A dynamic output feedback controller for a network controlled system based on delay estimates

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A Dynamic Output Feedback Controller for NCS based on Delay Estimates [*]

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Abstract

The stabilization of a network controlled system including a global time-varying delay is investigated in this note. This delay is considered to be unknown but it is assumed that a bounded error estimate is available. The exponential uncertainty induced by the time-varying delay is decomposed into a sum of a polytopic term and an uncertain bounded term. Sufficient conditions to design a dynamic output feedback controller depending on estimate of the time-varying delay are proposed as LMIs. An illustration shows how our methodology enlarges the design techniques of the literature.

Key words: Dynamic Output Feedback, Network Control System, Delay estimate.

1 Introduction

The Network Controlled Systems (NCS) are frequently encountered in practice for widespread fields of applications due to their suitable and flexible structure [1, 3]. Nevertheless some aspects of NCS such as time-varying delays, quantization, drop-out, imply that the stability analysis and control design become fundamentally more difficult to investigate [7]. They lead to a rich literature in automatic control particularly in what concerns the stability aspects of systems with time-varying delays [5, 9, 11, 23, 24].

In order to achieve stability requirements, controller synthesis methodologies have been proposed. Among of them the use of Lyapunov functions is the most popular for the stability analysis [5, 23] and control synthesis [6]. Robust controllers independent on the time-varying delays may be designed to stabilize the NCS but such strategies are generally source of conservatism. In order to reduce such a conservatism for NCS for which the current time-varying delay is available, controllers depending on the time-varying delays may be proposed.

Recently output dynamic feedback design for NCS has been proposed using Linear Matrix Inequalities (LMI), based on the design procedure introduced by C. Scherer in [21] or on the elimination lemma. The output dynamic feedback may be independent with respect to the time-varying delay [8,10,18], or time-varying delay dependent: [22] considers two delays (back and forth), [17] a global time-varying delay less than the sampling period and [16] a global and bounded time-varying delay multiple of the sampling period.

Knowing the value of the current time-varying delay is in practice difficult, not to say impossible. Nevertheless tools allowing to have a time-varying delay estimate are provided in the literature (see for more details [4, 20]). The goal of this paper is not to enrich this mature literature, but to make the best use of the information conveyed by the time-varying delay estimate in the stabilizing controller design. Such an idea was investigated in [13] for designing a state-feedback controller dependent on this estimate by considering an additional uncertain term in the exponential uncertainty.

This note aims at extending the results in [13] by relaxing the assumption of the state availability and by proposing a dynamic output feedback controller dependent on an estimate of the time-varying delay. The main result

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is allowed by considering the exponential uncertainty related to the estimate of the time-varying delay decomposed into a polytopic term and an uncertain term. Via the Petersen's Lemma and changes of variables similar to the one proposed by C. Scherer, we obtain sufficient conditions formulated as LMIs leading to the controller design ensuring the closed-loop stability of the NCS.

The note is organized as follows. In Section 2, the problem of dynamic output feedback controller design dependent on an estimate of the time-varying delay is formulated. Some preliminaries are proposed in Section 3 to allow the proof of the main result presented in Section 4. A numerical illustration is given in Section 5 to highlight the efficiency of our result compared with the literature, before some concluding remarks in Section 6.

Notation. $(.)'$ and $\star$ denote respectively the transpose and the symmetric block in a symmetric matrix. $\mathrm{diag}(A, B)$ denotes the diagonal matrix composed by matrices A and B. The hermitian operator is given by $\mathrm{He}(M) = M + M'$, for any square matrix M. $\sigma_{\max}(M)$ denotes the maximum singular value of the matrix M. Matrices $I_{n \times m}$ and $0_{n \times m}$ are respectively Identity matrix of size $n \times m$ and the null matrix of size $n \times m$. The shortcuts $I_n = I_{n \times n}$ and $0_n = 0_{n \times n}$ will be used. The symbol $\bullet$ denotes an irrelevant block in a matrix.

2 Problem Description

Consider the following continuous-time system,

$$\dot{x}(t) = Ax(t) + Bu(t), \quad \forall t \geq 0, \tag{1}$$
$$y(t) = Cx(t), \tag{2}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^p$ are respectively the continuous-time state, input and output of the system. The state $x(t)$ is sampled with a sampling period T to obtain the sampling states $x_k = x(kT)$ at time kT, $(k \in \mathbb{N})$. Due to the NCS structure, the continuous-time input $u(t)$ applied to the system is delayed by a global time-varying delay τ_k which is assumed to be unknown and to verify $0 \leq \tau_k \leq T$, that is,

$$u(t) = \begin{cases} u_{k-1}, & \forall t \in [kT, kT + \tau_k), \\ u_k, & \forall t \in [kT + \tau_k, (k+1)T). \end{cases} \tag{3}$$

Remark 1 *The assumption $0 \leq \tau_k \leq T$ may be relaxed into $0 \leq \tau_k \leq \ell T$, with ℓ an integer greater than one by an event-based approach [12] or into $(\ell-1)T \leq \tau_k \leq \ell T$ by using an extended state [24]. For the sake of clarity, we maintain in the sequel the assumption that the delays are lower than the sampling interval. Furthermore, this assumption is in accordance with NCS with real-time requirements, where reliable network environment with deterministic protocols are used.*

In such a framework, the discrete-time system writes

$$x_{k+1} = A_\mathrm{d} x_k + \Omega(\tau_k) B u_k + (B_\mathrm{d} - \Omega(\tau_k)B) u_{k-1}, \tag{4}$$
$$y_k = C x_k, \tag{5}$$

where $A_\mathrm{d} = e^{AT}$, $B_\mathrm{d} = \int_0^T e^{As} \mathrm{d}s B$ and $\Omega(\tau) = \int_0^{T-\tau} e^{As} \mathrm{d}s$, with $\tau \in [0, T]$. By introducing the extended state $\eta_k = \begin{bmatrix} x_k \\ u_{k-1} \end{bmatrix} \in \mathbb{R}^{n+m}$, one gets by following [13] the reformulation of the system (4):

$$\eta_{k+1} = \overline{A}(\tau_k)\eta_k + \overline{B}(\tau_k)u_k, \tag{6}$$
$$y_k = \overline{C}\eta_k, \tag{7}$$

where $\overline{A}(\tau_k) = \begin{bmatrix} A_\mathrm{d} & (B_\mathrm{d} - \Omega(\tau_k)B) \\ 0_{m \times n} & 0_m \end{bmatrix}$; $\overline{B}(\tau_k) = \begin{bmatrix} \Omega(\tau_k)B \\ I_m \end{bmatrix}$; $\overline{C} = \begin{bmatrix} C & 0_{p \times m} \end{bmatrix}$.

As it has been mentioned, the current delay τ_k is not exactly known and we consider the following assumption.

Assumption 2 *An estimate $\hat{\tau}_k \in [0, T]$ of τ_k is available and is characterized by $\delta\tau_{min} \leq \delta\tau_k = \tau_k - \hat{\tau}_k \leq \delta\tau_{max}$, where by definition $-T \leq \delta\tau_{min} \leq \delta\tau_{max} \leq T$.*

We thus introduce the reformulation

$$\eta_{k+1} = \left(\overline{A}(\hat{\tau}_k) + \begin{bmatrix} 0_n & -W_k B \\ 0_{m \times n} & 0_m \end{bmatrix} \right) \eta_k$$
$$+ \left(\overline{B}(\hat{\tau}_k) + \begin{bmatrix} W_k B \\ 0_m \end{bmatrix} \right) u_k, \tag{8}$$

where $W_k = \Omega(\tau_k) - \Omega(\hat{\tau}_k) = \Omega(T - \delta\tau_k)[I_n + A\Omega(\hat{\tau}_k)]$. Moreover, we relax the state availability assumption in [13] and assume here what follows.

Assumption 3 *Only the output y_k of the system is available, instead of its state x_k.*

The paper aims at solving the following problem.

Problem 4 *Consider the system (8). Under Assumptions 2 and 3, determine a full order dynamic output feedback, in the form*

$$\zeta_{k+1} = A_c(\hat{\tau}_k)\zeta_k + B_c(\hat{\tau}_k)y_k, \tag{9}$$
$$u_k = C_c \zeta_k + D_c y_k, \tag{10}$$

where $\zeta_k \in \mathbb{R}^{n+m}$, which asymptotically stabilizes the system (8).

Remark 5 *The model (10) involves only constant matrices C_c and D_c. It allows to consider in the sequel a structure of the output dynamic feedback which is linear with respect to the polytopic weighting gains.*

3 Preliminaries

By defining $0 \le \rho_k = 1 - \frac{\tau_k}{T} \le 1$, we can decompose $\Omega(\tau_k)$ into an uncertain polynomial parameter dependent matrix and an uncertain bounded remainder $\Delta\Omega_k$:
$\Omega(\tau_k) = T \int_0^{\rho_k} e^{ATs'} \mathrm{d}s' = \sum_{i=1}^h \frac{A^{i-1}T^i}{i!}\rho_k^i + \Delta\Omega_k$. Let introduce the following lemma, proven in [14].

Lemma 6 *The uncertain polynomial parameter dependent matrix $L(\rho) = L_0 + \rho L_1 + \rho^2 L_2 + \cdots + \rho^h L_h$, with $0 \le \rho \le 1$, can be rewritten as a convex polytope with $h + 1$ vertices, with $\mathcal{H} = \{1; \cdots; (h+1)\}$: $L(\rho) = \sum_{i\in\mathcal{H}} \mu_i(\rho)V_i$, with $V_i = \sum_{j=0}^{i-1} L_j; \forall i \in \mathcal{H}$ and $\mu_{h+1} = \rho^h; \mu_i = \rho^{i-1}(1-\rho), \forall i \in \{1, \cdots, h\}$, which verify $\sum_{i\in\mathcal{H}} \mu_i(\rho) = 1$, and $\mu_i(\rho) \ge 0, \forall i \in \mathcal{H}. \square$*

By identifying $L_0 = 0_n$; and $L_i = \frac{A^{i-1}T^i}{i!}, \forall i = 1, \cdots, h$, we have, with the definition of V_i given in Lemma 6: $\Omega(\tau_k) = \sum_{i\in\mathcal{H}} \mu_i(\hat{\rho}_k)V_i + (W_k + \Delta\Omega_k)$, and the following decompositions

$$\overline{A}(\tau_k) = \sum_{i\in\mathcal{H}} \mu_i(\hat{\rho}_k)\overline{A}_i + \begin{bmatrix} I_n \\ 0_{m\times n} \end{bmatrix}(W_k + \Delta\Omega_k)\begin{bmatrix} 0_n & -B \end{bmatrix};$$

$$\overline{B}(\tau_k) = \sum_{i=1}^{h+1} \mu_i(\hat{\rho}_k)\overline{B}_i + \begin{bmatrix} I_n \\ 0_{m\times n} \end{bmatrix}(W_k + \Delta\Omega_k)B;$$

$$\overline{A}_i = \begin{bmatrix} A_{\mathrm{d}} & (B_{\mathrm{d}} - V_iB) \\ 0_{m\times n} & 0_m \end{bmatrix}; \overline{B}_i = \begin{bmatrix} V_iB \\ I_m \end{bmatrix}.$$

In order to cope with the time-varying term $(W_k + \Delta\Omega_k)$ in the decomposition of $\Omega(\tau_k)$, we remind the following lemma provided in [19].

Lemma 7 *Given any positive scalar $\lambda > 0$ and any matrix Γ such that $\Gamma'\Gamma \le \gamma^2 I$, for any matrices H and Q of appropriated dimensions, we have $H\Gamma Q + Q'\Gamma'H' + \lambda HH' + \gamma^2\frac{Q'Q}{\lambda} \ge 0. \qquad \square$*

The main idea is to consider for the dynamic output feedback controller a polytopic structure depending on the parameters $\mu_i(\hat{\rho}_k)$, with $\hat{\rho}_k = 1 - \frac{\hat{\tau}_k}{T}$. That is we impose $A_{\mathrm{c}}(\hat{\tau}_k) = \sum_{i\in\mathcal{H}} \mu_i(\hat{\rho}_k)A_{\mathrm{c},i}; B_{\mathrm{c}}(\hat{\tau}_k) = \sum_{i\in\mathcal{H}} \mu_i(\hat{\rho}_k)B_{\mathrm{c},i}$.

The controlled system, with state $z_k = \begin{bmatrix} \eta_k \\ \zeta_k \end{bmatrix}$ becomes

$$z_{k+1} = \sum_{i=1}^{h+1} \mu_i(\hat{\rho}_k)(\mathbb{A}_i + E\Gamma_kF)z_k = \mathbb{A}_{\mathrm{cl},k}z_k, \quad (11)$$

where $F = \begin{bmatrix} 0_n & -B & 0_{n\times(n+m)} \end{bmatrix} + \begin{bmatrix} BD_{\mathrm{c}}\overline{C} & BC_{\mathrm{c}} \end{bmatrix}$;

$$\mathbb{A}_i = \begin{bmatrix} \overline{A}_i + \overline{B}_iD_{\mathrm{c}}\overline{C} & \overline{B}_iC_{\mathrm{c}} \\ B_{\mathrm{c},i}\overline{C} & A_{\mathrm{c},i} \end{bmatrix}; E = \begin{bmatrix} I_n \\ 0_{(n+2m)\times n} \end{bmatrix};$$

$$\Gamma_k = W_k + \Delta\Omega_k = \Omega(\tau_k) - \Omega(\hat{\tau}_k) + \Delta\Omega_k.$$

The equation (11) exhibits a polytopic part and an uncertain part, where W_k and $\Delta\Omega_k$ are gathered in Γ_k. We should point out that there exists an upper bound of the maximal singular value of matrix $\Gamma_k = \Omega(T - \delta\tau_k)[I_n + A\Omega(\hat{\tau}_k)] + \Delta\Omega_k$. It has been already shown in [13] that the uncertainty $\Omega(T - \delta\tau_k)$ is norm-bounded on a compact $\delta\tau_k \in [\delta\tau_{\min}; \delta\tau_{\max}]$. The matrix $\Delta\Omega_k$ is also bounded on the range $0 \le \hat{\rho}_k \le 1$. Thus there exists a scalar γ such that, with $\mathcal{T}(\tau_k) = [\tau_k - \delta\tau_{\max}; \tau_k - \delta\tau_{\min}] \cap [0; T]$

$$\sup_{\tau_k\in[0;T]; \hat{\tau}_k\in\mathcal{T}(\tau_k)} \sigma_{\max}(\Gamma_k) \le \gamma. \quad (12)$$

Remark 8 *We can obtain numerically an upper bound of the maximal singular value of Γ_k by computing offline the singular value of Γ_k on a sufficiently tight grid verifying $\tau_k \in [0; T]$ and $\hat{\tau}_k \in \mathcal{T}(\tau_k)$.*

The fundamental preliminary result is presented in the following proposition.

Proposition 9 *If there exist a matrix $U \in \mathbb{R}^{2(n+m)\times 2(n+m)}$, symmetric matrices $S_i \in \mathbb{R}^{2(n+m)\times 2(n+m)}$, matrices $A_{c,i}, B_{c,i}, (i\in\mathcal{H}), C_c$ and D_c such that the inequality*

$$\begin{bmatrix} U + U' - S_i & \star \\ (\mathbb{A}_i + E\Gamma_kF)U & S_j \end{bmatrix} > 0_{4(n+m)}, \forall(i,j)\in\mathcal{H}\times\mathcal{H} \quad (13)$$

is verified, then the controller (9)-(10) stabilizes the NCS (8).

Proof 10 *Assuming that the inequality (13) is verified, this implies that $U + U' - S_i > 0_{2(n+m)}$ and $S_i > 0_{2(n+m)}$, which induces that U is of full rank and invertible. Moreover $(U - S_i)'S_i^{-1}(U - S_i) \ge 0_{2(n+m)}$ allows the inequality, $\forall(i,j)\in\mathcal{H}\times\mathcal{H}$*

$$\begin{bmatrix} U'S_i^{-1}U & \star \\ (\mathbb{A}_i + E\Gamma_kF)U & S_j \end{bmatrix} > 0_{4(n+m)}. \quad (14)$$

3

By pre-multiplying the inequality (14) by $diag((U')^{-1}; S_i^{-1})$, and by post-multiplying it by $diag(U^{-1}; S_i^{-1})$, we have, with the change of variables $P_i = S_i^{-1}$, $\mathcal{L}_{ij} = \begin{bmatrix} P_i & \star \\ P_j(\mathbb{A}_i + E\Gamma_k F) & P_j \end{bmatrix} > 0_{4(n+m)}, \forall (i,j) \in \mathcal{H} \times \mathcal{H}$.

By denoting $P(\hat{\tau}_k) = \sum_{i \in \mathcal{H}} \mu_i(\hat{\rho}_k) P_i$ and by summing the last inequality and finally applying a Schur complement, one gets

$$\mathbb{A}'_{cl,k} P(\hat{\tau}_{k+1}) \mathbb{A}_{cl,k} - P(\hat{\tau}_k) < 0_{2(n+m)}. \qquad (15)$$

The positive definiteness of the matrices P_i and the inequality (15) allow to prove that the function $V(z_k, \hat{\tau}_k) = z_k' P(\hat{\tau}_k) z_k$ is a Lyapunov function for the closed-loop system (11), which is thus asymptotically stable.

The next section will provide sufficient conditions designing the dynamic output feedback (9).

4 Designing the dynamic output feedback controller

The inequality (13) is not linear with respect to the gain matrices of the controller, but via a change of variables, originally proposed in an LTI case in [21], this is reformulated into LMIs.

Proposition 11 *Let us consider the predefined system (8) and a scalar γ verifying the inequality (12). Assume that there exists matrices $X, Y, K \in \mathbb{R}^{(n+m)\times(n+m)}$, matrices $\hat{A}_i \in \mathbb{R}^{(n+m)\times(n+m)}$, $\hat{B}_i \in \mathbb{R}^{(n+m)\times p}$, matrices $\hat{C} \in \mathbb{R}^{m\times(n+m)}$ and $\hat{D} \in \mathbb{R}^{m\times p}$ and symmetric positive definite matrices $\hat{S}_i \in \mathbb{R}^{2(n+m)\times 2(n+m)}$, $(i \in \mathcal{H})$ such that the following LMIs are verified, $\forall (i,j) \in \mathcal{H} \times \mathcal{H}$*

$$\begin{bmatrix} \mathcal{X}_1 - \hat{S}_i & \star & \star & \star \\ \mathcal{X}_{2,i} & \hat{S}_j & \star & \star \\ 0_{n\times 2(n+m)} & \mathcal{X}_3 & I_n & \star \\ \gamma \mathcal{X}_4 & 0_{n\times 2(n+m)} & 0_n & I_n \end{bmatrix} > 0_{(6n+4m)}, \qquad (16)$$

where

$$\mathcal{X}_1 = \begin{bmatrix} Y + Y' & I_{(n+m)} + K \\ I_{(n+m)} + K' & X + X' \end{bmatrix}; \qquad (17)$$

$$\mathcal{X}_{2,i} = \begin{bmatrix} Y'\overline{A}_i + \hat{B}_i \overline{C} & \hat{A}_i \\ \overline{A}_i + \overline{B}_i \hat{D}\overline{C} & \overline{A}_i X + \overline{B}_i \hat{C} \end{bmatrix}; \qquad (18)$$

$$\mathcal{X}_3 = \begin{bmatrix} \begin{bmatrix} I_n & 0_{n\times m} \end{bmatrix} Y \begin{bmatrix} I_n & 0_{n\times m} \end{bmatrix} \end{bmatrix}; \qquad (19)$$

$$\mathcal{X}_4 = \begin{bmatrix} 0_n & -B \end{bmatrix} \begin{bmatrix} I_{(n+m)} & X \end{bmatrix} + \begin{bmatrix} B\hat{D}\overline{C} & B\hat{C} \end{bmatrix}. \qquad (20)$$

By choosing any invertible matrix $M \in \mathbb{R}^{(n+m)\times(n+m)}$, let us set the matrix $N = (M')^{-1}(K - Y'X)$. N is invertible and the controller with gain matrices

$$D_c = \hat{D}, \qquad (21)$$

$$C_c = (\hat{C} - \hat{D}\overline{C}X)N^{-1}, \qquad (22)$$

$$B_{c,i} = (M')^{-1}(\hat{B}_i - Y'\overline{B}_i \hat{D}), \qquad (23)$$

$$A_{c,i} = (M')^{-1}(\hat{A}_i - Y'(\overline{A}_i + \overline{B}_i \hat{D}\overline{C})X$$
$$- Y'\overline{B}_i C_c N - M'B_{c,i}\overline{C}X)N^{-1}, \qquad (24)$$

stabilizes the closed-loop system (11).

Proof 12 *The proof is based on a suitable congruence transformation and a change of variables allowing to obtain the inequality (13) in Proposition 9. Such a transformation has been initially proposed in [21] and extended in [2, 15]. The time-dependency of the Lyapunov function implies that the decomposition provided in [21] should be applied on the multiplier matrix U instead of the matrices S_i $(i \in \mathcal{H})$ and that U should be time-independent. Let us decompose the matrices U and U^{-1} as $U = \begin{bmatrix} X & \bullet \\ N & \bullet \end{bmatrix}$; $U^{-1} = \begin{bmatrix} Y & \bullet \\ M & \bullet \end{bmatrix}$. The matrix $M \in \mathbb{R}^{(n+m)\times(n+m)}$ appearing U^{-1} is assumed to be invertible. This invertibility property will be induced thereafter by the constraints about matrix U. Let us introduce the matrix $\Pi = \begin{bmatrix} Y & I_{(n+m)} \\ M & 0_{(n+m)} \end{bmatrix}$. We have the following relations $U\Pi = \begin{bmatrix} I_{(n+m)} & X \\ 0_{(n+m)} & N \end{bmatrix}$; $\Pi'U\Pi = \begin{bmatrix} Y' & K \\ I_{(n+m)} & X \end{bmatrix}$; by noting $K = (Y'X + M'N)$, as introduced in the proposition. Using the structure of U allows to reformulate $\Pi'(U + U')\Pi = \mathcal{X}_1$; $\Pi'\mathbb{A}_i U\Pi = \mathcal{X}_{2,i}$; $E'\Pi = \mathcal{X}_3$; $FU\Pi = \mathcal{X}_4$ where the changes of variables are used*

$$\hat{D} = D_c; \qquad (25)$$

$$\hat{C} = \hat{D}\overline{C}X + C_c N; \qquad (26)$$

$$\hat{B}_i = M'B_{c,i} + Y'\overline{B}_i \hat{D}; \qquad (27)$$

$$\hat{A}_i = M'A_{c,i}N + Y'(\overline{A}_i + \overline{B}_i \hat{D}\overline{C})X$$
$$+ Y'\overline{B}_i C_c N + M'B_{c,i}\overline{C}X. \qquad (28)$$

In addition, we will use in the sequel the change of variables $\hat{S}_i = \Pi'S_i\Pi$. By applying the Schur complement to the LMI (16) and due Lemma 7, we have $(\lambda = 1)$,

$$\begin{bmatrix} \Pi'(U + U' - S_i)\Pi & \star \\ \Pi'(\mathbb{A}_i + E\Gamma_k F_i)U\Pi & \Pi'S_j\Pi \end{bmatrix} > 0_{4(n+m)}. \qquad (29)$$

By considering M invertible, the matrix Π is invertible. Pre-multiplying the inequality (29) by $diag((\Pi')^{-1}, (\Pi')^{-1})$

and post-multiplying it by $diag(\Pi^{-1}, \Pi^{-1})$ leads to inequality (13). We have only to prove that N defined by $N = (M')^{-1}(K - Y'X)$ is invertible to allow the change of variables in Equations (22) and (24) and to end the proof. Due to Inequality (16), $\mathcal{X}_1$ is invertible and also $\Pi'U\Pi$. The product

$$\begin{bmatrix} I_{(n+m)} & -Y' \\ 0_{(n+m)} & I_{(n+m)} \end{bmatrix} \begin{bmatrix} Y' & K \\ I_{(n+m)} & X \end{bmatrix} = \begin{bmatrix} 0_{(n+m)} & K - Y'X \\ I_{(n+m)} & X \end{bmatrix}$$

is then invertible, as $(K - Y'X)$ and N.

The proof of Proposition 11 considers the particular case $\lambda = 1$ in Lemma 7. In fact setting $\lambda = 1$ is with no loss of generality but allows the linearization of the condition (16). The main result provided by Proposition 11 is based on the feasibility of LMIs (16) for a fixed value of γ. The question then arises to know what range of uncertainty $[\delta\tau_{\max}; \delta\tau_{\min}]$ for the dynamic output feedback controller ensures the closed-loop stability.

Proposition 13 *Considering that LMIs (16) are feasible for a value $\gamma = \gamma^* > 0$, then they are feasible for any value $0 < \gamma \leq \gamma^*$.*

Proof 14 *The property is obvious by applying on LMIs (16), with $\gamma = \gamma^* > 0$, the change of basis $diag(I_{5n+4m}; (\gamma^*)^{-1}I_n)$.*

Based on Proposition 13, a numerical line-search allows to compute a threshold γ^* of LMIs (16)-feasibility. By defining $\delta = \delta\tau_{\max} = -\delta\tau_{\min}$, an abacus based on the increasing function $\delta \to \gamma(\delta)$ can be computed. This abacus is thereafter used to obtain $\delta = \delta^*$ related to γ^*.

5 Illustration

In this section, we consider an example coming from [6] and revisited in [13], for which $n = 2$, $m = 1$, $A = \begin{bmatrix} 103.5 & 0 \\ 0 & -43.5 \end{bmatrix}$, $B = \begin{bmatrix} 33.6 \\ -5.1 \end{bmatrix}$. It has been pointed out that the design of a delay-independent state-feedback controller by applying the design method proposed by [6] failed with $T = 0.005$, nevertheless a design of a state-feedback controller dependent on an estimate of the delay with $\delta = 0.0015 = 3T/10$ succeeded via the method provided in [13]. Nevertheless such a method is not suitable when the state is not fully accessible. Let us consider $C = [10\ 1]$ to define the output of the system. γ is given in Table 1, as function of h. h should be chosen to reach a compromise between the improvement of γ and the size of the LMIs in Proposition 11. Here we choose $h = 1$ to apply Proposition 11, which leads to a solution.

By considering $\delta = \delta\tau_{\max} = -\delta\tau_{\min}$, one can plot the function $\delta \to \gamma(\delta)$ for distinct values of h on Figure 1.

h	1	2	3	4	5
γ	0.0039	0.0026	0.0024	0.0023	0.0023

Table 1
Upper bound γ associated with Γ_k in function of h.

For a fixed value of δ, when h increases, the upper bound of the uncertainty decreases by definition. For our simu-
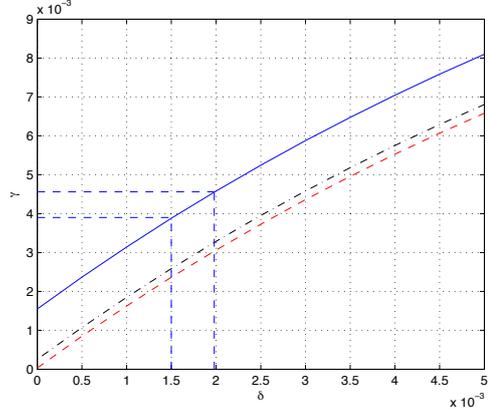

Fig. 1. Values of the bound γ in function of $\delta = \delta\tau_{\max} = -\delta\tau_{\min} \in [0; T]$ for $h = 1$ (blue solid line), $h = 2$ (black dashdot line) and $h = 3$ (red dashed line).

lation, the time-varying delay τ_k is randomly generated in $[0, T]$ and $\hat{\tau}_k$ in $\mathcal{T}(\tau_k)$. When choosing $M = I_{(n+m)}$, our approach leads to the state- and input-trajectories plotted in Figure 2, converging to the origin. Via the zero order holder, the continuous time system is simulated via TrueTime on Matlab-Simulink. The resulting continuous-time trajectories are superposed on the Figure 2. A numerical line-search allows to determine the
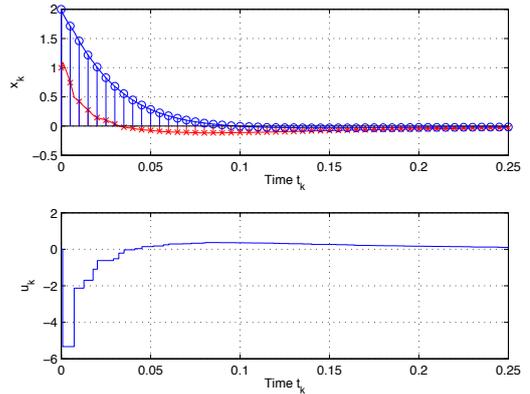

Fig. 2. State x_k ($\times$ for $x_{1,k}$ and $\circ$ for $x_{2,k}$) and the input u_k, in function of time t_k.

thresholds γ^* in function of h (given in Table 2). By using Figure 1, we can assert that our designed dynamic output feedback stabilizes the system for an estimate error less than $\pm 1.98 \times 10^{-3}s$.

5

h	1	2	3
γ^*	4.56×10^{-3}	4.32×10^{-3}	4.28×10^{-3}

Table 2

Threshold γ^* of LMIs (16)-feasibility in function of h.

6   Conclusion

The design of a dynamic output feedback controller for a network controlled system based on the knowledge of time-varying delay estimate has been studied in this note. As a preliminary result, the exponential uncertainty is rewritten into a sum of a polytopic term and an uncertain bounded term. With the help of the Petersen's lemma and a change of variables inspired by the Scherer's one, a sufficient condition formulated as LMIs has been provided to design a dynamic output feedback controller stabilizing the network controlled system. A illustrative example has been proposed.

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