velocity) projectiles onto the lunar nearside, but these are not numerous enough to influence the global distribution.

<sup>4</sup> The lateral cratering variations depend on the crater size since the size<sup>5</sup> frequency distribution of impactors s(> d) is not a simple power law, and
<sup>6</sup> the impact conditions are not everywhere the same. Nevertheless, the maxi<sup>7</sup> mum/minimum cratering ratio varies only by about 5% for D ranging from
<sup>8</sup> 30 m to 300 km. Consequently, for the following discussion, we shall consider
<sup>9</sup> that ΔC(> D, λ, φ) ≃ ΔC(> 1, λ, φ).

For smaller Earth-Moon distances  $a_m$ , the apex/antapex effect increases as the lunar velocity increases. Between 20 and 60 Earth radii, this dependency is fit by the simple equation

$$C(\text{apex})/C(\text{antapex}) = 1.12 \ e^{-0.0529 \frac{d_m}{R_e}} + 1.32 \ ,$$
 (19)

where  $R_e$  is the Earth radius. Over this range of Earth-Moon separations, latitudinal variations and the nearside/farside effect are found to vary by less than 1%. These calculations assume that the lunar obliquity stayed equal to its present value in the past.

Figure 6 plots the relative cratering rate  $\Delta C$  as a function of the angular distance from the apex of motion for the present Earth-Moon distance, and compares this with the counts of rayed craters with diameters greater than 5 km given in Morota et al. (2005). Rayed craters are younger than about 1 Ga (Wilhelms et al., 1987), which should corresponds to an Earth-Moon separation distance very close to the present one (Sonett and Chan, 1998; Eriksson and Simpson, 2000). As is seen, the model compares favorably to the

1 data.

2

[Fig. 6 about here.]

We note that the impact rate exhibits nearly the same behavior as the cra-3 tering rate, but with a reduced amplitude. The pole/equator ratio is 0.90. 4 whereas the apex/antapex ratio is 1.29. The latitudinal cratering variations 5 are enhanced with respect to the impact rate variations, as the mean impact 6 angle and impact velocity are smaller at the poles than at the equator (re-7 spectively by 2.5 degrees and 500 m/s), requiring a larger projectile to create 8 the same crater size. As large projectiles are less numerous than small ones, 9 the impact rate at the poles is smaller than the cratering rate (Le Feuvre and 10 Wieczorek, 2008). This is also true for the apex/antapex asymmetry, as the 11 average impact velocity is 500 m/s higher at the apex than at the antapex. 12 However, the increase is moderate, as the mean impact angle is only about 13 1.5 degree smaller. 14

It is seen in figure 7 that our predicted apex/antapex effect differs from that of Zahnle et al. (2001). These authors describe their variations of the impact rate as a function of  $\gamma$ , the angular distance to the apex, as

$$\Delta\phi(\gamma) = \left(1 + \frac{v_m}{\sqrt{2v_m^2 + \bar{V}^2}}\cos\gamma\right)^2 \,, \tag{20}$$

<sup>18</sup> where  $\bar{V} \simeq 19$  km s<sup>-1</sup> is the mean encounter velocity with the Earth-Moon <sup>19</sup> system. We are able to reproduce their analytical solution, but only under <sup>20</sup> the condition where we force the encounter inclinations with respect to the <sup>21</sup> lunar orbit plane to be isotropic in space. These authors used Öpik equations <sup>22</sup> (Shoemaker and Wolfe, 1982) for hyperbolic orbits that were assumed to pre-<sup>23</sup> cess uniformly inside the planet-moon system. We nevertheless point out that

<sup>1</sup> Zahnle et al. (2001) applied equation (20) to the moons of Jupiter, where this

<sup>2</sup> approximation might be valid.

3

We next provide analytical solutions for the relative variations of both the impact and cratering rates on the Moon. We also give solutions for the latitudinal variations presented on the terrestrial planets in Le Feuvre and Wieczorek
(2008). Two values are quoted for Mars, one that corresponds to its present
obliquity and eccentricity and the other to averaged results using variations
over 3 Ga as given in Laskar et al. (2004) (see Le Feuvre and Wieczorek, 2008).
Spatial variations in the impact flux and cratering rate are parameterized by
a sum a spherical harmonic functions

$$\Delta C(\lambda,\varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} C_{lm} Y_{lm}(\lambda,\varphi) , \qquad (21)$$

where  $Y_{lm}$  is the spherical harmonic function of degree l and order m,  $C_{lm}$  is the corresponding expansion coefficient, and  $(\lambda, \varphi)$  represents position on the sphere in terms of latitude  $\lambda$  and longitude  $\varphi$ , respectively. The real spherical harmonics are defined as

$$Y_{lm}(\lambda,\varphi) = \begin{cases} P_{lm}(\sin\lambda)\cos m\varphi & \text{if } m \ge 0\\ \\ P_{l|m|}(\sin\lambda)\sin |m|\varphi & \text{if } m < 0, \end{cases}$$
(22)

<sup>16</sup> and the corresponding unnormalized Legendre functions are listed in Table 4. <sup>17</sup> Many of the expansion coefficients are nearly zero since the cratering rate <sup>18</sup> is symmetric about both the equator and the axis connecting the apex and <sup>19</sup> antapex of motion. Only the most significant coefficients are listed in Tables 5 <sup>20</sup> and 6, which for most cases reproduce the data to better than 0.2%. The

<sup>1</sup> coefficients for the latitudinal variations on the terrestrial planets are listed in

 $_{2}$  Tables 7 and 8.

3 [Table 4 about here.]
4 [Table 5 about here.]
5 [Table 6 about here.]
6 [Table 7 about here.]
7 [Table 8 about here.]

#### **5** Crater chronology

Neukum et al. (2001a) (see also Neukum, 1983, 186 pp.; Strom and Neukum,
1988; Neukum and Ivanov, 1994) established empirically the following relationship between the number of craters with diameters greater than 1 km and
the age of the geologic unit:

$$\bar{N}(D > 1, t) = a \left( e^{bt} - 1 \right) + c t ,$$
 (23)

where  $\bar{N}(D > 1, t)$  is given per 10<sup>6</sup> km<sup>2</sup>, t is the age expressed in Ga, and 13  $a = 5.44 \times 10^{-14}$ , b = 6.93 and  $c = 8.38 \times 10^{-4}$ . This relationship is essentially 14 linear over the last 3.3 Ga (constant cratering rate in time) and approximately 15 exponential beyond. The data used to construct this empirical curve are ob-16 tained from radiometric ages of the Apollo and Luna rock samples, compared 17 to the crater density covering the associated geologic unit. We emphasize that 18 no agreed upon calibration data exist between 1 and 3 Ga and beyond 3.9 19 Ga (Stöffler and Ryder, 2001). We also note that equation (23) was originally 20

<sup>1</sup> obtained using age estimates of the highland crust and Nectaris impact basin,

<sup>2</sup> both of which are disputed and considerably older than 3.9 Ga.

Accounting for our calculated spatial variations, we first convert the measured crater density at a given site,  $N(D > 1, \lambda, \varphi)$ , into the corresponding spatially

<sup>5</sup> averaged quantity:

$$\bar{N}(D>1) = N(D>1,\lambda,\varphi)/\Delta C(D>1,\lambda,\varphi) .$$
(24)

The cratering asymmetry  $\Delta C$  is given in table 2, and we use the function that corresponds to the present Earth-Moon separation for cratered surfaces that are less than 1 Ga (consistent with the tidal deposit data of Sonett and Chan 8 (1998)). We choose a Earth-Moon separation of 40 Earth radii for units that 9 are older than 3 Ga (based on the tidal deposits data of Eriksson and Simpson 10 (2000)). This lunar semimajor axis value corresponds to a lunar orbital velocity 11 twice as large as the present one. We further assume that the lunar obliquity 12 was equal to its present value (nearly zero) for the entire time between 3.9 Ga 13 and the present. 14

The data used to calibrate the crater density versus age relationship are listed 15 in table 9, along with their corrections accounting for spatial variations in the 16 cratering rate. We use the crater density and ages values quoted in Stöffler and 17 Ryder (2001). We did not attempt to fit crater distributions with our model 18 production function to re-estimate N(D > 1) values, since our model already 19 reproduces very well the Neukum production function that was used to esti-20 mate this quantity. For the case of very young calibration surfaces, we suspect 21 that a thinner megaregolith might change the crater distribution with respect 22 to the Neukum production function at sizes larger than one kilometer, but, 23 for small exposure times, the largest observed craters are below this diameter

value. Based on the data and interpretations of Norman (2009), we exclude 1 the Nectaris basin from consideration, as it is possible that samples assigned 2 to this basin have instead an Imbrium provenance. We thus assume that the Descartes formation is not Nectaris ejecta, but rather Imbrium ejecta with an age of 3.85 Ga. We also exclude the Crisium basin from the fit, due to the uncertain provenance of samples dated at the Luna 20 site. Finally, we use 6 the recent crater counts performed on Copernicus deposits by Hiesinger et al. 7 (2010, abstract no 1533) with high-resolution Lunar Reconnaissance Orbiter 8 images, which agree with a constant impact flux during the last 800 Ma, in 9 contrast to previous studies. 10

As seen in table 9, the spatial correction is in general moderate, since the cali-11 bration terrains are located in the central portion of the nearside hemisphere, 12 far from the extrema of the spatially-dependent cratering rate. Nevertheless, 13 we point out that after correction, the Apennines, Fra Mauro and Descartes 14 formations, which are all Imbrian in age, exhibit nearly the same globally av-15 eraged crater density, which is consistent with our assignment of an Imbrium 16 origin to the Descartes formation, as recently suggested by Norman (2009) 17 (see also Haskin et al., 1998). 18

We perform a new fit of the calibration point, using the same functional form
as in equation 23. Our resulting best parameters are

$$a' = 1.893 \times 10^{-26}$$
  
 $b' = 14.44$  (25)  
 $c' = 7.960 \times 10^{-4}$ .

<sup>21</sup> and our proposed curve is shown in figure 8, along with the curves proposed by

Neukum et al. (2001a) and Marchi et al. (2009). It is seen that our model favors
a longer period of nearly constant impact flux (up to about 3.5 Ga), and that
the agreement with the calibration points is improved. Crisium basin is also
plotted (but was not included in the fit) for the tentative age of 3.895 given by
Swindle et al. (1991, abstract no 1229), showing an excellent agreement with
our model.

We note that the only calibration point that is not well fit to our relationship is the "young" group of basalts at the Apollo 11 landing site (point 9 in figure 8). As discussed in Stöffler and Ryder (2001), four different flow units having distinct compositions and ages were sampled at this site, with ages of 3.58, 3.7, 10 3.8, and 3.85 Ga. Previously measured crater densities were assigned to the 11 3.58 and 3.8 Ga age groups, and we acknowledge the possibility that one, or 12 perhaps both, of these assignments could possibly be in error. As an example, 13 we note that if the 3.7 Ga age were assigned to the younger crater density 14 that it would lie directly on our best fit line. In contrast, if the older crater 15 density were assigned the 3.7 Ga age, this would require a different form to 16 the fit between 3.41 and 3.75 Ga. Given the discrepancy of the 3.58 Ga crater 17 density, ages derived from our crater chronology between 3.41 and 3.75 Ga 18 should be used with caution. 19

Figure 8 also shows the range of bias that could occur if spatial variations are not taken into account in the crater chronology method. Radiometric ages obtained from future missions, or crater count data from images, that are far from the Apollo and Luna sampling sites might need to have their associated local crater densities corrected by a factor up to  $\sim 25\%$  to obtain the corresponding globally averaged value. Since the age/density relationship is approximatively linear for the last 3.5 Ga, ages could be biased by the same
<sup>1</sup> amount as the spatial variations. Maximal variations in apparent ages would

<sup>2</sup> correspond to geologic units located at the apex or at  $(\pm 65^{\circ}N, 90^{\circ}E)$ . If one

- <sup>3</sup> geologic unit were located at  $(0^{\circ}N, 90^{\circ}W)$  and another at  $(\pm 65^{\circ}N, 90^{\circ}E)$ , and
- $_{4}$  if both surfaces formed at 2 Ga, there would be about 900 Ma separating their

5 apparent ages.

6

7

#### [Fig. 8 about here.]

[Table 9 about here.]

 $_{*}$  The number of craters larger than D that accumulate on a given planet, at

• the location  $(\lambda, \varphi)$ , and over a period of t Ga, is

$$N(>D,\lambda,\varphi,t) = C(>D,\lambda,\varphi) \times T(t) , \qquad (26)$$

<sup>10</sup> where the temporal dependency is

$$T(t) = \frac{a'\left(e^{b't} - 1\right) + c't}{a'\left(e^{b'} - 1\right) + c'},$$
(27)

and a', b' and c' are given in equation 25. The dating of a planetary geologic unit is performed by finding the value of t that allows the best fit between  $N(D, \lambda, \varphi, t)$  and the data.

At a given location  $(\lambda, \varphi)$  (note that  $\varphi$  plays a role only in the lunar case and that, apart from Mars where the cratering rate is expected to be 30 % less at the equator than at the pole, the latitudinal effect on the terrestrial planets is less than 10%), the best age is found by solving the linear least-square problem as a function of T(t):  $N_{obs}(>D_k) = C(>D_k, \lambda, \varphi) \times T(t)$ , where  $N_{obs}(>D_k)$ are the K measured cumulative crater densities with  $k \in [1, K]$ . Each data possesses an uncertainty  $\sigma_k = \sqrt{N_{obs}(>D_k)}/A$ , where A is the area where

<sup>1</sup> craters have been counted. The solution (e.g., Tarantola 2001) is given by

$$T(t) = \frac{\sum_{k=1}^{K} C(>D_k, \lambda, \varphi) \ N_{obs}(>D_k) / \sigma_k^2}{\sum_{k=1}^{K} C^2(>D_k, \lambda, \varphi) / \sigma_k^2}$$
(28)

<sup>2</sup> The a posteriori uncertainty is given by

$$\delta T(t) = \left(\sum_{k=1}^{K} \frac{C^2(>D_k,\lambda,\varphi)}{\sigma_k^2}\right)^{-\frac{1}{2}} .$$

(29)

<sup>3</sup> The best age t (in Ga) and its uncertainty are finally determined from  $T(t) \pm$ 

4 
$$\delta T(t)$$
.

We next show an application of our procedure by estimating new ages of key geologic units on the Moon, Earth, Venus and Mercury. Figure 9a shows our 6 synthetic lunar SFD fitted to crater counts performed over the Orientale basin 7 (data from Neukum et al. (2001a)). The crater SFD is shown in R-plot form,  $R(>D) = D^3 \frac{dN(>D)}{dD}$ , where changes in slope are emphasized with respect 9 to a reference cumulative distribution having a power-law slope of -2. We use 10 a best-fitting megaregolith thickness of 500 m, and exclude in the fit craters 11 smaller than 1.5 km in diameter that have reached saturation level. We find 12 an age of  $3.73 \pm 0.01$  Ga. The difference with Neukum et al. (2001a), who 13 quoted an age of 3.70, lies principally in the updated coefficient values of 14 the calibration curve. Without accounting for spatial cratering variations, the 15 age would have been estimated to be 3.75 Ga. The bias in age from spatial 16 cratering variations is here moderate, as we are in the exponential part of the 17 calibration curve. 18

<sup>19</sup> On Earth, Hughes (2000) used a nearest neighbors technique and known crater <sup>20</sup> ages to calculate the size-dependent formation rate of craters over the last <sup>21</sup>  $125 \pm 20$  Ma. The best fit between our model production function and the <sup>22</sup> estimates of Hughes (2000) is for an age of  $138 \pm 6$  Ma (figure 9b), in excellent

agreement with a constant impact flux over this period. Craters smaller than 1 15 km in diameter were not included in the fit, being depleted as a consequence 2 of erosional processes (note that the megaregolith thickness was obviously set to 0 in this calculation). Our result implies that craters larger than about 10-20 km have not been erased by erosional processes over the last 100 Ma. We 5 also fit our model to the crater distribution of a number of North American 6 and European Phanerozoic cratons that have been independently estimated by Grieve and Dence (1979) to be 375 Ma old on average. Neukum and Ivanov 8 (1994) noted that the crater chronology, once applied to this crater population, 9 returns an age of about 700 Ma. Our best fit corresponds to an age of  $390 \pm 75$ 10 Ma, in agreement with Grieve and Dence (1979). 11

Using crater counts derived from the Magellan mission, the average age of 12 the Venusian surface has generally been estimated to be between 650 and 13 750 Ma (Neukum and Ivanov, 1994; McKinnon et al., 1997; Korycansky and 14 Zahnle, 2005). We fit our model distribution to the data, excluding craters 15 smaller than 25 km in diameter: their depletion with respect to the model is 16 attributed to atmospheric shielding. Our estimate, based on the use of the 17 non-porous scaling alone, is shown in figure 9c, and corresponds to an age 18 of  $240 \pm 13$  Ma (figure 9c), significantly younger than other estimates. The 19 difference lies principally in the impactor population used in the calculations. 20 In particular, both Korycansky and Zahnle (2005) and McKinnon et al. (1997) 21 used the Venus-crossing population estimated by Shoemaker et al. (1990), 22 which produces an impact rate that is significantly higher than our value 23 calculated from the NEO model of Bottke et al. (2002). This high impact rate 24 is compensated in their study by a very efficient atmospheric shielding model 25 that has the effect of decreasing the proportion of small craters. In particular,

the number of 30 km craters is reduced by about a factor 5 between their 1 airless and atmosphere-shielded simulations in order to fit the data, whereas our airless model fits the crater distribution at that size, suggesting that the atmosphere is of negligible effect. Neukum and Ivanov (1994) assumed that the mean impact velocity on Venus is  $19 \text{ km s}^{-1}$  (we calculate  $25 \text{ km s}^{-1}$ ), and that 5 the projectile flux at infinity was the same for Venus and the Moon. Moreover, 6 our venusian size-frequency distribution is not of the same shape as in Neukum 7 and Ivanov (1994), because it was not constructed from the lunar production 8 function, where porous megaregolith decreases the proportion of small craters. 9 but rather directly from the impactor size distribution. We finally note that 10 our inferred age is more similar to the estimate of Strom et al. (1994), who 11 calculated the global surface of Venus to be 290 Ma old. 12

13

#### [Fig. 9 about here.]

Figure 9d shows crater counts performed on Mercury, both on plains interior 14 to the Caloris basin and on the lineated Caloris basin sculpture (Fassett et al., 15 2009). Our best fits are respectively for megaregolith thicknesses of 250 m and 16 900 m, and for ages of  $3.30 \pm 0.3$  and  $3.73 \pm 0.2$  Ga. According to Fassett 17 et al. (2009), the size-frequency distribution of craters on the lineated sculp-18 ture (resulting from the basin formation) should be more representative of the 19 Caloris basin formation time than the distribution measured on the Caloris 20 rim, that exhibits a loss of small craters due to erosional processes and bad 21 lighting conditions for crater counts, both being the consequences of the rim 22 slope. Therefore, we attribute the age of 3.69 Ga to the Caloris basin. Our pre-23 ferred values are consistent with the interpretation that older surfaces present 24 a thicker layer of megaregolith than younger ones. Moreover, the megaregolith 25 appears to be thicker on Mercury than on the Moon, which is consistent with

the fact that Mercury has both impact rate and mean impact velocity that
are about twice as great as for the Moon. Finally, we suggest the possibility
that differences in crater size-frequency distribution between pre- and postLHB surfaces may partially be explained by megaregolith thickness variations
rather than changes in the impactor size distribution, the latter hypothesis
being supported by Strom et al. (2005).

#### 7 6 Discussion

Here we compare our calculations with published results, and discuss the sen-8 sitivity of our model to various parameters. Our present Earth/Moon impact 9 ratio and mean lunar impact velocity are calculated to be, respectively, 1.58 10 and 19.7 km s<sup>-1</sup>. This is comparable to the values of Stuart and Binzel (2004) 11 (respectively 1.61 and 19.3 km s<sup>-1</sup>), who used Opik calculations for an isolated 12 Moon possessing an Earth-like orbit about the Sun. These authors used the 13 NEO population of Stuart (2001), estimated from the debiased observations of 14 the LINEAR survey, and calculated the cratering rate on the Earth and Moon 15 using various scaling laws. They did not account for a porous megaregolith. 16

Gallant et al. (2009) used numerical simulations to estimate lunar asymme-17 tries. From the NEO population of Bottke et al. (2002), they calculated the 18 probability of encounter with the Earth-Moon system with Öpik probabilities, 19 and used a 4-body (projectile, Moon, Earth and Sun) numerical approach to 20 calculate the encounter trajectory, starting at 0.02 AU from the Earth. They 21 reported a mean lunar impact velocity of 20 km s<sup>-1</sup>. Our analytical approach 22 yields consistent values with their predicted apex/antapex cratering ratio as 23 a function of the Earth-Moon distance: our leading/trailing hemispheric ratio 24

is 1.17 for the present lunar semi-major axis  $a_m = 60$  Earth radii, and 1.20 1 for  $a_m = 40$  Earth radii, while Gallant et al. (2009) gave respectively 1.15 2 and 1.18. Their latitudinal effect is found to be 10%, whereas ours is 20%. The difference is due to the crater scaling employed: Gallant et al. (2009) did not include the effect of the impact angle on the crater diameter, while we consider that only the vertical component of the impact velocity contributes 6 to the crater size. By ignoring the effect of impact angle, we indeed obtain a similar latitudinal effect of  $\sim 10\%$  (see Le Feuvre and Wieczorek, 2008). 8 The "vertical component" scaling appears to be the safest assumption for a 9 single target body, though the impact angle dependence of the average crater 10 efficiency may vary from planet to planet (see Collins et al. 2009, Elbeshausen 11 2009). Nevertheless, this angle dependence is of negligible influence in the cal-12 culation of planetary cratering rate ratios with respect to the Moon, and only 13 affects the amplitude of the cratering asymmetries on a given body. Gallant 14 et al. (2009) also estimated the terrestrial impact rate as a function of local 15 time (see figure 9 of their paper). Although it is not the purpose of this paper, 16 we note here that our semi-analytical approach reproduces their results. 17

Recently, Marchi et al. (2009) proposed a revised crater chronology. The main
differences with our approach (excluding the assumption of spatially uniform
cratering rates in the latter) are the following:

• We use the orbital distribution of near-Earth objects of Bottke et al. (2002), modified for Mars, which is assumed to be in steady state and independent of bolide size. Marchi et al. (2009), in contrast, use a subset of this model based on orbital integrations of test particles coming from only the 3:1 and  $\nu_6$  resonances. Furthermore, the relative importance of these two sources is

<sup>1</sup> not the same as in the final debiased model of Bottke et al. (2002).

<sup>2</sup> • We assume that the size frequency distribution of objects impacting the

planets is the same for all planets and that the probability of an object

<sup>4</sup> impacting a planet is independent of size. In contrast, in Marchi et al. (2009),

<sup>5</sup> the size frequency distribution differs for each planet and is dependent on

how much time the impactors spend in the asteroid belt. Furthermore, the
 impact probabilities are assumed to depend upon the projectile size.

In the impact crater scaling relations, we use K=1.17 for impacts in water
for the gravity regime, whereas Marchi et al. (2009) use K=0.93 for impacts
in wet soils and rock in the strength regime (though employed in the general
scaling relationship that contains both strength and gravity terms).

When converting transient crater diameters to final crater diameters, we
use a multiplicative factor of 1.56 as suggested by Melosh (1989, 253 pp.)
and Melosh (1998), whereas Marchi et al. (2009) assume that the transient
crater diameter is equivalent to the final simple crater diameter for their
preferred impact scaling law that is based on the equations in Holsapple
and Housen (2007).

Both studies treat the case of impact crater scaling in the porous megare-18 golith differently. We use a linear transition from porous (i.e., "sand or co-19 hesive soils" from Holsapple and Housen (2007)) to nonporous scaling to fit 20 either the Neukum or Hartmann production functions. In contrast, Marchi 21 et al. (2009) place the transition between porous and nonporous scaling at 22 projectile diameters of 0.5 km (corresponding to a crater diameter of about 23 5 km), and additionally use an average strength parameter in their impact 24 crater scaling relationship that depends on crater size. 25

<sup>26</sup> It is difficult to quantify how each of these differences affect the final crater

size-frequency distribution on a planetary object, and hence the derived ages
of a surface. Nonetheless, we note that the different bolide size-frequency distributions and the different crater scaling laws could be significant. As an
example, if Marchi et al. (2009) were to have used the same crater scaling
constants and to have multiplied their transient crater diameters by the same
factor as us, this would have increased their predicted crater densities by about
a factor of 5. Excluding these factors from our analysis would have decreased
our crater densities by the same factor.

Regardless of these differences, we note that our model production function is in closer agreement with the Neukum production function than that proposed 10 by Marchi et al. (2009). In particular, our crater densities differ by a factor 11 of 1.1 and 0.8 at crater diameters of 1 and 50 km, respectively (1.3 and 0.7 12 with respect to the Hartmann production function), whereas for Marchi et al. 13 (2009) the respective factors are about 1.5 and 0.3 (1.7 and 0.2 with respect 14 to the Hartmann production function). Though a factor of 5 increase in crater 15 densities would improve their fit to the Neukum production function at large 16 crater diameters, the misfit would be significantly worse for small craters. 17

It should be noted that, according to Stewart and Valiant (2006), morphologi-18 cal analysis of young Martian craters combined with theoretical crater scaling 19 relationships implies a factor of about 1.4 for modification of the transient cav-20 ity by wall slumping and rim uplift (in addition to gravity collapse of complex 21 craters), whereas Melosh (1989, 253 pp.) suggested a factor of 1.56. Using the 22 Stewart and Valiant (2006) factor would make our crater densities differ at 23 most by 40 % from the Neukum production function for crater sizes between 24 100 m and 300 km. Using an impactor density of 2000 instead of 2700 kg m<sup>-3</sup> 25 would produce the same effect. Given the uncertainties in both the transient

cavity modification factor and the impactor densities, we favor our previous
estimate, that is in closer agreement with the observed production functions.

The density and porosity of the lunar megaregolith are not known with any 3 certainty. It is likely that porosity reduces the density of the first few hundred meters of the lunar crust. Nevertheless, the subsurface study of the Ries crater on Earth (Pohl et al., 1977) indicates that the density reduction is moderate. We used a value of 2500 kg m<sup>-3</sup> for the upper density of the megaregolith in the calculation of the impact crater sizes, based on Bondarenko and Shkuratov 8 (1999, abstract no. 1196). Regardless, we note that taking an extreme value 9 of 2000 kg  $m^{-3}$  would still produce a broad agreement with the Neukum 10 production function, with a maximum discrepancy of 50 % for crater sizes 11 between 500 m and 300 km. 12

We test the sensitivity of our calculated lunar cratering asymmetries by using 13 the NEO orbital distribution given by telescopic observations only, either for 14 diameters larger than 1 km or 4 km, instead of the orbital model provided 15 by Bottke et al. (2002). Using the d > 4 km population leads to a strong 16 enhancement of lunar latitudinal asymmetries. The pole/equator is in this 17 case equal to 0.56, due to the high proportion of these objects in the ecliptic 18 plane. For the d > 1 km population, latitudinal variations are less than 1 % 19 with respect to the average. These observations suggest that the observation of 20 low inclination objects with d < 1 km is incomplete, which is not unexpected since this type of detection is more difficult than for high inclination objects. 22 Regardless, in both cases, the apex/antapex ratio does not change significantly. 23

The porous regime is required so that the calculated lunar production rate for craters smaller than about 10 km matches the Hartmann and Neukum

production functions. As reported by Strom et al. (2005), counts of post-LHB 1 lunar craters seem to be adequately fit by the observed NEO size-frequency 2 distribution and the sole use of the non-porous cratering regime. An alternative possibility is that young geologic units that are less impacted, possess a thinner porous megaregolith layer than older units, and that this has acted to change the apparent size-frequency distribution of lunar craters with time. 6 The evolution of megaregolith thickness with time remains to be determined, as well as the equivalent thickness on Mercury and Mars, for which the distri-8 bution of craters smaller than a few kilometers is still to be better understood 9 before our model can be confidently used: Mercury may exhibit an abundance 10 of large secondaries, whereas the presence of subsurface ice may significantly 11 reduce the upper porosity of the martian crust. 12

Finally, we caution that our model may be inaccurate for periods prior to 3.5 13 Ga, both because of the Late Heavy Bombardment and a possible reorientation 14 of the Moon (Wieczorek and Le Feuvre, 2009). Our study is generalizable 15 to different impactor populations or planetary reorientations, but the affects 16 of these events are difficult to assess without a better knowledge. Waiting 17 in particular for a suitable dynamical model for small bodies in the early 18 solar system, we have applied our cratering model to times prior to 3.5 Ga, 19 based on the following considerations. First, the size-frequency distributions 20 of craters on the oldest lunar terranes are potentially explained by varying the 21 megaregoltih thickness, without the need for a different impactor population. 22 Second, even if the cratering asymmetries would have been modified by these 23 events, the influence of spatial variations on the age estimates is very moderate 24 for times prior to 3.5 Ga (which correspond to the exponential part of the 25 chronology curve). 26

#### 1 7 Conclusions

We have presented a complete set of equations that allows to calculate the 2 cratering rate as a function of crater size and location, on the Moon and 3 inner planets. Spatial variations of the cratering rate are calculated semi-4 analytically, hence considerably faster than by the use of a fully numerical 5 method. The approach is generalizable to any other planet or moon, providing 6 the orbital and size distributions of impactors are known. For our purpose, the NEO orbital distribution model of Bottke et al. (2002) has been used, and the size distribution of bolides has been fitted to a compilation of various estimates (Rabinowitz et al., 2000; Morbidelli et al., 2002; Harris, A. W., 2002; Stuart 10 and Binzel, 2004; Halliday et al., 1996; ReVelle, 2001; Brown et al., 2002). 11

Significant cratering asymmetries, both latitudinal and longitudinal, are found 12 on the lunar surface. These are the result of both the high proportion of low 13 inclination encounters and the synchronous rotation of the satellite. If the 14 resulting bias in the estimate of absolute ages is in general moderate with re-15 spect to other uncertainties associated with the crater chronology method, and 16 therefore should not invalidate most of the published ages, this systematic ef-17 fect could be as large as 500 Ma for geologic units of 2 Ga close to  $(0^{\circ}N, 90^{\circ}W)$ 18 or  $(\pm 65^{\circ}N, 90^{\circ}E)$ , and should be accounted for when determining relative ages 19 between different surfaces.

Our synthetic size-frequency distribution of lunar craters matches the standard production functions (see Neukum et al., 2001a), providing that a porous megaregolith is accounted for at crater diameters smaller than a few kilometers. The absolute number of craters formed during the last 3 billion years

<sup>1</sup> is consistent with a constant impact flux and the present day cratering rate.

<sup>2</sup> The shape of the size-frequency distribution is predicted to be time-dependent,

<sup>3</sup> since older surfaces would possess a thicker layer of fractured megaregolith.

We have provided an updated crater chronology curve that excludes some debated calibration points based on geologic considerations, and accounts for the spatially-dependent cratering rate at the calibration units. The proposed curve favors a longer period of constant impact flux, extending up to 3.5 Ga ago. We have also given polynomial fits for the crater size-frequency distributions calculated on Mercury, Venus, the Earth and Mars, allowing the easy calculation of age estimates on these planets.

Our model is consistent with the terrestrial cratering records (Hughes, 2000; 11 Grieve and Dence, 1979) for the last 400 Ma. For the Moon, we find that 12 the age of the Orientale impact basin is 3.73 Ga, which is 30 Ma older than 13 previous estimates. This corresponds to a shorter Late Heavy Bombardment, 14 since it is generally considered that the Caloris basin marks the end of this 15 event. On Mercury, we attribute the same age to the Caloris basin. We finally 16 estimate the venusian surface to be only 240 Ma old on average, which is 2–3 17 times younger than most previous estimates. Such an age is similar to the 18 oldest oceanic crust on Earth and implies that volcanic resurfacing occurs on 10 Venus at only a slightly reduced rate as on Earth. 20

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and Smith (1991).

#### 6 A Derivations

Here we describe in detail how we compute the lunar and planetary cratering 7 rates. We start by recalling how to obtain orbital elements in the 2-body 8 problem, which is required for calculating hyperbolic trajectories of objects as 9 they encounter a planet or Moon. Second, we adapt Opik impact probabilities, 10 originally intended for planets, to the case of a moon in synchronous rotation. 11 Third, we derive equations to calculate the impact rate as a function of position 12 on the target surface (planet or moon). Finally, we recall the equations that 13 allow us to calculate crater diameters from impactor diameters and impact 14 conditions. 15

#### 16 A.1 Orbital elements

<sup>17</sup> Consider a massless particle (our projectile) orbiting around a massive body <sup>18</sup> M (our target) isolated in space (two-body problem), described in a Cartesian <sup>19</sup> coordinate system (oxyz) whose origin is the center of the target, with  $\hat{\mathbf{x}}$ ,  $\hat{\mathbf{y}}$  and <sup>20</sup>  $\hat{\mathbf{z}}$  the associated unit vectors. The plane (oxy) is chosen as the orbital plane of <sup>21</sup> the target body. All the following definitions can be found in standard celestial <sup>22</sup> mechanics textbooks (e.g., Murray and Dermott, 2000, 606 pp.). The distance

<sup>1</sup> of the projectile to the center of the target is

$$r = \frac{p}{1 + e\cos f} , \qquad (A.1)$$

- where p is the semilatus rectum, e the eccentricity and f the true anomaly.
- <sup>3</sup> The pericentre distance  $(f = \pi)$  is given by

$$q = \frac{p}{1-e}$$

<sup>4</sup> The semilatus rectum is

$$p = \frac{h^2}{GM} , \qquad (A.3)$$

(A.2)

- $_{5}$  where M is the mass of the target body, and the specific angular momentum,
- <sup>6</sup> constant and perpendicular to the projectile orbital plane, is

$$\mathbf{h} = \mathbf{r} \times \dot{\mathbf{r}} \ . \tag{A.4}$$

7 The eccentricity vector is

$$\mathbf{e} = \frac{\dot{\mathbf{r}} \times \mathbf{h}}{GM} - \frac{\mathbf{r}}{r} , \qquad (A.5)$$

- <sup>8</sup> and the eccentricity is simply  $e = ||\mathbf{e}||$ . Alternatively, the eccentricity can be
- $_{9}$  calculated from the particle's specific energy  $\xi$  as

$$e = \sqrt{2\xi \left(\frac{h}{GM}\right)^2 + 1} . \tag{A.6}$$

The inclination of the orbit with respect to the (oxy) plane is calculated from

$$\cos i = \frac{\mathbf{h} \cdot \hat{\mathbf{z}}}{h} , \qquad (A.7)$$

<sup>11</sup> where  $i \in [0, \pi]$ . The nodal vector, that points towards the ascending node, is

$$\mathbf{n} = \hat{\mathbf{z}} \times \frac{\mathbf{h}}{h} , \qquad (A.8)$$

- <sup>1</sup> and the longitude of the ascending node, measured from (ox), is calculated
- <sub>2</sub> from

₃ and

5 with

4

The argument

$$\cos \Omega = \frac{\mathbf{n} \cdot \hat{\mathbf{x}}}{n} , \qquad (A.9)$$

$$\sin \Omega = \frac{\mathbf{n} \cdot \hat{\mathbf{y}}}{n} . \qquad (A.10)$$
of pericentre  $\omega$  is given by
$$\cos \omega = \frac{\mathbf{n} \cdot \mathbf{e}}{n \ e} , \qquad (A.11)$$

$$\omega \in [0, \pi] \text{ if } \mathbf{e} \cdot \hat{\mathbf{z}} \ge 0 , \qquad (A.12)$$

$$\omega \in ]\pi, 2\pi[ \text{ otherwise } .$$

- <sup>6</sup> The true anomaly is calculated from

$$\cos f = \frac{\mathbf{e} \cdot \mathbf{r}}{e \ r} \ , \tag{A.13}$$

7 with

$$f \in [0, \pi]$$
 if  $\mathbf{r} \cdot \dot{\mathbf{r}} \ge 0$ ,  
 $f \in ]\pi, 2\pi[$  otherwise.  
(A.14)

Finally, defining Θ as the angle that the projectile makes with respect to the
line of nodes, we have

$$\Theta = f + \omega , \qquad (A.15)$$

and the projectile crosses the target's orbital plane at  $\Theta = 0$  and  $\Theta = \pi$ , respectively on an ascending and descending trajectory.

#### <sup>1</sup> A.2 Encounter probabilities with the Moon

Let us call  $\mathbf{V}$  the encounter conditions with the Earth-Moon system, calcu-2 lated as explained in section 3.1, and expressed in a right-handed coordinate 3 frame where the origin is at the center of the Earth, the x-axis point towards 4 the Sun, (xy) is the Earth orbital plane, and the z-axis points upward. The 5 symmetry induced by the lunar rotation about the Earth makes the problem only dependent on V and  $V_z$ . Let us take  $V_y = 0$  and  $V_x \leq 0$ . Consider also 7 the frame (x'yz'), where the x'-axis and V are antiparallel. This frame is use-8 ful to express the specific angular momentum and eccentricity vector, since 9 it avoids complications due to the initial position of the projectile at infinity. 10 The rotation matrix from (x'yz') to (xyz) is therefore 11

$$M = \begin{pmatrix} \frac{\sqrt{V^2 - V_z^2}}{V} & 0 & \frac{V_z}{V} \\ 0 & 1 & 0 \\ -\frac{V_z}{V} & 0 & \frac{\sqrt{V^2 - V_z^2}}{V} \end{pmatrix} .$$
(A.16)

The encounter velocity and initial position of the projectile on the cross section disk, perpendicular to x', expressed in (x'yz') as

$$\mathbf{V} = \begin{pmatrix} -V \\ 0 \\ 0 \end{pmatrix}_{(x'yz')}, \quad \mathbf{r} = \begin{pmatrix} \infty \\ b\cos\alpha \\ b\sin\alpha \end{pmatrix}_{(x'yz')}, \quad (A.17)$$

where b is the impact parameter and  $\alpha$  defines the angular position on the ross section disk with respect to the y-axis.

- <sup>1</sup> From these initial conditions, the specific angular momentum of the projec-
- $_{2}$  tile and eccentricity vector are easily obtained by equations (A.4) and (A.5),
- and expressed in the (xyz) frame by the use of the rotation matrix M (equa-
- <sup>4</sup> tion A.16). The norm of the eccentricity vector is

$$E = \sqrt{1 + \left(\frac{b}{a_m}\right)^2 \left(\frac{V}{v_m}\right)^4} , \qquad (A.18)$$

 $_{\scriptscriptstyle 5}\,$  where  $a_m$  and  $v_m$  are the lunar semi-major axis and orbital velocity. The

6 inclination with respect to the system orbit plane (xy) is (equation A.7)

$$I = \cos^{-1}\left(\sqrt{1 - \frac{V_z^2}{V^2}}\cos\alpha\right) \,. \tag{A.19}$$

Note that the eccentricity E and inclination I of the hyperbolic trajectory have
been capitalized in order to distinguish them from the orbital elements e and
i for elliptic orbits about the Sun. The pericenter distance q is (equation A.2)

$$q = a_m (E-1) / \left(\frac{V}{v_m}\right)^2 , \qquad (A.20)$$

the nodal vector is calculated with equation (A.8), and the argument of pericenter is finally expressed by equation (A.11) as

$$\cos \omega = \frac{\frac{V_z}{V}\sqrt{E^2 - 1} - \sqrt{1 - \frac{V_z^2}{V^2}}\sin \alpha}{E\sin I} .$$
 (A.21)

For an intersection to occur between the projectile and Moon, the projectile must cross the lunar orbit plane at a distance  $a_m$  from the Earth. Since  $V_z$  is indifferently positive or negative (Öpik, 1951), the symmetry with respect to the system orbit plane allows to restrict ourselves to one of the nodes, here chosen to be the descending node. In this case,  $\Theta = \pi$  implies (equation A.15)

$$\cos f = -\cos\omega , \qquad (A.22)$$

<sup>1</sup> and  $r = a_m$  gives (equation A.1)

$$\cos f = \frac{q/a_m(1+E) - 1}{E}$$
 (A.23)

<sup>2</sup> By using the two above equations, with E, q and  $\cos \omega$  expressed as a function

 $_{3}$  of **V**, b and  $\alpha$  from equations (A.18), (A.20) and (A.21), it is found that the

<sup>4</sup> value of the impact parameter that allows intersection is

$$b_m = \frac{a_m}{2} \left( \sqrt{\left(\frac{V_z}{V \sin I}\right)^2 + 4\left(1 + \frac{\tan \alpha}{\tan I}\right)\left(\frac{v_m}{V}\right)^2} - \frac{V_z}{V \sin I} \right).$$
(A.24)

and that there is only one value of b that allows intersection on the descending

6 node for a given angular position on the lunar orbit cross section  $\alpha$ .

<sup>7</sup> For hyperbolic orbits, the relative encounter velocity with the Moon U is given <sup>8</sup> in Shoemaker and Wolfe (1982) as a function of q, e and I. For the appropriate <sup>9</sup> value of the impact parameter that allows collision, we rewrite these equations <sup>10</sup> as

$$\begin{cases} U^{2} = V^{2} + v_{m}^{2} - 2v_{m} U_{Y} , \\ U_{X}^{2} = V^{2} \left( 1 - \left( \frac{b_{m}}{a_{m}} \right)^{2} \right) + 2v_{m}^{2} , \\ U_{Y} = V \frac{b_{m}}{a_{m}} \cos I - v_{m} , \\ U_{Z}^{2} = V^{2} \left( \frac{b_{m}}{a_{m}} \right)^{2} \sin^{2} I . \end{cases}$$
(A.25)

where **U** is expressed in the frame (XYZ) attached to the center of the Moon, where the X-axis point towards the Earth, the Y-axis points to the antapex of motion, (XY) defines the Lunar orbital plane, and the Z-axis points upward. Since we have restricted ourselves to the descending node,  $U_Z$  is negative. But symmetry requires that  $U_Z$  is positive with the same probability. The sign of  $U_X$  is to be discussed in the following.

<sup>1</sup> Öpik probabilities assume that the argument of pericenter  $\omega$  takes any value <sup>2</sup> between 0 and  $2\pi$  with an equal probability. While this is appropriate for <sup>3</sup> elliptic orbits that precess about the Sun, the  $\omega$  of our hyperbolic trajectory is <sup>4</sup> constrained by the geometry of encounter with the Earth-Moon system. The <sup>5</sup> encounter probability with the Moon can be expressed as follows

$$P_m = \frac{\Delta(b^2)}{\tau'^2} \times \frac{\Delta t}{T_m},\tag{A.26}$$

<sup>6</sup> where  $\Delta(b^2)$  is the interval around  $b_m^2$  that allows the minimum distance be-<sup>7</sup> tween the projectile's and target's orbits to be smaller than the gravitational <sup>8</sup> cross section radius of the Moon,  $\tau_m$ . We are interested in the square of *b* here, <sup>9</sup> whose probability distribution on the lunar orbit cross section is uniform. The <sup>10</sup> ratio  $\frac{\Delta(b^2)}{\tau'^2}$  corresponds to the fraction in space allowing the collision, since  $b^2$ <sup>11</sup> can take values between 0 and  $\tau'^2$ . Similarly, the term  $\frac{\Delta t}{T_m}$  is the fraction of <sup>12</sup> time that allows the collision,  $T_m$  being the orbital period of the Moon. This <sup>13</sup> latter term is given by (see Öpik, 1951):

$$\frac{\Delta t}{T_m} = \frac{\tau_m}{4a_m} \frac{U}{\sqrt{U^2 - U_Y^2}}.$$
(A.27)

At this point we need to express  $\Delta(b^2)$  in terms of the relevant variables of our problem. At the node,  $\Theta = \pi$ ; denoting  $L_{max}$  the distance between  $r_{\Theta=\pi}$ and  $a_m$  such as the minimum distance between the two orbits equals  $\tau_m$ , we have

$$\frac{2L_{max}}{\Delta(b^2)} \simeq \left(\frac{\partial r_{\Theta=\pi}}{\partial b^2}\right)_{b=b_m} , \qquad (A.28)$$

18 with

$$L_{max} = \tau_m \sqrt{1 + U_X^2 / U_Z^2} .$$
 (A.29)

<sup>19</sup> Equations (A.27) and (A.29) come from the assumption that the projectile <sup>20</sup> and Moon follow straight lines trajectories in the vicinity of the node, and

<sup>1</sup> are demonstrated in Öpik (1951). Expressing  $\left(\frac{\partial r_{\Theta=\pi}}{\partial b^2}\right)_{b=b_m}$  as a function of V

<sup>2</sup> and  $\alpha$ , inserting equation (A.29) into equation (A.28), and equations (A.28),

 $_{3}$  (A.27) and (2) into equation (A.26), we finally get the impact probability with

4 the Moon as

$$P_m = \frac{\tau_m^2}{a_m^2} \frac{U}{|U_Z|} \frac{v_m^2}{2v_m^2 + V^2} \frac{1}{\sin I} \frac{\left(\sin I + \sqrt{1 - \left(\frac{V_z}{V}\right)^2} \sin \alpha - \frac{V_z |U_Z|}{v_m^2 \sin I}\right)^2}{2\sin I + 2\sqrt{1 - \left(\frac{V_z}{V}\right)^2} \sin \alpha - \frac{V_z |U_Z|}{v_m^2 \sin I}} .$$
 (A.30)

If  $U_X \leq 0$  and  $q \leq R_e$  simultaneously,  $R_e$  being the radius of the Earth, a collision with the planet occurs, and  $P_m$  is set to 0 (the Moon cannot be impacted, as the sign of  $U_X$  indicates that the projectile has already cross the Earth). Denoting  $b_e$  the impact parameter corresponding to  $q = R_e$  we have from equation A.20:

$$b_e = \sqrt{2R_e a_m \left(\frac{v_m}{V}\right)^2 + R_e^2} , \qquad (A.31)$$

and  $q \leq R_e$  corresponds to  $b_m \leq b_e$ . The sign of  $U_X$  is known from

$$U_X = -\sqrt{U_X^2} \text{ for } \sqrt{E^2 - 1} \sin \alpha / \tan I \ge 1$$
$$U_X = \sqrt{U_X^2} \quad \text{for } \sqrt{E^2 - 1} \sin \alpha / \tan I < 1 \tag{A.32}$$

<sup>11</sup> where the first and second cases correspond respectively to an argument of <sup>12</sup> pericenter comprised between 0 and  $\pi$  and between  $\pi$  and  $2\pi$  (equation A.12). <sup>13</sup> For instance, in the first case, the descending node is reached after the min-<sup>14</sup> imum distance to Earth, and the projectile is moving away from the Earth. <sup>15</sup> Finally, the impact probability with Earth is given by

$$P_e = \frac{1}{2} \frac{b_e^2}{\tau'^2} - P_m , \qquad (A.33)$$

where the factor 1/2 comes from the restriction to one of the two possible

<sup>1</sup> nodes. For a given encounter **V** with the Earth-Moon system,  $P_m$ , **U** and <sup>2</sup>  $P_e$  are calculated for  $\alpha \in [0, 2\pi]$ . The probability distribution of the lunar <sup>3</sup> encounter conditions is

$$p(\mathbf{U}) = \frac{p'(\mathbf{U})}{\int_{\mathbf{U}} p'(\mathbf{U}) \, \mathrm{d}\mathbf{U}} \;,$$

(A.34)

4 with

$$p'(\mathbf{U}) = \int_{\alpha=0}^{2\pi} \int_{\mathbf{U}'=0}^{\infty} \int_{\mathbf{V}} P_m(\alpha, \mathbf{V}) \ p(\mathbf{V}) \ \delta(\mathbf{U}(\alpha, \mathbf{V}) - \mathbf{U}') \ \mathrm{d}\mathbf{V} \ \mathrm{d}\mathbf{U}' \ \mathrm{d}\alpha \ , \ (A.35)$$

<sup>5</sup> where  $p(\mathbf{V})$  is the probability distribution of the encounter distribution with <sup>6</sup> the Earth-Moon system and  $\delta$  is the Kronecker function. Finally, the Earth/Moon <sup>7</sup> impact ratio is

$$r_e = \frac{\int_{\alpha=0}^{2\pi} \int_{\mathbf{V}} P_m(\alpha, \mathbf{V}) \ p(\mathbf{V}) \ \mathrm{d}\mathbf{V} \ \mathrm{d}\alpha}{\int_{\alpha=0}^{2\pi} \int_{\mathbf{V}} P_e(\alpha, \mathbf{V}) \ p(\mathbf{V}) \ \mathrm{d}\mathbf{V} \ \mathrm{d}\alpha} \ . \tag{A.36}$$

#### 8 A.3 Impact rate as a function of position

Each encounter U generates an infinity of hyperbolic trajectories towards the 9 target (moon or planet), initially uniformly distributed over the gravitational 10 cross section at infinity. Let us call b the impact parameter and  $\beta$  the angular 11 distance between a point on the target surface and the radiant. For a given 12 target mass M and radius R, one value of b corresponds to one value of  $\beta$ . 13 The impact rate at  $\beta$  is proportional to the area of the annulus  $2\pi b db$ 14 divided by the elementary target surface  $2\pi R^2 \sin\beta d\beta$ . The total impact rate is proportional to the gravitational cross sectional area,  $\pi \tau^2$ , divided by the 16 total area of the target surface  $4\pi R^2$ . The relative impact rate, normalized to 17 the global average, is therefore 18

$$\delta\phi = \frac{(2\pi \ b \ \mathrm{d}b)/(2\pi R^2 \sin\beta \ \mathrm{d}\beta)}{(\pi\tau^2)/(4\pi R^2)} = \frac{4}{\tau^2} \frac{b \ \mathrm{d}b}{\sin\beta \ \mathrm{d}\beta} \ . \tag{A.37}$$

- <sup>1</sup> We now have to express  $b \, db$  as a function of  $\beta$ . By noting that  $\beta = f_i f_{\infty}$ ,
- $_2$   $\,$  where  $f_i$  is the projectile's true anomaly at the time of impact, and  $f_\infty$  is the
- $_{\scriptscriptstyle 3}$   $\,$  initial true anomaly at infinity, we have

1

$$\begin{cases} \cos \beta = \cos(f_i) \cos(f_\infty) + \sin(f_i) \sin(f_\infty) ,\\ \sin \beta = \sin(f_i) \cos(f_\infty) - \cos(f_i) \sin(f_\infty) . \end{cases}$$

- An impact occurs when r = R. The corresponding cosine of the true anomaly
- <sup>5</sup> is (equation A.1)

$$\cos f_i = \frac{p/R - 1}{e} . \tag{A.39}$$

(A.38)

Since the distance between the projectile and target is initially infinite, the
cosine of the initial true anomaly is (equation A.1)

$$\cos f_{\infty} = -1/e. \tag{A.40}$$

<sup>8</sup> The true anomalie at infinity is negative (since  $\mathbf{r}_{\infty} \cdot \mathbf{U}_{\infty} < 0$ ). This is also true <sup>9</sup> for  $f_i$ , as the collision occurs necessarily before the projectile is at pericenter <sup>10</sup> (f = 0). Therefore,

$$\sin f_i = -\sqrt{1 - \cos^2 f_i} \quad \text{and} \quad \sin f_\infty = -\sqrt{1 - \cos^2 f_\infty} \ . \tag{A.41}$$

<sup>11</sup> The specific angular momentum is given by equation (A.4)

$$h = b \ U \ . \tag{A.42}$$

 $_{12}$  This allows to express the parameter of the conic as (equation (A.3),

$$p = \frac{b^2 U^2}{GM} \ . \tag{A.43}$$

 $_{\scriptscriptstyle 1}$   $\,$  From equation (A.6), with an specific energy  $\xi = U^2/2$  , the eccentricity is

$$e = \sqrt{1 + \left(\frac{b \ U^2}{GM}\right)^2} \ . \tag{A.44}$$

<sup>2</sup> Using equations (A.44)–(A.38), the impact parameter b is found to be

$$b = R \frac{\tan(\beta/2)}{1 + \tan^2(\beta/2)} \left( 1 + \sqrt{1 + \Gamma(1 + \tan^2(\beta/2))} \right) , \qquad (A.45)$$

 $_{\scriptscriptstyle 3}~$  and the maximum value of  $\beta$  is given for  $b=\tau$  as

$$\cos\beta_{max} = -\frac{\Gamma}{2+\Gamma} \ . \tag{A.46}$$

- <sup>4</sup> Calculating  $db/d\beta$  from the above equation, we get the relative impact flux as
- <sup>5</sup> a function of  $\beta$  from equation (A.37):

$$\delta\phi(\beta, \mathbf{U}) = \begin{cases} \frac{1+\cos\beta}{2(1+\Gamma)} \frac{1+\mu}{\mu} \times \\ \left[\mu^2 + \mu - (1+\mu) \frac{1-\cos\beta}{1+\cos\beta}\right] & \text{if } \beta \le \cos^{-1}\left(-\frac{\Gamma}{2+\Gamma}\right), \\ 0 & \text{if } \beta > \cos^{-1}\left(-\frac{\Gamma}{2+\Gamma}\right), \end{cases}$$

$$\mu = \sqrt{1+\Gamma + \Gamma \frac{1-\cos\beta}{1+\cos\beta}}. \qquad (A.48)$$

6 with

<sup>7</sup> The angular distance  $\beta$  is then simply expressed as a function of latitude  $\lambda$ <sup>8</sup> and longitude  $\varphi$  by calculating the scalar product between a given position <sup>9</sup> on the target and the radiant (equation A.44). Note that the dependency in <sup>10</sup> longitude  $\varphi$  corresponds to a target body in synchronous rotation. When it is <sup>11</sup> not the case, the flux is longitudinally uniform (the obliquity of the target is

<sup>1</sup> taken into account at the end of this section). For a zero obliquity, we have

$$\cos \beta = (\cos \lambda \cos \varphi , \ \cos \lambda \sin \varphi , \ \sin \lambda) \times \frac{\mathbf{U}}{||\mathbf{U}||} . \tag{A.49}$$

- $_{\rm 2}$   $\,$  We finally need the impact velocity u and incidence angle with respect to the
- <sup>3</sup> surface,  $\theta$ . The conservation of energy implies

4 giving

$$\frac{U^2}{2} = \frac{u^2}{2} - \frac{GM}{R} , \qquad (A.50)$$
$$u = U\sqrt{1+\Gamma} , \qquad (A.51)$$

<sup>5</sup> The specific angular momentum at the time of impact is (equation A.4)

$$h = R \ u \ \sin(\theta + \pi/2) \ . \tag{A.52}$$

<sup>6</sup> The conservation of h allows identification with equation (A.42), giving

$$\cos \theta = b/\tau \ , \tag{A.53}$$

 $_{7}~$  and the impact angle is finally given as a function of  $\beta$  as

$$\theta = \cos^{-1} \left( \frac{1+\mu}{2\sqrt{1+\Gamma}} \sqrt{1-\cos^2\beta} \right) . \tag{A.54}$$

<sup>8</sup> Let us finally account for the obliquity  $\epsilon$  of the target. The obliquity reduces <sup>9</sup> spatial variations of the impact rate, since the target's sphere is moved under <sup>10</sup> the projectile flux by precession of the spin axis. Let us denote  $\varphi_{\epsilon}$  the longitude <sup>11</sup> of the spin axis in the (XYZ) frame, and W the matrix

$$W = \begin{bmatrix} \cos^2 \varphi_{\epsilon} + \sin^2 \varphi_{\epsilon} \cos \epsilon & \cos \varphi_{\epsilon} \sin \varphi_{\epsilon} (1 - \cos \epsilon) & -\sin \varphi_{\epsilon} \sin \epsilon \\ \cos \varphi_{\epsilon} \sin \varphi_{\epsilon} (1 - \cos \epsilon) & \sin^2 \varphi_{\epsilon} + \cos^2 \varphi_{\epsilon} \cos \epsilon & \cos \varphi_{\epsilon} \sin \epsilon \\ \sin \varphi_{\epsilon} S_{\epsilon} & -\cos \varphi_{\epsilon} \sin \epsilon & \cos \epsilon \end{bmatrix}^{-1},$$
(A.55)

- <sup>1</sup> then the latitude and longitude  $(\lambda', \varphi')$  in the (XYZ) frame express as a
- $_{\rm 2}$   $\,$  function of the geographic coordinates  $(\lambda,\varphi)$  as

$$\sin\lambda' = W_{31}\cos\lambda\cos\varphi + W_{32}\cos\lambda\sin\varphi + W_{33}\sin\lambda , \qquad (A.56)$$

3 and

$$\cos\varphi' = \frac{W_{11}\cos\lambda\cos\varphi + W_{12}\cos\lambda\sin\varphi + W_{13}\sin\lambda}{\sqrt{1 - \sin^2\lambda'}} ,$$
  

$$\sin\varphi' = \frac{W_{21}\cos\lambda\cos\varphi + W_{22}\cos\lambda\sin\varphi + W_{23}\sin\lambda}{\sqrt{1 - \sin^2\lambda'}} .$$
(A.57)

<sup>4</sup> The impact rate is finally given as a function of the geographic position as

$$\delta\phi(\lambda,\varphi) = \frac{1}{2\pi} \int_0^{2\pi} \delta\phi(\lambda'(\lambda,\varphi,\epsilon,\varphi_\epsilon),\varphi'(\lambda,\varphi,\epsilon,\varphi_\epsilon)) \, \mathrm{d}\varphi_\epsilon \,, \qquad (A.58)$$

- s where the precession of the spin axis is simulated by  $\varphi_{\epsilon}$  taking uniform values
- <sup>6</sup> between 0 and  $2\pi$ .
- 7 A.4 Crater scaling

<sup>8</sup> For our purpose, we express the projectile diameter d required to create a <sup>9</sup> crater with a given final diameter D, under given impact conditions. Let us <sup>10</sup> call  $D_s$  the final diameter of a simple crater. Large craters collapse due to <sup>11</sup> gravity, such that  $D > D_s$ . According to Holsapple (1993),

$$\begin{cases} D_s = D & \text{if } D < D_* \ , \\ D_s = 0.98 \ D_*^{0.079} \ D^{0.921} \ \text{if } D \ge D_* \ , \end{cases}$$
(A.59)

where  $D_*$  is a transition diameter believed to be approximately inversely proportional to the surface gravity, and is about  $D_{*m} = 8.5$  km for the Moon,

1 that is

$$D_* = \frac{g_m}{g} D_{*m} , \qquad (A.60)$$

<sup>2</sup> where  $g_m$  is the lunar surface gravity.

<sup>3</sup> Before wall slumping and rim formation, the diameter of the transient cavity

- <sup>4</sup>  $D_T$  is smaller than the final diameter of the simple crater  $D_s$  by a factor
- $5 1.2 \times 1.3 = 1.56$  (Melosh, 1989, 253 pp.):

$$D_T = D_s / 1.56$$
 . (A.61)

<sup>6</sup> Denoting  $D_{\perp}$  the transient cavity for vertical impacts ( $\theta = \pi/2$ ) we assume

$$D_{\perp} = D_T (\sin \theta)^{-2\nu_1} , \qquad (A.62)$$

- that is, only the vertical component of the impact velocity has to be accounted
  for. Other assumed functional dependencies on the incidence angle can easily
  be used in place of equation (A.62).
- <sup>10</sup> In the gravity regime, where the tensile strength of rock is negligible, the <sup>11</sup> projectile diameter is finally given by Holsapple and Housen (2007) as

$$d = \left( K^{-1} D_{\perp} \ u^{-2\nu_1} g^{\nu_1} \left( \frac{\rho_0}{\rho} \right)^{\nu_2} \right)^{\frac{1}{1-\nu_1}} , \qquad (A.63)$$

with  $\rho$  and  $\rho_0$  respectively the density of the projectile and target, and g the surface gravity. The scaling parameter are taken from Holsapple and Housen (2007) as K = 1.17,  $\nu_1 = 0.22$  and  $\nu_2 = 0.31$  for the non-porous gravity scaling regime (impacts in water), and K = 1.03,  $\nu_1 = 0.17$  and  $\nu_2 = 0.332$ for the porous scaling. For a given crater diameter D, the impactor size d is known from the successive use of the above equations, from equation (A.63) to equation (A.59).

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Fig. 1. Impact probability per year on Earth, for impactors larger than a given diameter. Estimates come from atmospheric records or Öpik probabilities derived from telescopic observations (*black triangles*: Halliday et al. (1996); *black diamonds*: ReVelle (2001); grey triangles: Brown et al. (2002); grey squares and white triangles: Rabinowitz et al. (2000); grey circles: Harris, A. W. (2002); white circles: Morbidelli et al. (2002); black squares: Stuart and Binzel (2004)). Our compilation at large sizes is augmented by including the observed size-frequency distribution of Mars-crossing objects with sizes greater than 4 km, scaled to the terrestrial impact rates of Stuart and Binzel (2004) (grey diamonds). The red curve is the best fit polynomial.



Fig. 2. Model production function of lunar craters, for one year, in comparison with the Hartmann and Neukum measured production functions, and the model production function of Marchi et al. (2009). Respective megaregolith thicknesses of 700 and 250 m allow to fit either the Neukum or Hartmann production functions in the diameter range 2–20 km. The thin dotted red curve is obtained by using only the non-porous scaling relation.



Fig. 3. Planetary cratering ratios with respect to the Moon, for craters larger than a given diameter D. For illustrative purposes, craters with D < 10 km and D > 10km are respectively assumed to form in the porous and non-porous regimes. Curves are not shown for Venus and the Earth for D < 10 km, since the porous regime is not expected, and erosion or atmospheric shielding are known to be of significant influence at these sizes.



Fig. 4. Probability distribution of impact velocities for the Moon and terrestrial planets.



Fig. 5. Relative cratering rate on the Moon for the current Earth-Moon distance and for crater diameters larger than 1 km. This image is plotted in a Mollweide projection centered on the sub-Earth point.



Fig. 6. Relative lunar cratering rate as a function of angular distance from the apex of motion. Rayed crater data are for crater diameters greater than 5 km from Morota et al. (2005) that were counted between approximately [70°E, 290°E] in longitude and [-42°N, 42°N] in latitude. In comparison, the predicted apex/antapex cratering effect is shown over the same count area (solid black) and for the entire Moon (dotted black).



Fig. 7. Relative lunar impact rate as a function of the angular distance from the apex (solid black), in comparison with the equation from Zahnle et al. (2001) (solid grey). The latter is reproduced if the model forces latitudinal isotropy of encounter inclinations (dotted black).



Fig. 8. Lunar crater chronology calibration curve. Calibration points, listed in table 9, are shown before (grey circles) and after (red circles) accounting for the spatially-dependent cratering rate. Our proposed analytic fit (red line) is compared to the curves of Neukum et al. (2001a) (solid black) and Marchi et al. (2009) (dashed grey). Crisium basin, that was not included in the fit, is shown in blue. Also shown is the range of bias in age that could be produced by spatial variations in the cratering rate (purple area).



Fig. 9. Age estimates. Distributions plotted either in cumulative form or R-plot. Data represented by white circles are not included in the fit, either because of saturation (Moon), erosion (Earth) or atmospheric shielding (Venus). (a) Orientale basin:  $3.73 \pm 0.01$  Ga with a megaregolith thickness of 500 m. Data from Neukum et al. (2001a) (b) Terrestrial cratering record:  $138\pm 6$  Ma for the estimates of Hughes (2000) (circles),  $390\pm75$  Ma for the Phanerozoic cratons investigated by Grieve and Dence (1979) (squares) (c) Venusian surface:  $240 \pm 13$  Ma. Data from the Magellan mission (d) Caloris basin lineated sculpture (grey):  $3.73\pm0.2$  Ga with a megaregolith thickness of 250 m. Data from Fassett et al. (2009).

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Table 2

Planetary crater size-frequency distributions for 1 year (km<sup>-2</sup>): log  $\bar{C}(>D)$  $\sum_{i=0}^{10} C_i (\log D_{\rm km})^i, D \in [0.1 - 1000]$  km.

- 0	_0	( = ) ·				
		Moon	Moon	Mercury	Mercury	Venus
		non-porous	porous	non-porous	porous	non-porous
	$C_0$	-0.1049E + 02	-0.1206E + 02	-0.9939E + 01	-0.1159E+02	-0.1073E + 02
	$C_1$	-0.4106E + 01	-0.3578E + 01	-0.3994E + 01	-0.3673E+01	-0.4024E + 01
	$C_2$	-0.8715E + 00	$0.9917E{+}00$	-0.1116E + 01	0.9002E + 00	-0.4503E-01
	$C_3$	0.1440E + 01	$0.7884E{+}00$	$0.1269E{+}01$	0.9609E + 00	$0.1374E{+}01$
	$C_4$	0.1000E + 01	-0.5988E + 00	0.1272E+01	-0.5239E+00	$0.2433E{+}00$
	$C_5$	-0.8733E+00	-0.2805E + 00	-0.8276E+00	-0.3622E+00	-0.7040E + 00
	$C_6$	-0.2725E+00	$0.1665E{+}00$	-0.3718E+00	0.1508E+00	-0.3962E-01
	$C_7$	0.2373E + 00	0.3732E-01	0.2463E + 00	0.5224 E-01	$0.1541E{+}00$
	$C_8$	0.9500 E-02	-0.1880E-01	0.2091E-01	-0.1843E-01	-0.8944E-02
	$C_9$	-0.2438E-01	-0.1529E-02	-0.2756E-01	-0.2510E-02	-0.1289E-01
	$C_{10}$	0.3430 E-02	0.7058E-03	0.3659E-02	0.8053E-03	0.2102 E-02
		Earth	Mars (long term)	Mars (long term)	Mars (today)	Mars (today)
		non-porous	non-porous	porous	non-porous	porous
	$C_0$	-0.1099E + 02	-0.1089E + 02	-0.1213E + 02	-0.1082E + 02	-0.1207E + 02
	$C_1$	-0.3996E + 01	-0.4068E + 01	-0.3124E + 01	-0.4072E + 01	-0.3134E + 01
	$C_2$	0.2334E + 00	0.2279E + 00	$0.1295E{+}01$	0.2157E + 00	$0.1293E{+}01$
	$C_3$	0.1333E + 01	0.1422E + 01	0.1542E + 00	0.1426E + 01	$0.1713E{+}00$
	$C_4$	0.2286E-02	0.2470 E-01	-0.7519E + 00	0.3330E-01	-0.7518E + 00
	$C_5$	-0.6476E + 00	-0.7150E + 00	0.3125 E-01	-0.7171E + 00	0.2232E-01
	$C_6$	0.2875E-01	0.2056E-01	$0.1779E{+}00$	0.1856 E-01	$0.1786E{+}00$
	$C_7$	0.1313E + 00	$0.1535E{+}00$	-0.2311E-01	$0.1540E{+}00$	-0.2129E-01
	$C_8$	-0.1410E-01	-0.1526E-01	-0.1369E-01	-0.1510E-01	-0.1396E-01
	$C_9$	-0.1006E-01	-0.1252E-01	0.2623E-02	-0.1255E-01	0.2492 E-02
	$C_{10}$	0.1810E-02	0.2247E-02	0.5521E-05	0.2246E-02	0.3212E-04

5
Table 3
Impact rate, mean impact velocity and surface gravity for the inner planets, nor-
malized to the Moon's.

	Moon	Mercury	Venus	Earth	Mars	Mars
					(long term)	(today)
impact rate ratio	1	1.82	1.75	1.58	2.76	3.20
mean velocity ratio	1	2.16	1.28	1.04	0.53	0.54
surface gravity ratio	0 1	2.2	5.3	5.9	2.2	2.2
Rock						

Table 4 Unnormalized associated Legendre functions  $P_{lm}(\sin\lambda)$ l,m1 0, 0 $\cos \lambda$ 1, 1 $\frac{1}{2}(3\sin^2\lambda - 1)$ 2,0 $3\cos^2\lambda$ 2,2 $\tfrac{3}{2}(5\sin^2\lambda-1)\cos\lambda$ 3, 1 $\frac{1}{8}(35\sin^4\lambda - 30\sin^2\lambda + 3)$ 4, 0

Table 5

C

Spherical harmonic coefficients of the lunar relative impact flux  $\Delta \phi(\lambda, \varphi)$  for Earth– Moon separations of 30, 45, and 60 Earth radii.

Scalt

$C_{lm}$	30 Earth radii	45 Earth radii	60 Earth radii
$C_{0,0}$	1	1	1
$C_{1,-1}$	$-1.7779020 \times 10^{-1}$	$-1.4591830 \times 10^{-1}$	$-1.2670400\times 10^{-1}$
$C_{2,0}$	$-6.3209891 \times 10^{-2}$	$-6.2923420 \times 10^{-2}$	$-6.2755592\times 10^{-2}$
$C_{2,2}$	$-1.7937283 \times 10^{-3}$	$-1.2524090 \times 10^{-3}$	$-9.8998262\times 10^{-4}$
$C_{4,0}$	$-9.9735381 \times 10^{-3}$	$-1.0141450\times 10^{-2}$	$-1.0222370\times 10^{-2}$

Table 6 Spherical harmonic coefficients of the lunar relative cratering rate  $\Delta C(D > 1, \lambda, \varphi)$  for Earth–Moon separations of 30, 45, and 60 Earth radii.

SCRIP

		- ,	
60 Earth radii	45 Earth radii	30 Earth radii	$C_{lm}$
1	1	1	$C_{0,0}$
$-1.6092760\times 10^{-1}$	$-1.8571530 \times 10^{-1}$	$-2.2715950 \times 10^{-1}$	$C_{1,-1}$
$-1.3831914 \times 10^{-1}$	$-1.3874170 \times 10^{-1}$	$-1.3954110 \times 10^{-1}$	$C_{2,0}$
$-1.7822560\times 10^{-3}$	$-2.2729362 \times 10^{-3}$	$-3.3499412\times 10^{-3}$	$C_{2,2}$
$1.9948010 \times 10^{-3}$	$2.3263713 \times 10^{-3}$	$2.9040180 \times 10^{-3}$	$C_{3,-1}$
$2.8083000 \times 10^{-3}$	$2.7863080 \times 10^{-3}$	$2.7412530 \times 10^{-3}$	$C_{4,0}$

Table 7
Spherical harmonic coefficients of the relative impact flux $\Delta \phi(\lambda)$ for the terrestrial
planets.

*				
	Planet	$C_{00}$	$C_{20}$	$C_{40}$
	Mercury	1	$3.7395410 \times 10^{-2}$	$-7.9170623\times10^{-3}$
	Venus	1	$7.3546990 \times 10^{-3}$	$-6.0267052\times10^{-3}$
	Earth	1	$-2.6165971 \times 10^{-2}$	$-1.8682412\times 10^{-3}$
	Mars (today)	1	$1.6254980 \times 10^{-1}$	$-1.0738801 \times 10^{-2}$
	Mars (long-term average)	1	$8.7425552 \times 10^{-2}$	$4.7442493 \times 10^{-3}$

Table 8

Spherical harmonic coefficients of the relative cratering rate  $\Delta C(D > 1, \lambda)$  for the terrestrial planets.

SCRIP

Planet	$C_{00}$	C <sub>20</sub>	$C_{40}$
Mercury	1	$4.5850560 \times 10^{-2}$	$2.4749320 \times 10^{-3}$
Venus	1	$-1.8545722 \times 10^{-3}$	$3.5865970 \times 10^{-4}$
Earth	1	$-7.6586370 \times 10^{-2}$	$2.4353234 \times 10^{-4}$
Mars (today)	1	$3.3900970  imes 10^{-1}$	$2.2655340 \times 10^{-3}$
Mars (long-term average)	1	$1.7986312 \times 10^{-1}$	$-4.3484250\times 10^{-4}$

#### Table 9

Lunar crater chronology calibration points used in this study, and their spatial correction.

1

Geologic unit	Location	Age	Measured	Globally averaged
		_	crater density	crater density
		(Ga)	$(10^{-4} \text{ km}^{-2})$	$(10^{-4} \text{ km}^{-2})$
1 Cone crater	-3.7°N, -17.5°E	0.025	0.21	0.169
2 North Ray crater	$-9^{\circ}N$ , $-15.6^{\circ}E$	0.053	0.44	0.390
3 Tycho crater	$-43.4^{\circ}N$ , $-11.1^{\circ}E$	0.109	0.90	0.824
4 Copernicus	$9.7^{\circ}N$ , $-20.1^{\circ}E$	0.80	7.15	5.77
5 Ocean Procellarum (A12)	$18.4^{\circ}N$ , -57.4°E	3.15	36	29.7
6 Mare Crisium (L24)	$17.0^{\circ}N, 59.1^{\circ}E$	3.22	30	30.2
7 Mare Imbrium (A15)	$32.8^{\circ}N,-15.6^{\circ}E$	3.30	32	27.7
8 Mare Fecunditatis (L16)	-7.8°N,51.3°E	3.41	33	32.4
9 Mare Tranquilitatis (young) (A11)	$8.5^{\circ}N,31.4^{\circ}E$	3.58	64	60.1
10 Mare Serenitatis (A17)	$28.0^\circ\mathrm{N},\!17.5^\circ\mathrm{E}$	3.75	100	93.6
11 Mare Tranquilitatis (old) (A11)	$8.5^{\circ}N,31.4^{\circ}E$	3.80	90	84.5
12 Imbrium/Apennines	$18.9^{\circ}\mathrm{N}, -3.7^{\circ}\mathrm{E}$	3.85	350	301
13 Fra Mauro (A14)	$-3.7^{\circ}N, -17.5^{\circ}E$	3.85	370	298
14 Descartes (A16)	$-9.0^{\circ}N, 15.6^{\circ}E$	3.85	340	306