

Regime classification and planform scaling for internally heated mantle convection

Christian Hüttig, Doris Breuer

▶ To cite this version:

Christian Hüttig, Doris Breuer. Regime classification and planform scaling for internally heated mantle convection. Physics of the Earth and Planetary Interiors, 2011, 10.1016/j.pepi.2011.03.011. hal-00759562

HAL Id: hal-00759562 https://hal.science/hal-00759562

Submitted on 1 Dec 2012

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Accepted Manuscript

Regime classification and planform scaling for internally heated mantle convection

Christian Hüttig, Doris Breuer

 PII:
 S0031-9201(11)00069-0

 DOI:
 10.1016/j.pepi.2011.03.011

 Reference:
 PEPI 5401

To appear in: *Physics of the Earth and Planetary Interiors*

Received Date:13 April 2010Revised Date:16 March 2011Accepted Date:25 March 2011



Please cite this article as: Hüttig, C., Breuer, D., Regime classification and planform scaling for internally heated mantle convection, *Physics of the Earth and Planetary Interiors* (2011), doi: 10.1016/j.pepi.2011.03.011

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

1 Regime classification and planform scaling for internally heated mantle 2 convection

Christian Hüttig and Doris Breuer

DLR, Institute of Planetary Research, Rutherfordstrasse 2, 12489 Berlin

7

3

4

5

6

8 Abstract:

9 Internally heated 3-D mantle convection models in a spherical shell with temperature and 10 pressure dependent viscosity have been performed to provide new insights into the various 11 convection regimes, the transition from steady state convection to time-dependent convection 12 and the associated convection pattern. The analysis of a total of 91 simulations reveals four 13 regime types, i.e., a mobile-lid regime, a sluggish regime, a low-degree regime, and a 14 stagnant-lid regime. The occurrence of these regimes depends on the viscosity contrast and 15 the internal Rayleigh number. The low-degree regime occurs close to the boundary of the 16 stagnant-lid regime in case of temperature dependent viscosity. In case of additional pressure 17 dependence, the range of the low-degree regime is smaller and a narrow range of the sluggish-18 lid regime exists in the weakly convecting part. Furthermore, the transition to the stagnant-lid 19 regime occurs at a lower viscosity contrast. For the stagnant-lid regime we have derived a 20 scaling law describing the heat transport. Similar scalings could not be obtained for the other 21 regimes as this seems to require also a correlation of the convective pattern with the internal 22 Rayleigh number. Such a relation is only given for the stagnant-lid regime in case of 3D 23 spherical geometry. The stagnant-lid cases in steady state show a minimal possible degree of 24 the convective pattern that is independent on the pressure dependence of viscosity and 25 remains constant until time-dependent convection sets in with increasing Ra. At this stage, the 26 dominant degree of the convective pattern increases with increasing internal Ra but the slope 27 varies with the pressure dependence of the viscosity.

28

1. Introduction

30 The temporal and spatial evolution of mantle convection strongly depends on the mantle 31 rheology, and in particular the viscosity (e.g., Christensen, 1983; Christensen, 1984a; 32 Christensen and Harder, 1991; Giannandrea and Christensen, 1993; Hansen and Yuen, 1993; 33 Ratcliff et al., 1995; Roberts and Zhong, 2006; Tackley, 2000a; Tackley, 2000b; Weinstein, 34 1995; Zebib, 1993; Zhu and Feng, 2005). One first order effect is the temperature dependence 35 of the viscosity. For temperature variations typical in the terrestrial planets the viscosity varies 36 by several orders of magnitude. The consequences of this strong viscosity variation on the 37 mantle dynamic has been examined with experimental and numerical studies for about three 38 decades (e.g., Christensen, 1984a; Davaille and Jaupart, 1993; Giannandrea and Christensen, 39 1993; Grasset and Parmentier, 1998; Honda, 1996; Moresi and Solomatov, 1995; Morris and 40 Canright, 1984; Reese et al., 2005; Reese et al., 1999; Richter, 1978; Richter et al., 1982; 41 Solomatov, 1995). One important finding is the change in surface mobility and the 42 development of three different regimes depending on the viscosity contrast (e.g., Ratcliff et 43 al., 1996a; Ratcliff et al., 1997; Solomatov, 1995):

The **mobile-lid regime** is typical for iso-viscous convection and convection with extremely low viscosity contrasts where the surface is fully mobile. The **sluggish-lid regime**, sometimes called transitional regime, is typical for convection with moderate viscosity contrasts (10^2 - 10^4). The surface velocity is strongly reduced due to high viscosities of the cold material close to the surface. The **stagnant-lid regime** occurs with strong temperature-dependent viscosity (for viscosity contrasts larger than ~ $10^4 - 10^5$) as the surface completely stagnates and does not participate in convection. The heat transport within this lid is only by conduction.

Transition into the different regimes has also been associated with changes in the degree of convective pattern. For instance, for a moderate viscosity contrast, i.e., the sluggish-lid regime, the convection structure can become large-scale (e.g., McNamara and Zhong, 2005; Tackley, 1996b; Yoshida and Kageyama, 2006) whereas convection under the stagnant lid is typically characterized by small-scale convection (Reese et al., 2005; 1999).

56 In this paper, we present a systematic study of internally heated convection in a spherical shell 57 with varying viscosity contrast and Rayleigh number for either purely temperature dependent 58 viscosity or temperature and pressure dependent viscosity. To our knowledge, the only studies 59 for internally heated three-dimensional spherical shells have been performed by Reese *et al.* 60 (2005; 1999), and cover only a narrow parameter range within the stagnant-lid regime and do

not study the transition between the regimes or structural aspects in detail. Here, we have performed 91 simulations to provide new insight into the transition to the stagnant-lid regime, the transition from stationary to time-dependent convection, the associated convection patterns and the ratio of lateral to radial viscosity contrast. For the stagnant-lid regime, we have derived a scaling law for the heat transport and the dominant degree of the convective pattern depending on the internal Rayleigh number as well as a scaling law for the lateral viscosity contrast depending on the radial viscosity contrast.

68

69 2. Numerical Model

The numerical model is based on a simulation code by (Hüttig and Stemmer, 2008a; 2008b) that is able to handle fully spatial viscosity variations up to several orders of magnitude. The mantle convection is numerically treated as thermal convection in a 3-D spherical shell (with a core-to-surface radius ratio of 0.55) of a Boussinesq fluid with infinite Prandtl number. We apply free-slip boundary conditions, volumetric heating and surface cooling. The set of conservation equations then reads:

$$\nabla \cdot \vec{v'} = 0 \tag{0}$$

77
$$-\nabla p' + \nabla \cdot \left[\eta' \left(\nabla \overrightarrow{v'} + \left\{ \nabla \overrightarrow{v'} \right\}^T \right) \right] + Ra_0 T' \overrightarrow{e_r} = 0$$
(0)

78

$$\frac{\partial T'}{\partial t'} + \vec{v'} \nabla T' = \nabla^2 T' + 1 \tag{0}$$

The above parameters with an apostrophe are non-dimensional scaled parameters, where $\vec{v'}$ is velocity, T' is temperature, η' is viscosity, $\{ \}^T$ is vector transpose, $\vec{e_r}$ is the radial unity vector, t' is time, p' is pressure and Ra₀ is the Rayleigh number at the reference viscosity η_0 (here it is the surface viscosity). This Rayleigh number is defined as

$$Ra_0 = \frac{\rho g \alpha H d^3}{\kappa k \eta_0} \tag{0}$$

84 where ρ is the density, g is the acceleration of gravity, κ is the thermal diffusivity, k is the 85 thermal conductivity, H is the volumetric heat production rate, and d is the layer thickness. 86 The following scaling has been used to derive the non-dimensional parameters: $\vec{v'} = \frac{d}{\kappa}\vec{v}$,

87
$$p' = \frac{d^2}{\eta_0 \kappa} p$$
, $\eta' = \frac{\eta}{\eta_0}$, $t' = \frac{\kappa}{d^2} t$ and $T' = \frac{T - T_s}{\rho H d^2 / k}$ with T_s the surface temperature

88

The Rayleigh number together with the varying viscosity acts as a driving term for thermal convection. The viscosity itself is described by the Arrhenius law (Karato and Li, 1992; Karato and Wu, 1993) and here approximated by a linearized Frank-Kamentskii approach (Frank-Kamenetskii, 1969). This approach is based on the constant rheological gradient γ that describes the dependence on T' and - in case of pressure dependence - a pressure dependent part $\Delta \eta_p$

 $\eta' = \exp(-\gamma T' + \ln(\Delta \eta_p)(r_0' - r'))$ (0)

96 where r' is the radius (r'=r/d) and r_0 is the outer radius.

97 The rheological gradient is defined as

98

$$\gamma = -\frac{\partial \ln \eta'}{\partial T'} \tag{0}$$

In the present study, Ra_0 , defined at the surface, and γ have been varied between 1 and 10^6 99 and 0 and 120, respectively, with either $\Delta \eta_p = 1$ (without pressure dependence) or $\Delta \eta_p = 100$ 100 101 (pressure dependent viscosity contrast of 100). While the viscosity contrast due to pressure 102 $\Delta \eta_{v}$ is directly specified, the contrast due to temperature is an output parameter. Both setups, 103 i.e., with or without pressure dependence, took approximately half of all runs. In the 104 following, models with only temperature-dependent viscosity have the symbol TC and models 105 with temperature and a pressure dependence viscosity throughout the shell of two orders of 106 magnitude TPC. The appendix provides a detailed overview on all cases.

All runs were simulated until they reached the quasi-steady state, meaning that surface heat flow and mean velocities oscillate around a constant value for a sufficiently long time. The discrete base is a grid with 32-48 shells (depending on the strength of convection) and laterally divided icosahedra with 5 iterations, leading to 10.240 nodes per shell.

111 The following diagnostic (output) parameters for the present study have been calculated for112 each model:

113 1) The internal temperature T_i , which is defined as the laterally averaged temperature in the 114 convecting mantle. It is best described by the laterally averaged temperature at the depth of 115 the maximal radial velocity with

116

$$T_{i}' = \langle T' \rangle_{lat}(r') \quad \text{with}$$

$$r' = \left\{ r' \in \left[r_{c}', r_{0}' \right] : \left\langle \left| \overrightarrow{v_{r}} \right| \right\rangle_{lat}(r') = \max\left(\left\langle \left| \overrightarrow{v_{r}} \right| \right\rangle_{lat}(r') \right) \right\}$$

$$(0)$$

117 with r_c the inner radius and v_r the radial velocity.

118 2) The viscosity contrast in a convecting system is described by the viscosity of the119 convecting interior

120

 $\Delta \eta = \frac{\eta_0}{\eta_i} \tag{0}$

121

122 where η_i is the viscosity at the interior temperature at T_i . Note that, in contrast to bottom

123 heated convection, the viscosity contrast is an output parameter.

124 3) The internal Rayleigh number, based on the viscosity of the convecting interior

125
$$Ra_{H,i} = \frac{\rho g \alpha H d^5}{\kappa k \eta_i}$$

126

127 4) The structural complexity of convection can be obtained by transforming the temperature 128 field of a certain radius into spherical harmonics. The strength or energy of each degree of 129 convective pattern ℓ , referred to as 'degree' here after, represents the temperature variation 130 corresponding for this specific wavelength on the sphere. The higher the energy in high 131 degrees, the more complex is the convective pattern. This in turn reflects the amount of up-132 and down-wellings in the spherical shell. The resulting power spectrum λ_1 determines the energy of each degree of the convective pattern ℓ at radius r. The sum in radius of these 133 134 power spectra provides a one-dimensional spectrum of degrees that reflects the complexity of 135 convection of the entire spherical shell and is given by:

136

$$\lambda_{\ell} = \sum_{r} \lambda_{r,\ell} \tag{0}$$

(0)

137 The dominant degree, or the degree with the highest energy, ℓ is usually considered to 138 classify the structural regime and is the degree with the maximum energy. However, to obtain 139 a single parameter to describe the structural complexity of the convecting system other than 140 the dominant degree, a weighted degree ω is also calculated to account for multiple 141 occurrences of strong degrees. This method is similar to the robust-localization method 142 (Kertz, 1973; Buttkus, 2000):

143

$$\boldsymbol{\varpi} = \frac{\sum \ell \lambda_{\ell}}{\sum \lambda_{\ell}} \tag{0}$$

5) The thermal boundary layer (TBL) is the region within the convecting system that determines the transition from conductive to convective heat transport. A bottom-heated convection system consists of two such thermal boundary layers; purely internally heated convection as in our case has only the top boundary layer. The details like thickness and depth

148	of this laye	er also determine the heat-flow of the convecting system; the depth of the center of									
149	this region	(δ_0) is directly related to the surface heat flow, expressed as the Nusselt number,									
150	while the temperature difference across the TBL reflects the rheological gradient γ (Grasset										
151	and Parme	ntier, 1998). The following parameters have been obtained for each run. Their									
152	definition i	s illustrated in figure 1.									
153											
154	$\delta_{_0}$	Depth of the center of the TBL									
155	$\delta_{_b}$	Depth of the bottom boundary of the TBL									
156	δ_l	Depth of the upper boundary of the TBL equivalent to the thickness of the stagnant-									
157		lid, determined with the method illustrated in fig. 1b									
158	$\delta_{_{rh}}$	Thickness of the TBL, $\delta_l - \delta_b$									
159	$\delta^{*}_{{}_{rh}}$	Reduced thickness of the TBL, centered around $\delta_0: \delta_{rh}^* = 2(\delta_0 - \delta_b)$									

160 3. **Results**

The convection pattern and its temporal evolution change with increasing internal Rayleigh 161 162 number or viscosity contrast. Two different categories can be distinguished. One category 163 classifies the temporal evolution and differentiates between stationary convection and time-164 dependent / turbulent convection and is called time-dependence of the flow pattern. The 165 central parameter of influence is the Rayleigh number. We further distinguish the spatial properties of the flow such as structure and surface mobility. The spatial properties are 166 167 influenced mainly by the viscosity contrast and the applied rheology. This is called 168 convective regime.

169

170 **Time-dependence of the flow pattern:**

171 To achieve convection for a given viscosity contrast, the internal Rayleigh number must be 172 larger than a critical value. For Rayleigh numbers just above the critical value, convection sets 173 in and reaches a stationary in which the convection pattern stays stationary. With increasing 174 Rayleigh number, time-dependent convection sets in where all measurable output parameters 175 fluctuate over time (e.g., Bottaro et al., 1992; Busse, 1978; Craik, 2000; Hansen and Ebel, 176 1984; Hansen and Ebel, 1988; Hansen and Yuen, 1990; Hansen and Yuen, 1993; Hansen et 177 al., 1990; Hansen et al., 1992a; Hansen et al., 1992b; Krishnamurti, 1970; Tang and Tsang, 178 1997; Travis et al., 1990). If the output parameters are independent of the initial condition and

fluctuate around their temporally averaged value, it is called the quasi-stationary or time-dependent convection.

181 The time dependence of the flow depends mostly on the internal Rayleigh number as it 182 determines the vigor of convection. To classify the time dependence of convection, a criterion 183 is required that is sensible to temporal fluctuations of control variables, i.e., the maximal 184 velocity v_{max} . The (quasi-) stationary part of a simulation was used to calculate the mean 185 maximal velocity $\overline{v}_{\text{max}}$ and finally the M_0 indicator that determines the standard deviation of 186 v_{max} relative to $\overline{v}_{\text{max}}$: $M_o = \sigma \left(\frac{\overline{v}_{\text{max}}}{v_{\text{max}}}\right)$. This is equivalent to the average fluctuation in percent of

187 v_{max} .

Cases with a value of M_0 smaller than 1.5% are considered as stationary. For values of M_0 188 189 larger than 1.5%, the convective pattern is time-dependent. Between the stationary and time-190 dependent flows, we observe some intermediate cases in which the flow pattern is periodic. 191 We included these cases as time-dependent because they have values of M_0 larger than 1.5%. 192 A sharp boundary for the transition from steady state to time-dependent convection could only be identified for stagnant-lid cases (see classification of the stagnant-lid regime below) and is 193 at $Ra_{H,i} \sim 2.10^6$ for TC, similar to observation by Hansen et al. (1990), and $\sim 6.10^5$ for TPC 194 (Figure 2). The boundary for non-stagnant-lid cases is rather wide and between 10^5 and $2 \cdot 10^6$ 195 196 for all cases.

197 **Convection regimes**

198 To distinguish the convection regimes, in particular the transition to the stagnant-lid regime, 199 most studies have used the averaged surface velocity v_{top} (e.g., Davaille and Jaupart, 1993;

200 Hansen and Yuen, 1993; Solomatov, 1995; Tackley, 2000c; Trompert and Hansen, 1998).

201 Two such mobility definitions are $M = \frac{v_{top}}{v_{rms}}$ (Tackley, 2000c) where v_{rms} is the root mean

square velocity and $S = \frac{\delta_{rh}^2 v_{top}}{\kappa d}$ (Solomatov and Moresi, 1997) with δ_{rh} being the thermal boundary layer thickness. Here, we propose an alternative approach based on a well-defined correlation between the Nusselt number and the heat flux in the mobile-lid regime in the form of $\delta_{rh} = Nu^{-1}$ (Solomatov, 1993). Figure 3 shows the distinct behavior between Nu and δ_{rh} for the stagnant and non-stagnant-lid regime. Scaling laws, as those presented in the next section, highlight the differences between those regimes.

208 The correlation of the boundary layer thickness in relation to the Nusselt number can be used for an alternative mobility definition in the form of $M_1 = \delta_{rb} N u$, as it defines a useful 209 210 boundary for the stagnant-lid regime with $M_1 < 0.6$. The advantage of this definition is the 211 absence of any velocity component. Figure 4 shows the three different definitions derived 212 from Solomatov and Moresi (1997) and Tackley (2000c) as well as the just described M_1 for 213 all cases. A comparison of the three definitions indicates that the transition to stagnant-lid 214 convection can be best distinguished with our new mobility definition M_1 . According to this 215 definition, the stagnant-lid regime is observed for a viscosity contrast larger than $1.3 \ 10^4$ 216 within the convecting system for purely temperature-dependent viscosity cases. The relative 217 error on this value is less than 5% because the transition is valid for all relevant parameter 218 values. The transition into the stagnant-lid regime for pressure dependent viscosity starts at a 219 lower viscosity contrast of about 10^3 (with the exception of case #83 that will be discussed 220 below). It is interesting to note that in contrast to the bottom-heated case, our experiments 221 indicate that for an internally heated fluid, the critical viscosity contrast for the stagnant-lid 222 regime is smaller when the pressure dependence of viscosity is accounted for as well (Bunge 223 et al., 1996; 1997). We will demonstrate and discuss below that the transition into the 224 stagnant-lid regime is accompanied and can also be identified with an increase in the 225 dominant degree of convection and a rapid decrease of the ratio of lateral to radial viscosity.

For lower values of viscosity contrast, i.e., lower than the critical value at which stagnant-lid convection occurs, we distinguish three other regimes, i.e., the mobile-lid regime, the sluggish regime and the low-degree regime (Fig. 5). This is different to earlier studies that only distinguish between the mobile and the sluggish regime (Solomatov and Moresi, 1997). In the present classification of the regimes we consider the dominant degree of convection in addition to surface mobility to highlight structural differences as well.

232 Mobile-lid regime: This regime is characterized by a full mobile surface and a dominant 233 degree of the convective pattern larger than two. It includes isoviscous convection and 234 convection with low viscosity contrasts up to 10. The absence of low degree convection in the 235 present isoviscous cases is due to the chosen radius ratio. An increased core-to-surface radius 236 ratio leads to higher minimal degrees (Jarvis, 1993; Jarvis, 1994; Jarvis et al., 1995; 237 Travnikov et al., 2000). An increase in the surface Rayleigh number generally results in an 238 increase of the degree, although no tendency could be observed for increased internal 239 Rayleigh numbers with increasing γ . The velocity increases and the flow become turbulent 240 while maintaining a relatively low dominant degree. Sluggish-lid regime: This regime is 241 characterized by strongly reduced velocity at the surface and a dominant degree which is

242 larger than two. In this study, it has been only observed in a small parameter range of the TPC 243 cases. Due to the relatively low Rayleigh numbers and viscosity contrast, the patterns are 244 comparable with laboratory and theoretical experiments in the plain layer (Busse and Frick, 245 1985; Oliver and Booker, 1983; White, 1988). Low-degree regime: This regime is 246 characterized by a full mobile surface (in few cases close to the transition of the stagnant-lid 247 regime a reduced surface velocity has also been observed) and a dominant low degree of the convective pattern of one or two. In both cases, TC and TPC, starting from isoviscous 248 249 convection, an increase in the viscosity contrast leads to a drop in the dominant degree for 250 $Ra_0 < 10^5$ and increases rather abruptly while passing the stagnant-lid boundary. Within the Ra_{H_i} and $\Delta \eta$ parameter space, this low-degree region has the shape of a trapezoid and 251 252 therefore depends on both parameters, as illustrated in figure 5 with a degree-marker over the 253 observed parameter space.

For TPC (Fig. 5b), one case (#83 in the appendix, also L in figure 6) produces a convective pattern of degree one without a stagnant lid but with a viscosity contrast larger than the critical value of 10^3 . This case indicates that the transition to the stagnant-lid regime is not purely bound by the viscosity contrast, as indicated already by earlier studies (Grasset and Parmentier, 1998; Solomatov and Moresi, 1997). We have therefore considered in Figure 5b a small area of the low-degree regime that is located above the critical viscosity contrast although the exact position is not well constrained due to lack of data.

For all four regimes, a typical convection structure, temperature power-spectra for degrees up to 30 over depth and a velocity profile are presented in figure 6.

263

264 Scaling laws

265 Wavelength scaling

266 The dominant degree of the convective pattern indicates the structural complexity and is 267 directly associated with the number of convective up- and down-wellings. Within the 268 stagnant-lid regime, and only in the stagnant-lid regime, a strong correlation exists between 269 the degree of convective pattern and the internal Rayleigh number (Fig. 7). This was already 270 expected from previous studies for both bottom and internally heated convection (e.g., 271 Ratcliff et al., 1996a; Reese et al., 2005; Stemmer et al., 2006; Zhong et al., 2008) because it 272 was observed that the degree of convective pattern increases with increasing driving forces. 273 Due to the large number of cases in our study, we derive for the first time a scaling law to 274 describe the correlation between the convective pattern and the internal Rayleigh number.

We use the weighted degree ϖ instead of the dominant degree as for the former the correlation between the internal Ra and the convective pattern is stronger. The following fitting formula has been derived:

278
$$\boldsymbol{\varpi} = P(Ra_{S-T}, Ra_{H,i}) \ m \left(\ln(Ra_{H,i}) - \ln(Ra_{S-T}) + \frac{\Delta \boldsymbol{\varpi}_{step}}{m} \right) + \boldsymbol{\varpi}_{min} \tag{0}$$

where Ra_{s-T} is the internal Rayleigh number at which the transition to the time-dependent convection occurs, *m* is the slope in the time-dependent part, $\Delta \sigma_{step}$ is the observed increase at $Ra_{H,i} = Ra_{s-T}$, σ_{min} is a minimal possible degree, and P is the incomplete gamma function (Furman and Zitikis, 2008) to describe the transition from stationary to time-dependent convection with a step function.

The minimal possible degree ϖ_{\min} occurs for the stationary cases, i.e., small Rayleigh 284 285 numbers, and remains constant until the transition to time-dependent convection. This 286 minimal degree has a relatively large error because multiple possible degrees may exist for 287 the same Ra_{H_i} in the stationary convection depending on the initial condition of the temperature field (e.g., Ratcliff et al., 1995; Ratcliff et al., 1996b; Ratcliff et al., 1997; Zhong 288 289 et al., 2008). To account for this uncertainty, an error of plus-minus two degrees is expected for ϖ_{\min} . Note that the minimal degree also depends on the assumed core-to-surface radius 290 291 ratio.

The transition from stationary to time-dependent convection is accompanied by a step $\Delta \sigma_{step}$ towards higher degrees, i.e., the complexity of the flow pattern increases. At this transition, the degree increases linearly with increasing Rayleigh number (Fig. 7a). The fitting parameters for the stagnant-lid regime are summarized in table 1.

For the models with pressure dependent viscosity (TPC), the slope is less steep than for the TC cases resulting in lower degrees of the convective pattern, as predicted by Bunge *et al.* (1996). However, the minimal degree ϖ_{min} for stationary convection is not influenced by the pressure dependence of the viscosity.

As a consequence of the increase in the degree after the transition from stationary to timedependent convection, the observed temporal standard deviation of the velocity (M_o indicator) has to show a significant increase for $\sigma > \sigma_{\min} + \Delta \sigma_{step}$, as demonstrated in figure 7b. This leads to the conclusion that beyond $\sigma \ge 10$, convection is always time-dependent in the stagnant-lid regime.

305

306 Lateral viscosity contrast

307 Figure 8 shows the relation between the lateral to radial viscosity contrast of the convecting 308 system for the purely temperature-dependent cases and the pressure-dependent cases. For 309 non-stagnant-lid cases with purely temperature-dependent viscosity, the lateral viscosity 310 contrast is similar to the radial viscosity contrast $(\Delta \eta_{lat} \sim \Delta \eta)$. In contrast for non-stagnant-311 lid cases with pressure-dependent viscosity, the lateral viscosity contrast is much larger than 312 the radial viscosity contrast and a relation of $\Delta \eta_{lat} \approx 3\Delta \eta^2$ has been identified. For all stagnant-lid cases, $\Delta \eta_{lat}$ remains approximately constant. For TC this value is about 313 30 and for TPC it increases to 130 (Fig. 8). This increase in $\Delta \eta_{lat}$ depends solely on the 314 pressure dependence of the viscosity $\Delta \eta_{p}$. To obtain a relationship between the lateral and 315 316 radial viscosity contrast for temperature and pressure dependent viscosity models, we have considered three additional stagnant-lid cases assuming $\Delta \eta_p = 10^5$ (case #89-91). These 317

- models show a lateral viscosity contrast of $\Delta \eta_{lat} \sim 3.10^3$ and the following linear relationship could be established:
- 320

$$\Delta \eta_{lat} \approx \exp(2.95 + 0.43 \ln \Delta \eta_{n}) \tag{0}$$

321

322 Rheological constant

In the stagnant-lid regime, the convective system is separated into two parts, i.e., the convecting interior and the stiff upper part (the stagnant-lid) that is mainly controlled by conductive heat transport. Part of the convecting interior and just beneath the stagnant lid is the upper thermal boundary layer (TBL). The temperature drop within the TBL can be described by the rheological gradient γ and the constant a_{rh} :

328

$$a_{rh} = \Delta T_{rh} \gamma$$

$$\Delta T_{rh} = T_i - T_L$$
(0)

where $T_L = T(\delta_l)$ is defined as the temperature at the base of the stagnant lid and $T_i = T(\delta_b)$ is the internal temperature at the bottom of the TBL. This relation has been already suggested by Solomatov (1993; 1995), later proven in laboratory experiments for internally heated convection by Davaille and Jaupart (1993) and further confirmed by Grasset and Parmentier (1998) and Reese et al. (2005; 1999) with numerical simulations.

The thickness of the stagnant lid δ_l is not clearly defined and different approaches exist in the literature to constrain its value. Some refer to a non-eroded stagnant lid (Deschamps and Sotin, 2000), where advective forces are low enough to guarantee that material in the lid

337 remains within this region. This has been approximated by a threshold in the radially averaged 338 velocity profile. The thickness of the lid is then equal to the depth where the velocity profile 339 reaches the value of the expected numerical error in the velocity. Even though this is the most 340 plausible definition of a stagnant lid as it really defines a 'stagnant' lid, it is also very volatile 341 because the velocity profile has a rather smooth logarithmic transition towards zero, so small 342 variations in the threshold value have a great impact on the thickness of the lid. Another 343 widely used method consists of exploiting the tangent through the inflexion point of the 344 velocity profile within the TBL (Grasset and Parmentier, 1998), as illustrated in figure 1b.

345 Assuming that a_{rh} is constant, we have benchmarked the different methods of determining 346 the lid thickness to check which method leads to the least standard deviation of a_{rh} . Our 347 results show that the method using the tangent through the inflexion point of the velocity 348 profile is most suitable to obtain the least standard deviation. The average value is $a_{rh} \approx 2.88$ 349 \pm 0.04 (table 2). The discrepancy between our value and previous estimates of 3.1 (Reese et 350 al., 2005), 2 (Davaille and Jaupart, 1993), 2.2 (Davaille and Jaupart, 1994; Grasset and 351 Parmentier, 1998) and 2.4 (Solomatov and Moresi, 2000) is mainly due to the insensitivity of 352 this parameter within the heat-flow scaling law as explained in the next section.

353

354 Nusselt number

The convecting interior below the stagnant lid consists of small lateral and radial viscosity variations (see lateral viscosity scaling above) and can be considered as isoviscous (e.g., Grasset and Parmentier (1998). The Nu-Ra scaling relation for isovisous convection is given by

359

$$Nu = a R a^{\beta} \tag{0}$$

where *a* and β are constants. This relation is only valid for the convecting region, therefore, the Rayleigh number, which is based on the interior viscosity η_i and the internal temperature T_i , need to be rescaled with the effective non-dimensionalized thickness of the convecting interior $(1-\delta_i)$:

364

$$Ra_1 = Ra_0 \left(1 - \delta_l\right)^5 \eta_i^{-1} \tag{0}$$

365 as well as the internal temperature:

366
$$T_1 = \frac{T_i - T_L}{(1 - \delta_l)^2}$$
(0)

For purely internally heated convection, the internal temperature T_i is directly related to the Nusselt number and therefore related to the heat-flow (Reese et al., 1999). The Nusselt number is the ratio of the theoretical non-dimensionalized maximal conductive temperature

370 T_c to the internal temperature

388

$$371 Nu = \frac{T_C}{T_c}$$

372 This relation allows modifying the equation 15 to

$$T_1 = a R a_1^{-\beta}$$

374 The constants a and β have been obtained by fitting the internal temperature to the calculated 375 stagnant-lid cases for both the stationary and the time dependent convection (table 3). Figure 9 shows further the relation between T_1 and Ra_1 for all the stagnant-lid cases with time 376 dependent convection using the constants in table 3. The fitting constants of the time-377 378 dependent part of TC are very similar (less than 3e-3 difference in β) to those observed by Parmentier et al. (1994), who studied isoviscous three-dimensional Cartesian internally heated 379 380 convection with no-slip boundaries. Our results are also close to the results obtained by 381 Grasset and Parmentier (1998) in their 2D study (see table 3). This confirms the assumption 382 that convection beneath the stagnant lid can be treated as isoviscous (Davaille and Jaupart, 383 1993; Grasset and Parmentier, 1998; Reese et al., 2005; Solomatov and Moresi, 1996) and 384 further that the boundary to the stagnant lid is close to a no-slip boundary condition.

To derive a complete scaling law, the non-dimensionalized temperature (scaled with $\frac{\rho H d^2}{k}$) at the base of the lid can be derived from the conductive temperature profile for a volumetric heat source in spherical geometry with r_i as the inner and r_0 as the outer radius:

 $T(r) = \frac{1}{6} \left(-r^2 + r_i^3 \left(\frac{-2}{r} + \frac{2}{r_o} \right) + r_o^2 \right)$ (0)

389 This equation also leads to the maximal conductive temperature $T_c = T(r_i)$ in the spherical 390 shell, which is needed to obtain the Nusselt number. Inverting equation 20 yields the radius 391 for the temperature within the conducting lid that can be exploited to retrieve the lid depth 392 based on a known lid temperature with $\delta_i = d(T_L)$.

393 Besides using the parameter a_{rh} obtained from the tangent method (table 2) and the 394 associated parameters a and β (table 3), the large number of runs allows to fit all three 395 parameters directly. To provide a better understanding of the dependence of all three

(0)

396 parameters, a was varied over a reasonable range between 1 and 10 while the remaining two 397 parameters were fitted. The parameter fit is shown in figure 10a and the standard deviation in 398 figure 10b for time-dependent TC cases. It is interesting to note that for a wide range of 399 parameters the error remains almost constant. While a is in the range between ~ 1 and ~ 4.5 , the resulting standard deviation is very small and does not change significantly. This puts a_{th} 400 401 in a possible range between 0.5 and 7, and β between 0.21 and 0.26, respectively. The values 402 for β are approximately in between the two theoretical values for internally heated 403 convection with free-slip (-0.25) and no-slip (-0.2) boundaries (Schubert et al., 1990a). This is evident from the definition of a_{rh} itself; a higher value yields a more eroded stagnant-lid that 404 405 gives the convection below an interface definition towards a no-slip upper boundary while 406 lower values provide a more free-slip boundary through the shear effects of an increased local 407 radial viscosity contrast. Therefore, it is possible to choose the erosion level with the choice of a_{rh} and of the associated best-fits for a and β . However, the lowest error was found for 408 the time-dependent stagnant-lid regime at a = 3.57, $a_{rh} = 5.64$, $\beta = -0.22$, which is marked by 409 410 the vertical dashed line in figure 10.

411 Summary and conclusion

In the present parameter study, we performed internally heated 3-D mantle convection models in a spherical shell with temperature and pressure dependent viscosity. We have identified and classified four convection regimes, i.e., mobile-lid regime, sluggish regime, low-degree regime, and stagnant-lid regime that develop depending on the viscosity contrast and the internal Rayleigh number. In contrast to earlier work (Moresi and Solomatov, 1995; Solomatov and Moresi, 1997; Tackley, 1993; Tackley, 1996a), we consider also the dominant degree of the convective pattern to highlight structural differences.

419 A significant difference to other publications is the absence of the typical sluggish regime for 420 temperature-dependent viscosity (TC cases) and only a narrow range of the sluggish regime 421 for temperature and pressure-dependent viscosity (TPC cases). The sluggish regime should occur after Christensen (1984b) and Moresi and Solomatov (1995) for a viscosity contrast 422 between 10^2 and 10^4 , and is further confirmed from other studies in 2D and 3D boxed 423 424 simulations with internal heating and/or bottom heating (e.g., Carey and Mollendorf, 1980; 425 Hansen and Yuen, 1993; Hirayama and Takaki, 1993; Ogawa et al., 1990). The absence of the 426 sluggish regime for our TC cases and the small range observed for the TPC cases is mainly a 427 consequence of the full 3D spherical treatment of the convection. This is the first study in a

spherical shell to simulate mantle convection with purely internal heating for a largeparameter range.

430

431 For the transition into the stagnant-lid regime we have proposed an alternative mobility 432 definition in the form of $M_1 = \delta_{rh} N u$, which defines the boundary into the stagnant-lid regime 433 with $M_l < 0.6$. According to this mobility definition the stagnant-lid regime is observed for a 434 viscosity contrast larger than $1.3 \cdot 10^4$ within the convecting system for purely temperature-435 dependent viscosity cases. The transition into the stagnant-lid regime for pressure dependent viscosity starts at a lower viscosity contrast of about 10^3 . An alternative boundary to the 436 437 stagnant-lid regime that depends not only on the viscosity contrast can possibly be obtained if 438 the surface Rayleigh number exceeds the critical Rayleigh number of the convecting system. 439 This has been suggested already by Stengel et al. (1982), Richter et al. (1983) and Deschamps 440 and Sotin (2000). We have also identified one specific case for which the transition occurs 441 indeed for a larger viscosity contrast (figure 5b). A detailed study of this alternative boundary 442 requires, however, much more model runs with high viscosity contrasts and Rayleigh 443 numbers that is beyond the scope of our paper.

444

445 For the stagnant-lid regime, we have derived a scaling law for the Nusselt number in the 446 convecting interior similar to Grasset and Parmentier (1998). We confirm the assumption that 447 the convection below the stagnant-lid can be treated and scaled as an isoviscous fluid. The exponent β in the Nu-Ra scaling is 0.23 and consistent to earlier studies assuming internal 448 449 heating (Parmentier et al., 1994). This study also shows results for the rheological constant 450 that associates the temperature drop in the TBL with the rheological gradient. A complete 451 inversion of the three parameters, that was possible due to the large number of cases, shows 452 the weak dependence of the rheological constant on the two remaining fitting parameters, i.e. 453 a and β . A Nu-Ra scaling law for non-stagnant-lid cases as given by Moresi and Solomatov 454 (1995) could not be established. This is most likely due to the used spherical geometry, which 455 results in a large variation in the degree of the convective pattern. The scaling laws of Moresi 456 and Solomatov (1995) have been constrained from 2D boxed simulations. These models do 457 not show a large variation in the degree of the convective pattern because the degree is limited 458 by the chosen geometry. Our finding suggests that not only the Rayleigh number and the 459 applied rheology account for the cooling behavior of a convecting system but also the degree 460 of the convective pattern. This conclusion is confirmed by the wavelength scaling presented: 461 only for the regime where a relation exists between the input parameters and the degree of

462 convective pattern, i.e., the stagnant-lid regime, a scaling relation for the heat flow can be 463 obtained. For the mobile-lid and the sluggish-lid regime the dominant degree of the 464 convective pattern is unpredictable. Furthermore the influence of the convective pattern on the 465 heat-flow is expected to be greatest if the degree is low. Therefore, small changes within the 466 low-degree regime have a large effect on the heat flow and hence prevent scaling laws for the 467 low-degree regime.

468

The spectral analysis especially for the time-dependent low-degree cases reveals embedded high-degree convection within the low degree. Thus, the 'overlap' of the dominant low degree with the embedded high-degree convection could further explain the lack of simple scaling relations. Furthermore, this specific pattern of the low-degree convection let us suggest that the stagnant-lid regime could also be seen as embedded high-degree convection but within outer "degree-0" convection that transports heat solely by conduction (spherically symmetric flow does not affect radial heat transport and is here a synonym for convection-free).

476

477 For time-dependent cases in the stagnant-lid regime, a relation between the internal Rayleigh 478 number and a structural indicator, the weighted degree of the convective pattern, has been 479 identified: The weighted degree increases linearly with the internal Rayleigh number. This 480 behavior is similar for both models with and without pressure dependence in the viscosity. 481 However, the increase of the degree is weaker for the pressure dependence, i.e., for equivalent 482 internal Rayleigh number the degree is smaller in contrast to cases with purely temperature 483 dependent viscosity. In comparison to isoviscous convection, the same internal Rayleigh 484 number produces in the presence of a stagnant lid flow structures with higher degrees. This 485 seems to be valid for all heating modes (e.g., Reese et al., 2005; Stemmer et al., 2006; Zhong 486 et al., 2008). For stationary convection, degree five seems to be the lowest possible degree 487 beneath the stagnant lid. This limit is not affected by the presence of pressure dependence in 488 the viscosity but is bound to the used core-to-surface radius ratio of 0.55 and is likely to be 489 lower with a smaller core (e.g., McNamara and Zhong, 2005). The reason for the lower limit 490 of degree five in the stagnant-lid regime is due the reduced thickness of the convecting layer 491 and an indirect change in the upper boundary condition. The stagnant lid alters the surface 492 free-slip boundary condition towards a no-slip boundary condition, further hindering low-493 degree convection to develop. Tackley (1993) observed low degrees caused by bottom heated 494 convection in a 3D box and ascribed it to the influence of the boundary condition. He argued 495 that with a free-slip boundary condition the flow would choose a preferably long wavelength

496 in this scenario. The transition of the boundary condition from free-slip in the mobile and 497 low-degree regime to the no-slip boundary condition of the stagnant-lid regime is probably 498 responsible for the observed minimal degree. These results differ from the results of Bunge et 499 al. (1996; 1997) and Harder (2000). These authors have shown that a modest increase in the 500 mantle viscosity with depth has a remarkable effect on the convection pattern, resulting in a 501 long-wavelength structure up to degree-one flow. However, in their models the viscosity was 502 only dependent on pressure and the temperature dependence was neglected, i.e., they did not 503 consider the stagnant-lid regime.

504 The convection pattern described by its dominant degree is an important parameter to 505 constrain convection models of the terrestrial planets as it can be observed for instance by 506 seismic tomography and also indirectly from the structures at the planet's surface. 507 Interestingly, for Earth, Mars and the Moon a low-degree convection pattern seems likely. For 508 the Earth, this has been indicated by seismic tomography models (Grand et al., 1997; Masters 509 et al., 1996). Moreover, the formation of supercontinents on the Earth suggests that an even 510 lower degree flow structure than observed today may have once existed in the past (Gurnis, 511 1988; Zhong and Zhang, 2007). For both Mars and the Moon, even a degree-one mantle flow structure (consisting of one upwelling and one downwelling) may have existed on these 512 513 bodies, resulting in the observed hemispherical crustal dichotomy and the Tharsis rise on 514 Mars (Roberts and Zhong, 2006; Schubert et al., 1990b; Wise et al., 1979; Zhong and Zuber, 515 2001) and the hemispherical distribution of Mare basalts on the Moon (Stegman et al., 2003; 516 Zhong et al., 2000). Considering that the Moon and Mars are in the stagnant-lid regime, the 517 convection patterns underneath the stagnant lid obtained with the present models seem to be 518 inconsistent with the planetary observations.

519 In case of the Moon, the core-to-surface radius ratio of the convecting mantle of around 0.2 520 (Wieczorek et al., 2006) may be sufficiently small to obtain a low-degree convection to 521 explain the hemispherical distribution of Mare basalts on the Moon. For Mars, with a radius 522 ratio of about 0.5 (e.g., Sohl and Spohn, 1997), the situation is different. According to our 523 results, the convection pattern below the stagnant lid is larger than about degree five, even for 524 a strong pressure dependence of the viscosity. Thus, possible low-degree convection requires 525 additional constraints on the parameters influencing the convection pattern. For instance a 526 stratified viscosity profile can result in a low degree convection structure (Keller and Tackley, 527 2009; Roberts and Zhong, 2006; Zhong and Zuber, 2001), but phase transitions in the Martian 528 mantle may cause a low degree pattern as well (Harder, 1998; Harder and Christensen, 1996; 529 Weinstein, 1995).

530 4. **Appendix**

The table A1 summarizes the most important measurements conducted with each case. Allvalues are or base on non-dimensionalized measurements. The symbols have the following

- 533 meaning:
- *Case* A number to identify the case following two characters to classify the case.
- 535 First character: Time-dependency, S = stationary, T = time-dependent
- 536 Second character: Convective regime: S = stagnant-lid, L = low-degree, M = mobile,
- 537 U = sluggish
- $Ra_{H,0}$ The surface Rayleigh number (input parameter)
- γ The rheological gradient (input parameter)
- $\Delta \eta_P$ The pressure (or depth) dependent viscosity variation (input parameter)
- 541 Nu The Nusselt number
- T_i The temperature in the convecting interior
- $\Delta \eta$ The viscosity contrast
- $Ra_{H,i}$ The internal Rayleigh number
- $|Ra_{H}|$ The volume-averaged Rayleigh number
- |T| The volume-averaged temperature
- v_{rms} The root-mean-square velocity
- λ The dominant (with the maximal power) degree
- ω The power-weighted degree
- δ_l The lid-thickness in percent of the shell-thickness
- a_{rh} The rheological constant
- M_0 Standard-deviation of the maximal velocity
- M_1 Stagnant-lid indicator as described in section 3
- $v_{r,\min}$ The highest downwelling velocity
- $v_{r,\text{max}}$ The highest upwelling velocity
- $T_{r,\min}$ The lowest temperature at mid-radius
- $T_{r,\text{max}}$ The highest temperature at mid-radius
- $\sigma(\delta_l)$ The spatial standard-deviation of the lid-thickness

- 559 $\Delta \eta_{lat}$ The maximal lateral viscosity contrast
- 560
- 561 Acknowledgments. We wish to thank the editor, Mark Jellinek, Dave Stegman, and one
- anonymous reviewer for their fruitful comments. They helped to improve significantly the
- 563 manuscript. This research has been supported by the Helmholtz Association through the
- Acceleration 564 research alliance "Planetary Evolution and Life".

Figure 1.a) Typical temperature and heat-flow profile (case #3 with $Ra_{H,0}=1$ and $\gamma=60$,). The 565 566 dashed line shows the theoretical conductive heat flow while the dotted black line shows the 567 convective heat flow. The straight black and red lines are the calculated heat flow and 568 temperature profiles, respectively. The upper two horizontal lines mark the two described 569 different lid measures: "S. Lid V 2%" marks the boundary where the horizontally averaged 570 velocity reached more than 2% of the maximal velocity. The "S. Lid V curve" marks the depth 571 at which the tangent through the inflexion point of the velocity profile crosses zero. The latter 572 has been identified to represent best the depth of the stagnant lid, as illustrated in 1b and 573 discussed in the rheological constant section. The orange dotted line ("Equal HF") indicates 574 where the conductive heat-flow equals the convective heat-flow. The bottom dotted line (T_i) 575 indicates the depth at which the thermal boundary layer reaches the internal temperature. b) 576 The method of the tangent through the inflexion point applied to the velocity profile to 577 determine the stagnant-lid thickness. Shown are the horizontal average velocity profile (solid 578 black line), the gradient of the velocity profile (solid orange line), and the tangent of through 579 the inflexion point (solid green line). The vertical red dashed line indicates the position where 580 the velocity gradient is zero and the red solid line indicates the thickness of the stagnant lid.

581

Fig. 2 The indicator for the time-dependence of convection, M_0 , as a function of the viscosity 582 583 contrast and the internal Rayleigh number $Ra_{H,i}$. M_0 is symbolized as an integer for all cases 584 of TC (left) and TPC (right). Values of M_0 smaller than two indicate stationary convection 585 and $M_0 \ge 2$ time-dependent convection. The connected lines correspond to constant values 586 of $Ra_{H,0}$ in the order as they appear in the appendix table (top to bottom equals lowest to 587 highest $Ra_{H,0}$) the horizontal dashed lines indicate the boundary to the stagnant-lid regime 588 and the vertical lines the boundary to time-dependent convection in the stagnant regime. For 589 the non stagnant-lid cases the transition from stationary to time-dependent convection 590 varies over a broader range of $Ra_{H,i}$. The viscosity contrast is the total measured contrast.

591 Fig. 3 The boundary layer thickness δ_{rh} as a function of the Nusselt number for all cases of 592 TC. Crosses represent non stagnant-lid cases and diamonds stagnant-lid cases. The dashed 593 line follows $\delta_{rh} = Nu^{-1}$.

594

595 Fig. 4: Three different mobility definitions of convection are shown as function of the viscosity 596 contrast; left: M, (Tackley, 1996a), middle: S (Solomatov and Moresi, 1997), and right M₁ 597 (definition derived in this study). Each figure shows all cases; crosses for purely temperature 598 dependent viscosity (TC) and diamonds for temperature and pressure dependent viscosity 599 (TPC). The horizontal line indicates the threshold of M1 equal to 0.6. Below this threshold we 600 find cases in the stagnant-lid regime and above the cases in the non stagnant-lid regimes. 601 The vertical lines indicate the required viscosity contrast above which the stagnant-lid regime 602 is present; dashed line for TPC and dotted line for TC. The viscosity contrast is the total 603 measured radial contrast.

604

Fig. 5 Position of the different convective regimes in the $\mathbb{Ra}_{H,i}$ and $\Delta \eta$ parameter space for cases of a) TC and b) TPC. M indicates the mobile regime, L the low-degree regime, S the stagnant-lid regime and U the sluggish regime. Crosses mark degree-one cases, diamonds degree-two and stars higher degrees. The viscosity contrast is the total measured radial contrast.

610

Fig. 6 Typical cases of the four distinguished convective regimes. From top to bottom: the mobile regime M (case #88), the sluggish regime U (case #70), the low-degree regime L (case #83) and the stagnant-lid regime S (case #75), respectively. From left to right: a) the velocity profile, where the green lines represent the radial, red lines the lateral velocity components, and the black line the norm of the velocity; b) a map projection of the temperature field of the middle shell; c) the temperature power-spectra up to degree 30 over depth; d) an halfslice iso-surface (top and bottom) or a residual iso-surface depending on what representation

618 highlights best structural differences.

619

Fig. 7 a) Correlation of the internal Rayleigh number to the weighted degree in the stagnantlid regime and the associated fitting line. ,Crosses represent cases of TC and diamonds of TPC.; b) Correlation of the weighted degree to the standard deviation of the maximal velocity $(M_0 \text{ indicator})$ in the stagnant-lid regime. Lines connect similar surface Rayleigh numbers and

624 the numbers at the cases show the rheological input parameter γ .

625

Fig. 8 The lateral viscosity contrast is shown as function of the radial viscosity contrast for all cases of TC (left) and TPC (right). The colored lines connect models with the same surface Rayleigh number. The numbers indicate the dominant degree of the convective pattern. The vertical dashed lines mark the transition into the stagnant-lid regime and the diagonal lines mark an one-to-one relation between lateral and radial viscosity contrast.

Fig. 9 The isoviscous temperature (equation 17) is shown as a function of the quasi-isoviscous Rayleigh number Ra_1 (equation 16) for all time-dependent cases in the stagnant-lid regime. Crosses represent cases of TC and diamonds TPC. The dotted and dashed line shows the

- 634 *correlation function from the fit.*
- Fig. 10. a) The best fit of a, β and a_{rh} for all time-dependent cases of TC. The red line shows
- 636 the values of β and the black line of a_{rh} . b) The standard deviation of the corresponding
- 637 best fit. The dashed vertical lines indicates the position with the smallest error and the
- 638 *horizontal lines the corresponding value of* β (red dashed line) and a_{rh} (black dashed line).

639 REFERENCES FROM TABLE2 (just inserted here for EndNote to appear in the bib)

Reese et al. (2005) Davaille and Jaupart (1993) Davaille and Jaupart (1994) Grasset and Parmentier (1998) Solomatov and Moresi (2000)

- 640
- 641

642	
643	Bottaro, A., Metzener, P. and Matalon, M., 1992. Onset and two-dimensional patterns of
644	convection with strongly temperature-dependent viscosity. Physics of Fluids A, 4(4):
645	655-663.
646	Bunge, H.P., Richards, M.A. and Baumgardner, J.R., 1996. Effect of depth-dependent
647	viscosity on the planform of mantle convection. Nature, 379(6564): 436-438.
648	Bunge, H.P., Richards, M.A. and Baumgardner, J.R., 1997. A sensitivity study of three-
649 650	dimensional spherical mantle convection at 108 Rayleign number. Effects of depth-
651	Geophysical Research B: Solid Earth 102(6): 11991-12007
652	Busse FH 1978 Non-linear properties of thermal convection Reports on Progress in
653	Physics, 41(12): 1929-1967.
654	Busse, F.H. and Frick, H., 1985. SQUARE-PATTERN CONVECTION IN FLUIDS WITH
655	STRONGLY TEMPERATURE-DEPENDENT VISCOSITY. Journal of Fluid
656	Mechanics, 150: 451-465.
657	Carey, V.P. and Mollendorf, J.C., 1980. Variable viscosity effects in several natural
658	convection flows. International Journal of Heat and Mass Transfer, 23(1): 95-109.
659	Christensen, U., 1983. Convection in a variable-viscosity fluid: Newtonian versus power- law
660	rheology. Earth & Planetary Science Letters, 64(1): 153-162.
661	Christensen, U., 1984a. Convection with pressure- and temperature-dependent non-
662	Newtonian rheology. Geophysical Journal-Royal Astronomical Society, 77(2): 343-
663	384.
664	Christensen, U. and Harder, H., 1991. 3D convection with variable viscosity. Geophys. J. Int.,
665	104: 213-226.
666	Christensen, U.R., 1984b. Heat transport by variable viscosity convection and implications for
667	the Earth's thermal evolution. Physics of the Earth & Planetary Interiors, 35(4): 264-
660	Craik A D.D. 2000 Transitions between roll patterns in thermal convection Eluid Dynamics
670	Research 26(5): 305-323
671	Davaille A and Jaupart C 1993 Transient high Rayleigh number thermal convection with
672	large viscosity variations. Journal of Fluid Mechanics. 253: 141-166
673	Davaille, A. and Jaupart, C., 1994. Onset of thermal convection in fluids with temperature-
674	dependent viscosity: application to the oceanic mantle. Journal of Geophysical
675	Research, 99(B10).
676	Deschamps, F. and Sotin, C., 2000. Inversion of two-dimensional numerical convection
677	experiments for a fluid with a strongly temperature-dependent viscosity. Geophysical
678	Journal International, 143(1): 204-218.
679	Frank-Kamenetskii, D.A., 1969. Diffusion and heat transfer in chemical kinetics. Plenum
680	Press, New York.
681	Furman, E. and Zitikis, R., 2008. A monotonicity property of the composition of regularized
682	and inverted-regularized gamma functions with applications. Journal of Mathematical
683	Analysis and Applications, 348(2): 971-976.
684 687	Giannandrea, E. and Christensen, U., 1993. Variable viscosity convection experiments with a
685	stress-free upper boundary and implications for the heat transport in the Earth's
080 697	mantie. Physics of the Earth and Planetary Interfors, 78(1-2): 139-152.
007 688	Shano, S.F., Van Der Hist, K.D. and Withyanitoro, S., 1997. Global seisinic tomography: A snapshot of convection in the earth GSA Today $7(4)$
680	Shapshot of convection in the call. USA 100ay, $7(4)$. Grasset O and Parmentier F.M. 1008 Thermal convection in a volumetrically heated
690	infinite Prandtl number fluid with strongly temperature_dependent viscosity
070	minute France number num subligity temperature-dependent viscosity.

691	Implications for planetary thermal evolution. Journal of Geophysical Research B:
692	Solid Earth, 103(8): 18171-18181.
693	Gurnis, M., 1988. Large-scale mantle convection and the aggregation and dispersal of
694	supercontinents. Nature, 332(6166): 695-699.
695	Hansen, U. and Ebel, A., 1984. Numerical and dynamical stability of convection cells in the
696	Rayleigh number range 103-8.105. Annales Geophysicae, 2(3): 291-302.
697	Hansen, U. and Ebel, A., 1988. Time-dependent thermal convection - a possible explanation
698	for a multiscale flow in the Earth's mantle. Geophysical Journal, 94(2): 181-191.
699	Hansen, U. and Yuen, D.A., 1990. Heat transport in strongly chaotic thermal convection,
700	American Society of Mechanical Engineers, Heat Transfer Division, (Publication)
701	HTD, pp. 43-46.
702	Hansen, U. and Yuen, D.A., 1993. High Rayleigh number regime of temperature-dependent
703	viscosity convection and the Earth's early thermal history. Geophysical Research
704	Letters, 20(20): 2191-2194.
705	Hansen, U., Yuen, D.A. and Kroening, S.E., 1990, Transition to hard turbulence in thermal
706	convection at infinite Prandtl number. Physics of Fluids A. 2(12): 2157-2163.
707	Hansen, U., Yuen, D.A. and Kroening, S.E., 1992a. Mass and heat transport in strongly time-
708	dependent thermal convection at infinite Prandtl number. Geophys. Astro. Fluid, 63:
709	67-89.
710	Hansen, U., Yuen, D.A. and Maleysky, A.V., 1992b, Comparison of steady-state and strongly
711	chaotic thermal convection at high Rayleigh number. Physical Review A, 46(8): 4742-
712	4754
713	Harder, H., 1998. Phase transitions and the three-dimensional planform of thermal convection
714	in the Martian mantle. Journal of Geophysical Research E: Planets, 103(E7): 16775-
715	16797.
716	Harder, H., 2000. Mantle convection and the dynamic geoid of Mars. Geophysical Research
717	Letters. 27(3): 301-304.
718	Harder, H. and Christensen, U.R., 1996. A one-plume model of martian mantle convection.
719	Nature, 380(6574): 507-509.
720	Hiravama, O. and Takaki, R., 1993. Thermal convection of a fluid with temperature-
721	dependent viscosity. Fluid Dynamics Research, 12(1): 35-47.
722	Honda, S., 1996. Local Rayleigh and Nusselt numbers for cartesian convection with
723	temperature-dependent viscosity. Geophysical Research Letters, 23(18): 2445-2448.
724	Hüttig, C. and Stemmer, K., 2008a. Finite volume discretization for dynamic viscosities on
725	Voronoj grids. Physics of the Earth and Planetary Interiors, 171(1-4); 137-146.
726	Hüttig, C. and Stemmer, K., 2008b. The spiral grid: A new approach to discretize the sphere
727	and its application to mantle convection. Geochemistry Geophysics Geosystems, 9:
728	13.
729	Jarvis, G.T., 1993. Effects of curvature on two-dimensional models of mantle convection:
730	cylindrical polar coordinates. Journal of Geophysical Research, 98(B3): 4477-4485.
731	Jarvis, G.T., 1994. The unifying role of aspect ratio in cylindrical models of mantle
732	convection with varying degrees of curvature. Geophysical Journal International.
733	117(2): 419-426.
734	Jarvis, G.T., Glatzmaier, G.A. and Vangelov, V.I., 1995. Effects of curvature, aspect ratio and
735	plan form in two- and three- dimensional spherical models of thermal convection.
736	Geophysical & Astrophysical Fluid Dynamics, 79(1-4): 147-171.
737	Karato, S.I. and Li, P., 1992. Diffusion creep in perovskite: Implications for the rheology of
738	the lower mantle. Science, 255(5049): 1238-1240.
739	Karato, S.I. and Wu, P., 1993. Rheology of the upper mantle: A synthesis. Science.
740	260(5109): 771-778.

741	Keller, T. and Tackley, P.J., 2009. Towards self-consistent modeling of the martian
742	dichotomy: The influence of one-ridge convection on crustal thickness distribution.
743	Icarus., 202(2): 429.
744	Krishnamurti, R., 1970. On the transition to turbulent convection, Part 2: The transition to
745	time-dependent flow. J. Fluid Mech., 42: 309-320.
746	Masters, G., Johnson, S., Laske, G. and Bolton, H., 1996. A shear-velocity model of the
747	mantle. Philosophical Transactions of the Royal Society A: Mathematical, Physical
748	and Engineering Sciences, 354(1711): 1385-1411.
749	McNamara, A.K. and Zhong, S., 2005. Degree-one mantle convection: Dependence on
750	internal heating and temperature-dependent rheology. Geophysical Research Letters.
751	32(1): 1-5.
752	Moresi, L.N. and Solomatov, V.S., 1995, Numerical investigation of 2D convection with
753	extremely large viscosity variations. Physics of Fluids, 7(9): 2154-2162.
754	Morris, S. and Canright, D., 1984. A boundary-layer analysis of Benard convection in a fluid
755	of strongly temperature-dependent viscosity. Physics of the Earth and Planetary
756	Interiors, 36(3-4): 355-373.
757	Ogawa, M., Schubert, G. and Zebib, A., 1990. Numerical simulations of 3-D thermal
758	convection of a strongly temperature-dependent viscosity fluid. American Society of
759	Mechanical Engineers, Heat Transfer Division, (Publication) HTD, pp. 1-5.
760	Oliver, D.S. and Booker, J.R., 1983, PLANFORM OF CONVECTION WITH STRONGLY
761	TEMPERATURE-DEPENDENT VISCOSITY, Geophysical and Astrophysical Fluid
762	Dynamics. 27(1-2): 73-85.
763	Parmentier, E.M., Sotin, C. and Travis, B.J., 1994, Turbulent 3-D thermal convection in an
764	infinite Prandtl number, volumetrically heated fluid: implications for mantle
765	dynamics, Geophysical Journal International, 116(2): 241-251.
766	Ratcliff, J.T., Schubert, G. and Zebib, A., 1995. Three-dimensional variable viscosity
767	convection of an infinite Prandtl number Boussinesq fluid in a spherical shell.
768	Geophysical Research Letters, 22(16): 2227-2230.
769	Ratcliff, J.T., Schubert, G. and Zebib, A., 1996a. Effects of temperature-dependent viscosity
770	on thermal convection in a spherical shell. Physica D: Nonlinear Phenomena, 97(1-3):
771	242-252.
772	Ratcliff, J.T., Schubert, G. and Zebib, A., 1996b. Steady tetrahedral and cubic patterns of
773	spherical shell convection with temperature-dependent viscosity. Journal of
774	Geophysical Research B: Solid Earth, 101(B11): 25473-25484.
775	Ratcliff, J.T., Tackley, P.J., Schubert, G. and Zebib, A., 1997. Transitions in thermal
776	convection with strongly variable viscosity. Physics of the Earth and Planetary
777	Interiors, 102(3-4): 201-212.
778	Reese, C.C., Solomatov, V.S. and Baumgardner, J.R., 2005. Scaling laws for time-dependent
779	stagnant lid convection in a spherical shell. Physics of the Earth and Planetary
780	Interiors, 149(3-4): 361-370.
781	Reese, C.C., Solomatov, V.S., Baumgardner, J.R. and Yang, W.S., 1999. Stagnant lid
782	convection in a spherical shell. Physics of the Earth and Planetary Interiors, 116(1-4):
783	1-7.
784	Richter, F.M., 1978. Experiments on the stability of convection rolls in fluids whose viscosity
785	depends on temperature. J Fluid Mech, 89(pt 3): 553-560.
786	Richter, F.M., Daly, S.F. and Nataf, H.C., 1982. A parameterized model for the evolution of
787	isotopic heterogeneities in a convecting system. Earth and Planetary Science Letters,
788	60(2): 178-194.
789	Richter, F.M., Nataf, H.C. and Daly, S.F., 1983. HEAT-TRANSFER AND
790	HORIZONTALLY AVERAGED TEMPERATURE OF CONVECTION WITH

791	LARGE VISCOSITY VARIATIONS. Journal of Fluid Mechanics, 129(APR): 173-
792	192.
793 704	Roberts, J.H. and Zhong, S., 2006. Degree-1 convection in the Martian mantle and the origin
794 705	Schubert, C. Demonici, D. and Cleterusian, C.A. 1000a, Martle demonica in Mars and
795 707	Schubert, G., Bercovici, D. and Giatzmaier, G.A., 1990a. Mantie dynamics in Mars and
/96	Venus: influence of an immobile lithosphere on three-dimensional mantle convection.
797	Journal of Geophysical Research, 95(B9).
798	Schubert, G., United States. National, A. and Space, A., 1990b. Origin and thermal evolution
799	of Mars. [NASA contractor report], NASA-CR-188095. National Aeronautics and
800	Space Administration], Washington, D.C.
801	Sohl, F. and Spohn, T., 1997. The interior structure of Mars: Implications from SNC
802	meteorites. Journal of Geophysical Research E: Planets, 102(E1): 1613-1635.
803	Solomatov, V.S., 1993. Parameterization of temperature- and stress-dependent viscosity
804	convection and the thermal evolution of Venus. Flow and creep in the solar system:
805	observations, modeling and theory: 131-145.
806	Solomatov, V.S., 1995. Scaling of temperature- and stress-dependent viscosity convection.
807	Physics of Fluids, 7(2): 266-274.
808	Solomatov, V.S. and Moresi, LN., 1996. Stagnant lid convection on Venus. Journal of
809	Geophysical Research E: Planets, 101(E2): 4737-4753.
810	Solomatov, V.S. and Moresi, LN., 2000. Scaling of time-dependent stagnant lid convection:
811	Application to small-scale convection on the Earth and other terrestrial planets. J.
812	Geophys. Res., 105: 21795-21818.
813	Solomatov, V.S. and Moresi, L.N., 1997. Three regimes of mantle convection with non-
814	Newtonian viscosity and stagnant lid convection on the terrestrial planets. Geophysical
815	Research Letters, 24(15): 1907-1910.
816	Stegman, D.R., Jellinek, A.M., Zatman, S.A., Baumgardner, J.R. and Richards, M.A., 2003.
817	An early lunar core dynamo driven by thermochemical mantle convection. Nature,
818	421(6919): 143-146.
819	Stemmer, K., Harder, H. and Hansen, U., 2006. A new method to simulate convection with
820	strongly temperature- and pressure-dependent viscosity in a spherical shell:
821	Applications to the Earth's mantle. Physics of the Earth and Planetary Interiors, 157(3-
822	4): 223-249.
823	Stengel, K.C., Oliver, D.S. and Booker, J.R., 1982. ONSET OF CONVECTION IN A
824	VARIABLE-VISCOSITY FLUID. Journal of Fluid Mechanics, 120(JUL): 411-431.
825	Tackley, P.J., 1993. Effects of strongly temperature-dependent viscosity on time- dependent,
826	three-dimensional models of mantle convection. Geophysical Research Letters,
827	20(20): 2187-2190.
828	Tackley, P.J., 1996a. Effects of strongly variable viscosity on three-dimensional compressible
829	convection in planetary mantles. Journal of Geophysical Research B: Solid Earth,
830	101(2): 3311-3332.
831	Tackley, P.J., 1996b. On the ability of phase transitions and viscosity layering to induce long
832	wavelength heterogeneity in the mantle. Geophysical Research Letters, 23(15): 1985-
833	1988.
834	Tackley, P.J., 2000a. Mantle convection and plate tectonics: Toward an integrated physical
835	and chemical theory. Science, 288(5473): 2002-2007.
836	Tackley, P.J., 2000b. Self-consistent generation of tectonic plates in time-dependent, three-
837	dimensional mantle convection simulations, 1. Pseudoplastic yielding. Geochem.
838	Geophys. Geosyst., 1(8): doi:10.1029/2000GC000036.
839	Tackley, P.J., 2000c. Self-consistent generation of tectonic plates in time-dependent, three-
840	dimensional mantle convection simulations, 2. Strain weakening and asthenosphere.
841	Geochem. Geophys. Geosyst., 1(8): doi:10.1029/2000GC000043.

842 843	Tang, L.Q. and Tsang, T.T.H., 1997. Temporal, spatial and thermal features of 3-D Rayleigh- Be ² nard convection by a least squares finite element method. Computer Methods in
844	Applied Mechanics and Engineering 140(3-4): 201-219
845	Travis B Olson P and Schubert G 1990 Transition from two-dimensional to three-
846	dimensional planforms in infinite-Prandtl-number thermal convection Journal of Fluid
847	Mechanics 216: 71-91
848	Travnikov V Brasch W and Egbers C 2000 The natural convection in a spherical shell
849	ZAMM Zeitschrift für Angewandte Mathematik und Mechanik. 80(4 SUPPL, 3).
850	Trompert, R.A. and Hansen, U., 1998. On the Rayleigh number dependence of convection
851	with a strongly temperature-dependent viscosity. Physics of Fluids, 10(2): 351-360.
852	Weinstein, S.A., 1995. The effect of a deep mantle endothermic phase change on the structure
853	of thermal convection in silicate planets. Journal of Geophysical Research, 100(E6).
854	White, D.B., 1988. THE PLANFORMS AND ONSET OF CONVECTION WITH A
855	TEMPERATURE-DEPENDENT VISCOSITY. Journal of Fluid Mechanics, 191:
856	247-286.
857	Wieczorek, M.A. et al., 2006. The Constitution and Structure of the Lunar Interior. Reviews
858	in Mineralogy and Geochemistry, 60(1): 221-364.
859	Wise, D.U., Golombek, M.P. and McGill, G.E., 1979. Tharsis province of Mars: Geologic
860	sequence, geometry, and a deformation mechanism. Icarus, 38(3): 456-472.
861	Yoshida, M. and Kageyama, A., 2006. Low-degree mantle convection with strongly
862	temperature- and depth-dependent viscosity in a three-dimensional spherical shell.
863	Journal of Geophysical Research B: Solid Earth, 111(3).
864	Zebib, A., 1993. Linear and weakly nonlinear variable viscosity convection in spherical
865	shells. Theoretical and Computational Fluid Dynamics, 4(5): 241-253.
866	Zhong, J.Q. and Zhang, J., 2007. Modeling the dynamics of a free boundary on turbulent
867	thermal convection. Physical Review E - Statistical, Nonlinear, and Soft Matter
868	Physics, 76(1).
869	Zhong, S., Parmentier, E.M. and Zuber, M.T., 2000. A dynamic origin for the global
870	asymmetry of lunar mare basalts. Earth and Planetary Science Letters, 177(3-4): 131-
871	
872	Zhong, S. and Zuber, M.T., 2001. Degree-1 mantle convection and the crustal dichotomy on
873	Mars. Earth and Planetary Science Letters, 189(1-2): 75-84.
8/4	Zhong, S.J., McNamara, A., Tan, E., Moresi, L. and Gurnis, M., 2008. A benchmark study on
8/5	mantle convection in a 3-D spherical shell using Citcoms. Geochemistry Geophysics
8/0	Ceosystems, 9. Zhu T and Ears D 2005. The netterns of high degree thermal free convection and its
8// 070	Zhu, T. and Feng, K., 2005. The patterns of high-degree thermal free convection and its
070 870	reatures in a spherical shell. Acta Selsinologica Sinica, 27(1): 11-24.
880	
000	





Figure 2





Figure 4











A A













TC Reese <i>et al.</i> (2005) Davaille and Jaupart (1993) Davaille and Jaupart (1994) Grasset and Parmentier (1998) Solomatov and Moresi (2000)	convection 3.14 / 0.33 (5)	convection 2.88 / 0.04 (24) 3.1 / 0.1 2 2.2 2.2
TC Reese <i>et al.</i> (2005) Davaille and Jaupart (1993) Davaille and Jaupart (1994) Grasset and Parmentier (1998) Solomatov and Moresi (2000)	3.14 / 0.33 (5)	2.88 / 0.04 (24) 3.1 / 0.1 2 2.2 2.2
Reese <i>et al.</i> (2005) Davaille and Jaupart (1993) Davaille and Jaupart (1994) Grasset and Parmentier (1998) Solomatov and Moresi (2000)		3.1/0.1 2 2.2 2.2
Davaille and Jaupart (1993) Davaille and Jaupart (1994) Grasset and Parmentier (1998) Solomatov and Moresi (2000) TPC		2 2.2 2.2
Davaille and Jaupart (1994) Grasset and Parmentier (1998) Solomatov and Moresi (2000) TPC		2.2 2.2
Grasset and Parmentier (1998) Solomatov and Moresi (2000)		2.2
Solomatov and Moresi (2000)		
ТРС		2.4
	6.46 / 2.44 (3)	3.71 / 0.16 (17)

Case	$Ra_{H,0}$	γ	$\Delta \eta_P$	Nu	T _i	$\Delta\eta$	Ra _{H,i}	$ Ra_H $	 <i>T</i>	v _{rms}	λ	ω	δ_l	a _{rh}	M ₀	<i>M</i> ₁	$v_{r,min}$	$v_{r,max}$	T _{r,min}	T _{r,max}	$\sigma(\delta_l)$	$\Delta \eta_{lat}$
1 SS	1	40	1	1,115	0,314	2,74E+05	2,7E+05	8,1E+04	0,185	6,5	5	5,3	44,3%	3,49	0,0%	0,50	-0,7	3,3	0,230	0,296	4,05%	2,0E+01
2 SS	1	50	1	1,232	0,284	1,19E+06	1,2E+06	4,6E+05	0,179	18,1	4	5,5	44,1%	2,90	0,3%	0,34	-6,6	11,7	0,211	0,281	1,58%	3,9E+01
3 TS	1	60	1	1,381	0,253	3,71E+06	3,7E+06	1,7E+06	0,170	31,1	12	13,0	39,3%	2,85	2,7%	0,29	-36,6	50,9	0,210	0,259	1,80%	2,7E+01
4 TS	1	70	1	1,530	0,229	8,53E+06	8,5E+06	4,3E+06	0,160	46,8	17	16,9	35,2%	2,87	3,3%	0,27	-103,5	104,9	0,196	0,236	1,52%	3,5E+01
5 TS	1	80	1	1,672	0,209	1,77E+07	1,8E+07	9,7E+06	0,152	65,4	19	20,5	31,8%	2,96	1,4%	0,25	-212,6	139,2	0,188	0,217	1,33%	2,8E+01
6 TS	1	90	1	1,810	0,193	3,41E+07	3,4E+07	2,0E+07	0,144	85,7	25	23,4	29,3%	2,92	1,9%	0,23	-354,8	209,1	0,176	0,199	1,26%	3,9E+01
7 TS	1	100	1	1,963	0,178	5,23E+07	5,2E+07	3,3E+07	0,135	97,7	26	24,6	27,9%	2,95	4,7%	0,23	-403,7	284,2	0,165	0,182	1,22%	5,6E+01
8 SS	3	40	1	1,205	0,291	1,05E+05	3,1E+05	1,2E+05	0,181	10,1	4	5,0	42,1%	2,90	0,3%	0,41	-2,2	6,7	0,213	0,291	2,92%	2,7E+01
9 TS	3	50	1	1,311	0,267	5,61E+05	1,7E+06	7,1E+05	0,174	22,8	6	8,7	40,3%	2,85	1,3%	0,35	-17,0	23,9	0,209	0,271	1,72%	2,3E+01
10 TS	3	60	1	1,473	0,238	1,45E+06	4,4E+06	2,1E+06	0,164	39,5	14	15,0	35,8%	2,81	4,2%	0,30	-68,4	71,6	0,202	0,244	1,80%	2,1E+01
11 TS	3	70	1	1,627	0,215	3,31E+06	9,9E+06	5,2E+06	0,154	60,3	18	18,4	32,1%	2,92	4,0%	0,27	-175,8	136,8	0,189	0,224	1,56%	2,6E+01
12 TS	3	80	1	1,776	0,197	6,65E+06	2,0E+07	1,1E+07	0,146	88,7	21	22,0	29,5%	2,86	5,9%	0,25	-337,2	210,6	0,178	0,203	1,35%	2,5E+01
13 TS	3	90	1	1,921	0,182	1,26E+07	3,8E+07	2,3E+07	0,139	102,9	25	25,0	27,0%	2,91	4,0%	0,24	-463,1	239,8	0,168	0,187	1,26%	3,5E+01
14 TS	3	100	1	2,062	0,170	2,24E+07	6,7E+07	4,3E+07	0,132	132,6	29	27,5	25,3%	2,87	4,8%	0,23	-675,0	345,0	0,156	0,173	1,29%	5,4E+01
15 TS	3	110	1	2,208	0,159	3,56E+07	1,1E+08	7,2E+07	0,126	162,8	29	29,0	23,5%	2,90	4,4%	0,23	-825,8	428,8	0,146	0,163	1,26%	7,6E+01
16 TS	3	120	1	2,351	0,149	5,49E+07	1,6E+08	1,1E+08	0,120	199,4	30	30,0	22,0%	2,97	5,9%	0,22	-917,6	564,0	0,140	0,152	1,06%	9,7E+01
17 SS	10	30	1	1,106	0,316	1,29E+04	1,3E+05	4,0E+04	0,185	4,8	4	4,6	37,7%	3,52	0,0%	0,68	-0,9	5,9	0,225	0,315	4,97%	1,6E+01
18 SS	10	40	1	1,278	0,274	4,72E+04	4,7E+05	2,0E+05	0,176	14,1	5	6,0	38,1%	2,91	0,2%	0,41	-7,6	13,1	0,198	0,282	2,19%	3,0E+01
19 TS	10	50	1	1,426	0,245	1,95E+05	2,0E+06	9,2E+05	0,166	27,9	11	11,8	35,3%	2,85	1,3%	0,34	-40,1	44,8	0,196	0,253	2,04%	2,4E+01
20 TS	10	60	1	1,591	0,220	5,13E+05	5,1E+06	2,7E+06	0,156	45,9	14	16,4	31,9%	2,87	5,0%	0,31	-110,9	82,8	0,183	0,227	1,72%	3,2E+01
21 TS	10	70	1	1,752	0,200	1,12E+06	1,1E+07	6,3E+06	0,147	67,6	21	19,5	28,9%	2,88	2,6%	0,28	-231,3	143,9	0,176	0,206	1,54%	2,5E+01
22 TS	10	80	1	1,907	0,184	2,27E+06	2,3E+07	1,3E+07	0,139	93,2	21	22,6	26,5%	2,89	4,4%	0,27	-401,7	206,3	0,166	0,190	1,34%	3,3E+01
23 TS	10	90	1	2,056	0,170	4,32E+06	4,3E+07	2,/E+0/	0,132	125,6	26	26,8	24,8%	2,87	7,0%	0,25	-642,0	283,2	0,156	0,175	1,41%	4,5E+01
24 SL	100	20	1	1,189	0,294	3,46E+02	3,5E+04	1,3E+04	0,175	4,9	2	2,2	0,0%	5,89	0,0%	1,19	-1,4	13,2	0,197	0,351	2,50%	2,6E+01
25 SL	100	30	1	1,479	0,237	1,08E+03	1,1E+05	4,0E+04	0,159	14,5	1	1,1	0,0%	7,10	0,0%	1,04	-4,6	24,1	0,153	0,298	0,55%	1,2E+02
20 IL	100	40	1	1,582	0,221	3,15E+03	3,2E+05	1,5E+05	0,154	28,2	12	14.6	0,0%	0,00	2,0%	0,91	-54,3	42,4	0,117	0,254	0,99%	4,5E+02
27 13 29 TC	100	50	1	1,099	0,200	2,00E+04	2,0E+00	2,05+06	0,149	40,1 60 E	15	10 1	21,270	2,02	2,5%	0,57	-100,2	110 7	0,100	0,215	2,12%	2,5E+01
2813	100	00	T	1,001	0,180	0,071+04	0,72+00	3,91+00	0,140	00,5	17	10,1	24,770	2,09	0,078	0,34	-221,5	119,7	0,100	0,193	1,0378	2,31+01
						G																

Case	$Ra_{H,0}$	γ	$\Delta \eta_P$	Nu	T _i	$\Delta\eta$	Ra _{H,i}	$ Ra_H $	 <i>T</i>	v_{rms}	λ	ω	$\boldsymbol{\delta}_l$	a_{rh}	M ₀	M ₁	$v_{r,min}$	v _{r,max}	T _{r,min}	$T_{r,max}$	$\sigma(\delta_l)$	$\Delta \eta_{lat}$
29 TS	100	70	1	2,057	0,170	1,42E+05	1,4E+07	8,7E+06	0,132	84,2	21	21,4	23,0%	2,89	4,3%	0,31	-380,7	172,3	0,151	0,176	1,49%	3,0E+01
30 TS	100	80	1	2,226	0,157	2,79E+05	2,8E+07	1,8E+07	0,124	112,9	23	24,3	21,4%	2,90	5,0%	0,30	-540,4	263,1	0,143	0,161	1,41%	4,6E+01
31 SL	1000	10	1	1,581	0,221	9,12E+00	9,1E+03	6,0E+03	0,146	6,9	2	2,4	0,0%	2,21	0,3%	1,08	-5,4	14,9	0,119	0,301	0,08%	1,0E+00
32 SL	1000	20	1	2,122	0,165	2,71E+01	2,7E+04	1,9E+04	0,125	13,4	2	2,2	0,0%	3,30	0,1%	1,07	-9,0	24,7	0,092	0,228	0,04%	1,0E+00
33 TL	1000	30	1	2,221	0,158	6,77E+01	6,8E+04	4,3E+04	0,120	21,4	2	2,1	0,0%	4,73	1,6%	1,08	-16,8	35,7	0,076	0,197	0,05%	7,7E+01
34 SL	1000	40	1	2,288	0,153	1,14E+02	1,1E+05	6,9E+04	0,113	35,3	1	1,3	0,0%	6,12	0,0%	1,06	-23,6	42,5	0,057	0,175	0,03%	2,2E+02
35 TL	1000	50	1	2,292	0,153	4,47E+02	4,5E+05	2,4E+05	0,116	51,5	1	2,0	0,0%	7,64	6,2%	0,95	-128,9	59,1	0,046	0,166	0,09%	8,1E+02
36 TL	1000	60	1	2,362	0,148	4,71E+03	4,7E+06	2,4E+06	0,116	91,0	1	3,1	0,0%	8,89	3,1%	0,86	-302,2	150,0	0,057	0,157	0,59%	1,5E+03
37 TS	1000	70	1	2,501	0,140	1,72E+04	1,7E+07	1,1E+07	0,113	103,3	21	21,7	17,1%	2,94	4,6%	0,37	-500,4	205,7	0,122	0,145	1,66%	3,3E+01
38 TS	1000	80	1	2,685	0,130	3,40E+04	3,4E+07	2,3E+07	0,107	134,3	26	25,6	16,5%	2,85	13,1%	0,34	-707,0	307,0	0,116	0,134	1,37%	1,0E+00
39 SM	1e4	0	1	2,469	0,142	1,00E+00	1,0E+04	1,0E+04	0,108	11,5	4	4,1	0,0%	0,00	0,0%	1,06	-25,7	13,5	0,046	0,186	0,00%	1,0E+00
40 SM	1e4	10	1	2,672	0,131	3,50E+00	3,5E+04	2,7E+04	0,099	18,9	3	3,4	0,0%	1,31	0,1%	1,04	-40,2	20,7	0,032	0,154	0,00%	3,9E+00
41 SL	1e4	20	1	2,728	0,128	1,05E+01	1,0E+05	6,8E+04	0,096	29,7	2	2,4	0,0%	2,57	0,1%	1,04	-50,3	20,0	0,025	0,137	0,03%	1,2E+01
42 SL	1e4	30	1	2,876	0,122	2,26E+01	2,3E+05	1,3E+05	0,091	51,9	1	1,5	0,0%	3,65	0,0%	1,04	-49,9	51,8	0,022	0,130	0,00%	3,6E+01
43 SL	1e4	40	1	2,916	0,120	5,01E+01	5,0E+05	2,7E+05	0,089	61,2	1	1,8	0,0%	4,80	0,0%	1,04	-76,9	51,4	0,018	0,125	0,02%	9,7E+01
44 TL	1e4	50	1	3,123	0,112	9,82E+01	9,8E+05	5,3E+05	0,085	67,8	1	3,2	0,0%	5,60	14,6%	1,01	-126,4	69,2	0,018	0,117	0,06%	2,3E+02
45 TL	1e4	60	1	3,332	0,105	2,06E+02	2,1E+06	1,1E+06	0,083	81,6	1	3,4	0,0%	6,30	6,3%	1,01	-205,1	66,2	0,019	0,112	0,05%	4,8E+02
46 TL	1e4	70	1	3,509	0,100	2,37E+02	2,4E+06	1,3E+06	0,081	107,7	1	3,3	0,0%	6,98	5,5%	0,97	-335,0	86,1	0,016	0,105	0,19%	1,6E+03
47 TL	1e4	90	1	3,813	0,092	9,54E+02	9,5E+06	5,0E+06	0,078	180,7	1	2,9	0,0%	8,26	7,0%	0,89	-847,8	183,9	0,016	0,099	0,62%	8,1E+03
48 TL	1e4	110	1	4,000	0,087	1,36E+04	1,4E+08	8,1E+07	0,076	419,6	1	1,7	0,0%	9,62	7,7%	0,83	-1757,9	476,0	0,034	0,104	1,21%	3,3E+04
49 TM	1e5	0	1	3,367	0,104	1,00E+00	1,0E+05	1,0E+05	0,079	35,1	3	6,7	0,0%	0,00	6,8%	1,04	-177,5	28,5	0,017	0,112	0,12%	1,0E+00
50 TL	1e5	10	1	3,731	0,094	2,53E+00	2,5E+05	2,1E+05	0,073	46,3	2	7,4	0,0%	0,94	9,4%	1,02	-155,2	52,0	0,026	0,097	0,00%	2,4E+00
51 TL	1e5	20	1	4,042	0,087	5,39E+00	5,4E+05	3,9E+05	0,068	56,3	2	8,9	0,0%	1,73	5,8%	1,00	-247,9	53,1	0,013	0,090	0,06%	5,8E+00
52 TM	1e5	30	1	4,428	0,079	9,70E+00	9,7E+05	6,9E+05	0,064	55,4	9	12,6	0,0%	2,37	4,6%	0,98	-250,4	61,4	0,018	0,083	0,22%	1,1E+01
53 TM	1e5	40	1	4,657	0,075	1,79E+01	1,8E+06	1,2E+06	0,062	68,7	5	12,3	0,0%	3,01	3,4%	1,00	-296,9	74,4	0,020	0,081	0,14%	2,4E+01
54 TM	1e5	60	1	4,967	0,070	5,80E+01	5,8E+06	3,7E+06	0,059	93,4	6	10,7	0,0%	4,23	3,7%	1,02	-445,5	114,3	0,024	0,077	0,21%	9,3E+01
55TM	1e6	0	1	6,405	0,055	1,00E+00	1,0E+06	1,0E+06	0,045	69 <i>,</i> 8	13	16,3	0,0%	0,00	4,9%	1,01	-382,7	102,7	0,016	0,058	0,41%	1,0E+00
56 SS	1	50	100	1,099	0,318	1,07E+05	1,1E+05	4,0E+04	0,185	5,2	6	6,4	40,4%	5,36	0,2%	0,59	-1,1	13,1	0,231	0,319	5,26%	8,3E+01
						C		•														

Case	$Ra_{H,0}$	γ	$\Delta \eta_P$	Nu	T _i	$\Delta\eta$	Ra _{H,i}	$ Ra_H $	T	v_{rms}	λ	ω	$\boldsymbol{\delta}_l$	a _{rh}	M_0	<i>M</i> ₁	v _{r,min}	v _{r,max}	T _{r,min}	T _{r,max}	$\sigma(\delta_l)$	$\Delta \eta_{lat}$
57 TS	1	70	100	1,426	0,246	1,11E+06	1,1E+06	3,9E+05	0,167	22,4	7	8,8	36,0%	3,76	7,1%	0,33	-28,3	62,1	0,196	0,260	1,63%	1,2E+02
58 TS	1	90	100	1,697	0,206	9,48E+06	9,5E+06	2,8E+06	0,150	52,2	12	14,0	30,5%	3,62	3,3%	0,27	-157,5	191,1	0,179	0,213	1,29%	1,1E+02
59 TS	1	100	100	1,839	0,190	1,91E+07	1,9E+07	5,5E+06	0,143	76,0	15	17,0	27,9%	3,63	5,5%	0,26	-268,7	271,9	0,170	0,197	1,09%	1,0E+02
60 TS	1	110	100	1,969	0,178	3,67E+07	3,7E+07	1,0E+07	0,136	87,4	15	16,6	25,9%	3,68	6,4%	0,25	-339,8	380,3	0,160	0,182	1,11%	1,3E+02
61 TS	1	120	100	2,101	0,167	6,69E+07	6,7E+07	1,8E+07	0,130	117,4	20	19,3	24,5%	3,51	6,3%	0,23	-464,7	556,8	0,152	0,170	1,00%	1,5E+02
62 TS	3	80	100	1,667	0,210	1,41E+06	4,2E+06	1,3E+06	0,152	39,3	11	13,6	29,9%	3,72	3,5%	0,30	-100,4	143,9	0,177	0,217	1,37%	1,1E+02
63 TS	3	90	100	1,810	0,193	3,38E+06	1,0E+07	3,0E+06	0,144	57,1	14	15,4	27,5%	3,68	2,8%	0,28	-186,1	208,2	0,168	0,199	1,28%	1,0E+02
64 TS	3	110	100	2,090	0,167	1,33E+07	4,0E+07	1,1E+07	0,130	102,1	20	19,2	24,2%	3,54	4,4%	0,25	-415,8	466,1	0,151	0,171	0,98%	1,2E+02
65 TS	3	120	100	2,224	0,157	2,38E+07	7,1E+07	1,9E+07	0,124	130,3	21	20,8	22,8%	3,50	6,4%	0,24	-514,5	588,3	0,143	0,160	0,90%	1,6E+02
66 SS	10	50	100	1,309	0,267	1,12E+04	1,1E+05	5,2E+04	0,171	10,6	6	6,1	31,5%	4,77	0,0%	0,43	-6,1	35,6	0,198	0,288	3,80%	1,2E+02
67 TS	10	70	100	1,643	0,213	1,81E+05	1,8E+06	6,2E+05	0,153	30,5	9	11,4	28,8%	3,81	5,7%	0,34	-63,8	122,4	0,174	0,224	1,59%	1,1E+02
68 TS	10	80	100	1,793	0,195	5,22E+05	5,2E+06	1,6E+06	0,145	45,1	12	14,0	26,9%	3,70	3,0%	0,32	-132,5	170,9	0,164	0,202	1,37%	1,1E+02
69 TS	10	90	100	1,943	0,180	1,20E+06	1,2E+07	3,6E+06	0,137	63,5	14	15,7	25,0%	3,65	4,4%	0,29	-218,2	251,8	0,157	0,186	1,19%	1,1E+02
70 SU	100	30	100	1,134	0,309	1,60E+02	1,6E+04	8,0E+03	0,179	3,4	4	4,7	0,0%	9,26	0,1%	1,13	-1,4	21,7	0,206	0,335	4,15%	6,5E+01
71 TU	100	40	100	1,377	0.255	4,19E+02	4,2E+04	2,3E+04	0,163	8,6	6	6,0	0,0%	10,27	2,4%	0,88	-8,6	42,5	0,167	0,280	6,85%	2,4E+02
72 TS	100	50	100	1,607	0,218	1,55E+03	1,5E+05	7,2E+04	0,153	16,4	8	5,3	23,9%	4,21	1,9%	0,47	-24,6	47,4	0,149	0,244	4,99%	3,4E+02
73 TS	100	60	100	1,772	0,198	8,15E+03	8,1E+05	3,1E+05	0,145	26,2	9	9,4	23,7%	3,87	6,4%	0,41	-53,5	104,2	0,144	0,212	2,39%	2,5E+02
74 TS	100	70	100	1,934	0,181	2,85E+04	2,8E+06	9,4E+05	0,137	39,8	12	12,2	22,7%	3,71	2,4%	0,37	-104,4	144,5	0,145	0,189	1,43%	1,3E+02
75 TS	100	80	100	2,095	0,167	7,26E+04	7,3E+06	2,3E+06	0,129	57,1	13	14,3	21,3%	3,71	3,0%	0,35	-173,6	214,0	0,140	0,173	1,28%	1,2E+02
76 TS	100	100	100	2,412	0,145	2,99E+05	3,0E+07	8,6E+06	0,116	99,8	18	17,7	18,9%	3,70	4,7%	0,31	-347,5	400,5	0,127	0,148	1,04%	1,3E+02
77 TU	300	40	100	1,625	0,217	9,56E+01	2,8E+04	1,7E+04	0,145	10,9	4	4,3	0,0%	8,66	2,4%	0,86	-9,8	43,2	0,129	0,247	0,87%	3,8E+02
78 SM	300	50	100	1,825	0,192	4,93E+02	1,5E+05	6,9E+04	0,141	18,9	3	4,2	0,0%	9,59	0,8%	0,89	-30,8	58,9	0,121	0,219	1,79%	5,7E+02
79 SM	1000	20	100	1,307	0,268	4,66E+00	4,7E+03	3,4E+03	0,159	4,1	4	4,5	0,0%	5,36	0,5%	1,07	-3,2	18,3	0,153	0,305	0,15%	3,9E+01
80 SL	1000	40	100	2,049	0,171	1,59E+01	1,6E+04	1,1E+04	0,120	16,3	2	2,3	0,0%	6,83	0,6%	0,89	-11,9	49,2	0,100	0,212	0,17%	4,2E+02
81 TL	1000	60	100	2,464	0,142	8,23E+01	8,2E+04	4,8E+04	0,112	38,0	1	2,5	0,0%	8,52	1,5%	0,95	-58,6	103,3	0,074	0,171	0,14%	2,3E+03
82 TL	1000	80	100	2,716	0,129	2,78E+02	2,8E+05	1,6E+05	0,104	85,8	1	1,9	0,0%	10,31	1,9%	0,85	-191,3	238,6	0,055	0,144	0,34%	1,4E+04
83 TL	1000	100	100	2,895	0,121	3,15E+04	3,1E+07	9,3E+06	0,101	175,2	1	2,7	0,0%	3,27	9,0%	0,35	-484,5	655,4	0,096	0,127	2,67%	2,0E+03
						C																

Case	Ra _{H,0}	γ	$\Delta \eta_P$	Nu	T _i	$\Delta\eta$	Ra _{H,i}	$ Ra_H $	T	v_{rms}	λ	ω	δ_l	a_{rh}	M ₀	M ₁	$v_{r,min}$	v _{r,max}	T _{r,min}	T _{r,max}	$\sigma(\delta_l)$	$\Delta \eta_{lat}$
84 SM	1e4	20	100	2,111	0,166	1,82E+00	1,8E+04	1,0E+04	0,106	10,3	5	5,3	0,0%	3,32	0,4%	0,69	-9,7	29,5	0,075	0,194	0,00%	2,2E+01
85 SM	1e4	40	100	3,108	0,113	4,99E+00	5,0E+04	2,7E+04	0,084	20,1	4	4,7	0,0%	4,50	1,0%	0,84	-17,6	60,1	0,051	0,137	0,00%	9,7E+01
86 SL	1e4	80	100	3,854	0,091	1,95E+01	2,0E+05	1,0E+05	0,073	83,3	1	1,4	0,0%	7,27	0,7%	1,02	-51,1	140,3	0,034	0,103	0,05%	1,2E+03
87 TL	1e4	100	100	4,195	0,083	4,04E+01	4,0E+05	2,0E+05	0,070	114,1	1	1,5	0,0%	8,34	6,4%	0,93	-114,1	196,8	0,027	0,093	0,08%	9,3E+03
88 TM	1e5	120	100	5,909	0,059	7,24E+01	7,2E+06	2,6E+06	0,050	127,5	3	6,4	0,0%	7,11	3,5%	0,98	-252,8	167,7	0,012	0,061	0,06%	5,4E+03
89 TS	1	90	1e5	1,557	0,225	4,51E+05	4,5E+05	1,5E+05	0,154	25,7	6	8,0	27,5%	6,55	1,5%	0,34	-46,5	192,2	0,170	0,233	3,56%	9,0E+03
90 SS	10	50	1e5	1,067	0,328	3,61E+03	3,6E+04	1,5E+04	0,185	3,6	5	6,1	28,7%	8,50	0,2%	1,07	-3,3	56,9	0,232	0,336	18,55%	5,6E+02
91 SS	10	60	1e5	1,250	0,280	5,19E+03	5,2E+04	2,4E+04	0,169	10,4	6	6,9	23,7%	8,82	0,1%	0,52	-10,4	124,6	0,202	0,288	9,67%	5,4E+03
						6									9							

881	
-----	--

882 Highlights

- 883
- > Internally heated 3-D mantle convection models in a spherical shell with temperature and
- 885 pressure dependent viscosity have been performed to provide new insights into the various 886 convection regimes.
- 887 >We were able to predict the pattern of convection (dominant degree) for high Rayleigh
- 888 numbers in the stagnant-lid regime.

889 >A case study of 91 3D simulations helped to identify a low-degree regime close to the border

MAN

- to the stagnant-lid regime.
- 891 >We were able to determine the rheological constant through full inversion.
- 892
- 893