



Mathematical Model for Forecasting and Estimating of Market Demand

Dan Nicolae, Valentin Pau, Mihaela Jaradat, Mugurel Ionut Andreica, Vasile Deac

► To cite this version:

Dan Nicolae, Valentin Pau, Mihaela Jaradat, Mugurel Ionut Andreica, Vasile Deac. Mathematical Model for Forecasting and Estimating of Market Demand. Recent Advances in Applied Mathematics (ISBN: 978-960-474-150-2 / ISSN: 1790-2769) [Proceedings of the American Conference on Applied Mathematics (AMERICAN-MATH)], Jan 2010, Cambridge, United States. pp.629-634. hal-00768770

HAL Id: hal-00768770

<https://hal.science/hal-00768770>

Submitted on 23 Dec 2012

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Mathematical model for forecasting* and estimating of market demand

DAN NICOLAE^α, VALENTIN PAU^β, MIHAELA JARADAT^χ,

MUGUREL IONUT ANDREICA^δ, VASILE DEAC^δ

“Titu Maiorescu” University^{α,β} Bucharest, “Bogdan Vodă” University^χ Cluj Napoca,

Polytechnic University^δ of Bucharest, Academy of Economic Studies^δ Bucharest

ROMANIA

topmath2009@yahoo.com

*Nobel prize, 1972, John R.Hicks, Kenneth J.Arrow : „for their pioneering contributions to general economic equilibrium theory and welfare theory”.

Abstract : *The scientific study article(a monograph) , presents a forecast and estimate the evolution of the market demand.*

Key-Words: *fuzzy sets, forecasting, estimating, statistical extrapolation, market demand.*

1 Problem formulation

Methods for forecasting

1.1 Probabilistic Model [10]

The methods operate with certain statistical parameters.

The most frequently used is the mean square deviation, namely:

$$\sigma = \sqrt{\frac{\sum_{j=1}^n (x_j - \bar{x})^2}{n}} \quad (1)$$

We can use the statistical tables, if we know a certain distribution. Knowing the sequence of the likelihood coefficients k , we determine a probability

$P(x \geq \bar{x} + k\sigma)$, respectively $P(x \leq \bar{x} - k\sigma)$, if x is

maximized and a probability $P(x \geq \bar{x} - k\sigma)$,

respectively $P(x \leq \bar{x} + k\sigma)$ if x is minimized.

If the distribution function is not known, then we apply Cebâşev's formula:

$$P(\bar{x} - k\sigma \leq x \leq \bar{x} + k\sigma) \geq 1 - \frac{1}{k^2} \quad (2)$$

where k = the likelihood coefficient.

1.2 Fuzzy Model Used to Mean

Expanding [3], [13]

There are cases when mean expanding is useful.

For instance, it is possible that the mean not to be found in the excellence domain of the variable under research.

This case, after the mean value \bar{x} is determined, the membership degree [5], [6], $\mu(\bar{x}) = 1$ is assigned to it. In order to find an expanded mean, we proceed to "expanding" figure 1, for instance, on interval [0.95;1]. We calculate a value x' with the property that:

$$\mu(x') = 0,95 \quad (3)$$

If this value is single, then the expanded mean is the interval $[x', \bar{x}]$ or $[\bar{x}, x']$. If two values, x' and x'' would exist, such that:

$$\mu(x') = \mu(x'') = 0,95 \quad (4)$$

then, the expanded mean is given by the interval

$$[x', \bar{x}] \cup [\bar{x}, x''] = [x', x'']$$

1.3 Linguistic Model

For a linguistic procedure to be applied, it is necessary that the continuous parameter x (or the discrete parameter, but with a large number of restricted values) to be subject to a granulation operation. We consider the maximum and the minimum levels and among them, certain levels which are denominated by using words (verbs, adverbs) etc., as for instance:

if five levels must be introduced between a minimum weight piece and a maximum weight one, we shall use the following gradations :

- very light - for minimum;
- light - for immediately next level;
- average - for immediately next level;
- heavy - for immediately next level;

- very heavy - for immediately next level.

In order to find the average weight, if the frequency of gradations is symmetrical, we allow for the weights:

X_{\min} and X_{\max} , and we divide the level

$[X_{\min}, X_{\max}]$ into equal parts (four intervals exist among the five levels).

An interval I will have the length:

$$I = \frac{x_{\max} - x_{\min}}{4} \quad (5)$$

In this case, \bar{x} is determined by the relation:

$$\bar{x} = x_{\min} + 2I = x_{\max} - 2I \quad (6)$$

Practically, \bar{x} is lower rounded, and we obtain for mean x an interval under the form:

$$(\bar{x} - r, \bar{x} + r) \quad (7)$$

where r = the rounding value.

It is possible that this rounding not to be symmetrical, the obtained interval is:

$$[\bar{x} - r', \bar{x} + r''] \quad (8)$$

Where:

r' = rounding to left; r'' = rounding to right.

If the number of gradations is even, then, the number of intervals is odd, therefore the mean is already represented in an interval. If the frequency of gradations is not symmetrical, then each gradation is weighted with the interval centre (attached to each gradation). Thus, an average gradation representing an interval is obtained.

1.4 Mean Expanding Method

1.4.1 The Truncation Method

The steps of the method are:

Step 1. The value \bar{x}_i is truncated and a minimum j - level truncated value is obtained, according to the j -level of truncation: $y_{ij}^m = T_j \bar{x}_i$

Step 2. A unit for j -level is added, namely b^1 (where b is the numeration basis) and the maximum level is obtained.

$$y_{ij}^M = y_{ij}^m + b^j \quad (\text{for integer number}) \quad \text{and}$$

$$y_{ij}^M = y_{ij}^m + b^{-j} \quad (\text{for } j\text{-rank fractions})$$

j = decimals number.

1.4.2 The Rounding Method

The method consists in two steps:

Step 1. The number is lower rounded for n sequence,

yielding: x_n^m

Step 2. The number is upper rounded for the n level,

yielding: x_n^M

The differences are calculated in the same way:

$$\Delta_n = x_n^M - x_n^m \quad \text{with the property that:}$$

$$\lim_{x \rightarrow \infty} \Delta_n = 0$$

2 Models for estimating the evolution of market demand

2.1 Theory of demand

Quantitative theory of demand is based on the following *assumptions*:

I₁) If a stable income, the demand for certain goods decreases with increasing price, and vice versa.

This assumption allows the formulation of numerical methods with which to predict the increase (decrease) demand for the reduction (increase) the price in a defined proportion.

Graphic illustration where noted:

q_i = the demand volume for the product i ;

p_i = unit price for product i ,

(see fig. no.1 a, b, c).

The sensitivity of demand to price changes is illustrated by the *elasticity of demand coefficient* (c), of the price (p), which shows the percentage change (in reverse) application of a good, if its price changes by 1%.

The expression for calculation is:

$$E_{c/p} = \frac{\Delta C}{C} : \frac{\Delta p}{p} \quad (9)$$

ΔC , Δp = demand growth;

change (+/-)reference price in two periods.

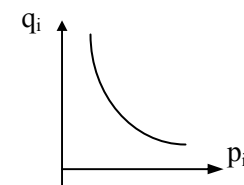


Figure (1a)

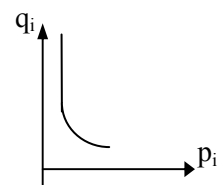


Figure (1b)

(1a) demand-price function curve ($q_i = (p_i)$);

(1b) a decrease in price causes a rapid increase in demand (luxury items);

(1c) demand is less sensitive to price changes

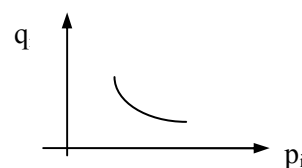


Figure (1c)

Fig. no.1.- Possible evolution of the price

I₂) If a variable income, demand for a good increases with income growth and decreases with increasing price. If we assume for each level of income other f_v demand function:

$$q_i = f_v(p_i), \quad (10)$$

where possible changes in demand can be represented by several successive demand curves as shown in fig.no.2.:

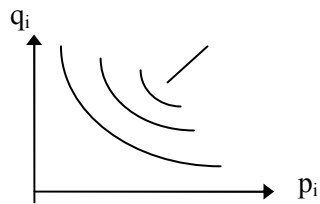


Fig.no.2.- Evolutions of demand

If the price is kept constant, the application may be described as a function of income:

$$q_i = f(v) \quad (11)$$

The coefficient of elasticity of demand (c) to income (v), shows the percentage increase in demand when income increases by 1%.

That is:

$$E_{c/v} = \frac{\Delta C}{C} : \frac{\Delta I}{I} \quad (12)$$

Changing prices determines travel demand curves. In the literature, curves expressing the dependence between demand and income are known as Engel curves. Dependence is linear, the demand for different products or product groups known saturation point and is influenced by the interchangeability of products. Demand-income dependence can be expressed primarily through the following types of functions:

a) $C_1 = \frac{m_1 V}{V + n_1}$ (for strictly necessary goods) (13)

b) $C_2 = \frac{m_2(V - p_2)}{V + n_2}$ (for everyday consumer goods) (14)

c) $C_3 = \frac{m_3 V(V - p_3)}{V + n_3}$ (for luxury goods) (15)

d) $C_4 = m_4 V - n_4 V^2$ (for goods that are out of use when a certain level of revenue) (16)

The significance of notations are:

C = demand for the product (or product group) considered;

V = income;

m, n, p = econometric parameters.

The interpretation of the four functions, is as follows:

a) C₁, first function, known as Törnquist function I, show that with increasing income, demand increases at a rate lower, and tends to be capped. The graph is shown in fig.4a.:

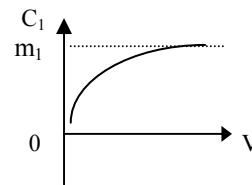


Fig.no.4a. -Törnquist I Function

When $V \text{ limit} \rightarrow \infty$ is m_1 , so the level toward which C₁ is M₁. The first derivative of the function is decreasing, tending to 0, when $V \rightarrow \infty$.

$$C_1 = \frac{\partial C_1}{\partial V} = \frac{m_1 n_1}{(V + n_1)^2} \quad (17)$$

The graph is shown in fig. no.4 b.:

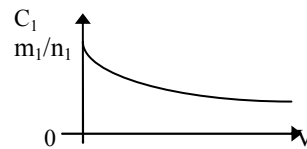


Fig. no. 4b.-Törnquist I Derivative function

b).C₂ function (Törnquist II) is similar to C₁ in terms of demand-income dependence, because it has a threshold of saturation (at m₂), which means that the growth of increasingly high on V, C₂ tend to m₂, which can be noticed by:

$$\lim_{V \rightarrow \infty} C_2 = \lim_{V \rightarrow \infty} \frac{m_2(V - p_2)}{V + n_2} = m_2 \quad (18)$$

The demand for this group of products or services begins to manifest only after $V > p_2$, meaning that p₂ parameter is negative, after calculations.

For $V = 0$:

$$C_2 = \frac{m_2 p_2}{n_2} \quad (19)$$

C₂ value for $V = 0$ is not negative, as $p_2 < 0$.

The graph is shown in fig. no.4c.:

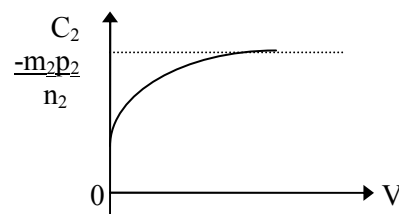


Fig. no.4c.-Törnquist II Function

First derivative function is also decreasing (fig.no.4d.):

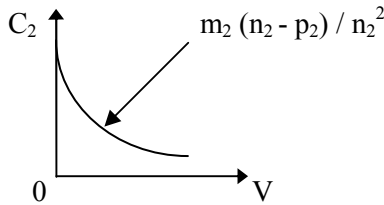


Fig.no.4d.-Törnquist II Derivative function

$$C_2' = \frac{\partial C_2}{\partial V} = \frac{m_2(n_2 + p_2)}{(V + n_2)^2} \quad (20)$$

c) C_3 function (Törnquist III) is specified for the products that required a continuous increase with increasing income. The curve presents an oblique asymptote to the branch to $+\infty$.

$$m = \lim_{V \rightarrow \infty} \frac{C_3}{V} = m_3 \quad (21)$$

$$n = \lim_{V \rightarrow \infty} (C_3 - m_3 V) = -m_x(p_3 + n_3) \quad (22)$$

Therefore, $C = m_3 V - m_3(p_3 + n_3)$ is the asymptotically wanted. For $n_3 < 0$ demand occurs when $-n_3 < p_3$. For graphic representation, we determine the intersection with the abscissa:

$$C = 0 \rightarrow V = p_3 + n_3$$

For the asymptote, and

$$C_3 + 0 \Rightarrow m_3 V \frac{V - p_3}{V + n_3} = 0 \Rightarrow V = p_3 \quad (23)$$

for the function.

As $n_3 < 0 \Rightarrow p_3 + n_3 < p_3$ and C_3 function graph is shown in fig.no. 4e.:

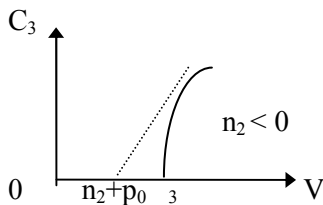


Fig.no.4e.-Törnquist III Function

First derivative of the function C_3 is increasing and has the expression:

$$C_3' = \frac{\partial C_3}{\partial V} = m_3 \left[1 - \frac{m_3^2 + n_3 p_3}{(V + n_3)^2} \right] \quad (24)$$

It may be noted that $C = m_3$ is a horizontal asymptote, and $V = -n_3$ is the vertical asymptote of C_3^I function. Given the fact that $n_3 < 0 \Rightarrow -n_3 > 0$, and the derivative graph is shown in fig.no. 4 f:

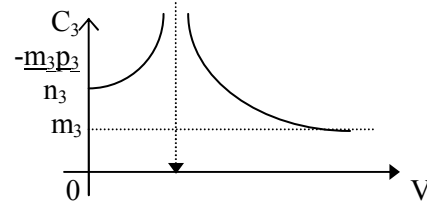


Fig.no.4 f.-Törnquist III Derivative function

For:

$$V = 0 \Rightarrow C_3' = \frac{-m_3 p_3}{n_3}; \frac{m_3 p_3}{n_3} > 0 \quad (25)$$

as $n_3 < 0$ and

$$m_3 < - (m_3 p_3) / n_3,$$

given that $-n_3 < p_3$.

d) C_4 function is typical for products or groups of product, which are out of use after a certain level of revenue.

So, the demand for such products are initially increases, reaches a maximum for $V = m_4/2n_4$, then start to decrease reaching zero, which means that for higher incomes than this limit, there is no demand for such products.

2 Problem Solution

Consumer demand is treated continuously in connection with the supply of goods, as among them there is a complex interaction.

An approach to the processes occurring on the market require prior research and a proper understanding of the real complexity of the categories of individual and social demand, potential and actual demand.

By research on a type of behavior, we can determine future structure of the supply / demand actually recorded in previous years ($t=1,2,\dots,-N$).

We note:

m_{ij} = demand of article j of family i ;

$i = \alpha$ (fixed), number of families;

$j = 1, 2, \dots, n$, number of articles of family i (may vary from year to year).

Assortment structure is given by the probability of producing an application of article j in m_{aj} quantity, the quantity demanded would be (26), (27):

$$m_{aj} = \sum_{j=1}^n m_y$$

$$P_{aj} = \frac{m_{aj}(t)}{\sum_{j=1}^n m_{aj}(t)}$$

Assume that requests for items j form a complete system of events.

For $t = 1, 2, \dots, N$ years, it is defined a stochastic matrix:

$$P_{jt} = \|P_{oj}(t)\|_{n \times n}; \sum_{j=1}^n P_{oj}(t) = 1 \quad (28)$$

As :

$$m_{oj}(t) > 0 \rightarrow \alpha(P_{ji}) - \max(\min P_{oj}(t) > 0) \quad (29)$$

$t = 1, \dots, N$;

$j = 1, \dots, n$. the matrix is Markov type.

We consider 2 consecutive years:

$$\Delta = P_{j,t+1} - P_{j,t}, (P_{j,t} = P_{nj}(t)) \quad (30)$$

If $\Delta > 0$, we note :

$$\Delta = \Delta_j^+ \text{ for } j = 1, 2, \dots, m$$

$$\Delta < 0, \Delta = \Delta_j^- \text{ } j = m+1, \dots, n$$

$$\Rightarrow \sum_{j=1}^m \Delta_j^+ = \Delta^+ = -\Delta^- = \sum_{j=m+1}^n \Delta_j^- \quad (31)$$

Starting from the definition of matrix $|P_{ij}|_{n \times n}$, it is defined a transition matrix:

$$T_g = \|T_{ij}\|_{n \times n}, T_{ij} = P_{ij} / \sum_{j=1}^n P_{ij} \quad (32)$$

Same for the following years $t = 1, 2, \dots, N$, we obtain $(N-1)$ matrixes type T_t . Matrix is built:

$$T^\Sigma = \sum_{t=1}^{N-1} T_g, \quad T^\Sigma = \|T_{ij}^\Sigma\| \quad (33)$$

from which to obtain the matrix:

$$M = \left\| \frac{T_{ij}^\Sigma}{\sum_{j=1}^n T_{ij}^\Sigma} \right\| \quad (34)$$

from which it can determine the structure of the vector elements of sales next year.

We note (35):

$$K_i^- = \frac{\Delta_i^-}{\sum_{i=m+1}^n \Delta_i^-} = \sum K_i^- = 1$$

It builds $|P_{ij}|_{n \times n}$ with elements defined as:

- if $i=j$ put $P_{ij} = \min(P_{ij}, P_{i,t+1})$

For i , for which Δ_i^+ put $P_{i,i} = \min(\dots, \dots)$,

and $P_{i,j} = 0$, for $j = 1$.

For i , for which Δ_i^- to do so:

- if j comes from elements for which:

$$\Delta_j^- \text{ put } P_{ij} = K_j^- \cdot \Delta_j^-$$

- if comes from elements for which:

$$\Delta_j^+ \text{ put } P_{ij} = K_j^+ \cdot \Delta_j^+$$

The practical application questions a number of years which is considered to obtain satisfactory results.

For a few years, there is a stochastic matrix (Markov).

From one year to another:

$$T_i^\Sigma = \sum_{i=1}^{N-2} T_i \quad (36)$$

The calculation for year N of matrix T_N is done by applying the recurrent relationship:

$$T_2^\Sigma = T_1^\Sigma + T_N$$

N is determined based on the matrix T_2 , then do

calculations to determine the structure vector

$(N+1)$ to determine the coefficients K_j^- and K_j^+ and

determine differences Δ_j , building on practical

application requirements. Observations must cover a

sufficient number of years, something that is

determined by simulation.

3 Conclusions

Given the dynamic link between demand and income, and its progress, the normal trend is diversification.

Philosophical connections

“Ubi Concordia, ibi Victoria!”

References:

- [1] Albu, L.I, Model to Estimate the Composite Index of Economic Activity in Romania, *Romanian Journal of Economic Forecasting*, Bucharest, Romania, no.2., vol .9, 2008.
- [2] Aluja, J. Gil, Investment Selection Base on Diversified Criteria, *Annals of Royal Academy of Economic And Financial Sciences*, Academic Year 1993/1994, pp 129-157.
- [3] Bellman, R.L., Zadeh, L.A., Decision-making in a Fuzzy Environment, *Management Science*, No.17, 1970.
- [4] Dubois, D.M, Sabatier, For a Naturalist Approach to Anticipation from Catastrophe Theory to Hyperincursive Modelling, in *CASYS International Journal of Computing Anticipatory System* (IJICAS), Liege, Belgium, D.M Dubois Ed., 1998.
- [5] Kauffmann, A., Aluja, J.G., *Las matematicas del azary de la incertidumbre*, Editorial Ceura, Madrid, Spain, 1990.
- [6] Negoitã, C.V. *Vag (Vague)*, Paralela 45 Publishing House, Pitesti, Romania, 2003.
- [7] Odobleja, St., *Psychologie consonnantiste*, Meloine Publishing House, Paris, France, 1938.
- [8] Ruxanda, Gh., Botezatu, A., Spurious Regression and Cointegration, Numerical Example Romania's M2 Money Demand, *Romanian Journal of Economic Forecasting*, no. 3, Bucharest, Romania, 2008.
- [9] Stoica, M., Andreica, M., Nicolae, D., Andreica, R., *Subtle Sets and their Applications*, Cibernetica Publishing House, Bucharest, Romania, 2008

- [10] Stoica, M., Andreica, M., Nicolae, D., Cantău, *Methods and Models of Economic Forecasting*, Universitara Publishing House, Bucharest, Romania, 2006.
- [11] Stoica, M., Nicolae, D., Ungureanu, M.A., Andreica, A., Andreica, M., Fuzzy Sets and their Applications, *Proceedings of the 9th WSEAS International Conference on Mathematics & Computers in Business & Economics* (MCBE'08), Bucharest, Romania, 2008.
- [12] Zadeh, L.A., Fuzzy Logic and its Applications to Approximate Reasoning,, *Information Processing* 3, IFIP Congress, Stockolm, Sweden, 1974.

- [13] Zadeh, L.A., Fuzzy Sets, *Info&Cth.*, Vol. I, 1965, pp. 338-353.
-

- [14] ** Oxford - Dictionary of Economics.
- [15] ** Economic Glossary.
- [16] ** Encyclopedia Britanica.