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# Integrated Optimal Design of a Passive Wind Turbine System: An Experimental Validation

Duc-Hoan Tran, Bruno Sareni, Xavier Roboam, *Member, IEEE*, and Christophe Espanet

**Abstract**—This work presents design and experimentation of a full passive wind turbine system without the active electronic part (power and control). The efficiency of such a device can be obtained only if the system design parameters are mutually adapted through an integrated optimal design method. This approach, based on multiobjective optimization, aims at concurrently optimizing the wind power extraction and the global system losses for a given wind speed profile while reducing the weight of the wind turbine generator. It allows us to obtain the main characteristics (geometric and energetic features) of the optimal Permanent Magnet Synchronous Generator (PMSG) for the passive wind turbine. Finally, experiments on the PMSG prototype built from this work show a good agreement with theoretical predictions. This validates the design approach and confirms the effectiveness of such a passive device.

**Index Terms**—Genetic algorithms, integrated optimal design (IOD), multiobjective optimization, passive wind turbine.

## NOMENCLATURE

$a_k$	Power coefficient factors.
$\alpha_m$	Electrical half pole width.
$B_a, B_r, B_y$	Airgap/Remanent/Yoke induction.
$C_p$	Wind turbine power coefficient.
$d_r, d_y$	Rotor/Stator yoke thickness.
$d_s$	Slot depth.
$E_{dc}, E_s$	DC equivalent electromotive force (EMF), phase EMF.
$E_w$	Wind energy.
$f_w$	Wind turbine damping coefficient.
$g$	Airgap width.
$g_i$	Optimization constraint $N^\circ i$ .
$I_s$	Phase current.
$I_{dc}$	DC current in the battery.

$J_s$	Current density.
$J_w$	Wind turbine inertia.
$K_{1b}$	Winding factor.
$K_C$	Carter coefficient.
$K_r$	Slot filling coefficient.
$l_m$	Magnet thickness.
$l_r$	Permanent Magnet Synchronous Generator (PMSG) active length.
$L_m, L_l, L_s$	Main/Leakage/Stator inductance.
$\lambda$	Tip speed ratio.
$\mu_r$	Magnet permeability.
$N_{cs}$	Number of conductors per slot.
$N_{spp}$	Number of slots per pole per phase.
$\Omega_b, \Omega_w$	Base/Wind turbine rotation speed.
$P_b, P_u$	Base/Useful power.
$P_c, P_i, P_j, P_m$	Conduction/Iron/Joule/Mechanical loss.
$p$	Number of pole pairs.
$\rho$	Air density.
$R_{dr}, R_{rl}$	Slot depth bore radius/Radius length ratio.
$R_{sc}$	Equivalent armature bore radius.
$R_w$	Blade radius.
$R_{dc}, R_s, R_{over}$	DC/Stator/Overlap resistance.
$r_d$	Diode resistance.
$r_s$	Bore radius.
$T_{em}, T_w$	Electromagnetic/Wind turbine torque.
$V_b, V_{dc}, V_s$	Base/dc/phase voltage.
$V_w$	Wind speed.
$\Phi_{dc}, \Phi_s$	DC equivalent/phase flux.
$\theta_e$	Opening angle of a slot.
$w_m, w_s, w_T$	Magnet/Slot/Tooth width.

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## I. INTRODUCTION

**C**LOSE to high-power wind turbines for onshore or offshore applications, small wind systems represent an interesting target for applications such as rural electrification and autonomous energy production networks for water

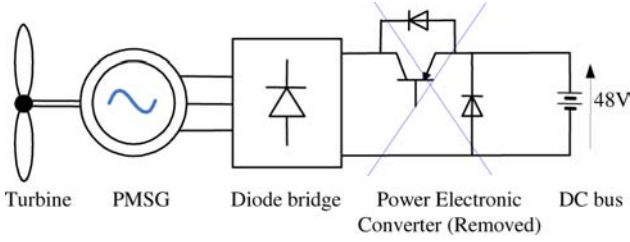


Fig. 1. Fully “Passive” wind turbine system.

pumping, desalination. Optimizing energy efficiency generally leads to adaptation of the load impedance and consequently the speed of the generator with the wind turbine operating conditions. Many active structures have been thus proposed [1]–[15] to allow tracking the maximum power operation through corresponding maximum power point tracking (MPPT) strategies.

However, for such an application frame, the system cost has to be drastically minimized, for instance, by simplifying the structure with PM synchronous generator feeding a diode rectifier associated with a battery bus. For grid-connected applications, impedance adaptation can be obtained through the grid inverter as in [13].

In earlier studies [16], [17], we proposed a very “low-cost” and more reliable structure for remote applications without an active control unit and with a minimum number of sensors (see Fig. 1). In fact, for such a device, a “natural” impedance adaptation can be achieved with the passive structure by optimizing the accordance between system parameters using an integrated optimal design (IOD) approach. This approach has been successfully applied in simulation for a small Savonius wind turbine of 300 W. In this paper, the same methodology is investigated on a passive system composed of 1.5-kW three-bladed wind turbine. The optimal Permanent Magnet Synchronous Generator (PMSG) resulting from the IOD process is validated from experiments on a real prototype.

## II. MODELING OF THE PASSIVE WIND TURBINE SYSTEM

### A. Wind Model

A typical wind cycle defined in terms of speed harmonic decomposition is taken for describing the wind speed behavior  $V_w$  [15]

$$V_w(t) = 10 + 0.2 \sin(0.1047t) + 2 \sin(0.2665t) + \sin(1.2930t) + 0.2 \sin(3.6645t). \quad (1)$$

### B. Wind Turbine Model

A three-bladed wind turbine of radius  $R_w$  is considered in this study. It provides a wind power defined by [18]

$$P_w = \frac{1}{2} C_p(\lambda) \rho \pi R_w^2 V_w^3 \quad (2)$$

where the  $C_p$  power coefficient varies as a function of the tip speed ratio  $\lambda = R_w \Omega_w / V_w$  according to Fig. 2. In particular, it can be interpolated by a seventh-order polynomial as follows:

$$C_p(\lambda) = \sum_{k=0}^7 a_k \lambda^k \quad (3)$$

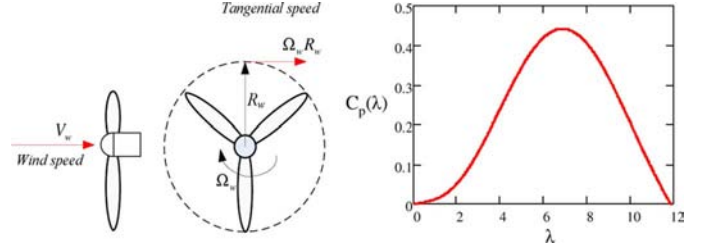


Fig. 2. Three-bladed wind turbine with its power coefficient.

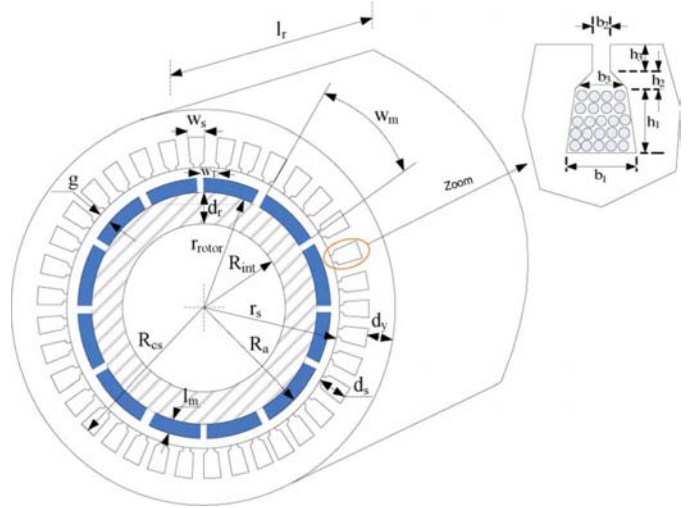


Fig. 3. PMSG geometric features.

where the  $a_k$  values are given in Appendix C. The dynamic model of the turbine is represented by the following relation:

$$T_w - T_{em} = J_w \frac{d\Omega_w}{dt} + f_w \Omega_w \quad (4)$$

where  $T_w, T_{em}$  are, respectively, the wind and electromagnetic torques,  $J_w$  and  $f_w$  being the total wind turbine inertia and viscous damping coefficient [17].

### C. PMSG Model

Two analytical models have been developed in order to characterize the electromagnetic behavior of the PMSG from its geometric features defined in Fig. 3.

- 1) *1st Model*: This model has been derived from [19] and is presented in more detail in [17]. It considers a 1-D distribution of the magnetic field in the airgap and the permanent magnet represented by a first-harmonic approximation. Because of its simplicity and its low computational cost, it is used in the IOD process to optimize the PMSG.
- 2) *2nd Model*: This model has been developed in [20]–[22]. It takes into account 2-D space harmonics of the magnetic field distribution in the airgap and in the permanent magnet. Because of its better accuracy, this model is taken as a reference to validate the results obtained with the previous simplified model.

Note that the electrical circuit parameters of the PMSG, i.e., phase flux, stator inductances, and stator resistances, can be obtained from both models as shown in Appendix A.

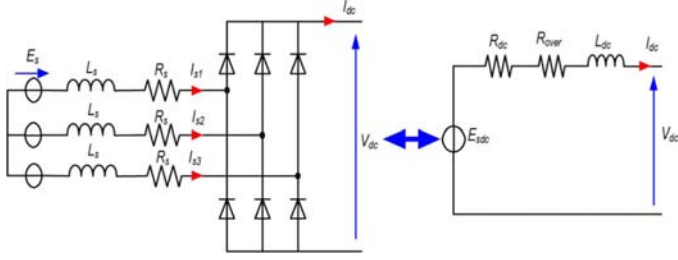


Fig. 4. Equivalent dc model of the PMSG rectifier association.

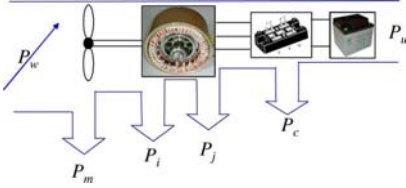


Fig. 5. Power balance in the passive wind turbine system.

#### D. Circuit Model of The Passive Wind Turbine System

In order to reduce the computing time of electrical values, a simplified dc equivalent model (called mixed-reduced model in [17]) has been proposed in [16] and [17] for representing the PMSG-Diode rectifier association (see Fig. 4). This model includes the armature reaction in the generator and the diode overlapping during the commutation interval. The correspondence between ac [root mean square (rms)] values and dc ones, in the PMSG circuit model is given in Appendix B.

### III. MULTIOBJECTIVE OPTIMIZATION OF THE PASSIVE WIND TURBINE SYSTEM

In this section, the IOD of the PMSG is presented, converting the IOD problem into a multiobjective optimization problem.

#### A. Design Variables, Objectives, and Constraints

1) *Design Objectives*: Because of the “naturally small” efficiency of the passive wind turbine, the first objective  $OF_1$  deals with the maximization of the average useful power  $P_u$  during the wind cycle defined in (1). This power consists of the wind power extracted from the wind turbine  $P_w$  reduced from all losses in the system. These losses include mechanical losses  $P_m$  in the turbine, conduction losses  $P_c$  in the diode rectifier, iron losses  $P_i$ , and Joule losses  $P_j$  in the PMSG. The power balance in the system is illustrated in Fig. 5. For more details about the calculation of these losses, we invite the reader to refer to [17]

$$OF_1 = \max\{P_u\} = \max\{P_w - P_m - P_j - P_i - P_c\}. \quad (5)$$

It should be noted that maximizing the useful power implies the maximization of the extracted wind power as well as the minimization of all losses in the system.

The second objective  $OF_2$  consists in the minimization of the total system mass  $M_{tot}$  which includes the wind turbine mass  $M_w$ , the PMSG mass  $M_{PMSG}$ , and the radiator mass  $M_{rad}$  associated with the diode rectifier

$$OF_2 = \min\{M_{tot}\} = \min\{M_w + M_{PMSG} + M_{rad}\}. \quad (6)$$

TABLE I  
DESIGN VARIABLES

Design Variables	Nature	Bounds
Radius/Length Ratio	Continuous	$R_{rl} \in [0.1, 5]$
Slot depth/ Radius Ratio	Continuous	$R_{dr} \in [0.03, 0.3]$
Yoke Induction [T]	Continuous	$B_y \in [1.2, 1.9]$
Number of Pole Pairs	Discrete	$p \in \{1, \dots, 30\}$
Current Density [A.mm <sup>-2</sup> ]	Continuous	$J_s \in [0.5, 4]$
Base Power [W]	Continuous	$P_b \in [300, 3000]$
Nbr of slots per pole per phase	Discrete	$N_{spp} \in \{1, \dots, 5\}$
Base Speed [rad.s <sup>-1</sup> ]	Continuous	$\Omega_b \in [25, 95]$

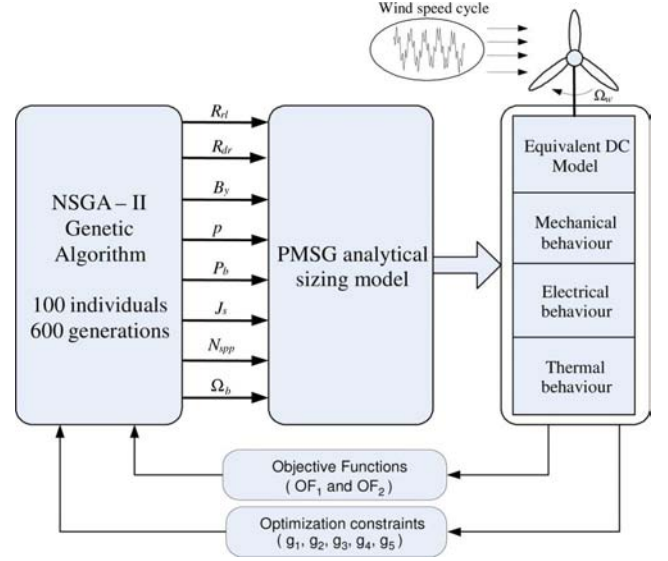


Fig. 6. IOD process based on multiobjective NSGA-II optimization.

The PMSG mass  $M_{PMSG}$  is evaluated from the mass of each components (iron, copper, magnet) and geometric features similar to [17] and [23]. The radiator mass of the diode rectifier is calculated from the radiator thermal resistance and the conduction losses according to [17].

2) *Design Variables*: Design variables and their associated bounds are mentioned in Table I. These variables should be optimized in the IOD process in accordance with the design objectives defined in the last subsection.

3) *Design Constraints*: Five constraints are introduced to ensure the system feasibility in relation to the parametric variation of design variables in the optimization process. These constraints concern the number of copper winding per slots (higher than one and limited by the minimal section of winding conductors), the maximum temperature associated with the copper windings in the PMSG, the demagnetization limit of the magnets, and the maximum temperature in the semiconductor junctions of the diode rectifier. All details about the calculation of these constraints and the computation of thermal values in the PMSG and in the diode rectifier can be found in [17] and [23].

#### B. IOD Process Based on Multiobjective Optimization

The IOD process based on multiobjective optimization is illustrated in Fig. 6. The nondominated sorting genetic algorithm (NSGA-II) [24] with a self-adaptive recombination procedure of genetic operators [25] is used for optimizing both objectives in

TABLE II  
NSGA-II CONTROL PARAMETERS

NSGA-II Control Parameters	Value
Population Size	100
Number of Generations	600
Crossover Probability	1.0
Design Variable Mutation Rate	0.125
X-gene Mutation Rate (see [25])	5%

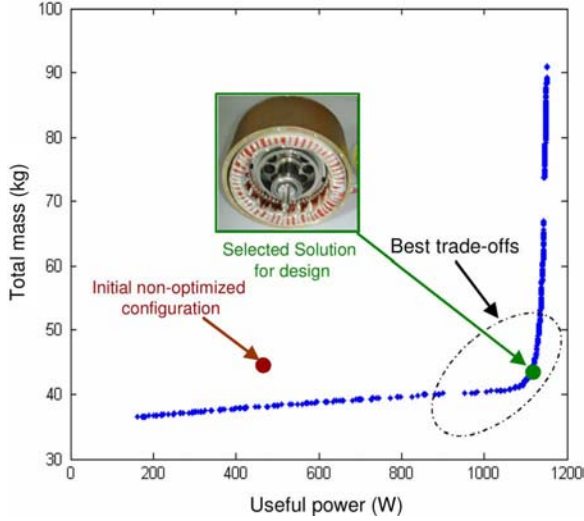


Fig. 7. Pareto-optimal configurations of the PMSG obtained from the IOD.

compliance with all constraints. These constraints are integrated in the algorithm by modifying the dominance rule according to [23]. Five independent runs are performed with the control parameters defined in Table II, in order to take into account the NSGA-II stochastic nature. Note that optimization is performed for the particular wind cycle defined in (1). However, it has been shown in [26] that results obtained with this IOD process are relatively insensitive to wind conditions.

### C. Results

Fig. 7 shows the global Pareto-optimal front obtained by merging all fronts associated with the five independent runs. In this figure, we present an optimal solution (“selected solution for design”) whose parameters are mentioned in Appendix C and compare it with an initial nonoptimized PMSG configuration [27]. This optimal solution is chosen in the Pareto-front elbow in order to ensure similar “weightings” between useful power and mass objectives. We also plot in Fig. 8 the wind power extraction of the passive wind turbine system for both solutions. It can be seen that wind power extraction of the optimized solution for various wind speeds is clearly better, nearby the ideal wind power extraction.

In order to validate the design of the optimal solution extracted from the Pareto-optimal front and obtained with the less accurate sizing, we compare the electromagnetic behavior of the PMSG with both analytical models presented in Section III and with the finite element method. Fig. 9 shows a comparison of the flux density in the PMSG airgap, obtained with the FEMM software [28], [29] and with the second analytical model. The main electromagnetic parameters of the PMSG determined with each

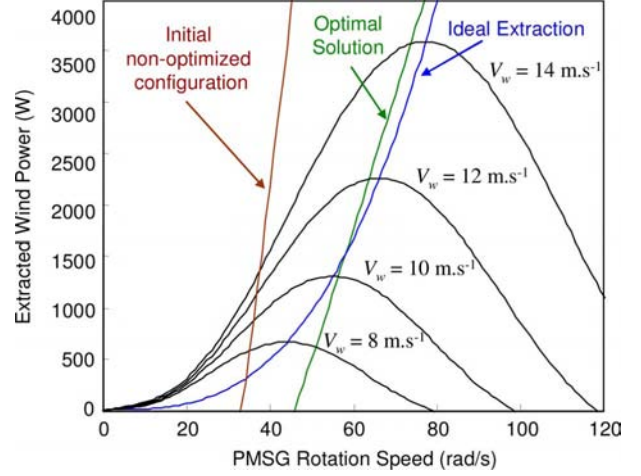


Fig. 8. Extracted wind power of two solutions (initial and optimal).

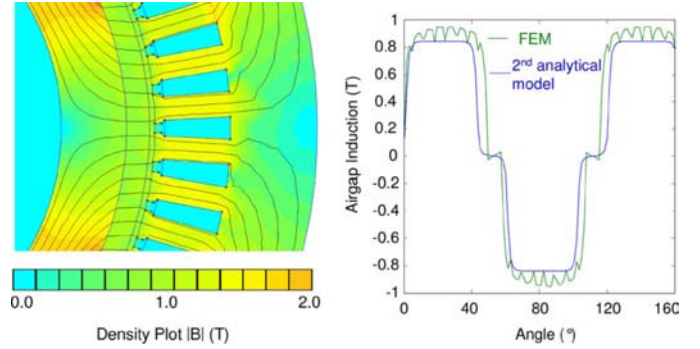


Fig. 9. Flux density in the PMSG airgap.

TABLE III  
ELECTROMAGNETIC PARAMETERS OF THE OPTIMAL PMSG

	$B_a(T)$	$R_s(\Omega)$	$L_s(mH)$	$\Phi_s(Wb)$
1 <sup>st</sup> analytical model	0.846	0.13	1.37	0.21
2 <sup>nd</sup> analytical model	0.844	0.13	1.34	0.22
Finite Elements	0.859	0.14	1.40	0.21
Actual Prototype - Measurements	-	0.14	1.42	0.20

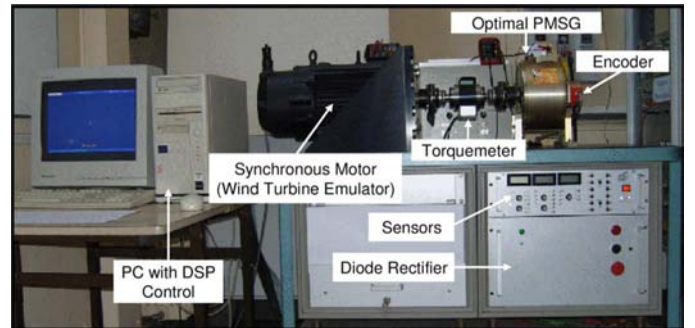


Fig. 10. Passive wind turbine system bench.

model are given in Table III. We also mention in Table III the values obtained from measurements on the actual PMSG prototype built from this study. It can be noted that there is a remarkable agreement between all data, and especially between the PMSG design and the corresponding prototype.

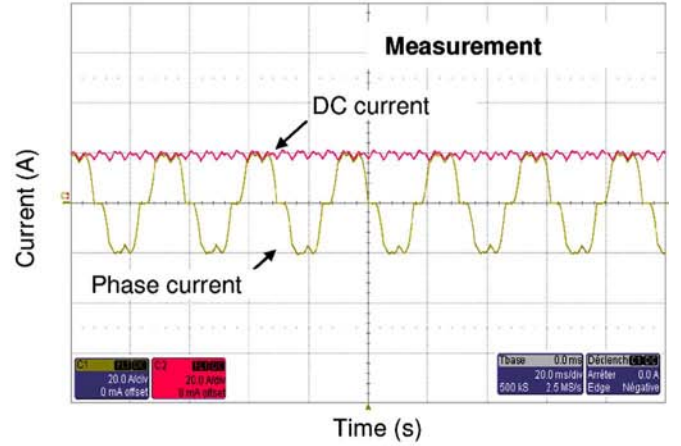
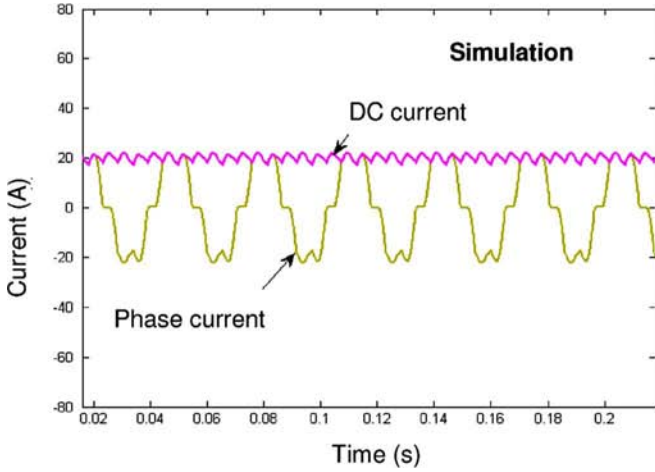


Fig. 11. Phase current in the PMSG and dc current in the battery.

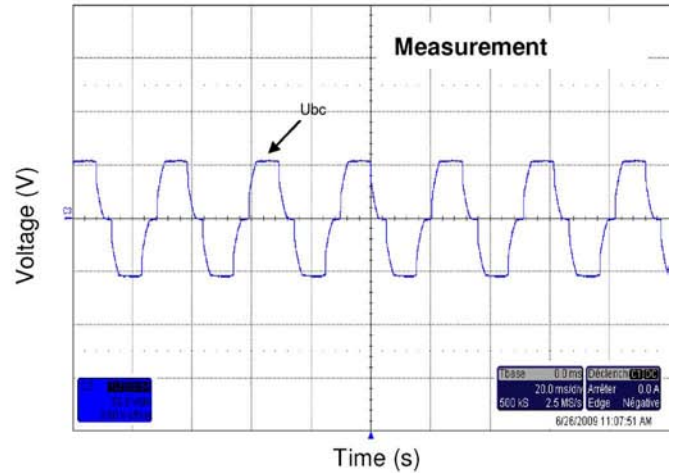
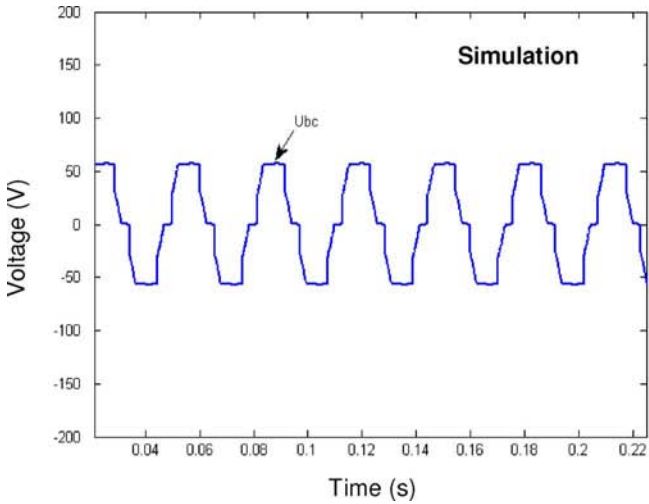


Fig. 12. Phase-to-phase voltage in the PMSG.

#### IV. EXPERIMENTAL VALIDATION OF THE PASSIVE WIND TURBINE SYSTEM

A test bench has been developed in order to experimentally validate the passive wind turbine system (see Fig. 10). It allows a “physical emulation” of the passive wind turbine system, the turbine being “simulated” by a servomotor which reproduces torque and inertia with respect to rotation and wind speeds ( $C_p(\lambda)$  curve). The servomotor is coupled through a torquemeter with the optimal PMSG prototype issued from the IOD approach. The PMSG is itself connected through a three-phase diode rectifier with battery of dc voltage  $V_{dc} = 48$  V. The system is controlled by a digital signal processor (DSP) processor (DS1102 board) which converts sensor information (voltage, current, position, and torque). We investigate the behavior of the passive wind turbine system by comparing experimental measurements with corresponding simulations. For that purpose, a complete three-phase model of the PMSG-diode rectifier and battery association has been developed. This model allows us to characterize phase voltage and phase current in the PMSG and in the diode rectifier as well as the dc voltage and current in the battery. We first compare experiment and

simulation electrical values at the input and output of the diode rectifier for a constant PMSG rotation speed close to the PMSG base speed, i.e.,  $\Omega_s = 60$  rad.s<sup>-1</sup>. Fig. 11 presents the PMSG phase current and dc current. For the same conditions, Fig. 12 gives the phase-to-phase voltage value in the PMSG. In order to validate the passive wind turbine system in wind variable operating, the wind cycle defined by (1) is imposed to the servomotor which emulates the wind turbine. Fig. 13 compares the electromagnetic power and the associated energy during the simulated wind cycle. The maximal power and energy corresponding to the case of an ideal wind extraction are also drawn to assess the wind power extraction quality of the passive wind turbine system. Note that differences are not important which confirms the system’s “natural” efficiency when its parameters have been optimized in relation to the wind turbine. Finally, the average useful power  $\langle P_u \rangle$  during the simulated wind cycle is given in Table IV and compared with average extracted wind power  $\langle P_w \rangle$  to estimate the global efficiency of the wind turbine system. We also mention in Table IV the average power associated with an ideal wind extraction. It can be seen from measurements that the wind power extraction efficiency related to the particular wind cycle is about 90%. The global efficiency

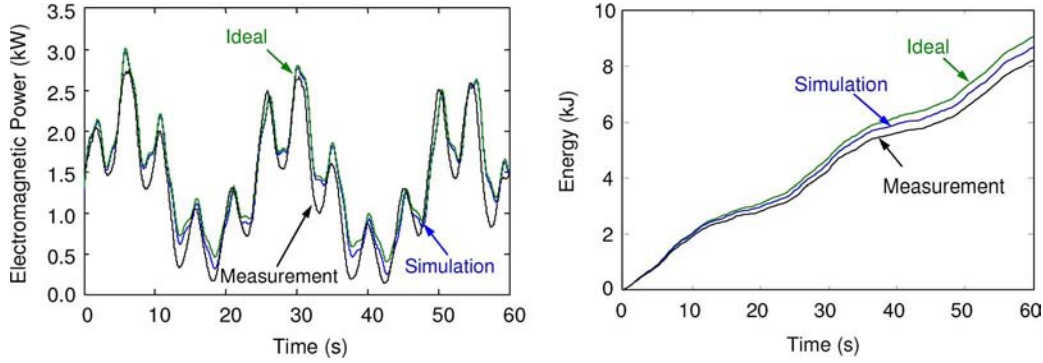


Fig. 13. Electromagnetic PMSG Power and energy during a wind cycle.

TABLE IV  
AVERAGE POWERS IN THE PASSIVE WIND TURBINE SYSTEM DURING  
A WIND CYCLE

	$\langle P_w \rangle$ [W]	$\langle \delta P \rangle$ [W]	$\langle P_u \rangle$ [W]
Ideal	1411	-	-
Simulation	1365	342	1023
Measurement	1310	308	1002

of the wind turbine system is close to 70%. Note also the very good agreement between simulated and experimental results for all investigated tests.

## V. CONCLUSION

In this paper, the design of a full passive wind power system based on a 1500-W three-bladed wind turbine has been investigated. Such a “low-cost” structure without active control and with a minimum number of sensors can be efficient only if the system design parameters are mutually adapted through an IOD approach which aims at optimizing the wind power extraction, the total system losses, and the total system mass. Issued from this study, an optimal PMSG prototype has been constructed and implanted in an experimental test bench. This test bench allows a “physical emulation” of the passive wind turbine system, the turbine being “simulated” by a servomotor which reproduces torque and inertia with respect to the rotation and wind speeds. All experimental tests have shown a very good agreement between the system simulations and corresponding measurements. Finally, a good global efficiency of the passive wind turbine system, close to 70%, has been noted from experiments, for the particular wind cycle profile used in the IOD process. All the results obtained in this study confirm the interest of such a “low-cost” passive structure (no control and power electronic devices, a minimum of sensors) sized from an IOD approach. In the outlooks of this work, the sensitivity analysis of the passive wind turbine efficiency according to design variable variations will be investigated in more detail. Robust design methods will be applied in order to take into account design variable uncertainties and their impact on design objectives or constraints.

## APPENDIX

### A. Analytical Models of the PMSG

This Appendix illustrates the calculation of the main electromagnetic parameters of the PMSG for both analytical models investigated in the paper (see Table V).

### B. DC Equivalent Model

A simplified equivalent dc model is used to represent the behavior of the PMSG-diode rectifier association in transient state [16], [17]. With this model, electrical values are determined from a quasi-static approach using analytical relations. Only thermal and mechanical modes (i.e., slow modes) are simulated by ODE equations. Electrical modes (i.e., fast modes) are neglected. Consequently, the computation time devoted to the simulation of all physical values (thermal, mechanical, and electric values) during a wind cycle is considerably reduced. With this model, the CPU time (on a 2-GHz standard PC computer) required for evaluating one individual (including geometric sizing, wind simulation, constraint, and objective computation) in the NSGA-II is about 0.6 s. This leads to a global CPU time of 10 hours for each NSGA-II run using 100 individuals and 600 generations. More details about the analysis of the CPU time devoted to the electrical simulation of the passive wind turbine can be found in [17].

The correspondence between synchronous generator circuit and the equivalent dc model is given in Table VI. The voltage  $V_{dc}$  can be calculated from Fig. 4 as detailed in [17]

$$V_{dc} = \sqrt{E_{sdc}^2 - (p\Omega_s L_{dc} I_{dc})^2} - (R_{dc} + R_{over}) I_{dc} \quad (7)$$

where  $R_{over} = 3L_s p \Omega_s / \pi$  models the diode overlapping during the commutation interval. From the above relation, we obtain the rectified current

$$I_{dc} = \sqrt{b^2 - c^2} - b \quad (8)$$

where

$$b = \frac{V_{dc}(R_{dc} + R_{over})}{(p\Omega_s L_{dc})^2 + (R_{dc} + R_{over})^2} \quad (9)$$

$$c = \frac{V_{dc}^2 - E_{sdc}^2}{(p\Omega_s L_{dc})^2 + (R_{dc} + R_{over})^2}. \quad (10)$$

### C. Wind Turbine System Parameters

The parameters of the wind turbine and of the optimized PMSG are listed below.

#### Wind turbine

Radius of wind turbine rotor:  $R_w = 1.25$  m;

Equivalent inertia:  $J_w = 1.5$  kg·m<sup>2</sup>;



TABLE V  
ELECTROMAGNETIC MODELS OF THE PMSG

Parameters	1 <sup>st</sup> Model [17], [19]	2 <sup>nd</sup> Model [20], [21], [22]
Airgap induction	$B_a = B_r \frac{l_m / K_C g}{\mu_r + l_m / K_C g}$	$B_{r,t}(r, \theta) = \sum_{n=1,3,5}^{\infty} 2\alpha_p \left( \frac{B_r}{\mu_r} \right) \frac{\sin \frac{n\pi\alpha_m}{2}}{n\pi\alpha_m} \frac{np}{(np)^2 - 1} R_a^{-(np-1)} \left[ r^{np-1} + r_s^{2np} r^{-(np+1)} \right]$ $\left\{ \frac{(np-1)R_a^{2np} + 2r_{rotor}^{np+1}R_a^{np-1} - (np+1)r_{rotor}^{2np}}{\frac{\mu_r+1}{\mu_r} [R_a^{2np} - r_{rotor}^{2np}] - \frac{\mu_r-1}{\mu_r} [R_a^{2np} - r_s^{2np} \left( \frac{r_{rotor}}{R_a} \right)^{2np}]} \right\} \cos np\theta$
Phase flux	$\Phi_s = 2K_b N_{spp} B_{1a} r_s l_r N_{cs}$	$\Phi_s = 4pN_{spp} N_{cs} l_r r_s B_r \sum_{n=1,3,5\dots}^{\infty} \frac{\sin \left( n\alpha_m \frac{\pi}{2} \right) \sin \left( np \frac{\theta_e}{2} \right) \sin \left( n \frac{\pi}{6} \right)}{n \frac{\pi}{2} \quad np \frac{\theta_e}{2} \quad n \frac{\pi}{6}} \frac{1}{(np)^2 - 1}$ $\left[ \frac{np+1 - (np-1) \left( \frac{R_a}{r_s} \right)^{2np} - 2 \left( \frac{R_a}{r_s} \right)^{np-1}}{1 - \left( \frac{R_{Sc}}{R_{cs}} \right)^{2np}} \right] \left( \frac{R_{Sc}}{R_a} \right)^{np}$
Main inductance	$L_m = \frac{4\mu_0 l_r r_s}{\pi \left( K_C g + \frac{l_m}{\mu_r} \right)} K_b^2 N_{spp}^2 N_{cs}^2$	$L_m = \sum_{n=1,3,5\dots}^{\infty} \left[ \mu_0 \frac{4p^2}{\pi} l_r \left( \frac{\sin \left( np \frac{\theta_e}{2} \right)}{np \frac{\theta_e}{2}} \right)^2 \frac{1}{np} \frac{1 + \left( \frac{r_{rotor}}{R_{Sc}} \right)^{2np}}{1 - \left( \frac{r_{rotor}}{R_{Sc}} \right)^{2np}} N_{spp}^2 N_{cs}^2 K_b^2(n) \right]$
Mutual Inductance	$M_s = -\frac{2\mu_0 l_r r_s}{\pi \left( K_C g + \frac{l_m}{\mu_r} \right)} K_b^2 N_{spp}^2 N_{cs}^2$	$M_s = \sum_{n=1,3,5\dots}^{\infty} \left[ -\mu_0 \frac{4p^2}{\pi} l_r \left( \frac{\sin \left( np \frac{\theta_e}{2} \right)}{np \frac{\theta_e}{2}} \right)^2 \frac{1}{np} \frac{1 + \left( \frac{r_{rotor}}{R_{Sc}} \right)^{2np}}{1 - \left( \frac{r_{rotor}}{R_{Sc}} \right)^{2np}} N_{spp}^2 N_{cs}^2 K_b^2(n) \sin \left( n \frac{\pi}{6} \right) \right]$
Leakage inductance	$L_f = 2\mu_0 l_r N_{spp}^2 N_{cs}^2 \left( \frac{2h_1}{3(b_1+b_3)} + \frac{2h_2}{b_2+b_3} + \frac{h_3}{b_2} \right)$	$L_f = 2\mu_0 l_r N_{spp}^2 N_{cs}^2 \left( \frac{2h_1}{3(b_1+b_3)} + \frac{2h_2}{b_2+b_3} + \frac{h_3}{b_2} \right)$
Stator inductance	$L_s = L_m - M_s + L_f$	$L_s = L_m - M_s + L_f$
Stator resistance	$R_s = \frac{2pN_{spp}}{\sigma_{cu}} \left( l_r + \frac{\pi(r_s + 0.5d_s)}{p} \right) \frac{N_{cs}^2}{S_{slot}}$	$R_s = \frac{2pN_{spp}}{\sigma_{cu}} \left( l_r + \frac{\pi(r_s + 0.5d_s)}{p} \right) \frac{N_{cs}^2}{S_{slot}}$

TABLE VI  
CORRESPONDENCE BETWEEN PMSG AND DC EQUIVALENT MODEL

Parameters	PMSG	DC Equivalent Model
Flux	$\Phi_s$	$\Phi_{dc} = \frac{3\sqrt{6}}{\pi} \Phi_s$
Inductance	$L_s$	$L_{dc} = 3 \left( \frac{\sqrt{6}}{\pi} \right)^2 L_s$
Resistance	$R_s$	$R_{dc} = 3 \left( \frac{\sqrt{6}}{\pi} \right)^2 R_s$
EMF	$E_s = \Phi_s p \Omega_s$	$E_{sdc} = \frac{3\sqrt{3}}{\pi} \Phi_s p \Omega_s$

Damping coefficient:  $f_w = 0.06$  N·m/rad;  
Ideal power coefficients:  $\lambda_{opt} = 6.9$ ,  $C_p(\lambda_{opt}) = 0.442$ ;

Interpolation factors of power coefficient:

$$\begin{aligned} a_7 &= -4.10^{-8}; a_6 = -4.2.10^{-6}; \\ a_5 &= 2.1.10^{-4}; a_4 = -3.1.10^{-3}; \\ a_3 &= 1.65.10^{-2}; a_2 = -1.8.10^{-2}; \\ a_1 &= 1.7.10^{-2}; a_0 = -1.9.10^{-3} \end{aligned}$$

Mass:  $M_w = 34$  kg

**PMSG**

Base speed:  $\Omega_b = 77$  rad·s<sup>-1</sup>

Base Power:  $P_b = 1753$  W

Yoke induction:  $B_y = 1.9$  T

Radius/length ratio:  $R_{r1} = 1.45$

Slot depth/radius ratio:  $R_{dr} = 0.3$

Current density:  $J_s = 3.9$  A/mm<sup>2</sup>

Number of pole pairs:  $p = 3$ ;

Number of slots per pole per phase:  $N_{\text{spp}} = 3$ ;  
 Resistance:  $R_s = 0.13 \text{ Ohm}$ ;  
 Inductance:  $L_s = 1.4 \text{ mH}$ ;  
 Flux:  $\Phi_s = 0.21 \text{ Wb}$ ;  
 Active length:  $l_r = 48.5 \text{ mm}$ ;  
 Bore radius:  $r_s = 70.5 \text{ mm}$ ;  
 Airgap width:  $g = 1.8 \text{ mm}$ ;  
 Yoke thickness of stator and rotor:  $d_y = d_r = 14 \text{ mm}$ ;  
 Slot, teeth width:  $w_s = w_T = 5.5 \text{ mm}$ ;  
 Active mass:  $M_{\text{PMSG}} = 9.8 \text{ kg}$ ;  
**Diode Rectifier** (IXYS- Vue 50)  
 Radiator mass:  $M_{\text{rad}} = 0.18 \text{ kg}$  (issued from optimization)  
 $M_{\text{rad}} = 0.24 \text{ kg}$  (prototype)  
**Battery**  
 NP38-12 I 12 V 38 Ah.

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