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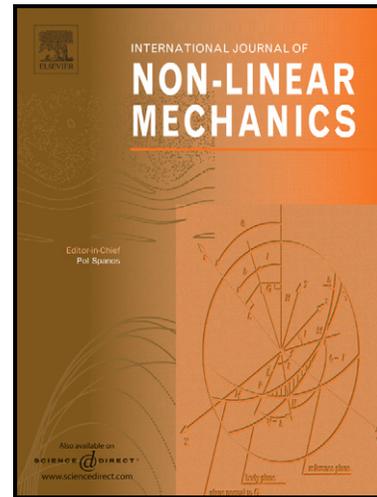
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Unsteady Inviscid Flowfields of 2-D Airfoils
by Non-linear Singular Integral Computational Analysis

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Abstract

A two-dimensional aerodynamics representation analysis is introduced for the investigation of inviscid flowfields of unsteady airfoils. The problem of the unsteady flow of a two-dimensional NACA airfoil is therefore reduced to the solution of a non-linear multidimensional singular integral equation, when the form of the source and vortex strength distribution is dependent on the history of the above distribution on the NACA airfoil surface. An application is given to the determination of the velocity and pressure coefficient field around an aircraft by assuming constant source distribution.

Key Words and Phrases

Non-linear Multidimensional Singular Integral Equations, Two-dimensional NACA airfoil, Non-linear Aerodynamics, Constant Source Distribution, Aircraft, Velocity & Pressure Coefficient Field.

1. Introduction

During the last years the non-linear singular integral equations have concentrated an increasing interest, because of their application to the solution of basic problems of aerodynamics and fluid mechanics, especially referring to unsteady flows. The theory and computational methods by non-linear singular integral equations consist of the latest high technology to the solution of generalized problems of solid and fluid mechanics. Hence, there is a big interest to the continuous improvement of such computational methods.

The new design aerodynamic problems are reduced to the solution of a non-linear singular integral equation, which is used for the determination of the velocity and pressure coefficient field around a NACA airfoil. Such an aerodynamic behaviour of the NACA airfoils is a very important element to the design of new generation aircrafts, with very high speeds. Therefore, special attention should be given to the new technology computational methods concentrated to the solution of the before mentioned aerodynamic and fluid dynamic problem.

A.M.O.Smith and J.L.Hess [1], were the first scientists who investigated aerodynamic panel methods for studying airfoils with zero lift. According to them, the airfoil was modeled with distributed potential source panels for nonlifting flows, or vortex panels for flow with lift. This method was further extended by R.H.Djojodihardjo and S.E.Widnall [2], P.E.Robert and G.R.Saaris [3], J.M.Summa [4], D.R.Bristow [5], D.R.Bristow and J.D.Hawk [6] and R.J.Lewis [7], for studying three-dimensional steady and unsteady flows, by combining source and vortex singularities. Also, the unsteady panel methods to the modeling of separated wakes using discrete vortices, were further extended by T.Sarpkaya and R.L.Schoaf [8].

Beyond the above, N.D.Ham [9], F.D.Deffenbaugh and F.J.Marschall [10], M.Kiya and M.Arie [11] and T.Sarpkaya and H.K.Kline [12] investigated some other flow models. According to them, the separating boundary layers were represented by an array of discrete vortices, emanating from a known separation point location on the airfoil surface.

On the other hand, during the past years, several scientists made extensive calculations by using unsteady turbulent boundary layer methods. Among them we mention: R.E.Singleton and J.F.Nash [13], J.F.Nash, L.W.Carr and R.E.Singleton [14], A.A.Lyrio, J.H.Ferzinger and S.J.Kline [15], W.J.McCroskey and S.I.Pucci [16] and J.Kim, S.J.Kline and J.P.Johnston [17].

Recently, non-linear singular integral equation methods were proposed by E.G.Ladopoulos [18] - [22] for the solution of fluid mechanics and aerodynamic problems and by E.G.Ladopoulos and V.A.Zisis [23], [24] for two-dimensional fluid mechanics problems applied to turbomachines.

In the present research, the aerodynamic problem of the unsteady flow of a two-dimensional NACA airfoil moving by a velocity U_A , is reduced to the solution of a non-linear multidimensional singular integral equation. This nonlinearity results because the source and vortex strength distribution are dependent on the history of the vorticity and source distribution on the NACA airfoil surface. Beyond the above, a turbulent boundary layer model is further proposed, based on the formulation of the unsteady behaviour of the momentum integral equation.

An application is finally given to the determination of the velocity and pressure coefficient field around an aircraft by assuming constant source distribution.

2. Unsteady Aerodynamics by Non-linear Singular Integral Equations

A new non-linear unsteady fluid mechanics representation analysis is investigated, for the aerodynamic problem of a two-dimensional NACA airfoil. This method consists to the generalization of all past methods, by reducing the problem to the solution of a non-linear multidimensional singular integral equation. The above nonlinearity results because of the general form given to the source and vortex strength distribution, while these are dependent on the history of the vorticity and source distribution on the NACA airfoil surface. In this case the airfoil is moving with a speed U_A . [18] – [22]

Hence, consider a two-dimensional airfoil moving in an homogeneous and inviscid fluid. (Fig.1).

Fig. 1

The airfoil with the wake comprise s complete lifting system in an irrotational flow through the ideal fluid. Because of the existence of such an irrotationality, then for the local fluid velocity \mathbf{U} one has:

$$\nabla \times \mathbf{U} = 0 \quad (2.1)$$

Furthermore, by replacing the fluid velocity with the total velocity potential H we have:

$$\mathbf{U} = \nabla H \quad (2.2)$$

while (2.2) can be further written as:

$$\mathbf{U} = \mathbf{U}_\infty + \nabla h \quad (2.3)$$

where \mathbf{U}_∞ denotes the outward velocity (Fig. 1) and h the potential due to the presence of the airfoil.

Beyond the above, the use of Green's theorem [25] results the following relation for the velocity potential $h(\mathbf{x}, t)$, with t the time, at any point \mathbf{x} in continuous, acyclic irrotational flow:

$$h(\mathbf{x}, t) = -1/2\pi \int_S \frac{g[\xi, t, h]}{r} dS + 1/2\pi \int_{S+W} \delta[\xi, t, h] \frac{\partial}{\partial n_1} \left(\frac{1}{r} \right) dS \quad (2.4)$$

in which S is the surface of the airfoil (Fig. 1), W the surface of the wake, \mathbf{n}_1 the surface normal at the source point ξ (Fig. 1), $g[\xi, t, h]$ the source strength distribution, $\delta[\xi, t, h]$ the vortex strength distribution and r the distance equal to:

$$r = |\mathbf{x} - \xi| \quad (2.5)$$

The velocity potential (2.4) can be further written as following, which denotes a two-dimensional non-linear singular integral equation:

$$h(\mathbf{x}, t) = -1/2\pi \int_S \frac{g[\xi, t, h]}{r} dS + 1/2\pi \int_{S+W} \frac{\delta[\xi, t, h]}{r^2} dS \quad (2.6)$$

The kinematical surface tangency condition on the surface of the airfoil must be valid: [26]

$$(1/|\nabla S(\mathbf{x}, t)|) \frac{\partial S(\mathbf{x}, t)}{\partial t} + \frac{\partial h}{\partial n_2} + \mathbf{U}_\infty \cdot \mathbf{n}_2 = 0 \quad (2.7)$$

in which \mathbf{n}_2 denotes the surface normal at the field point \mathbf{x} (Fig. 1).

This condition can be also written in the following form, for a body fixed coordinate system:

$$(1/|\nabla S(\mathbf{x}, t)|) \frac{\partial S(\mathbf{x}, t)}{\partial t} = -(\mathbf{U}_A + \boldsymbol{\omega}_A \times \mathbf{x}) \cdot \mathbf{n}_2 \quad (2.8)$$

where \mathbf{U}_A denotes the airfoil translation velocity and $\boldsymbol{\omega}_A$ the airfoil angular rotation.

From eqs (2.7) and (2.8) follows:

$$\frac{\partial h}{\partial n_2} + (\mathbf{U}_\infty - \mathbf{U}_A - \boldsymbol{\omega}_A \times \mathbf{x}) \cdot \mathbf{n}_2 = 0 \quad (2.9)$$

Finally, by inserting (2.9) into (2.6) follows a two-dimensional non-linear singular integral equation:

$$1/2\pi \int_S g[\xi, t, h] \frac{\partial}{\partial n_2} \left(\frac{1}{r} \right) dS + 1/2\pi \int_{S+W} \delta[\xi, t, h] \frac{\partial}{\partial n_2} \left(\frac{1}{r^2} \right) dS = -(\mathbf{U}_\infty - \mathbf{U}_A - \boldsymbol{\omega}_A \times \mathbf{x}) \cdot \mathbf{n}_2 \quad (2.10)$$

The non-linear singular integral equation (2.10) can be also written as:

$$1/2\pi \int_S \frac{g[\xi, t, h]}{r^2} dS + 1/\pi \int_{S+W} \frac{\delta[\xi, t, h]}{r^3} dS = (\mathbf{U}_\infty - \mathbf{U}_A - \boldsymbol{\omega}_A \times \mathbf{x}) \cdot \mathbf{n}_2 \quad (2.11)$$

Therefore, by solving the non-linear integral equation (2.11) with the corresponding boundary conditions, then the velocity at any field point will be determined through (2.7).

3. Non-linear Pressure Distribution Analysis

By the unsteady Bernoulli equation, which is valid at any point in an irrotational, ideal flow, will be determined the pressure distribution on the airfoil:

$$P = P_\infty - \rho \left[\frac{\partial H}{\partial t} + 1/2(\nabla H)^2 \right] \quad (3.1)$$

in which ρ denotes the fluid density.

Furthermore, by using the derivation of the previous section, then (3.1) reduces to the following form:

$$P = P_\infty - \rho \left[\frac{\partial h}{\partial t} + (\mathbf{U}_\infty - \mathbf{U}_A - \boldsymbol{\omega}_A \times \mathbf{x}) \cdot \nabla h + 1/2(\nabla h)^2 \right] \quad (3.2)$$

Also, (3.2) can be written as:

$$P = P_\infty - \rho \left[\frac{\partial H}{\partial t} + (\mathbf{U}_\infty - \mathbf{U}_A - \boldsymbol{\omega}_A \times \mathbf{x}) \cdot \nabla_s H + \frac{\partial H}{\partial n_1} (\mathbf{U}_\infty - \mathbf{U}_A - \boldsymbol{\omega}_A \times \mathbf{x}) \cdot \mathbf{n}_1 + 1/2 (\nabla_s H)^2 + 1/2 \left(\frac{\partial H}{\partial n_1} \right)^2 \right] \quad (3.3)$$

if replacing the ∇h , by the surface gradient $\nabla_s h$:

$$\nabla h = \nabla_s h + \frac{\partial h}{\partial n_1} \boldsymbol{\varepsilon}_{n_1} \quad (3.4)$$

Hence, because of (2.9), then (3.3) takes the form:

$$P = P_\infty - \rho \left[\frac{\partial H}{\partial t} + (\mathbf{U}_\infty - \mathbf{U}_A - \boldsymbol{\omega}_A \times \mathbf{x}) \cdot \nabla_s H - 1/2 \left\{ (\mathbf{U}_\infty - \mathbf{U}_A - \boldsymbol{\omega}_A \times \mathbf{x}) \cdot \mathbf{n}_1 \right\}^2 + 1/2 (\nabla_s H)^2 \right] \quad (3.5)$$

which will be used for the computations.

4. Laminar and Turbulent Boundary Layer Models for Aerodynamics

Several boundary layer models can be used for the laminar, the turbulent parts of the flow and the transition region between them, in order to determine the aerodynamic behaviour of the airfoils. These boundary layer models are the finite difference, finite element or integral models.

In the present research a turbulent boundary layer model is proposed, based on the formulation of the unsteady behaviour of the momentum integral equation. Hence, the unsteady momentum integral equation valid for both laminar and turbulent flow may be written as: (Fig. 2)

$$\frac{1}{u_b^2} \frac{\partial}{\partial t} (u_b d_1) + \frac{\partial d_2}{\partial S} + \frac{1}{u_b} \frac{\partial u_b}{\partial S} (2d_2 + S) = \frac{c_F}{2} \quad (4.1)$$

where u_b denotes the boundary layer edge velocity, t the time, d_1 the displacement thickness, d_2 the momentum thickness, S the surface distance and c_F the friction factor.

Fig. 2

Furthermore, consider the case of a laminar layer, so that the pressure gradient parameter μ is given by the following formula:

$$k = \frac{d_2}{u_b} R_d \left(\frac{\partial u_b}{\partial S} + \frac{1}{u_b} \frac{\partial u_b}{\partial t} \right) \quad (4.2)$$

where R_d denotes the Reynolds number based on u_b and d_2 .

By considering also some special relations between the parameters $c_F/2$, d_2 and d_1 , then we obtain a solution for the laminar formulation. Hence, for the wedge solutions following relations can be used:

$$\begin{aligned} \frac{c_F}{2} &= \frac{1.91 - 4.13D}{R_d} \\ N &= (0.68 - 0.922D)^{-1} \\ D &= 0.325 - 0.13kN^2 \end{aligned} \quad (4.3)$$

where N is the shape parameter, D the blockage factor d_1/d_b with d_b the boundary layer thickness and R_d the Reynolds number based on u_d and d .

Beyond the above, for the turbulent layer model following relation is valid:

$$\frac{1}{u_B} \frac{\partial}{\partial S} [u_B(d_B - d_1)] = A \quad (4.4)$$

and the function A is obtained by the formulas:

$$\begin{aligned} \frac{dA}{dS} &= 0.025(A_B - A)d_B \\ A_B &= 4.24K_B \left(\frac{B}{1-B}\right)^{0.916} \\ K_B &= 0.013 + 0.0038e^{-\phi/15} \\ \phi &= \frac{d_1}{\tau_w} \frac{dP}{dx} \end{aligned} \quad (4.5)$$

in which τ_w denotes the wall shear stress and dp/dx the streamwise pressure gradient.

The shape factor relationships are further obtained as following:

$$\begin{aligned} \frac{u}{u_B} &= 1 + \xi \ln\left(\frac{y}{d_B}\right) - f \cos^2\left(\frac{\pi y}{2d_B}\right) \\ \xi &= \frac{1}{0.41} \left(\operatorname{sgn} \frac{c_F}{2}\right) \left(\frac{c_F}{2}\right)^{1/2} \\ f &= 2(B - \xi) \\ \frac{c_F}{2} &= \frac{\tau_w}{\rho u_B^2} \end{aligned} \quad (4.6)$$

with u the velocity in the boundary layer at a distance y from the wall and ρ the fluid density.

Hence, the skin friction law is equal to:

$$\frac{c_F}{2} = 0.051 |1 - 2B|^{1.732} \left(\frac{R_d}{B}\right)^{-0.268} \operatorname{sgn}(1 - 2B) \quad (4.7)$$

5. Velocity Calculations for Constant Source Distribution (Airfoil with Velocity)

Consider the special case of a constant source distribution g . In this case the general non-linear problem presented in previous paragraphs, is much more simplified and is solved as a linear problem. The geometrical representation of the problem is shown in Fig. 3.

Fig. 3

In this special case the fluid velocity \mathbf{U} , is calculated as following:

$$\mathbf{U} = \int_{-A/2}^{A/2} \frac{g \, dr}{2\pi r} (\cos \phi \mathbf{i} + \sin \phi \mathbf{j}) \quad (5.1)$$

where \mathbf{i} , \mathbf{j} are the unit vectors on the x and y axes, respectively, and A denotes the separating wake (Fig. 3).

Hence, the fluid velocity \mathbf{U} can be determined as following, for the cases when $y_p \neq 0$ and $y_p = 0$:

$$\mathbf{U} = \begin{cases} g/2\pi \left[\ln \left| \frac{r_1}{r_2} \right| \mathbf{i} - (\phi_1 - \phi_2) \mathbf{j} \right], & y_p \neq 0 \\ g/2\pi \ln \left| \frac{r_1}{r_2} \right| \mathbf{i}, & y_p = 0 \end{cases} \quad (5.2)$$

Moreover, we consider the pressure coefficient C_p :

$$C_p = (P - P_\infty) / [1/2 \rho (U_\infty - U_A)^2] \quad (5.3)$$

where ρ denotes the fluid density and P_∞ the stream pressure.

By using Bernoulli's equation, then the coefficient C_p may be written as:

$$C_p = -U^2 / (U_\infty - U_A)^2 \quad (5.4)$$

which can be used for the computations.

6. Application of Unsteady Aerodynamics for New Generation Aircrafts

As an application of the previous mentioned two-dimensional unsteady aerodynamics theory, we will calculate the velocity field presented around an aircraft. The construction of new generation turbojet engines makes possible the design of very fast big jets. Beyond the above, the increasing evolution of aeroelasticity in aircraft turbomachines continues to be still improved, according to the needs of aircraft powerplant and turbine designers.

Hence, the Aeronautical Industries should achieve a competitive technological advantage in several strategic areas of new and fast developing advanced technologies, by which a bigger market share can be achieved, in the medium and longer terms. Such an increasing big market share includes the design of new generation large aircrafts with very high speeds.

In the present application the length of the aircraft under consideration is $c=50.0m$ and the airfoil section NACA 0021 (Fig. 3).

Moreover, it was supposed unit source distribution and therefore, the velocity field on the boundary of the airfoil was computed by (5.2). Also, the pressure coefficient C_p was calculated through (5.4) for several aircraft velocities U_A and wind velocity $U_\infty = 15m/sec$.

Figures 4, 5, 6 and 7 show the pressure distribution on the turbojet presented, for aircraft speeds $U_A = 1,2,3,4$ Mach respectively (1 Mach=332 m/sec). Also, Figs. 4a to 7a show the same pressure distribution on the airfoil, in three dimensional form.

Figs. 4 - 7 / Figs. 4a - 7a

As it is shown in the above Figures, for the up boundary points of the NACA airfoil the values of the pressure coefficient are increasing approximately up to $x/c = 0.25$, while they decreasing again up to $x/c = 1$. On the other hand, for the down boundary points the values of C_p are decreasing up to $x/c = 0.35$, and then increasing up to $x/c = 1$.

7. Conclusions

A general non-linear model has been proposed for the determination of the velocity and pressure coefficient field around a NACA airfoil moving by a velocity U_A in two-dimensional unsteady flow. Such a problem was reduced to the solution of a two-dimensional non-linear singular integral equation, which has to be solved by computational methods. The nonlinearity resulted because of the form of the general type of the source and vortex strength distribution.

Furthermore, a turbulent boundary layer model was proposed, based on the formulation of the unsteady behaviour of the momentum integral equation. The unsteady momentum integral equation which was studied, is valid for both laminar and turbulent flow, and was given as a general method for the determination of the aerodynamic behaviour of the airfoils.

On the other hand, by supposing constant source distribution, then the velocity and pressure coefficient field around an aircraft moving with several velocities, was determined. This method should be applied for the design of new generation large aircrafts with very high speeds.

Finally, the non-linear singular integral equation methods, will be in future of continuously increasing interest, as such methods will be very important for the solution of the generalized solid and fluid mechanics problems. Special attention should be therefore given to the amelioration of the non-linear singular integral equation methods, as many modern solid and fluid mechanics problems with considerable complicated forms, are recently reduced to non-linear forms.

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Figure Captions

- [1]: A two-dimensional airfoil of surface S in an homogeneous and inviscid fluid.
- [2]: Laminar and Turbulent Boundary Layer Model for Aerodynamics.
- [3]: Coordinate system for the 2D airfoil of an aircraft.
- [4]: Pressure distribution around the aircraft of Fig.3, for constant source distribution and speed 1 Mach.
- [5]: Pressure distribution around the aircraft of Fig.3, for constant source distribution and speed 2 Mach.
- [6]: Pressure distribution around the aircraft of Fig.3, for constant source distribution and speed 3 Mach.
- [7]: Pressure distribution around the aircraft of Fig.3, for constant source distribution and speed 4 Mach.
- [4a]: Pressure distribution around the aircraft of Fig.3, for constant source distribution and speed 1 Mach – 3D form.
- [5a]: Pressure distribution around the aircraft of Fig.3, for constant source distribution and speed 2 Mach – 3D form.
- [6a]: Pressure distribution around the aircraft of Fig.3, for constant source distribution and speed 3 Mach – 3D form.
- [7a]: Pressure distribution around the aircraft of Fig.3, for constant source distribution and speed 4 Mach – 3D form.

Non-linear Multidimensional Singular Integral Equation for 2-D Unsteady Aerodynamics

Determination of the Velocity and Pressure Coefficient Field around an Aircraft

Turbulent Boundary Layer Model based on the behavior of Momentum Integral Equation.

General type of the Source and Vortex Strength Distribution around NACA Airfoils

Unsteady Momentum Integral equation for both Laminar and Turbulent Flows

Accepted manuscript

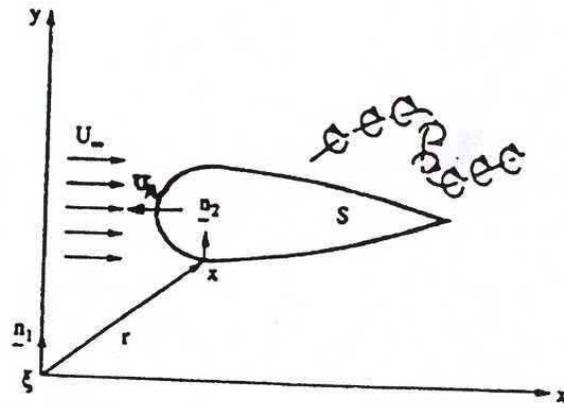


Fig. 1

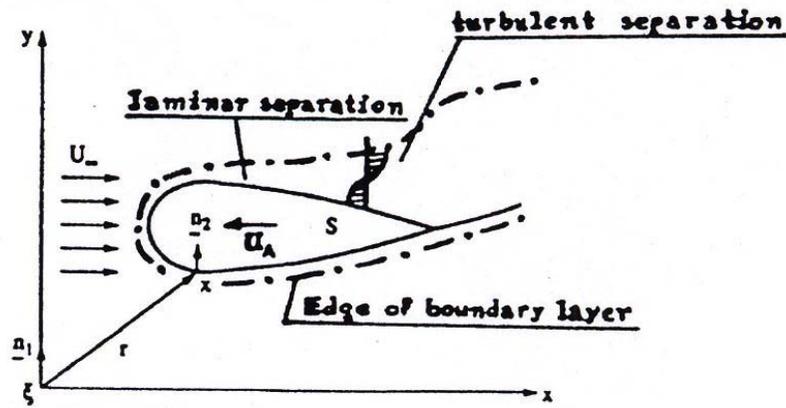


Fig. 2

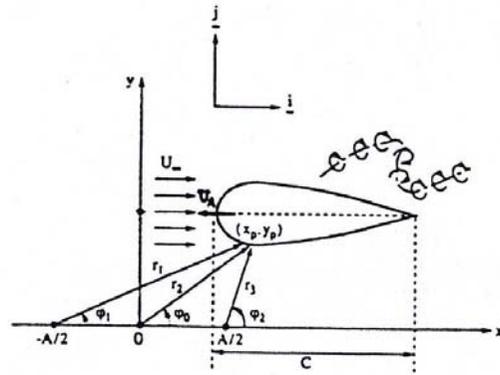


Fig. 3

Figure 4

Figure 4. Pressure distribution around the aircraft for constant source distribution and speed 1 Mach

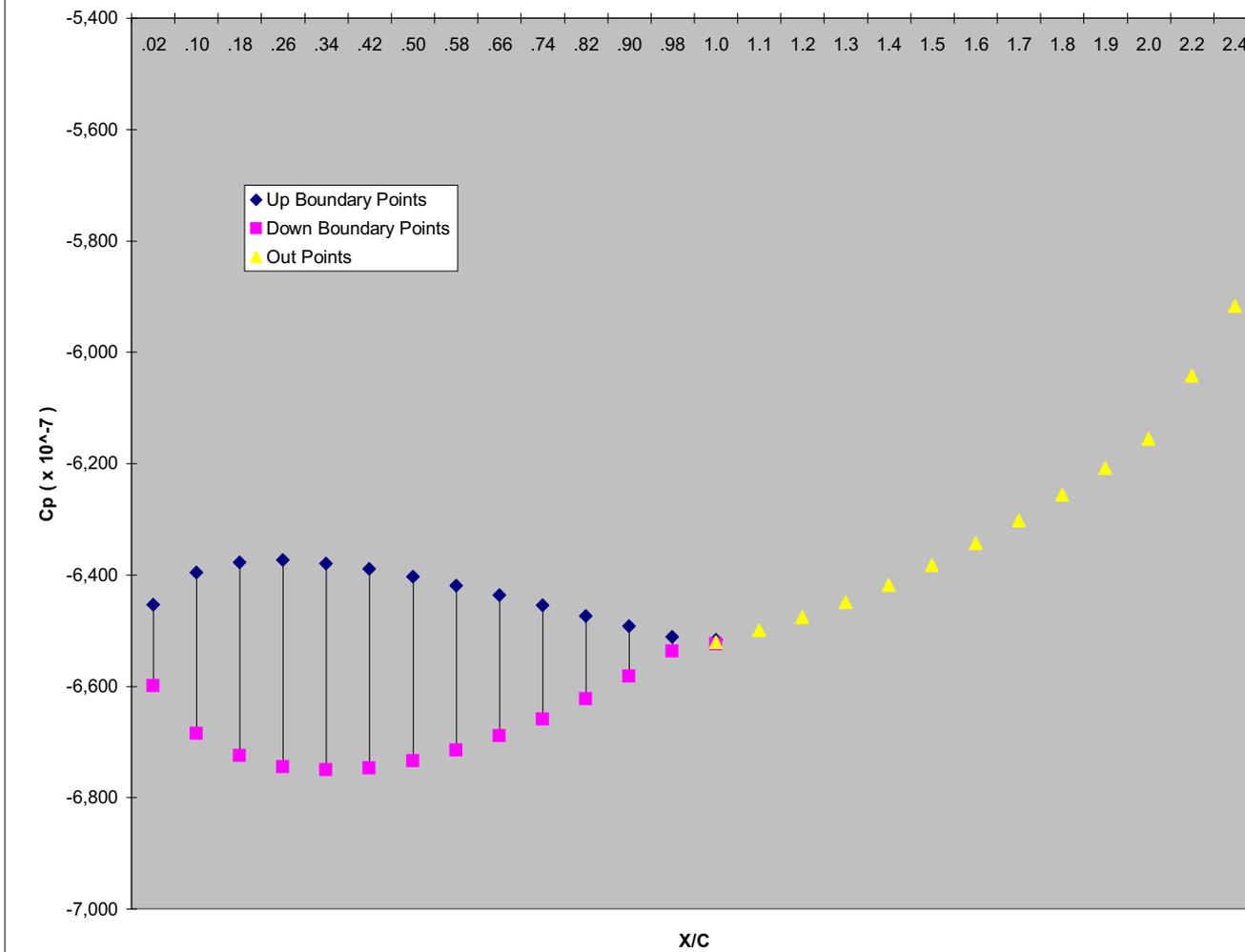


Figure 4a

Figure 4a. Pressure distribution around the aircraft for constant source distribution and speed 1 Mach

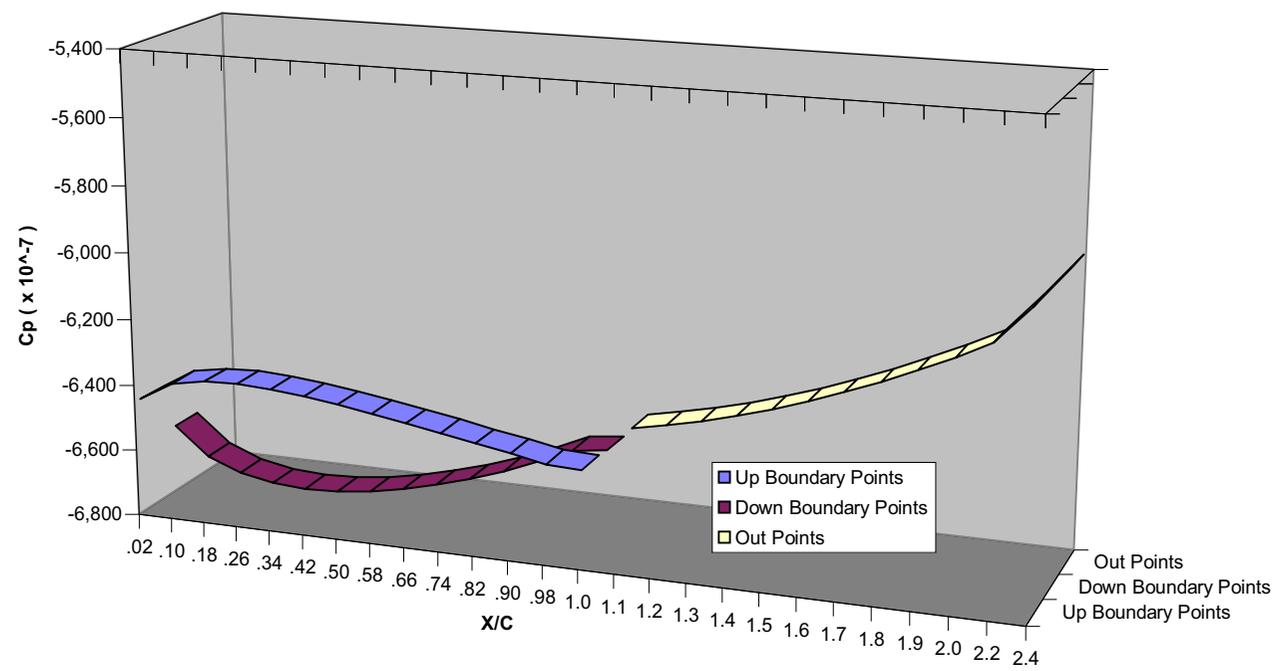


Figure 5

Figure 5. Pressure distribution around the aircraft for constant source distribution and speed 2 Mach

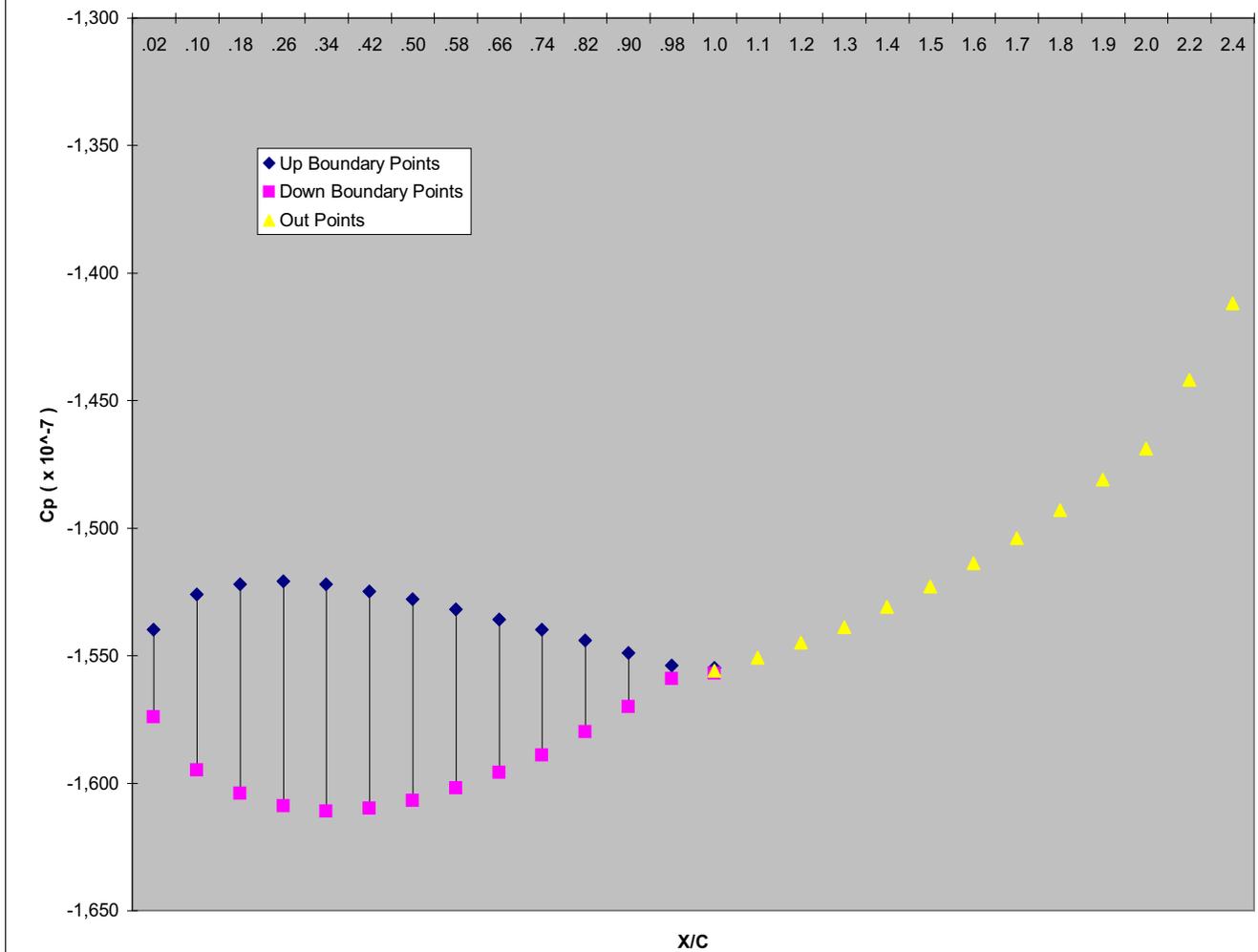


Figure 5a

Figure 5a. Pressure distribution around the aircraft for constant source distribution and speed 2 Mach

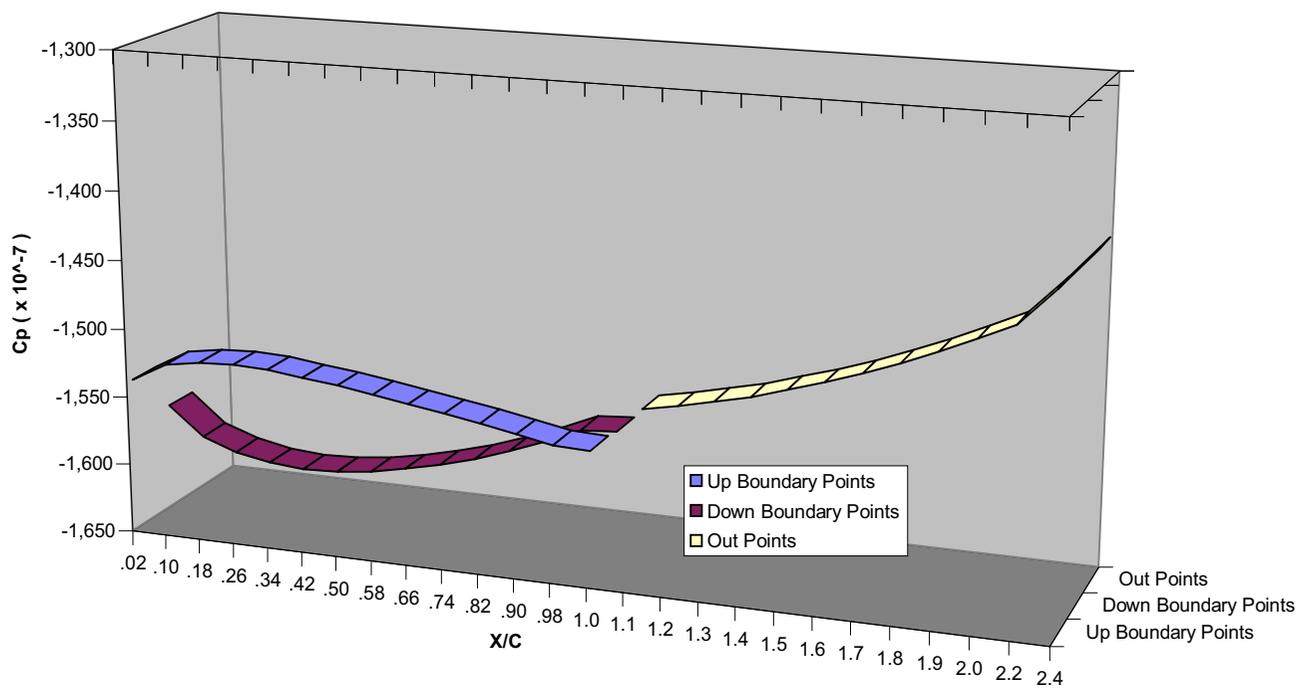


Figure 6

Figure 6. Pressure distribution around the aircraft for constant source distribution and speed 3 Mach

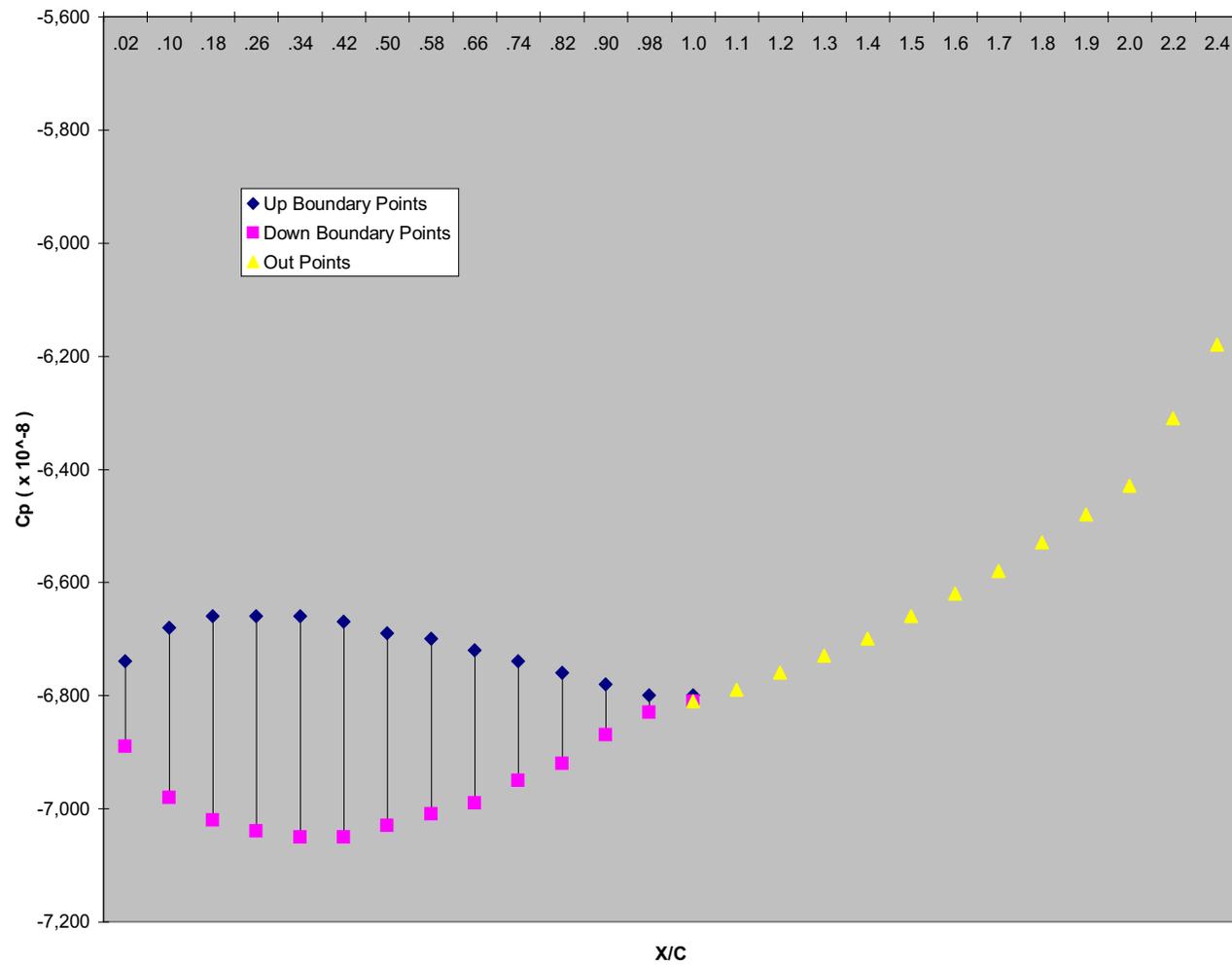


Figure 6a

Figure 6a. Pressure distribution around the aircraft for constant source distribution and speed 3 Mach

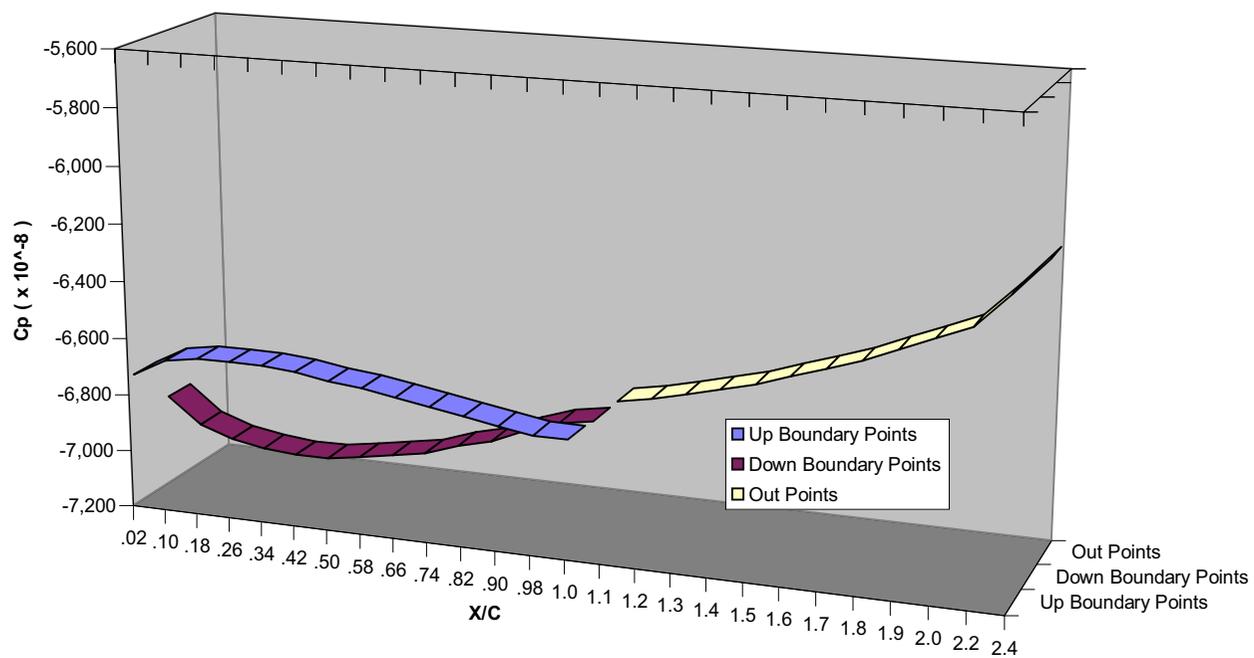


Figure 7

Figure 7. Pressure distribution around the aircraft for constant source distribution and speed 4 Mach

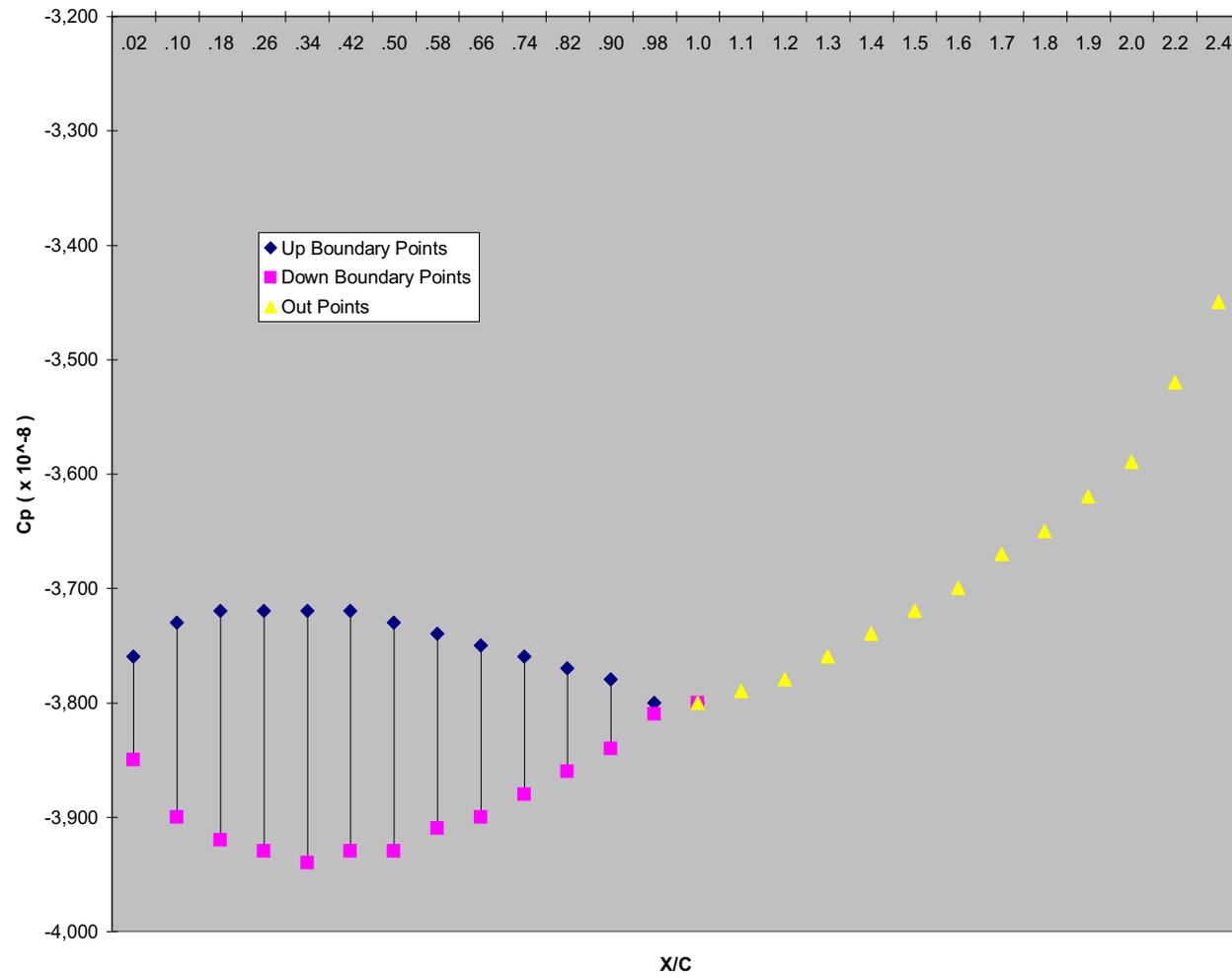


Figure 7a

Figure 7a. Pressure distribution around the aircraft for constant source distribution and speed 4 Mach

