A LPV/$H_\infty$ Global Chassis Controller for 
performances Improvement Involving 
Braking, Suspension and Steering Systems

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Abstract: This paper is concerned with the design of a multivariable Global Chassis Controller (GCC), using LPV/$H_\infty$ robust controllers for suspension, active steering and electro-mechanical braking actuators which aim at improving comfort and safety performance in critical driving situations. The proposed solution is to schedule the three control actions (braking, steering and suspension) according to the driving situation evaluated by a specific monitor. In emergency cases, the GCC provides a working hierarchical use of the 3 controllers, depending on the dangerousness of the driving situation. Simulations on a complex nonlinear full vehicle model, subject to critical driving situations, show the reliability and robustness of the solution.

Keywords: Vehicle dynamics, Braking, Suspension, Steering, LPV, $H_\infty$ control.

1. INTRODUCTION

Automotive light vehicles are complex systems involving different dynamics. On one side, vertical, roll and pitch behaviors are often related to comfort performances (even if roll is also linked to safety characteristics, Gáspár et al. (2007)), and are rather slow dynamics. On the other hand, safety performances are mainly characterized by the longitudinal, lateral and yaw dynamics. This paper aims at controlling all those dynamics with a GCC design strategy. The new trends in vehicle dynamic control (either commercial or heavy) are to synthesize multivariable controllers able to achieve both comfort and safety according to the vehicle driving situation (e.g. normal, dangerous or critical) and to enhance performances using the available actuators. This led to an increasing research in this area. Some interesting results have been already obtained. In Gáspár et al. (2005), a heavy vehicle Linear Parameter Varying (LPV) model was introduced with a scheduled robust control, involving suspensions and braking actions. In Chou and d’Andréa Novel (2005), an interesting nonlinear control law involving suspension and braking actuators for commercial cars was developed. More recently, Falcone et al. (2007) a model predictive approach (involving online optimization) using braking and steering actuators proposed. Following our previous studies (Doumiati et al. 2010, Poussot-Vassal et al. (2011a), Poussot-Vassal et al. (2011b)) a new LPV Global Chassis Control is developed in this paper, using EMB at the rear axle, AS at the front axle and active suspensions actuators. The main objective is to enhance the vehicle handling and safety properties during critical driving situations (eg, large lateral accelerations, ...). Thanks to two high level monitoring parameters, a coordinated control of the three kinds of actuators

\[ i = \{f, r\} \text{ and } j = \{l, r\} \]

(Active Steering, Electro-Mechanical Braking, and active suspension actuators) is obtained. Indeed, while the lateral dynamics (yaw control) is handled by the braking/steering control, the control of the vertical dynamics using the suspension actuators is actually scheduled according to the level of emergency situations, allowing to set (in real time) the suspension performance from "soft" (normal situation) to "hard" (critical situation).

This control approach is performed in the $H_\infty$/LPV framework. The interest of the proposed GCC is that, more than a simple controller, it provides a hierarchical use of the control actions. First, when a dangerous situation is detected through the braking monitor $R_b$, the braking action is limited accordingly in order to bring back the force into the linear stable zone of the tire characteristic. To efficiently handle critical situations, the active steering and the active suspensions are activated, thanks to the use of a scheduling parameter $R_s$ function of the braking monitor $R_b$. The whole strategy allows to improve the road handling and to save energy, thanks to a smart progressive activation and coordination of the braking, suspension and steering actuators, according to the driving situation.

The paper is structured as follows: Section 1 provides introductive elements and notations. Section 2 briefly introduces the models used for synthesis and validation purpose, together with their limitations. In Section 3, the main contribution of the paper, in addition to the monitoring strategy for braking, is to coordinate the three actuators (steering, braking and suspension) to enhance the vehicle performances. Performance analysis is done in Section 4 through time domain simulations performed on a complex nonlinear full vehicle model. Conclusions and discussions are given in the last Section. Throughout the paper, the following notation will be adopted: index $i = \{f, r\}$ and $j = \{l, r\}$ are used to identify vehicle front, rear and left, right positions re-
respectively. Then, index \{s,t\} holds for forces provided by suspensions and tires respectively. \{x,y,z\} holds for forces and dynamics in the longitudinal, lateral and vertical axes respectively. Then let \( u = \sqrt{v_x^2 + v_y^2} \) denote the vehicle speed, \( R_{ij} = \frac{v_x - (z_{us\ ij} - z_{rs\ ij})}{v_y} \) the effective tire radius, \( m = m_0 + m_{uz\ ij} + m_{uz\ fr} + m_{uz\ rr} \) the total vehicle mass, \( \delta = \delta_d + \delta^s \) is the steering angle (\( \delta_d \)), the driver steering input and \( \delta^s \), the additional steering angle provided by steering actuator, see Section 3) and \( T_{bi}, \) the braking torque provided by the braking actuator (see Section 3). The model parameters are those of a Renault Mégane Coupé (see Table 1). They were obtained during a collaborative study with the MIPS laboratory in Mulhouse, through identification with the real data.

### 2. VEHICLE MODELING

#### Dynamical equations:
In this paper, a full nonlinear vehicle model is involved. This model and the corresponding parameters are detailed in Poussot-Vassal et al. (2011b). It reproduces the vertical (\( z_i \)), longitudinal (\( x \)), lateral (\( y \)), roll (\( \phi \)), pitch (\( \psi \)) and yaw (\( \psi \)) dynamics of the chassis. It also models the vertical and rotational motions of the wheels (\( z_{w\ ij} \) and \( \omega_{w\ ij} \) respectively), the slip ratios (\( \lambda_{ij} = \frac{\max(0, v_{ij} - R_{ij}\omega_{w\ ij}\cos(\beta_{ij}))}{v_{ij}} \)) and the center of gravity side slip angle (\( \beta_{sy\ ij} \)) as a function of the tires and suspensions forces. The dynamical equations are given in equation (1), where \( F_{ix\ ij} = F_{ix\ ij} + F_{ix\ rr} \), \( F_{iy\ ij} = F_{iy\ ij} + F_{iy\ rr} \), \( F_{iz\ ij} = F_{iz\ ij} + F_{iz\ rr} \), and \( F_{sz\ ij} = F_{sz\ ij} + F_{sz\ rr} \), \( i = \{f,r\} \). This model will be used in simulation for validation purpose (see Section 4). The main interest of the full vehicle model is that it takes into account nonlinear load transfer, slipping and side slip angles that are essential phenomena entering in the tire force, and consequently, in the global chassis dynamics, especially in dangerous driving situations. **Suspensions model:** Suspensions are usually modeled by a spring and a damping element. In real vehicles, their characteristics are nonlinear (see e.g. Zin et al. (2008)). Here, as long as the main focus is on the longitudinal, lateral, yaw behaviors and since active suspensions are considered, without loss of generality, linear models are assumed for stiffness and damping as:

\[
F_{sz\ ij} = k_{ij}(z_{sz\ ij} - z_{us\ ij}) + c_{ij}(\dot{z}_{sz\ ij} - \dot{z}_{us\ ij}) + u_{rj}^H \tag{2}
\]

where \( k_{ij} \) : the stiffness coefficient, \( c_{ij} \) : the damping coefficient and \( u_{rj}^H \) : the suspension control. **Actuators dynamic:** In the paper, the considered actuators are modeled as first order low-pass transfer functions:

- The active suspension systems:

\[
\dot{F}_{sus\ ij} = \tau(F_{0} - F_{sus\ ij}) \tag{3}
\]

where \( \tau = 200 \text{rad/s} \) is the actuator cut-off frequency. \( F_{sus\ ij} \) and \( F_{sus\ ij} \) are the suspension controller and actuator outputs, respectively.
- The EMB actuators, providing the braking torque:

\[
\dot{T_{bi}} = \kappa(T_{0} - T_{bi}) \tag{4}
\]

where, \( \kappa = 70 \text{rad/s} \) is the actuator cut-off frequency, \( T_0 \) and \( T_{bi} \) are the rear braking controller and actuator outputs respectively. In this paper, only the rear braking system is used to avoid coupling phenomena occurring with the steering system and because it affects more the vehicle yaw behavior than the front one does (\( j = r \)).
- The AS actuator providing an additional steering angle:

\[
\dot{\delta}^s = \kappa(\delta^f - \delta^s) \tag{5}
\]

where, \( \kappa = 10 \text{rad/s} \) is the actuator cut-off frequency, \( \delta^f \) and \( \delta^s \) are the steering controller and actuator outputs respectively. This actuator is constrained between \([-5, +5] \text{deg}\).}

### 3. MAIN RESULT: GCC STRUCTURE, SYNTHESIS AND SUPERVISION

This section is devoted to the description of the main result of this paper, namely, the multivariable Global Chassis Controller (GCC) involving front active steering, rear braking and active suspension actuators (see Fig. 1).

The objective is to improve handling and safety in critical situations, first, by using the rear braking actuators, then to activate the steering action and set the active suspension dampers to “hard” in order to improve the car handling performances. Conversely, during normal situations, the steering is deactivated, the braking action is attenuated, and the suspension dampers are set to “soft” to improve passengers comfort. Fig. 1 emphasizes the coordination through the use of the parameters \( R_s \) and \( R_b \).

![Fig. 1. Global chassis control Implementation scheme.](image-url)

The idea is that the GCC uses the monitoring parameters to generate the braking torques that aim at improving handling performances and enhancing the passengers safety when critical driving situations appear. It also provides
\[
\begin{align*}
\dot{v}_x &= -(F_{tx_j} \cos(\delta) + F_{tx_r} + F_{ty_j} \sin(\delta))/m - \dot{\psi} v_y \\
\dot{v}_y &= -(F_{tx_j} \sin(\delta) + F_{ty_j} + F_{ty_r} \cos(\delta))/m + \dot{\psi} v_x \\
\dot{z}_s &= -(F_{sz_j} + F_{sz_r} + F_{dz})/m_s \\
\dot{z}_{us_j} &= (F_{sz_j} - F_{tx_j})/m_{us_j} \\
\dot{\theta} &= ((F_{sz_j} - F_{sz_r})t_r + (F_{sz_r} - F_{sz_j})t_f + mh v_x)/I_x \\
\dot{\phi} &= ((F_{sz_j} t_f - F_{sz_r} t_r - mh v_x)/I_y \\
\dot{\psi} &= (F_{tz_j} \sin(\delta) + F_{ty_j} \cos(\delta)) - (F_{tx_j} - F_{tx_r})t_f \cos(\delta) - (F_{tx_r} - F_{tx_j})t_r + M_{dz})/I_z \\
\dot{\omega}_ij &= (F_{ty_j} + F_{ty_r})/(mv_x) + \psi
\end{align*}
\]

(1)

an additive steering angle and sets the suspension active dampers to "hard", providing then the maximum help to the driver and avoiding emergency situations.

3.1 The monitoring strategy

The aim of the monitoring is first to evaluate the driving situation and then to tune brake, steer and suspension control objectives to overcome conflicting effects. This strategy, introduced by the authors in Poussot-Vassal et al. (2011a) to schedule braking and steering action, is extended here to the three control actions. In a real car, such a block may be much more complex, but here, since as far as attitude and yaw stability are concerned, the following strategy, based on the measurement of the longitudinal slip ratio of the rear wheels \(s_{rj}\) is efficient while being simple. As previously introduced, two monitor variables are computed:

1. **Braking monitor:** \(R_b = \min_{ij=t,l,r}(r_b)\), is a function of the absolute value of the slip ratio \(|s_{rj}|\). \(r_b\) is defined as a relay (hysteresis like) function: \(\rightarrow 0\) when 'on', \(\rightarrow 1\) when 'off'. The switch 'on' (resp. 'off') threshold is \(s^+\) (resp. \(s^-\)) (see Poussot-Vassal et al. (2011b)). When the slipping is low, the vehicle is in a normal situation, hence \(R_b \rightarrow 1\). When the slip ratio raises and becomes greater than \(s^+\), a critical situation is detected, then \(R_b \rightarrow 0\). As \(R_b\) is function of the slip ratio, the choice of \(s^+\) (resp. \(s^-\)) is done according to the tire friction curve. Here (and in a general case), \(s^+ = 9%\) and \(s^- = 8\%\), in order to delimitate the linear and peak tire friction force with the unstable part of the tire. See in Section 3.2.1 how the braking controller is tuned according to the \(R_b\) parameter and its role in the control strategy.

2. **Suspension and steering monitor:** \(R_s\) is defined as:

\[
R_s = \begin{cases} 
1 & \text{when } 1 > R_b > R^2_{crit} \\
R^2_{crit} - R^2_{crit} & \text{when } R^2_{crit} < R_s < R^2_{crit} \\
0 & \text{when } 0 < R_s < R^2_{crit}
\end{cases}
\]

(6)

when \(R_b > R^2_{crit}(= 0.9)\), i.e. when low slip \(< s^-\) is detected, the vehicle is not in an emergency situation and \(R_b\) is set to 1. When \(R_b < R^2_{crit}(= 0.7)\), i.e. when high slip occurs \(> s^+\), a critical situation is reached and \(R_b\) is set to 0. Intermediate values of \(R_b\) will give intermediate driving situations. Hence, as explained in Section 3.2.2, in the first case, the suspension dampers will be set to "soft" to enhance comfort and there is no need to use a steering assistance. In the second case, the suspension control is "hard" to improve road handling and the steering assistance is essential to enhance the car handling and passengers safety (intermediate performances will be reached for \(R_b\) values in between).

In the following, all controllers are derived thanks to the LPV/\(H\infty\) methodology in order to meet the monitor requirements. Such a synthesis makes it possible to smoothly change the control performances thanks to the parameters (here \(R_b\) and \(R_s\), guaranteeing internal stability (avoiding switching) and minimizing the \(H\infty\) norm.

3.2 Global Chassis Control Design

The GCC is developed in two steps. First the steering/braking controllers are designed using the linear bicycle model. The suspension controller is synthesized using the linear vertical full car model. In each case the LPV/\(H\infty\) controllers scheduled by the monitoring parameters \(R_b\) and \(R_s\), are developed using the approach dedicated to polytopic systems.

**Problem formulation for the design of the braking/steering control** Let introduce first the extended bicycle model described in Eq.(7).

The considered LPV/\(H\infty\) control problem is described in Fig(2) with the following scheduled weighting functions:

- \(W_{\psi \dot{\psi}} = 10^{s/\alpha_0+1} \), is used to shape the yaw rate error \(\dot{\psi}_{ref} - \dot{\psi}\)\(\psi = \dot{\psi}_{ref} - \dot{\psi}\)
- \(W_{\psi \dot{\psi}} = 10^{-3}\), attenuates the lateral acceleration
- \(W_{\psi \dot{\psi}}(R_b) = (1 - R_b) 10^{s/\alpha_0+1} \), attenuates the yaw moment control input
- \(W_{R_s}(R_s) = R_s 10^{s/\alpha_0+1} \), attenuates the steering control input according to value of \(R_s\)

where \(\alpha\) (or) is the braking (steering) actuator cut-off frequency.

- When \(R_b \rightarrow 1\), the tire is in the linear zone, there is no risk of locking; the weighting function gain of \(W_{\psi \dot{\psi}}\) is chosen to be low. Therefore, the braking control is activated.

- When \(R_b \rightarrow 0\), a high slip ratio is detected, and a critical situation is detected, the tire may lock, so the gain of the weighting function is set to be high. This allows to deactivate the braking signal and therefore, a natural stabilisation of the slip dynamic is achieved.

On the other hand, when the driving situation is dangerous and presents high risk for passengers, the steering control...
is activated through \( W_{iF}(R_b) \). The steering action depends on the varying parameter \( R_b \) which depends on the driving situation monitored by the parameter \( R_b \), with \( R_b(\cdot) \in \mathcal{P}_{R_b} \) and \( \mathcal{P}_{R_b} := \{ R_b \in \mathbb{R} : R_{s} \leq R_b \leq R_{s} \} \) (where \( R_{s} = 0.1 \) and \( R_{s} = 1 \)).

**Fig. 2.** Generalized plant for braking steering control synthesis.

The generalized plant corresponding to Fig 2 is LPV and can be modeled as,

\[
\Sigma(R(\cdot)) : \begin{bmatrix} \dot{x} \\ z \\ w \\ u \end{bmatrix} = \begin{bmatrix} A(R(\cdot)) & B_1(R(\cdot)) & B_2 \\ C_1(R(\cdot)) & D_{11}(R(\cdot)) & D_{12} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ w \\ u \end{bmatrix}
\]

where \( x \) includes the state variables of the system and of the weighing functions, \( w = F_{dy} \) and \( u = \theta \) are the exogenous and control inputs respectively; \( z = [z_1, z_2, z_3, z_4] = [W_{v}, \dot{\psi}, \tilde{w}, W_{\tilde{w}}(R_b)T_{b_{ij}}, W_{iF}(R_b)\theta] \) holds for the controlled output, and \( y = \psi_{ref}(v) - \psi \) is the controller input \( (\psi_{ref}(v) \text{ is provided by a reference bicycle model as the one described in (7))} \). Notice that the LPV model (8) is affine w.r.t parameters \( R_b \) and \( R_0 \) and can be described as a polytopic system, i.e. a convex combination of the systems defined at each vertex formed by \( \mathcal{P}_{R(\cdot)} \), namely \( \Sigma(R(\cdot)) \) and \( \Sigma(\mathcal{R}(\cdot)) \).

**Problem formulation for the design of the suspension control** The control of the vertical dynamics is ensured through the suspension system, in order to achieve frequency specification performances, see Poussot-Vassal et al. (2006) and Sammier et al. (2003). According to the driving situation given by \( R_b \) and as soon as a critical one is detected, the parameter \( R_b \) decreases . The steering control is therefore activated and the suspension control is tuned to enhance roadholding. This controller is tuned, thanks to the LPV/H∞ techniques using a full linear vertical model and the following generalized plant, (see Fig. 3) including parameterized weighting functions:

where \( W_{z_0}(R_b) = R_b \frac{z_0^2 + 2\zeta_0 \omega_0 + \omega_0^2}{\omega_0^2 + 2\zeta_0 \omega_0 \omega_0 + \omega_0^2} \) is shaped in order to reduce the bounce amplification of the suspended mass \( (z_0) \) between \([0,12]\), \( W_{\tilde{w}}(R_b) = (1 - R_b) \frac{z_2^2 + 2\zeta_2 \omega_2 + \omega_2^2}{\omega_2^2 + 2\zeta_2 \omega_2 + \omega_2^2} \) attenuates the roll bounce amplification in low frequencies. The parameters of these weighting functions are obtained using genetic algorithm optimization as in Do et al. (2010), \( W_u = 3.10^{-2} \) is set to shape the control signal.

According to Fig 3, the following parameter dependent suspension generalized plant \( \Sigma_{vy}(R_b) \) is obtained:

\[
\Sigma_{vy}(R_b) := \begin{bmatrix} \dot{\xi} = A(R_b)\xi + B_1\tilde{w} + B_2u \\ \tilde{w} = C_1\xi + D_{11}\tilde{w} + D_{12}u \\ y = C_2\xi + D_{21}\tilde{w} + D_{22}u \end{bmatrix}
\]

where \( \xi = [\chi_x \chi_x]^T; \tilde{z} = [z_1 \ z_2 \ z_3]^T; \tilde{w} = [\tilde{z}_{ij} F_{dy,x,y} M_{dy,y}]^T; y = z_{def}; u = u_{ij}^w; \chi_x \) are the vertical weighting functions states.

The LPV system (Eq.9) includes a single parameter and can be described as a polytopic system, i.e. a convex combination of the systems defined at each vertex of a polytope defined by the bounds of the varying parameter.

**4. SIMULATION RESULTS**

Some frequency domain plots are provided to analyse the performances of the lateral and vertical controllers. Time domain simulations are performed on the full nonlinear vehicle model given in Section 2, including nonlinear suspensions forces. In the sequel, the performances obtained by the proposed gain-scheduled controller, denoted as ‘LPV’, are analyzed and compared to the Renault Mégane Coupé car (without control, denoted as ‘Reference car’) and, for sake of completeness, with a simple LTI/H∞ controller (without scheduled gains), denoted as ’LTI’. Note that the LTI/H∞ controller is obtained by solving the previous
$H_\infty$ problems frozen with $R_s = 0.1$ and $R_b = 0.9$. The following scenario is used:

1. the vehicle runs at 130km/h in straight line,
2. 5cm bump on the left wheels (from $t = 0.5$ to 1s),
3. a double line change manoeuvre is performed (from $t = 2$ to 6s) by the driver,
4. lateral wind occurs at vehicle’s front, generating an undesirable yaw moment (from $t = 2.5$ to 3s),
5. 5cm bump on the left wheels, during the manoeuvre (from $t = 3$ to 3.5s).

The resulting monitored signals are obtained (see Fig. 4).

Fig. 4. Monitoring signals

Fig. 5. Yaw rate

In this scenario, the road is considered as wet, which reduces the road/tire adhesion and the lateral tire contact forces. A consequence, the uncontrolled vehicle drives away from the desired path.

The yaw rate and lateral speed in Fig 5 and 6 show that the proposed integrated control is very efficient and enhances the vehicle stability compared to the uncontrolled one.

The variation (see Fig. 4) of the scheduling parameters shows that the braking monitor $R_b$ (thanks to the slip ratio measure) attenuates the control gain of the braking controller (compared to LTI controller) in order to reduce the torque control (see Fig.9) and brings back the slip ratio close to the linear zone. It shows also that the maximum braking forces are not efficient enough to stabilize the vehicle when the critical situation appears. At this moment, the scheduling parameter $R_s$ takes the value that provides the necessary assistance to driver, either by giving an additional steering $\delta^+$ and setting the suspension dampers to “hard” to enhance roadholding. This control strategy allows to reduce lateral accelerations and to enhance yaw rate tracking and stability, compared to the non controlled one (see Fig. 5 and 6).

Good improvements of the chassis displacement and roll motion are shown in Fig. 7 and 8, thanks to the coordination between the three actuators, which allows to get a significant improvement of the desired performances.

Fig. 7. Vertical displacement of the chassis

Fig. 8. Roll angle ($\theta$)

Fig. 9. Rear right/left Breaking torque.
and LPV case. The steering control seems saturated, but Fig. 11 shows the steering control contribution for the LTI controller uses all the actuators simultaneously, without any coordination (i.e., more energy consumption in the LTI case then in the proposed LPV strategy since the actuators coordination (i.e, more energy consumption in the LTI case performances and stability, using the previously presented method). Furthermore, the LTI controller uses all the actuators simultaneously, without any coordination (i.e. more energy consumption in the LTI case then in the proposed LPV strategy since the actuators actions are activated only when necessary).

**5. CONCLUSION**

In this paper, a new LPV/H∞ GCC strategy is developed, including braking, steering and suspension actuators in order to improve the performances and to handle critical driving situations. Simulations of a consistent representative driving situation, performed on a complex nonlinear model, have shown the efficiency of the proposed approach. The authors stress that one of the advantages of the braking method used is that the exact knowledge of the tire force curve is not needed to guarantee good performances. As well, the strategy previously developed can be used with other braking strategies (in particular with the local ABS, see Tanelli et al. (2007)).

As a perspective, extensions to semi active suspension will be considered following the recent authors works in that field (see Savaresi et al. (2010)).

**REFERENCES**


