Accessibility for Plurals in Continuation Semantics

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Logic and Engineering of Natural Language Semantics 9 (LENLS 9)

November 30, 2012

Outline

Overview

Key Words

Plurality (Group, Individual), Dynamic Semantics, Continuation, DRT, Anaphoric Accessibility, Functional Programming

Main goals of the presentation:

1. Investigating two plurality formations (mostly based on [Kamp and Reyle, 1993])
2. Compositionally obtaining the semantic representation for plurality under dynamic semantics

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  - Some Terminologies: *cohesion*, *anaphor*, *antecedent*
  - Anaphora ties pieces of discourse into a "unified whole"

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Example (Anaphora)

(1)
a. *John*$_1$ has *a car*$_2$. *He*$_1$ likes *it*$_2$.
b. *John*$_1$ has a car. *His*$_1$ car is red.
c. John has *a car*$_1$. *The car*$_1$ is red.
d. John has a *cool car*$_1$. Mary has a *same one*$_1$.
e. *John drives to work everyday*$_1$. *It*$_1$ takes him half an hour.

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The Problem of Plurality

- The semantics of plurality is not a naïve quantitative extension of singularity

Example (Distributivity vs. Collectivity)

(2) a. John and Mary went to school.
b. John and Mary gathered in Paris.
c. John and Mary lifted a piano.

- Singular and Plural Pronouns
  - *he*, *she*, *I*: **individuals**
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Summation Sketch

Definition (Summation) [Kamp and Reyle, 1993]

The process of constructing plural referents (groups of individuals) out of explicit individuals.

Example (Summation Sketch)

(3) a. John went to Bill's party with Mary. They had a nice time.

b. John loves Mary. Bill also loves Mary. They have to find a solution.

- Plural referents (groups of individuals) do not need necessarily be explicitly mentioned in the context, e.g.,

In (3-a): John $\oplus$ Bill $\oplus$ Mary;

in (3-b): John $\oplus$ Bill, John $\oplus$ Bill $\oplus$ Mary

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Example (Summation Sketch Continued)

(4) John went to Paris. Bill and Mary gathered to Rome.

a. She enjoyed the historical monuments very much.

b. They planned the whole trip without telling her.

- Even plural referents are explicitly mentioned, the individual components can **be broken down** and **re-form** other plural referents, e.g.,

In (4-a): from Bill $\oplus$ Mary $\Rightarrow$ Mary;
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The process of constructing plural referents (groups of individuals) out of quantified noun phrases.

- Quantified NP: *quantifier* + *noun*
- Generalized quantifier: *every*, *all*, *none*, *most*, *few*, etc.

Example (Abstraction Sketch)

(5) a. Every farmer owns a donkey. *He is /They are rich.
b. Few students came on time. They were too lazy.

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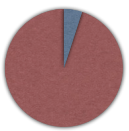
every ⇒  ; *few* ⇒ 

A New Approach to Dynamics [de Groote, 2006]

- A **pure** Montagovian framework for discourse dynamics
- Basic Types
  - ι (e): individuals/entities
  - o (t): propositions/truth values
  - γ: left context

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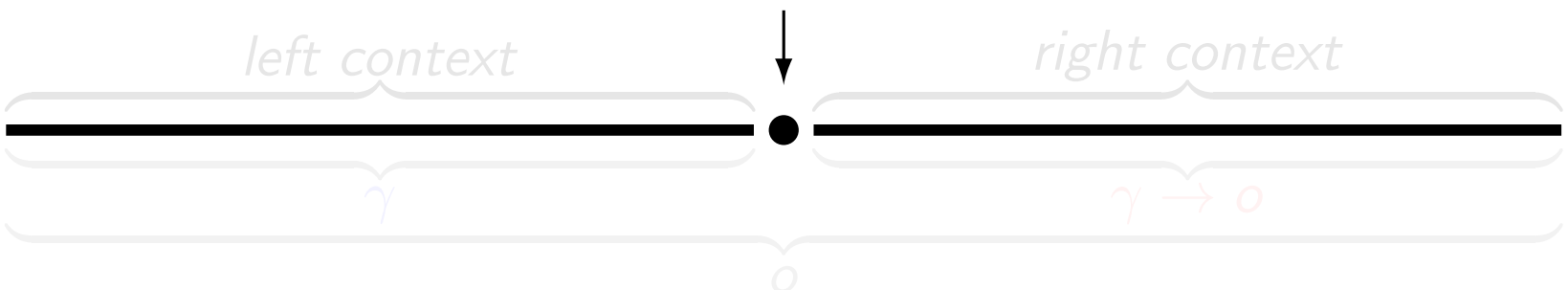

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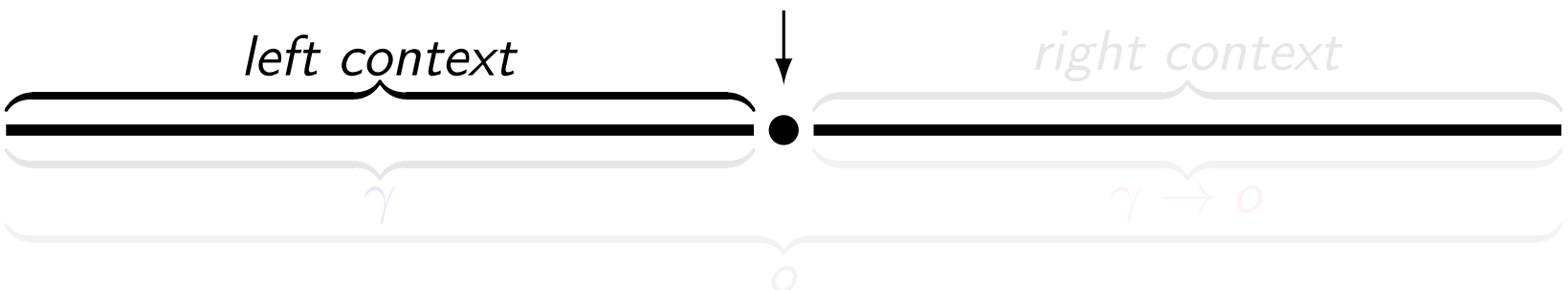

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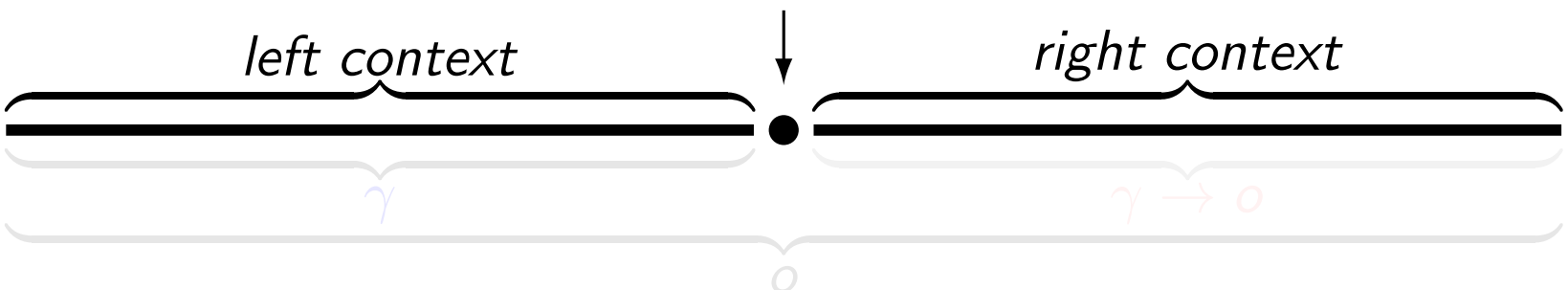

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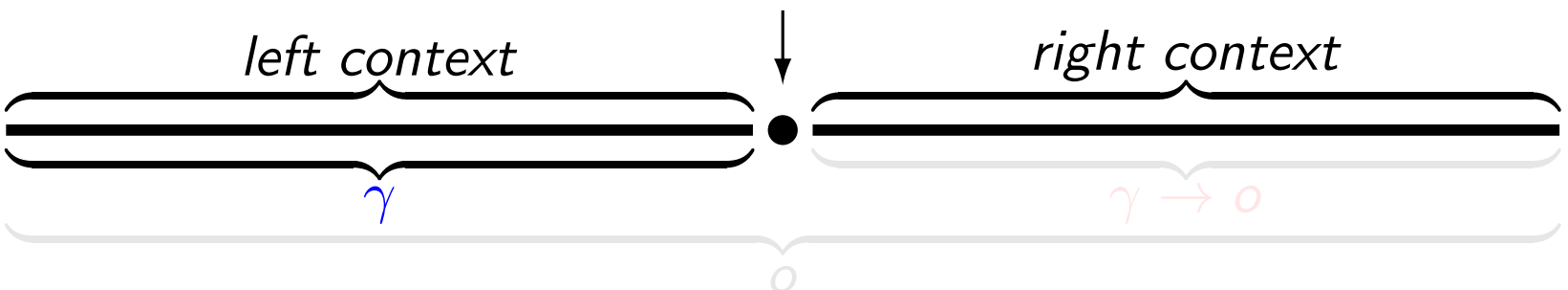

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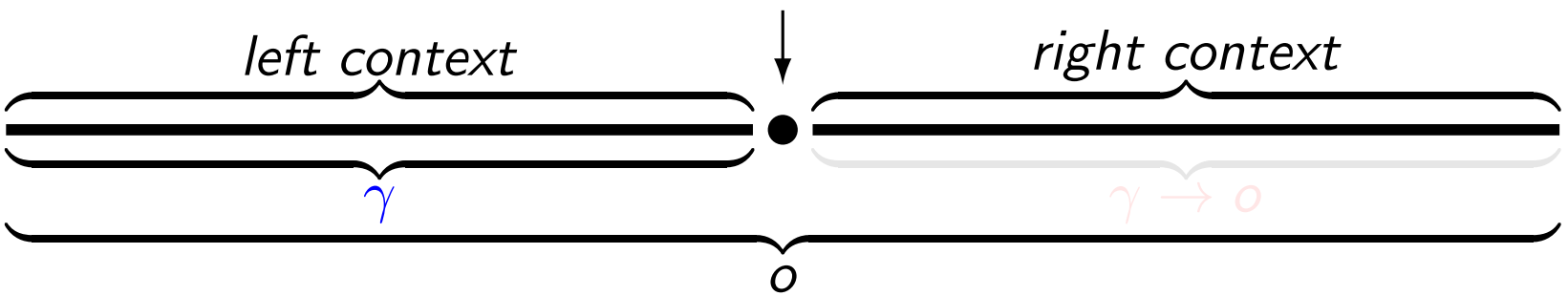

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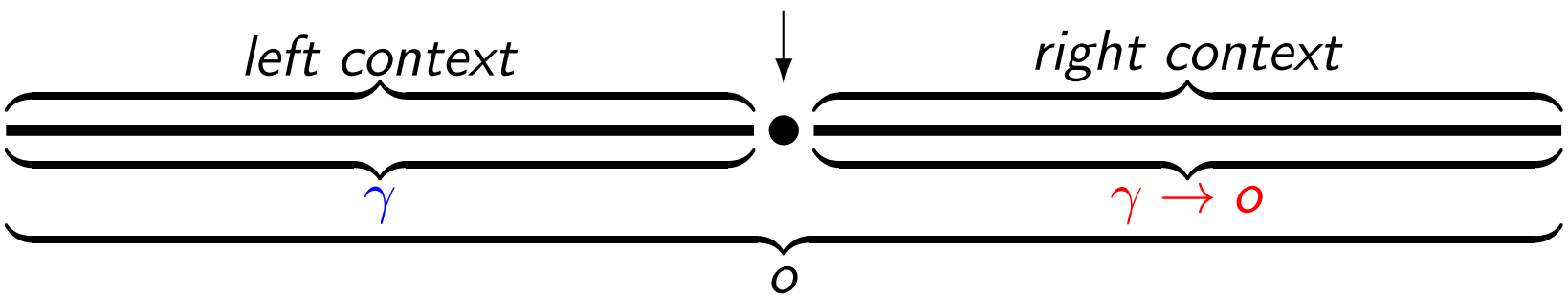

Type System & Composition

Typing Rules

$[\![s]\!]$	$\gamma \to (\gamma \to o) \to o$	o
$[\![n]\!]$	$\iota \to [\![s]\!]$	$\iota \to o$
$[\![np]\!]$	$(\iota \to [\![s]\!]) \to [\![s]\!]$	$(\iota \to o) \to o$

Discourse Composition

$$[\![D.S]\!] = \lambda e\phi.[\![D]\!]e(\lambda e'.[\![S]\!]e'\phi)$$

- A general DRS corresponds to:

$$\lambda e\phi.\exists x_1 \cdots x_n.C_1 \wedge \cdots C_m \wedge \phi e'$$

- e' is a left context made of e and the variables $x_1, x_2, x_3, \ldots$

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Lexical Entries

Lexicon	**λ-Expression**
John/Mary	$\lambda\psi e\phi.\psi\mathbf{j}/\mathbf{m}(\mathbf{j}/\mathbf{m} :: e)\phi$
she/they	$\lambda\psi e\phi.\psi(sel_{she/they}\, e)e\phi$
smiles	$\lambda s.s(\lambda xe\phi.Smile(x) \wedge \phi e)$
kisses	$\lambda os.s(\lambda x.o(\lambda ye\phi.Kiss(x, y) \wedge \phi e))$

- Remarks
  - "::" adjoins **accessible variables** in the selection list
  - "sel_{she}" selects the **correct variable** from the list

$\iota \to \gamma \to \gamma$

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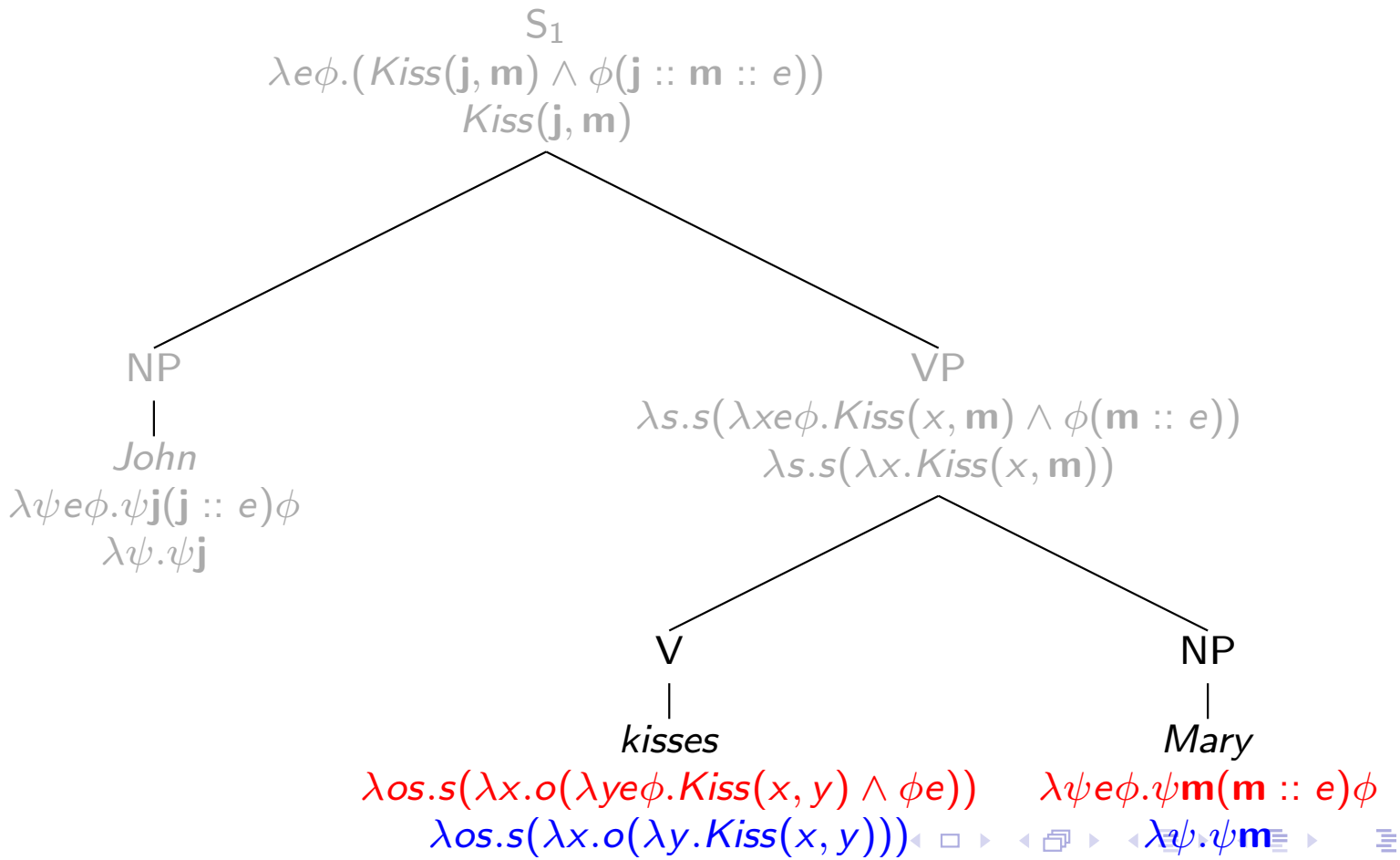
Compositional Example

(6) John kisses Mary. She smiles.

1. 

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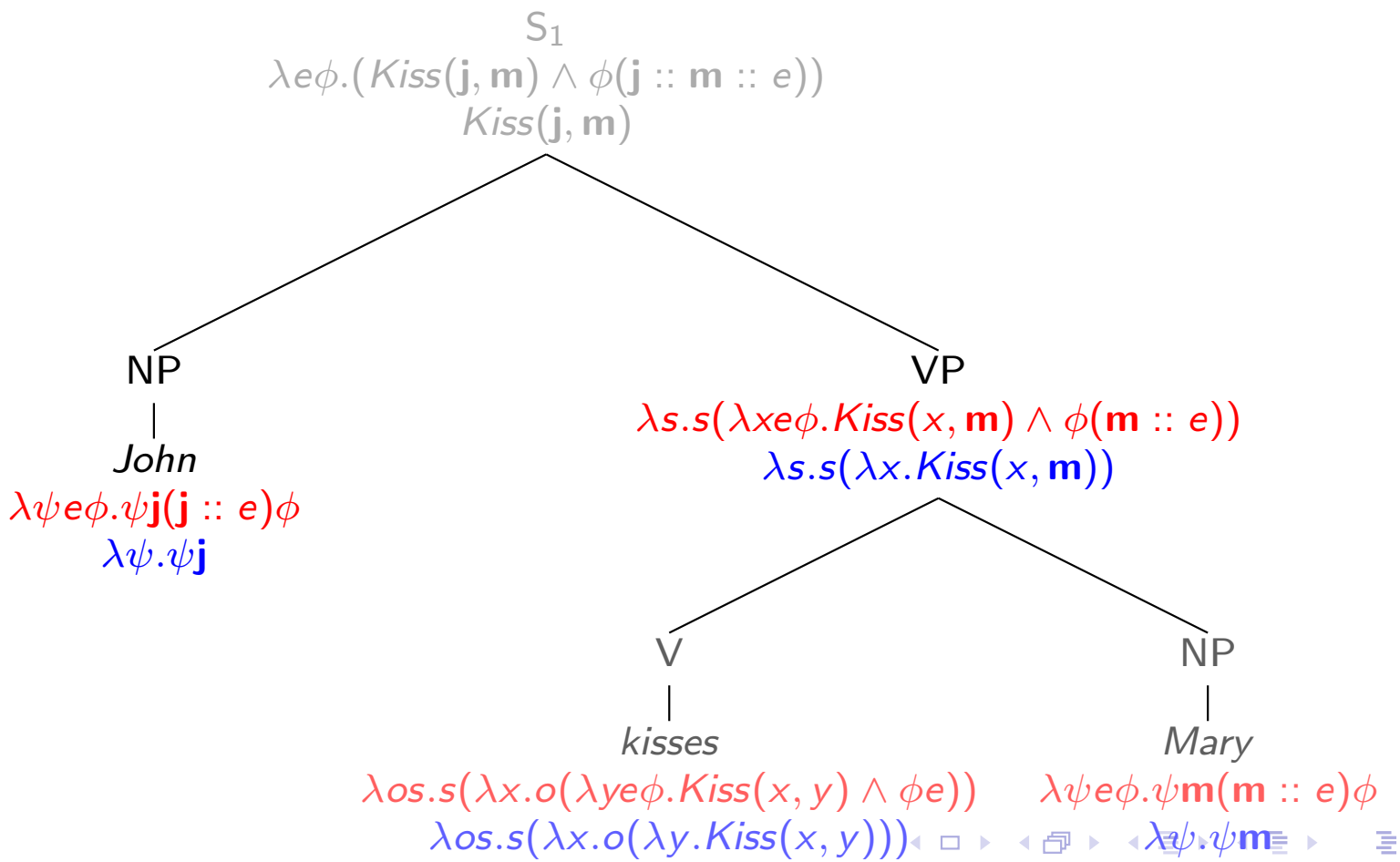

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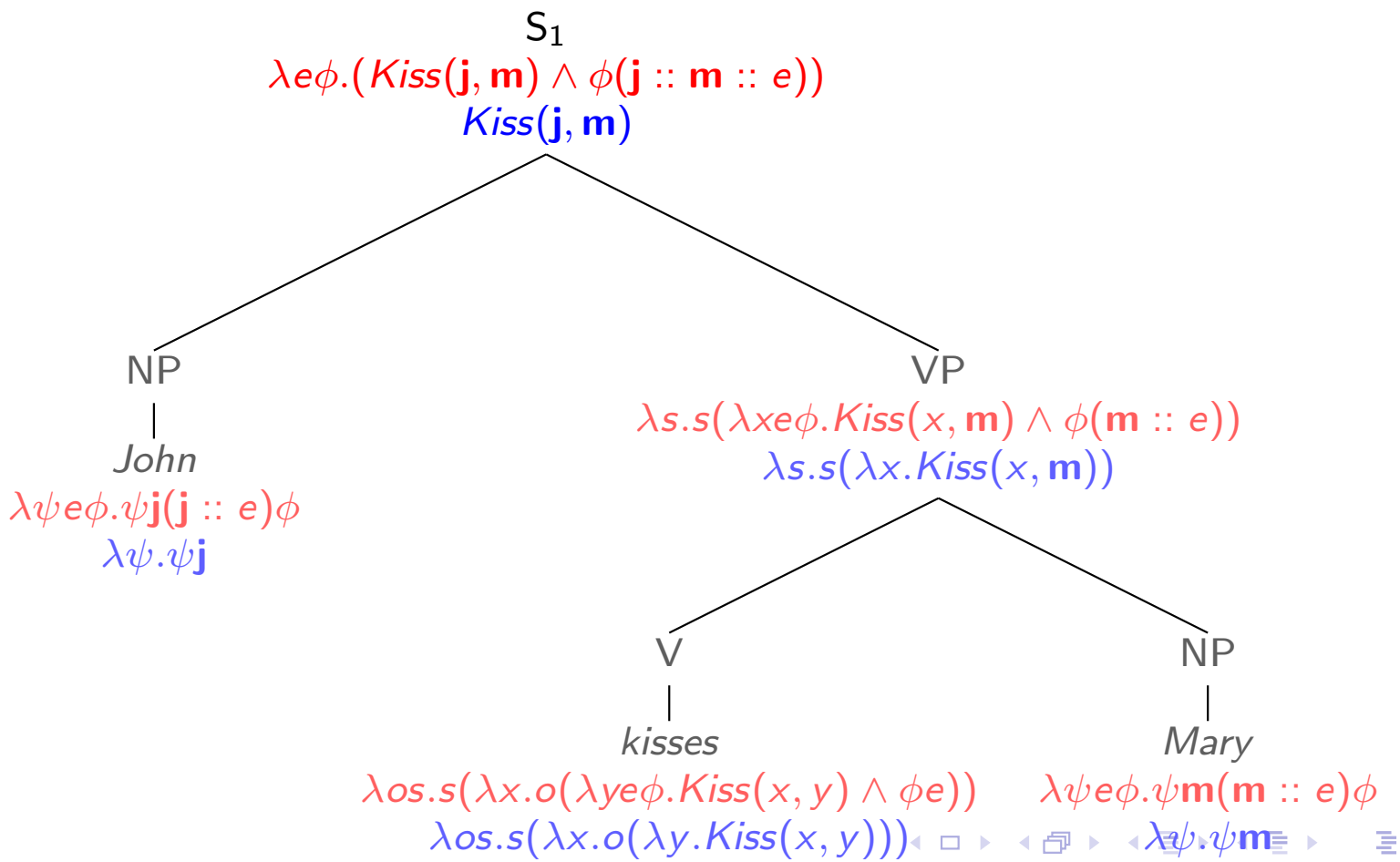

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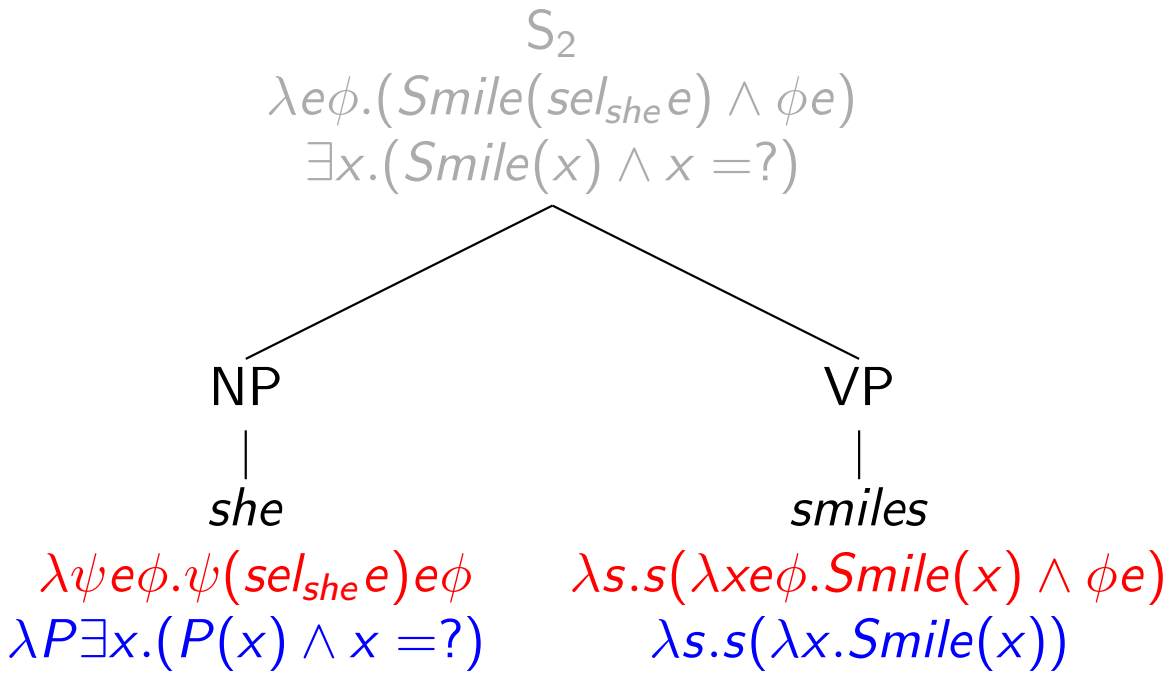
2. 

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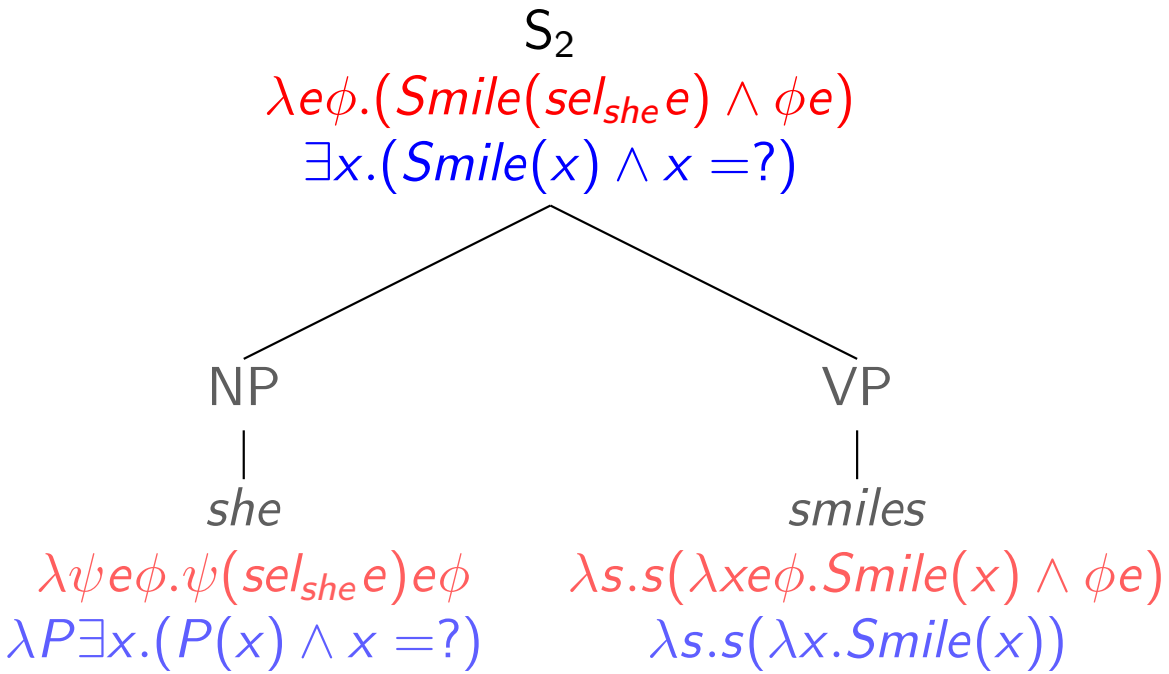

Compositional Example Continued

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$\lambda e\phi.(Kiss(\mathbf{j},\mathbf{m}) \wedge Smile(sel_{she}(\mathbf{j} :: \mathbf{m} :: e)) \wedge \phi(\mathbf{j} :: \mathbf{m} :: e))$

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More Observations

- Recall: explicit group could be broken down to form other valid referents (singular or plural) - Example (4)
- Supposition: all sub-groups consisted of accessible referents can be potential antecedents

Example (Summation - More Observations)

(7) John was in Paris. Bill was in Rome. Mary was in Barcelona.

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Generating All Sub-Groups

- Power Group
  - The set of all possible groups made up of any number of current accessible referents
  - A concept similar to power set in mathematics
- The summation function: $\mathfrak{Sum}$

Example (Performance of $\mathfrak{Sum}$)

- $\mathfrak{Sum}(j :: e) \Rightarrow (j :: e)$
- $\mathfrak{Sum}(m :: j :: e) \Rightarrow (m :: j :: j \oplus m :: e)$
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Generating All Sub-Groups

- Power Group
  - The set of all possible groups made up of any number of current accessible referents
  - A concept similar to power set in mathematics
- The summation function: $\mathfrak{Sum}$

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Two Supporting Functions

Definition (The Append Function $\mathfrak{App}$)

$\mathfrak{App}$ takes two lists l_1 and l_2 as arguments, $\mathfrak{App}(l_1, l_2)$ will be:

- l_2, if $l_1 = [\,]$ - the empty list;
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$\mathfrak{Sum}$ in Real Practice

Example (Natural Language Example for $\mathfrak{Sum}$)

(8) a. John and Mary went to Paris.
b. Bill and Lucy went to Rome.

- Necessary Lexical Entries
  - Proper Names
    - $[\![John]\!] = \lambda\psi e\phi.\psi \mathbf{j} \mathfrak{Sum}(\mathbf{j} :: e)\phi$
  - Conjunction "*and*"
    1. $[\![and]\!]_{dis} = \lambda AB\psi e\phi.A\psi e(\lambda e'.B\psi e'\phi)$
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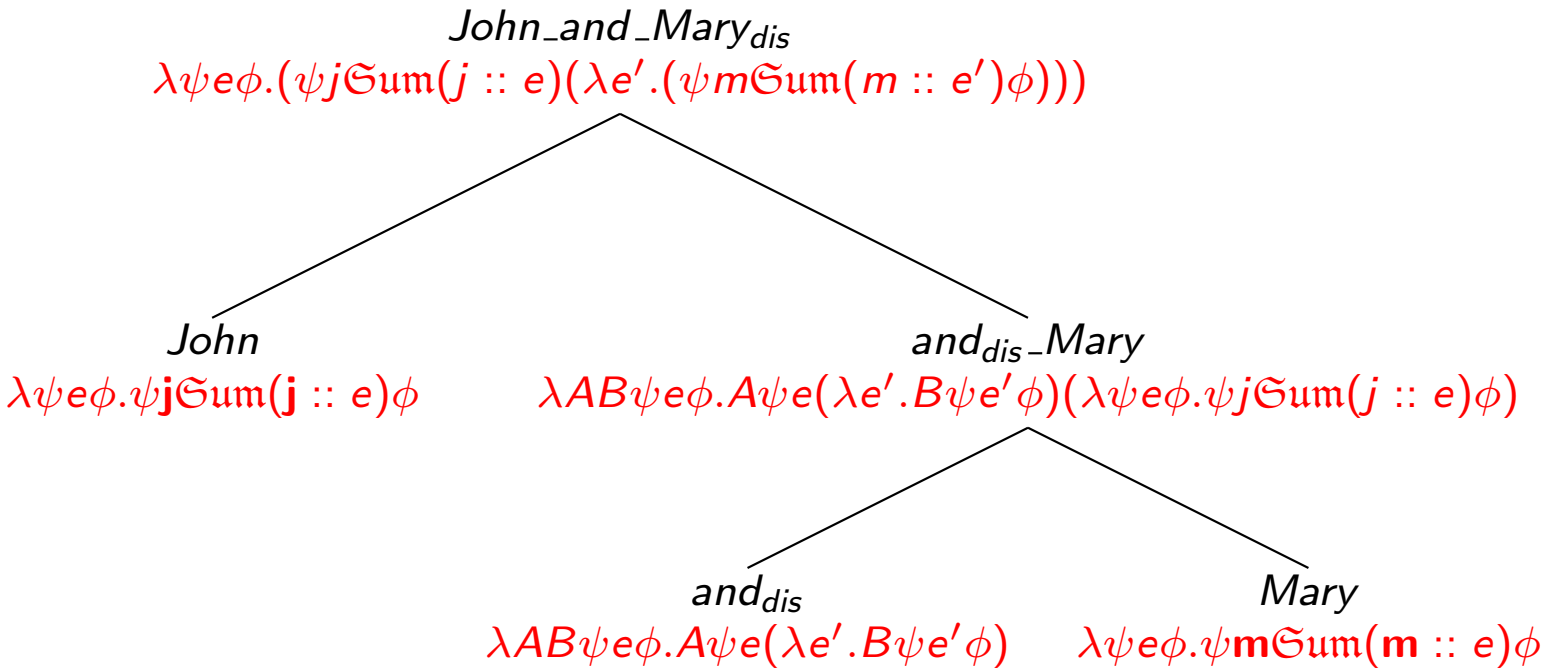
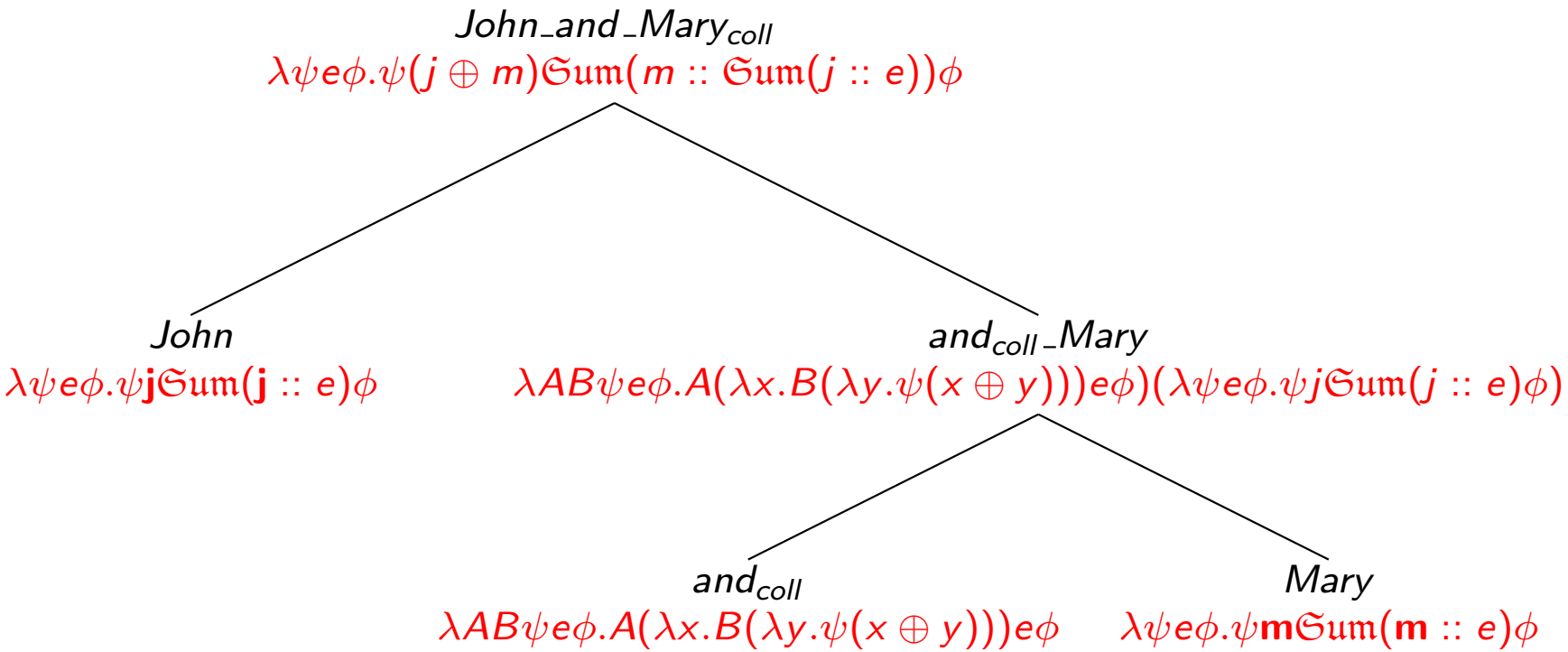
Distributive "and"



Collective "and"

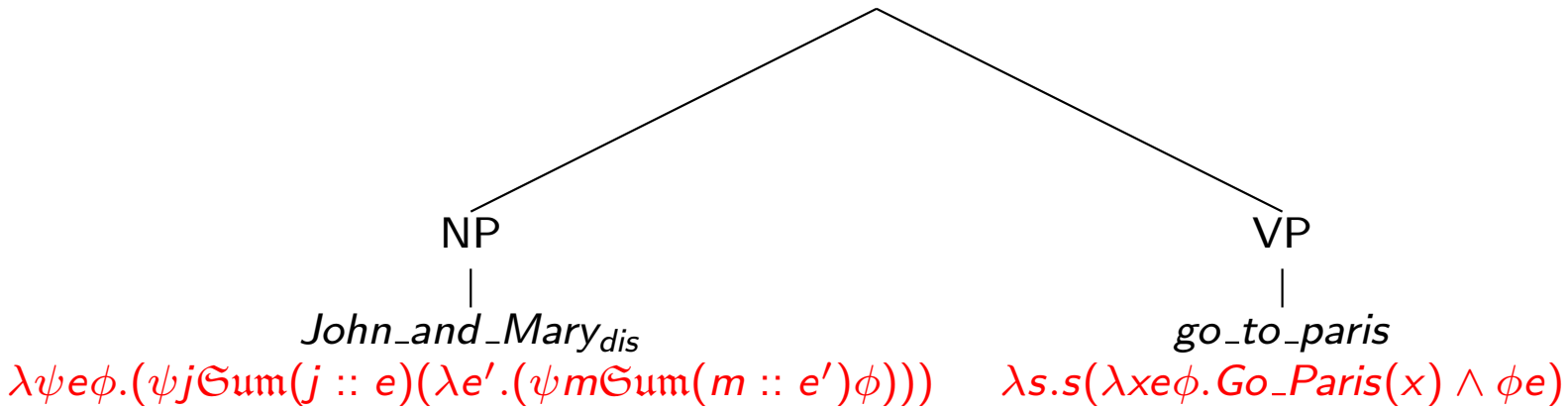


$\mathfrak{Sum}$ in Real Practice Continued

(8-a)

$\lambda e\phi.(Go\_Paris(j) \wedge Go\_Paris(m) \wedge \phi(j :: m :: j \oplus m :: e))$

$\lambda e\phi.(Go\_Paris(j) \wedge Go\_Paris(m) \wedge \phi(\mathfrak{Sum}(j :: \mathfrak{Sum}(m :: e))))$



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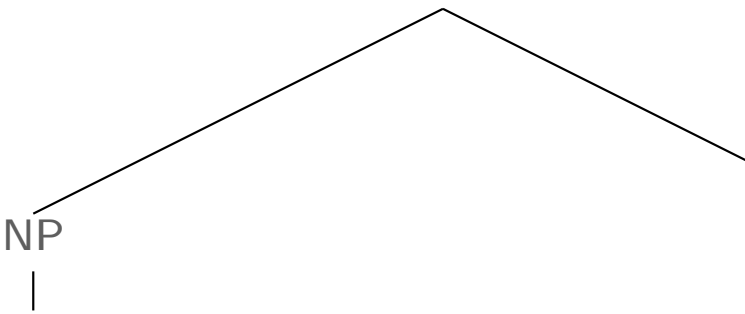

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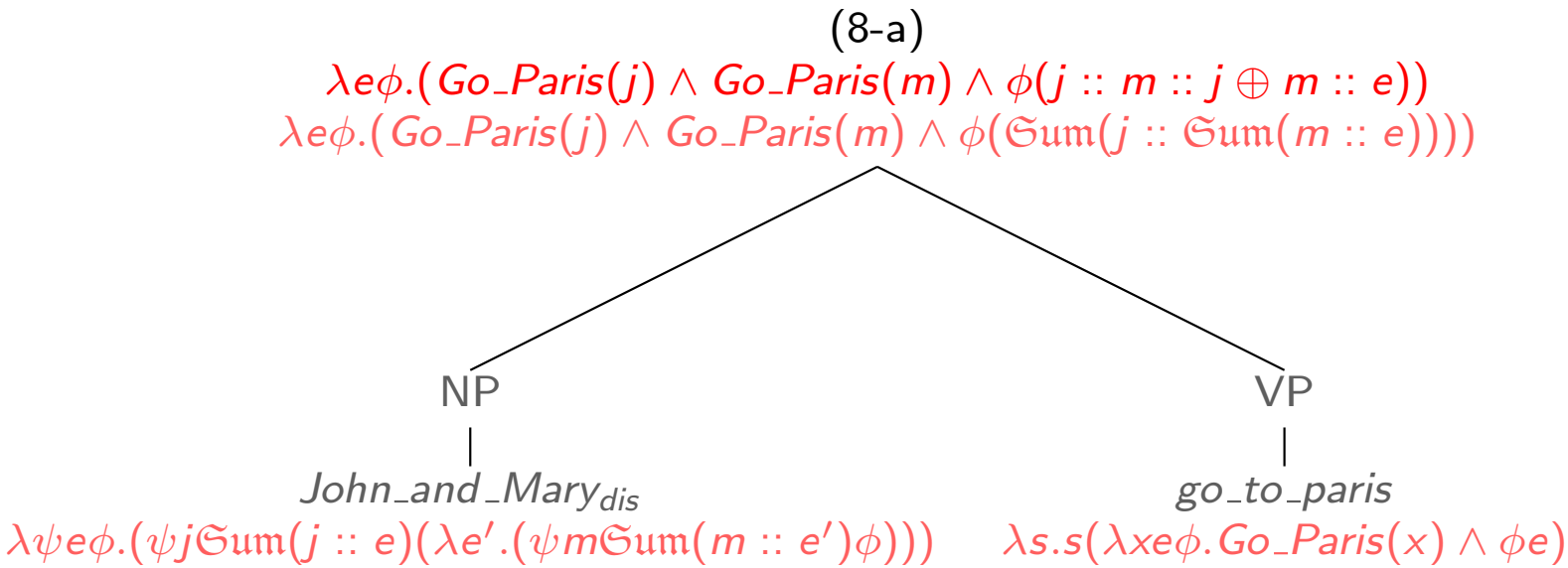
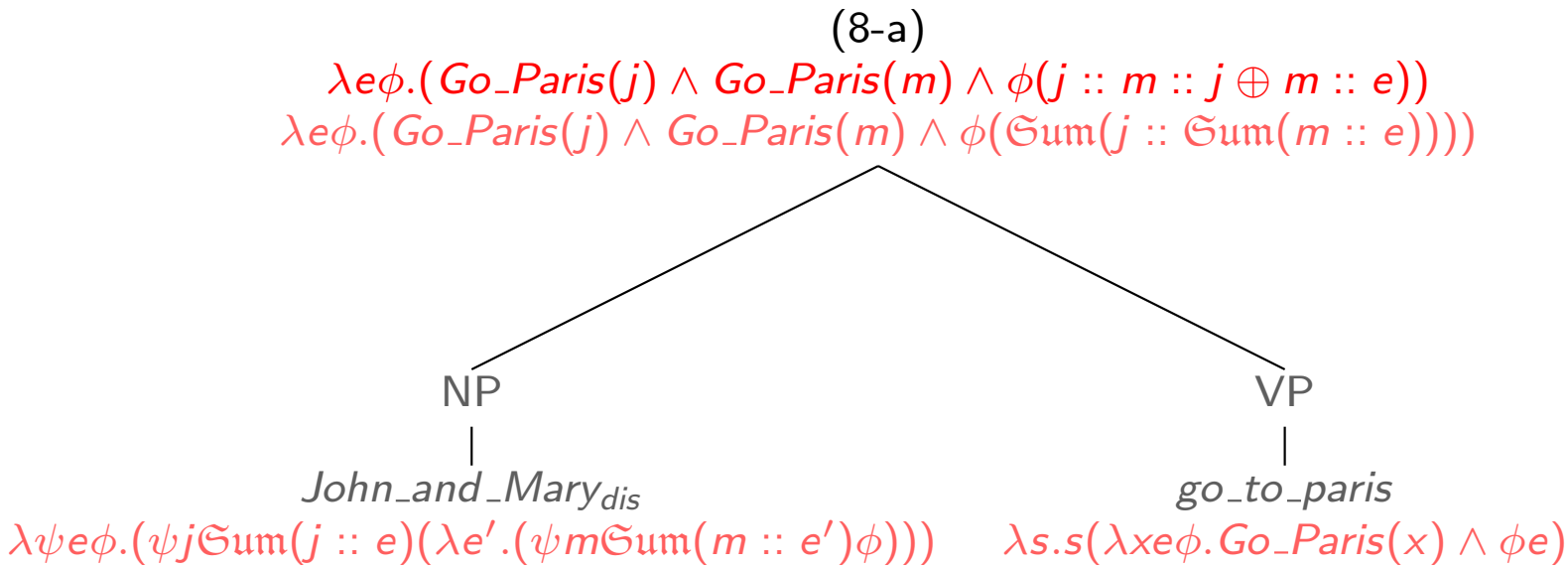


$\mathfrak{Sum}$ in Real Practice Continued



Similar for (8-b)

$\mathfrak{Sum}$ in Real Practice Continued

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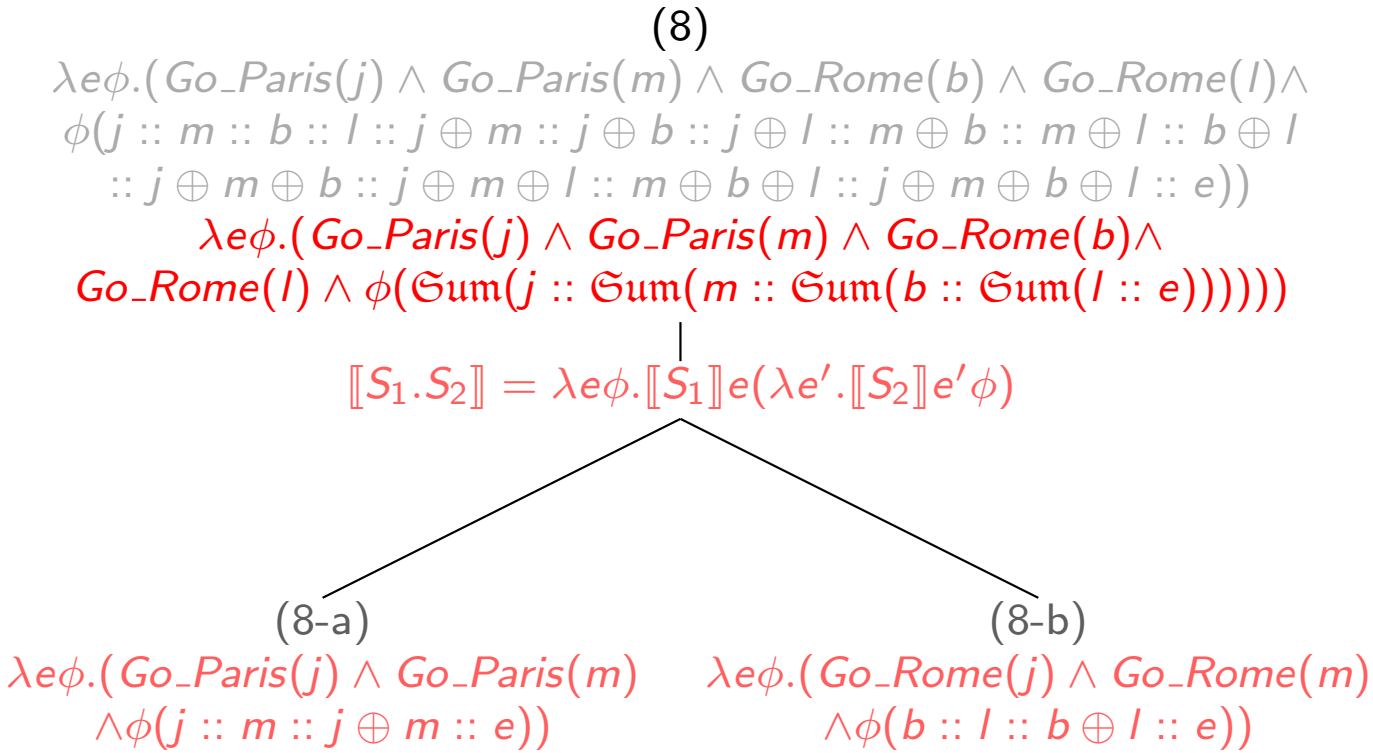
Sum in Real Practice Continued



$\mathfrak{Sum}$ in Real Practice Continued

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More Observations

Example (Abstraction - More Observations)

(9) Two of five students went to school.

a. They worked hard.

b. They had to hand in the homework by tomorrow.

- QNP: *Generalized Quantifier* + *Noun*
- More than one potential group referents are introduced by the same NP

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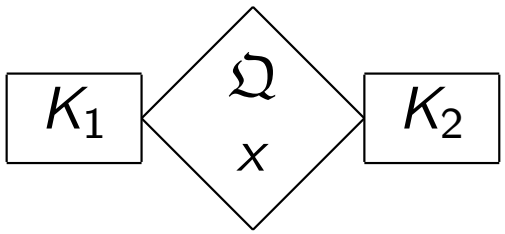
Abstraction in DRT [Kamp and Reyle, 1993]

- Duplex Condition: the relation between two sets, which is constrained by the property of QNP
- 

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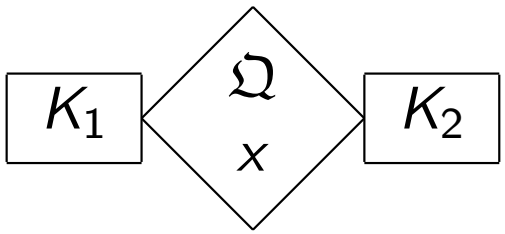
- 

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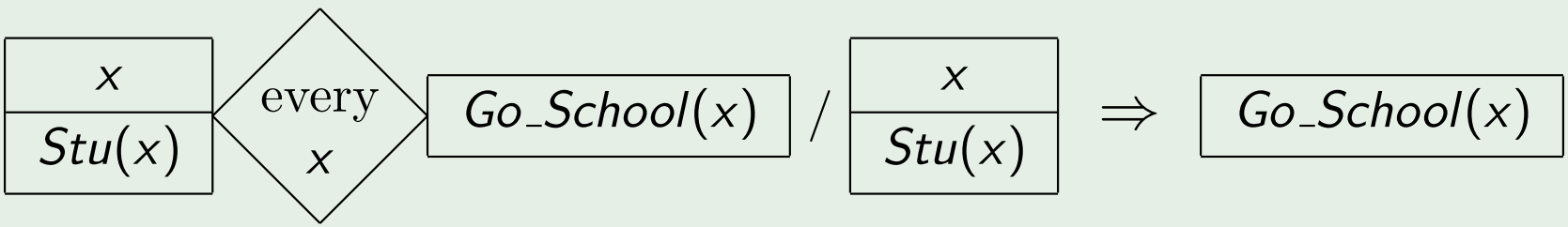

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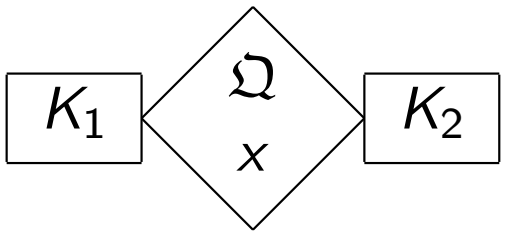
- 

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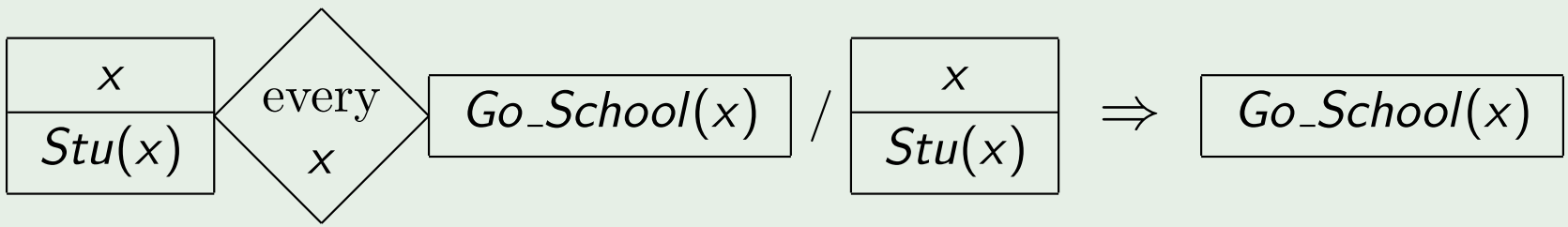

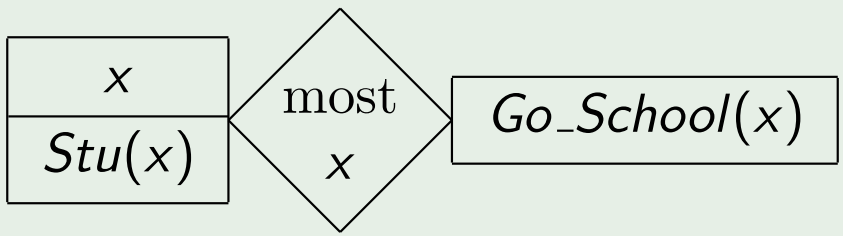

Three Groups

- Maximum Group
- Reference Group / Refset Anaphora
- Complement Group / Compset Anaphora

Figure: Structure Denoted by Generalized Quantifiers

Three Groups

- Maximum Group
- Reference Group / Refset Anaphora
- Complement Group / Compset Anaphora

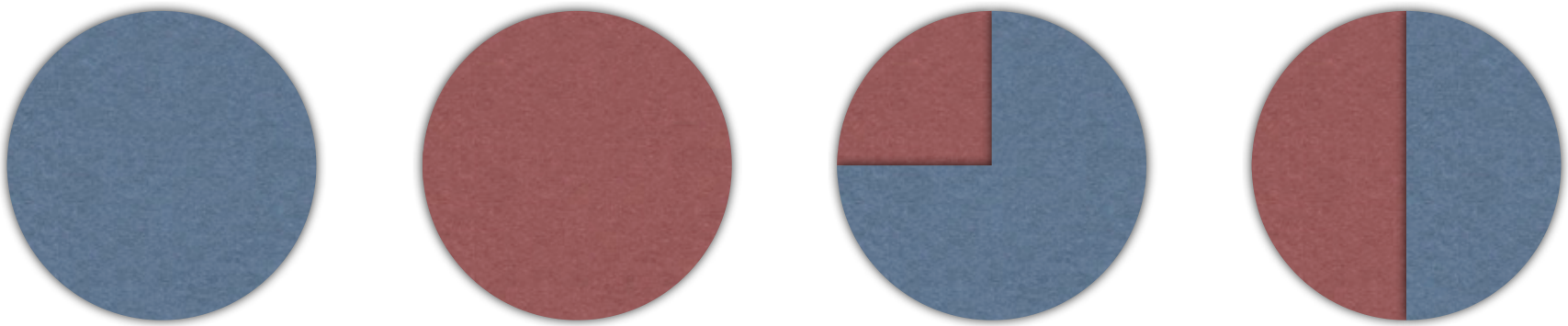

Figure: Structure Denoted by Generalized Quantifiers

Unveiling All Groups

- Proposition: to unveil all potential groups formed from abstraction
- Lexical Entry

Generalized Quantifier

$[\![GQ]\!] = \lambda\psi AB e\phi.Quan(\psi)x.((Axe\lambda e.\top)Rel(\psi)(Bxe\lambda e.\top)) \wedge \phi((\mathfrak{Abs}(\psi, x) :: e)$

- "*Quan*()" and "*Rel*()" are quantifier-sensitive
  - $Quan(every) = \forall$, $Quan(a) = \exists$
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Formal Definition for $\mathfrak{Abs}$

Definition (The Abstraction Function $\mathfrak{Abs}$)

$\mathfrak{Abs}$ takes two arguments: a generalized quantifier q and the related individual variable x. The output, namely $\mathfrak{Abs}(q, x)$ will be a left context consisting of two group referents R_i and C_i:

- R: the reference group of individuals denoted by the quantifier;
- C: the complement group of individuals denoted by the quantifier;
- i: the index that signifies the dependency of the two groups.

Example (Entry for "every")

$$[\![every]\!] = [\![GQ]\!](every)$$
$$\Rightarrow \lambda ABe\phi.Quan(every)x.(Axe\lambda e.\top Rel(every)Bxe\lambda e.\top) \wedge \phi(\mathfrak{Abs}(every, x) :: e)$$
$$\Rightarrow \lambda ABe\phi.\forall x.(Axe\lambda e.\top \rightarrow Bxe\lambda e.\top) \wedge \phi(\mathfrak{Abs}(every, x) :: e)$$

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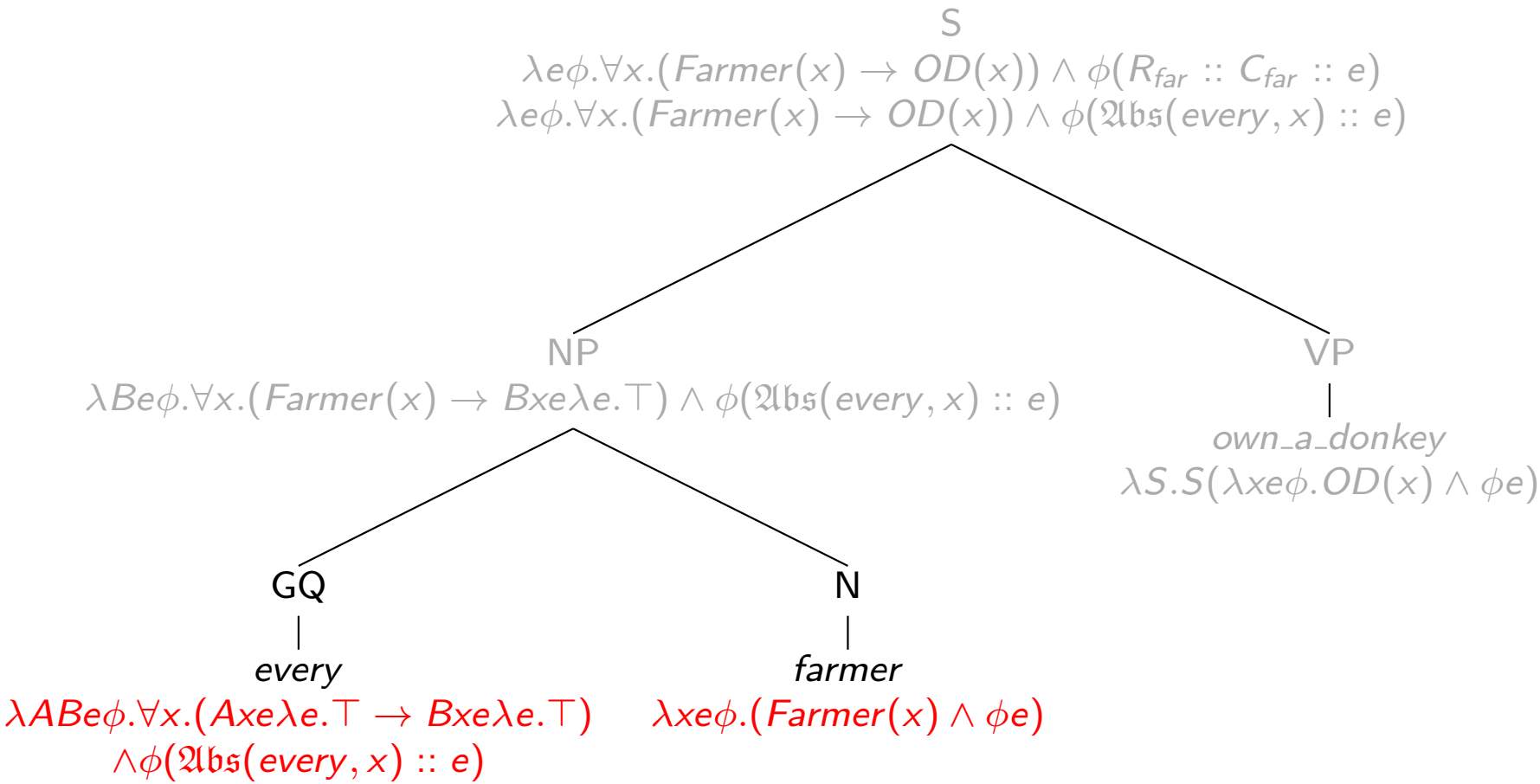
$\mathfrak{Abs}$ in Real Practice

(10) Every farmer owns a donkey.



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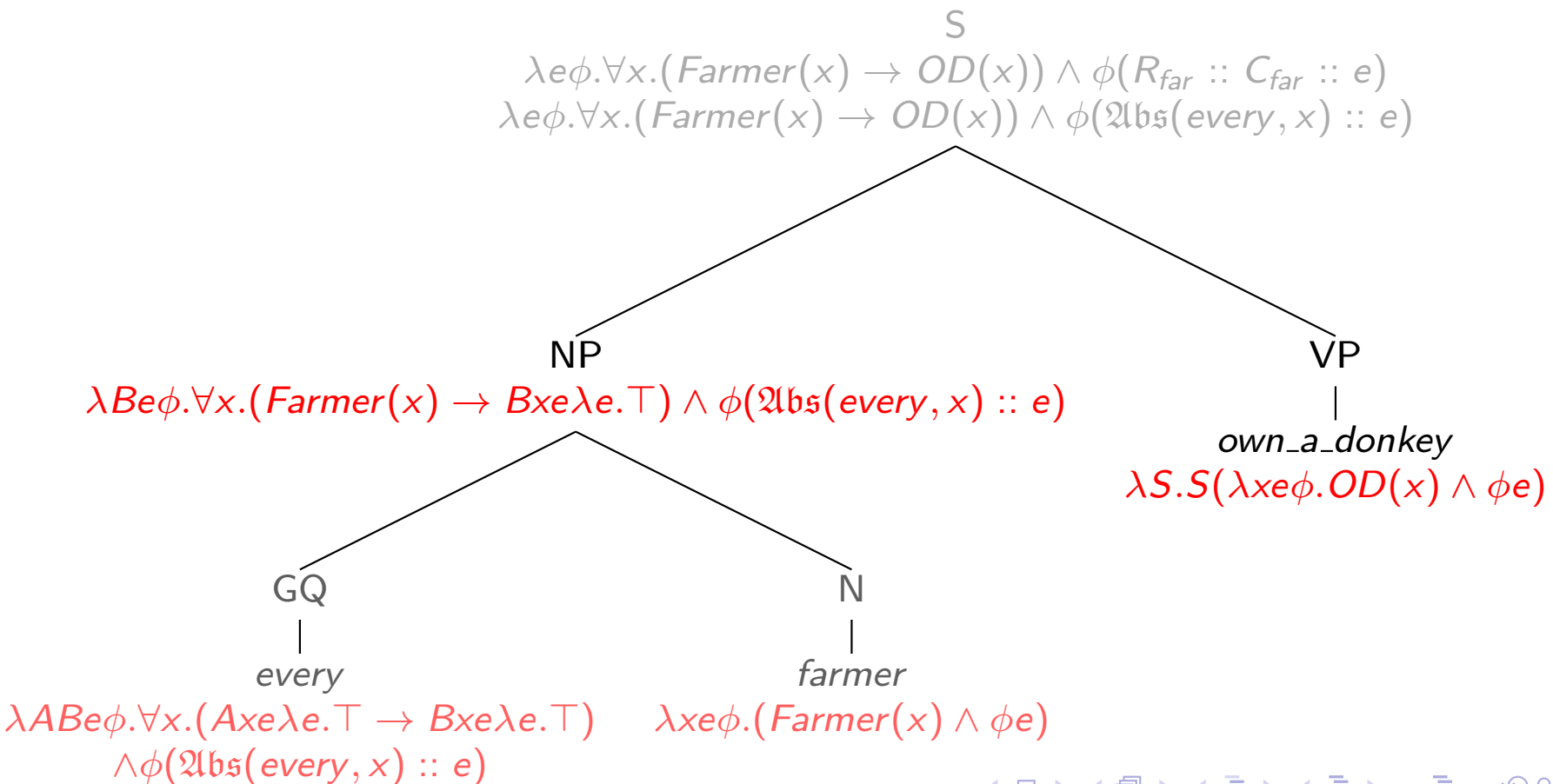

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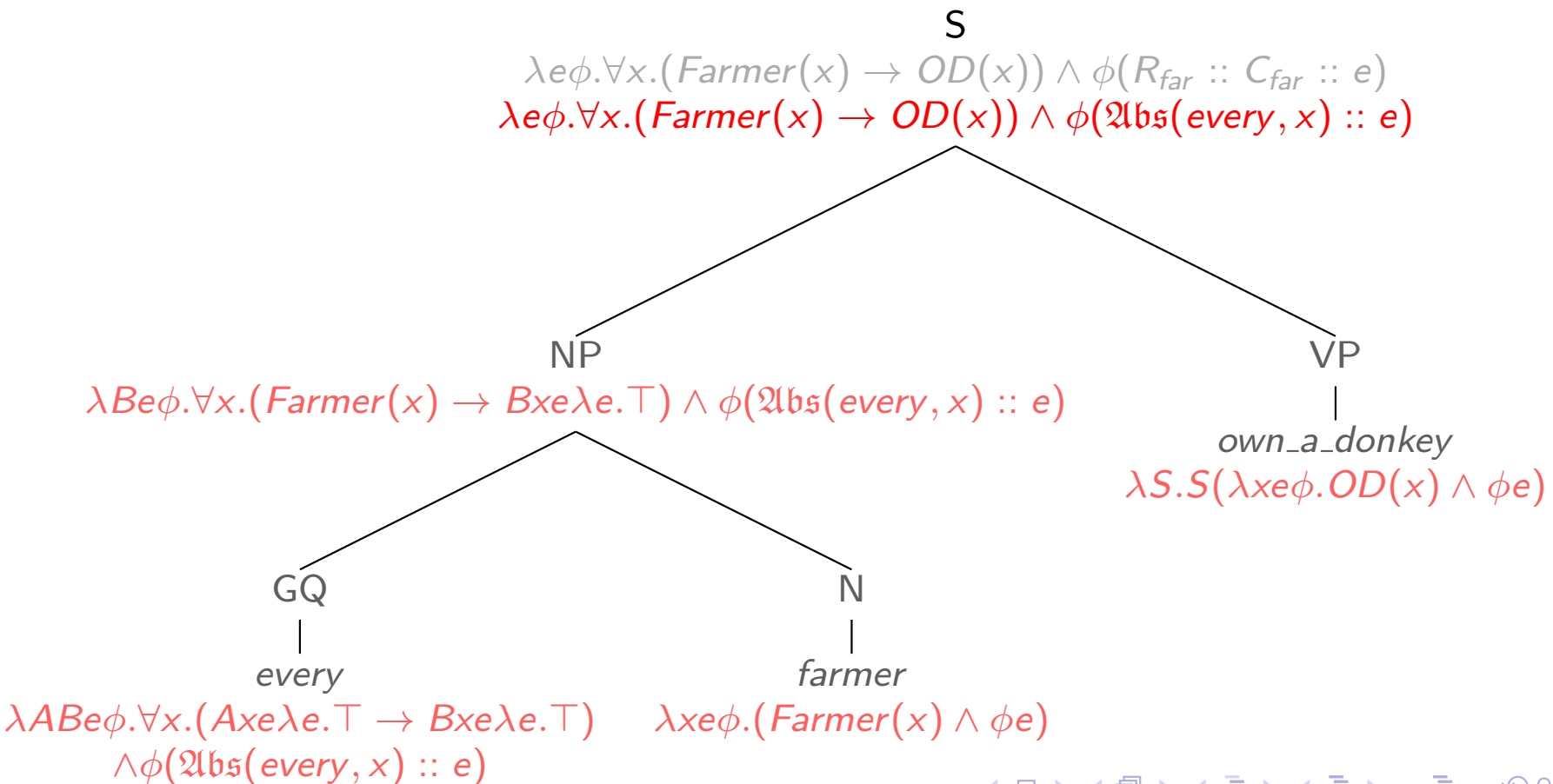

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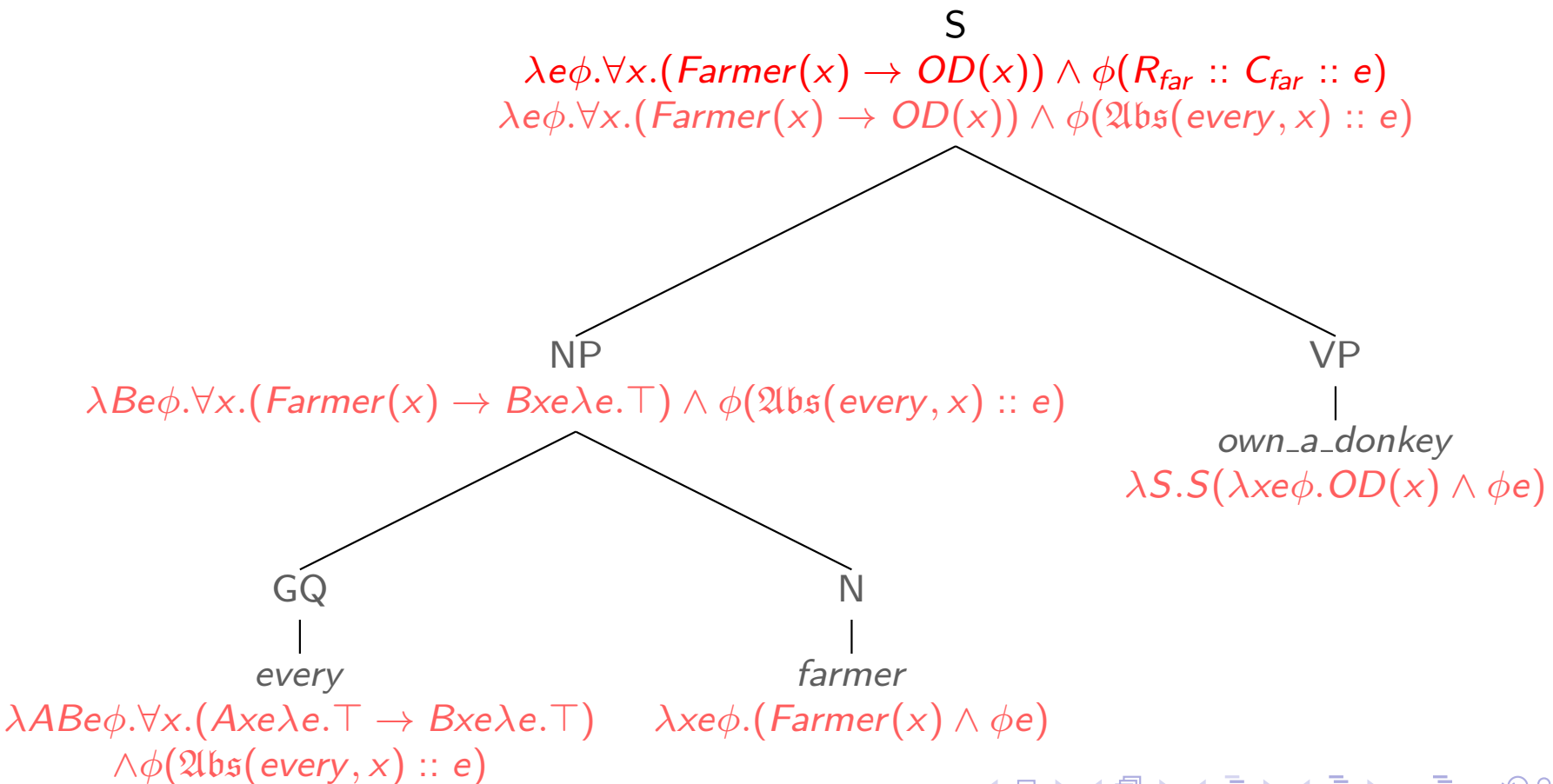

Summary

- Conclusion
  - Investigating plural anaphora within a new dynamic semantic framework
  - A potential list containing accessible plural referents is provided for summation and abstraction respectively
  - The framework is sound on the aspect of compositionality
  - The proposal is not responsible for the complete task of anaphora resolution
- Future Work
  - More elaborate definition on $\mathfrak{Sum}$ and $\mathfrak{Abs}$
  - Concern of over generation
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  - Combining with event semantics

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