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# Constitutive modelling of fibre-reinforced composites with unidirectional plies using a plasticity-based approach

G.M. Vyas<sup>a,\*</sup>, S.T. Pinho<sup>a</sup>, P. Robinson<sup>a</sup>

<sup>a</sup>*Department of Aeronautics, Imperial College London, Prince Consort Road, South Kensington, London SW7 2AZ, UK.*

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## Abstract

This paper presents the development of a constitutive model able to accurately represent the full non-linear mechanical response of polymer-matrix fibre-reinforced composites with unidirectional (UD) plies under quasi-static loading. This is achieved by utilising an elasto-plastic modelling framework. The model captures key features that are often neglected in constitutive modelling of UD composites, such as the effect of hydrostatic pressure on both the elastic and non-elastic material response, the effect of multiaxial loading and dependence of the yield stress on the applied pressure.

The constitutive model includes a novel yield function which accurately represents the yielding of the matrix within a unidirectional fibre-reinforced composite by removing the dependence on the stress in the fibre direction. A non-associative flow rule is used to capture the pressure sensitivity of the material. The experimentally observed translation of subsequent yield surfaces is modelled using a non-linear kinematic hardening rule. Furthermore, evolution laws are proposed for the non-linear hardening that relate to the applied hydrostatic pressure.

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\*Corresponding author

Email address: [gv04@imperial.ac.uk](mailto:gv04@imperial.ac.uk) (G.M. Vyas)

Multiaxial test data is used to show that the model is able to predict the non-linear response under complex loading combinations, given only the experimental response from two uniaxial tests.

*Keywords:* Polymer-matrix Composites (PMCs), Stress/strain curves, Non-linear behaviour, Modelling

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## 1. Introduction

### 1.1. Background

With the increasing use of fibre-reinforced composites with unidirectional plies as primary structural parts in aerospace, it is of increasing importance to predict how and when composites will fail. Precise failure predictions are only possible provided the stress state is accurately known. This requires an accurate representation of the constitutive response of the material under a variety of uniaxial and combined loading conditions, within both unidirectional and multidirectional laminates.

### 1.2. Constitutive Response of Materials

Hydrostatic pressure can cause significantly greater changes in the mechanical properties of polymers and polymer-matrix composites than in metals or rocks. Furthermore, polymers and polymer-matrix composites yield at different stresses in tension and compression, while the elastic modulus increases with hydrostatic pressure [1–3].

Although pressure can inhibit crack initiation in polymers, it also raises the yield stress, inhibiting local yielding [4]. As the yield and fracture mechanisms compete for failure, it has been reported that polymers undergo a ductile-brittle transition, as shown in figure 1. With further pressure increases, crack growth is suppressed and

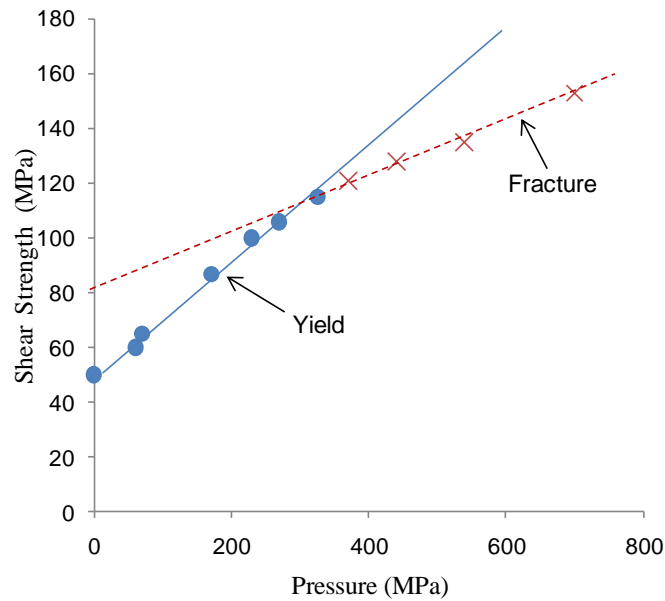


Figure 1: Shear strength as a function of hydrostatic pressure for a polymer (PMMA) [5]

additional transitions may be observed, as failure may again be caused by yielding [5].

The presence of fibres in composite materials alters the material dependence on hydrostatic pressure; the effect is still significant and, importantly, non-isotropic. Pressure increases interfacial normal and shear stresses, resulting in greater adhesion between fibres and matrix, whilst reducing the influence of flaws such as microcracks and voids [2]. The effects of hydrostatic pressure on the non-linear response of a UD composite are shown in figure 2a and b, indicating a change in the post-yield behaviour and an increase in elastic modulus.

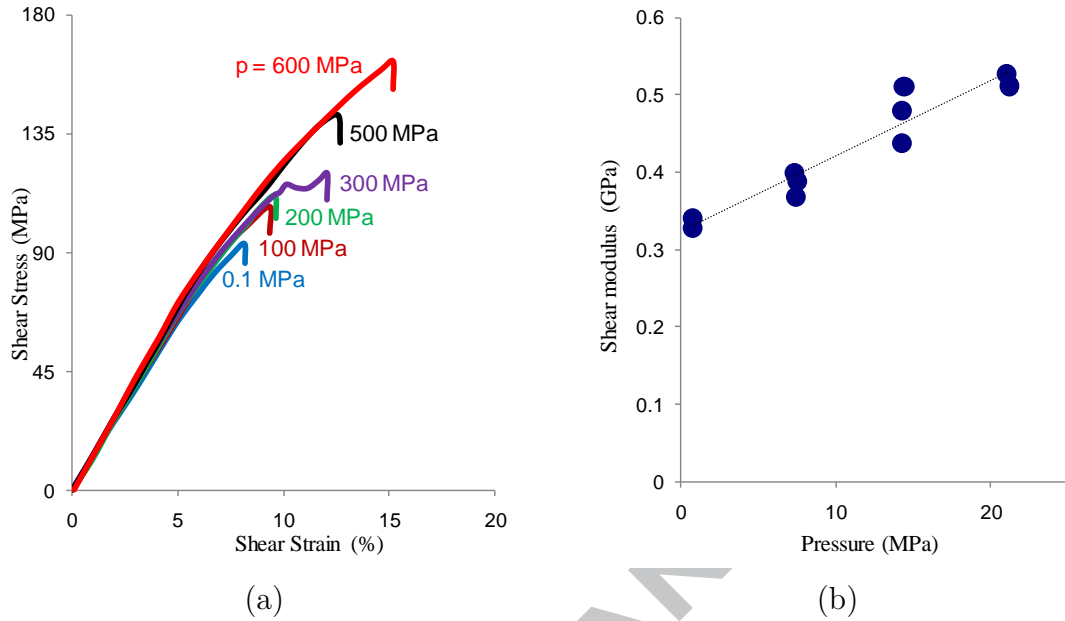


Figure 2: Effect of increasing hydrostatic pressure on shear response and modulus of a UD composite [3, 6]

Another significant effect is the interaction between different stress components under multiaxial loading also affects the material response. Figure 3 shows that stress components are coupled, as the shear response of a UD composite is influenced by the introduction of a stress in the transverse direction.

### 1.3. Plasticity Modelling

#### 1.3.1. Isotropic Yield Criteria

Several criteria have been proposed to model yielding in polymers. The yield function for the linear Drucker-Prager criterion, one of the most widely used in the literature, reads:

$$f = \sqrt{J_{2D}} + \mu\sigma_m - \sigma_0 \quad (1)$$

where  $\mu$  is the hydrostatic pressure sensitivity,  $\sigma_0$  is the yield stress,  $\sigma_m = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33})$  is the hydrostatic pressure, and  $J_{2D}$  is the second invariant of the deviatoric

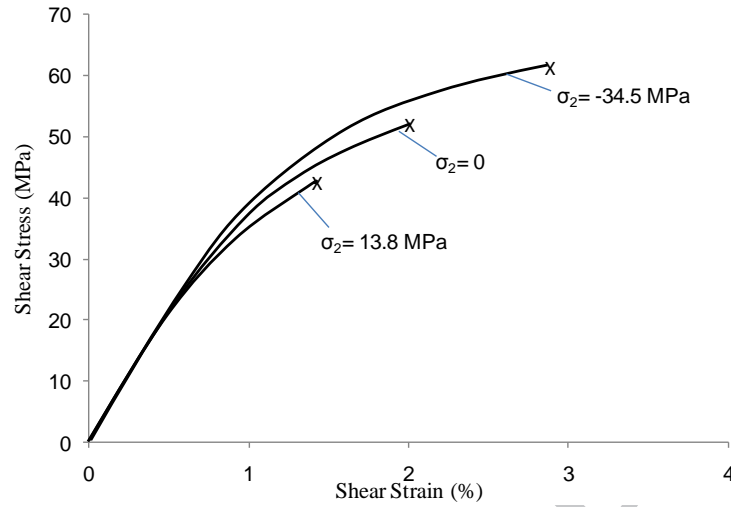


Figure 3: Effect of multiaxial loading on shear response [7, 8]

stress tensor. Yield is considered to occur when  $f \geq 0$ .

The linear Drucker-Prager criterion is widely used for computational implementation as it produces a smooth yield surface, avoiding the numerical instability that can arise from modelling the vertices of discontinuous criteria, such as the Mohr-Coulomb. In addition, it has been argued that these vertices are only appropriate for metals and are unsuitable for frictional and quasi-brittle materials [9]. Finally, the linear Drucker-Prager criterion is also considered to be more suitable than Mohr-Coulomb for modelling materials in which the deformation cannot be well represented by frictional sliding on failure planes [10].

The linear Drucker-Prager criterion predicts a linear increase in the shear yield stress with increasing hydrostatic pressure. However, experimental observations [11] suggest the dependence is slightly convex. The hyperbolic and exponent forms of the Drucker-Prager are able to predict this convexity, but require the calculation of many material constants [12]. However, the Raghava criterion [13, 14] produces a

similar yield surface to the hyperbolic Drucker-Prager and requires only two material constants. It is stated as:

$$f = J_{2D} + 2(C - T)(\sigma_{11} + \sigma_{22} + \sigma_{33}) - 2CT \quad (2)$$

where  $C$  and  $T$  are the absolute values of the compressive and tensile yield strengths respectively.

The Raghava criterion is not, however, considered suitable for modelling all materials; prompting the proposal of many yield criteria for different materials, some of which can also be applied to polymers. Zhang et al. [15] proposed a criterion similar to the Drucker-Prager criterion, but with a dependence on the square of the hydrostatic pressure. The criterion by Altenbach and Tushtev [1] is also suitable for modelling materials with different strengths in tension and compression. The criterion developed by Bigoni and Piccolroaz [9] is suitable for modelling a variety of different pressure sensitive, frictional, ductile and brittle-cohesive materials, as the yield surface is allowed to change shape. It can be considered as a generalisation of several different criteria, including Drucker-Prager and Mohr-Coulomb. The criterion used by Mahnken and Schlimmer [16] can also be reduced to the Drucker-Prager or Von Mises criteria, whilst Kolling and Haufe [17] use a yield criterion consisting of two Drucker-Prager yield surfaces.

As the criteria above are for isotropic materials, they are unsuitable for the modelling of fibre-reinforced composites. However, they can be modified to account for the presence of fibres. The linear Drucker-Prager and the Raghava appear to be the best choices for this as they are fairly simple, hence can be manipulated mathematically. More complex criteria such as that of Altenbach and Tushtev [1] are more difficult to handle computationally as their derivatives will be required in



a computational algorithm.

### 1.3.2. Yield criteria for Unidirectional Composites

Several authors have previously attempted to modify isotropic yield criteria for use with unidirectional composites. Xie and Adams [18] modified a quadratic yield criterion that does not consider hydrostatic pressure by imposing  $\frac{\partial f}{\partial \sigma_{11}} = 0$ , where 11 denotes the fibre direction, obtaining:

$$2f = F(\sigma_{22} - \sigma_{33})^2 + 2\sigma_{23}^2 + 2M(\sigma_{31}^2 + \sigma_{12}^2) \quad (3)$$

where  $F$  and  $M$  are material constants. It should be noted that using this approach with an associated flow rule implies that  $d\epsilon_{11}^p = 0$ , so that the response in the fibre direction remains linear elastic.

Goldberg et al. [19, 20] use laminate theory and a 'fibre structuring' approach for the micromechanical modelling of the composite. In this approach, the authors first characterise the matrix using the linear Drucker-Prager criterion, then the fibre, then the whole composite. This approach is used as it is easier to test a polymer than perform an equivalent test on a composite. The in-situ matrix properties and deformation response are assumed to be equivalent to those of the bulk polymer. Alternatively, the polymer properties can be adjusted such that test data from composite specimens matches the analytical predictions [19].

Cazacu et al. [21] have produced a transversely isotropic yield criterion for rocks, which can also be adapted for UD composites. If used to model an isotropic material, the Cazacu criterion reduces to a form similar to the Raghava criterion, but is not able to represent the linear elastic response of a UD composite when loaded in the fibre direction. For modelling composites, the criterion of Bigoni and Piccolroaz [9] is of a similar form to the Drucker-Prager criterion.

Spencer [22] has suggested two criteria for the yielding of transversely isotropic materials; the first is analogous to the von Mises criterion and the second is a maximum shear stress criterion analogous to the Tresca criterion. However, neither of the criteria are pressure dependent, although this has been used as a basis for modelling of UD composites by Vogler et al. [23].

As none of the criteria discussed are simultaneously hydrostatically sensitive and suitable to model unidirectional composites, this work will develop a yield criterion that can be used to capture the dependence of the yield stress on hydrostatic pressure, allow a linear elastic response of the fibres and account for the presence of multiaxial loading.

### 1.3.3. Flow Rule and Hardening

The flow rule determines the increment in plastic strain,  $d\varepsilon_p$ . It is stated as:

$$d\varepsilon_p = d\lambda \frac{\partial g}{\partial \sigma} \quad (4)$$

where  $d\lambda$  is the plastic multiplier and  $\frac{\partial g}{\partial \sigma}$  determines the direction of plastic flow, where  $g$  is the 'plastic potential' function. If the yield criterion is used as the plastic potential function, the flow is termed 'associated', else it is termed 'non-associated'. Many plasticity models in the literature use associated flow, but for the modelling of pressure sensitive materials the flow rule should be defined based on experimental observations [24]. Generally, the flow rule for hydrostatically sensitive materials is defined by modifying the value of the hydrostatic pressure sensitivity coefficient,  $\mu$  to  $\mu'$  [24–26].

The post yield stress-strain behaviour can be modelled using kinematic or isotropic hardening, or a combination of the two. In isotropic hardening, the yield surface is assumed to expand uniformly in all directions. Kinematic hardening represents a

translation of the yield surface [27], an effect which has been noted experimentally for composites [28, 29].

## 2. Constitutive Model for UD composites

One possible approach to modelling the constitutive response is through the use of a plasticity approach to generate the non-linear stress-strain curves observed experimentally for UD composites. Several papers in the literature use a viscoelastic approach in order to capture the time dependent behaviour of the material [30, 31]. This paper is concerned with loading under quasi-static conditions at the ply level and so an elastic-plastic constitutive law is used.

A model is hereby developed that is able to represent the full non-linear response of UD composites under quasi-static loading. This non-linearity may be the result of several different mechanisms, including plasticity, microcracking and other forms of damage. As with other models of this type [23], the modelling of the non-linearities does not account for the exact physical cause of the non-linearity. Rather, the model aims to faithfully reproduce the constitutive response under superimposed hydrostatic pressure or multiaxial loading.

### 2.1. Yield Criterion

A pressure dependent yield criterion for UD composites is formulated which satisfies the condition  $\frac{\partial f}{\partial \sigma_{11}} = 0$ . This is achieved by removing all terms and brackets containing  $\sigma_{11}$  from an existing yield criterion. As an example, the yield curves produced by modifying the linear Drucker-Prager and Raghava criteria in this way are plotted in figure 4, with experimental data for the failure of graphite tubes [3]. Although the experimental values are not necessarily for yield, they provide a useful first point of comparison.

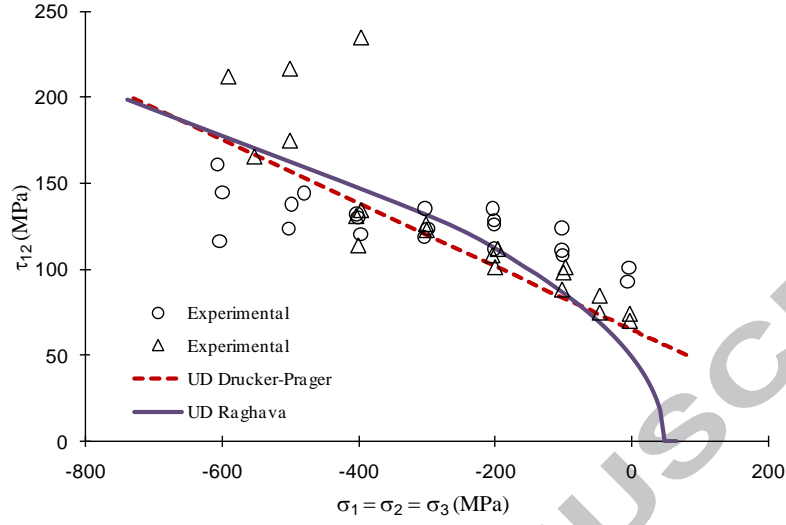


Figure 4: Predicted yield curves under hydrostatic pressure using Drucker-Prager and Raghava criteria modified for UD composites compared to experimental data for failure (matrix cracking or yield) [3]

The Raghava criterion, expressed in terms of  $\mu$  and  $\sigma_0$  instead of the tensile and compressive yield strengths, is proposed as it captures the yield in tension well. The criterion is:

$$f = \sqrt{\frac{1}{6}(\sigma_{22} - \sigma_{33})^2 + \sigma_{12}^2 + N\sigma_{23}^2 + \sigma_{31}^2 + \frac{\mu}{2}(\sigma_{22} + \sigma_{33})} - \sigma_0 \quad (5)$$

The constants  $\mu$  and  $\sigma_0$  in equation 5 are calculated using experimental data from any two uniaxial tests [25], as shown in table 1, where  $S$  is the shear yield stress. It should be noted that the numerical value of  $\mu$  differs depending on the yield stresses used for its calculation. As the material is assumed to be transversely isotropic, the constant  $N$  is included to account for the difference in shear properties out of plane. In most load combinations applied to composite laminates  $\sigma_{23}$  is negligible, therefore in this work it is assumed  $N = 1$ .

Table 1: Definitions of  $\mu$  and  $\sigma_0$  using data from different loading tests

Uniaxial test data	$\mu$ (MPa)	$\sigma_0$ (MPa)
Shear and Compression	$\frac{C}{3} - \frac{2S^2}{T}$	$S$
Shear and Tension	$\frac{T}{3} + \frac{2S^2}{C}$	$S$
Tension and Compression	$\frac{C-T}{3}$	$\sqrt{\frac{TC}{6}}$

## 2.2. Flow Rule

The plastic potential for the flow rule used is defined based on the chosen yield criterion in equation 5. In order to correctly reproduce the experimentally measured curves, it is necessary to alter the hydrostatic sensitivity when plastic flow occurs:

$$g = \sqrt{\frac{1}{6}(\sigma_{22} - \sigma_{33})^2 + \sigma_{12}^2 + N\sigma_{23}^2 + \sigma_{31}^2 + \frac{\mu'}{2}(\sigma_{22} + \sigma_{33})} \quad (6)$$

There are several methods for the calculation of the parameter  $\mu'$  ( $\neq \mu$ ) for non-associative flow. In an isotropic material, it can be related to the plastic Poisson's ratio  $\nu_p$ , assuming a uniaxial compression in the  $z$ -direction of the material. The derivation for an isotropic material is given by Zhang et al. [15], the resulting expression for  $\mu'$  is:

$$\mu' = \frac{9(1 - 2\nu_p)}{2(1 + \nu_p)} \quad (7)$$

Modifying this derivation so that there is no plastic deformation parallel to the fibres leads to an assumed definition of the volumetric plastic strain increment,  $d\varepsilon_{vp} = (1 - \nu_p)d\varepsilon_{zvp}$ , where  $d\varepsilon_{zvp}$  is the plastic strain increment in the through thickness direction. Following the derivation of equation 7 [15] with this volumetric plastic strain leads to an expression for  $\mu'$  considered suitable for unidirectional composite materials:

$$\mu' = \frac{9(1 - 2\nu_p)}{(2 + \nu_p)} \quad (8)$$

Equation 8 should be used with caution as it often requires non-physically sensible values of  $\nu_p$  to obtain suitable values of the non-associated hydrostatic pressure coefficient,  $\mu'$ , that match the observed experimental behaviour. However, the same comment can be made for the isotropic material expression in equation 7, in which plausible flow behaviour is only obtained if  $0 \leq \nu_p \leq 0.5$ , implying  $0 \leq \mu' \leq \frac{9}{2}$  [32].

The value of  $\mu'$  can also be calculated based on the volume change seen experimentally in a material under a given loading. However, as volume change is rarely measured, in this study  $\mu'$  is determined by matching experimental curves from two different uniaxial tests - for example shear and compression - using the effective stress.

### 2.3. Effective Stress

As the model developed is related to Drucker-Prager criterion based models [19], the same effective stress is used, defined as:

$$\sigma_e = \sqrt{3}f \quad (9)$$

This effective stress allows the data from different loading regimes to be matched when using the yield criterion in equation 5 with the non-associative flow rule, as shown in figure 5. The material used to generate this plot is IM7-8552, with  $\mu = 4$  MPa and  $\sigma_0 = 61$  MPa.

The incremental effective plastic strain,  $d\varepsilon_e^p$ , is defined as:

$$d\varepsilon_e^p = \sqrt{\frac{2}{3}d\boldsymbol{\varepsilon}_p : d\boldsymbol{\varepsilon}_p} \quad (10)$$

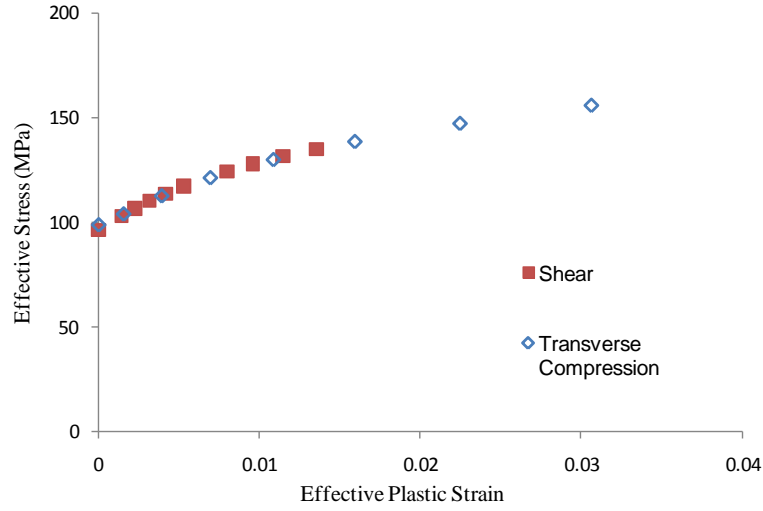


Figure 5: Effective stress-strain plot using equation 9

#### 2.4. Hardening

Kinematic hardening is implemented as it is able to reproduce the experimentally observed constitutive behaviour of polymers and unidirectional composites [28, 29] without the need for isotropic hardening and allows for the modelling of unloading. The kinematic hardening is included in the model using the back stress,  $\alpha$ . The yield criterion then becomes:

$$f = \sqrt{\frac{1}{6} (\xi_{22} - \xi_{33})^2 + \xi_{12}^2 + N\xi_{23}^2 + \xi_{31}^2 + \frac{\mu}{2} (\sigma_{22} + \sigma_{33}) - \sigma_0} \quad (11)$$

where  $\xi_{ij} = \sigma_{ij} - \alpha_{ij}$ .

The simplest and most commonly used non-linear hardening rule used in the literature is the Armstrong-Frederick rule [24, 26, 33]. The evolution of the back stress is given through the use of the material constants  $c$  and  $\gamma$ . In its uniaxial

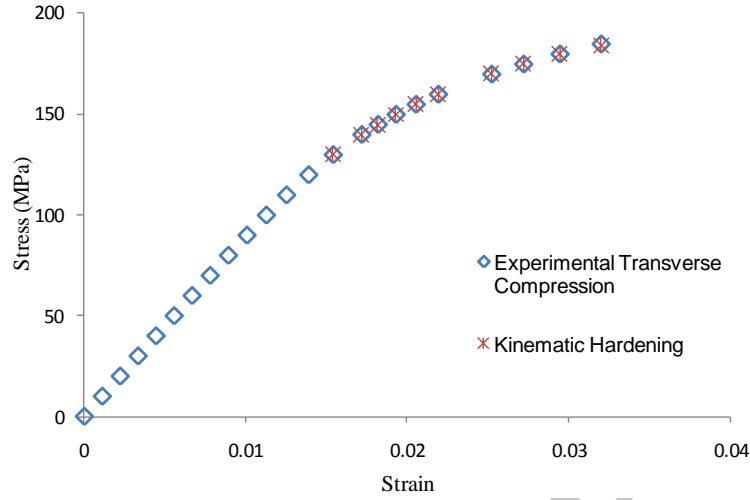


Figure 6: Evolution of hardening compared to experimental data

form, the hardening law is stated as:

$$\alpha = \frac{c}{\gamma}(1 - e^{(-\gamma\varepsilon_p)}) \quad (12)$$

To ensure that the Armstrong-Frederick rule is suitable for composites, the predicted hardening using equation 12 is compared with the experimental data from a transverse compression test for IM7-8552 [34], as shown in figure 6.

### 2.5. Calculation of the plastic multiplier

For non-associative flow, the plastic multiplier,  $d\lambda$ , is calculated by substituting Hooke's law and the hardening rule into the consistency condition. For the case of kinematic hardening, the consistency condition is given by:

$$\frac{df}{d\sigma}d\sigma + \frac{df}{d\alpha}d\alpha = 0 \quad (13)$$

The full derivation of the plastic multiplier follows the method found in Dunne and Petrinic [33] for an associative flow plastic multiplier. The end result for non-



associative flow is:

$$d\lambda = \frac{f_\sigma \mathbf{D}_e d\boldsymbol{\epsilon}}{f_\sigma \mathbf{D}_e g_\sigma + \gamma f_\alpha \boldsymbol{\alpha} - \left(\frac{2}{3}\right) c f_\alpha g_\sigma} \quad (14)$$

where  $f_\sigma = \frac{df}{d\sigma}$ ,  $f_\alpha = \frac{df}{d\alpha}$ ,  $g_\sigma = \frac{dg}{d\sigma}$  and  $\mathbf{D}_e$  is the elastic stiffness matrix.

## 2.6. Evolution of Hardening with Hydrostatic Pressure

Previous studies have indicated the elastic modulus of polymers and composites is greater when hydrostatic pressure is applied [3, 35]. The effect of hydrostatic pressure in the elastic region is modelled as by Pinho et al. [6], assuming a linear increase of  $E_{22}$ ,  $E_{33}$  and shear moduli with hydrostatic pressure.

As  $\gamma \rightarrow 0$ , the Armstrong-Frederick rule reduces to linear kinematic hardening. The value of  $\gamma$  must be positive to cause the modulus to drop in the plastic region. However, the experimental data (see figure 2) suggests that the value must decrease with increasing hydrostatic pressure. Two forms of this evolution are proposed; linear and exponential.

The linear form is stated as:

$$\gamma = \gamma_0 + \eta_\gamma \sigma_m \quad (15)$$

The exponential form is:

$$\gamma = \gamma_0 e^{\eta_\gamma \sigma_m} \quad (16)$$

If the linear form (equation 15) is used,  $\gamma > 0$  must be enforced. This is not required for the exponent version which tends to zero as the hydrostatic pressure is increased. In both versions, the value of the slope coefficient for  $\gamma$ ,  $\eta_\gamma$ , must be defined based on experimental observations. A comparison of the evolution of  $\gamma$  using the two forms with experimental data is shown in figure 7.

To identify the parameter  $\eta_\gamma$  and similarly, the effect of hydrostatic pressure in

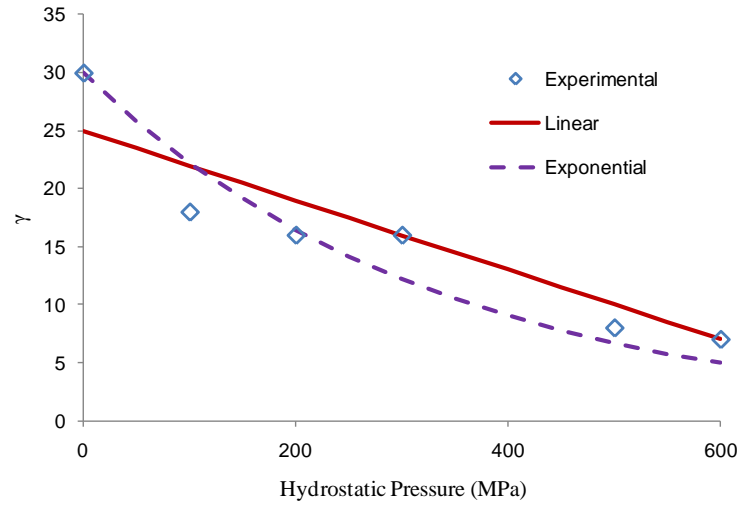


Figure 7: Comparison of linear and exponential evolution of  $\gamma$  with hydrostatic pressure using values of  $\gamma$  measured from the experimental results for UD composites in shear by Shin and Pae [3]

the elastic region, tests are required on UD composites with superimposed hydrostatic pressure to generate stress-strain curves such as figure 2. The parameters are easily identifiable from these stress strain curves. For the linear region, the change in slope of the curve is plotted against hydrostatic pressure to obtain slope coefficients for Young's modulus and shear modulus (figure 2b),  $\eta_E$  and  $\eta_G$ , respectively. For the non-linear region, the non-linear hardening law is plotted along with the experimental data at different values of  $\gamma$ . The parameter  $\eta_\gamma$  is determined from a plot of  $\gamma$  against hydrostatic pressure, such as that in figure 7, using either the linear or exponential evolution law.

### 3. Predictions

The model is implemented as a stand-alone FORTRAN code to simulate a uniaxial material loading. The exponential evolution of  $\gamma$  (equation 16) is used as it more closely reproduces the experimental data (see figure 7). The experimental data for

Table 2: Material properties for T300/PR319 and AS4/55A

Model Parameters	$\gamma^0$	$G_{12}$ (MPa)	$\eta_\gamma$	$\mu$ (MPa)	$\eta_G$	$c$	$\sigma_0$ (MPa)	$N$
T300/PR319	21	1300	0.0014	6.1	0.18	13000	64	1
AS4/55A	140	4200	0.007	7	0.4	50000	23	1

T300/PR319 was obtained using the high pressure torsion test apparatus, details of which can be found in the literature [3, 36].

Figure 8 shows a comparison of the predictions produced by the model to the change in shear response with increasing hydrostatic pressure for a T300/PR319 carbon-epoxy material system. The slope coefficient for shear,  $\eta_G$ , was calculated as a best fit to the data shown in figure 2b. The slope coefficient  $\eta_\gamma$  was calculated using the curves for  $p = 0$  MPa and  $p = 200$  MPa in figure 2a.

Figure 9 shows the experimental data for AS4/55A with differing transverse loading applied plotted with the predicted curves. In the experimental procedure the  $\sigma_2$  component is applied first, followed by the shear, with the curves being plotted during the shear loading phase [7]. Details of the procedure used to generate this data can be found in [8]. The required slope coefficients are calculated from the experimental data in figure 3 in the same way as for T300/PR319, using the data at  $\sigma_2 = 0$  MPa and  $\sigma_2 = -34.5$  MPa. The input data used to produce figures 8 and 9 is shown in table 2.

#### 4. Discussion

The proposed yield criterion shown in equation 5 is pressure dependent and able to produce a similar shape to the hyperbolic Drucker-Prager and the Raghava criteria described previously, whilst remaining relatively simple and allowing for linear material response in the fibre direction. Thus, it is considered suitable for

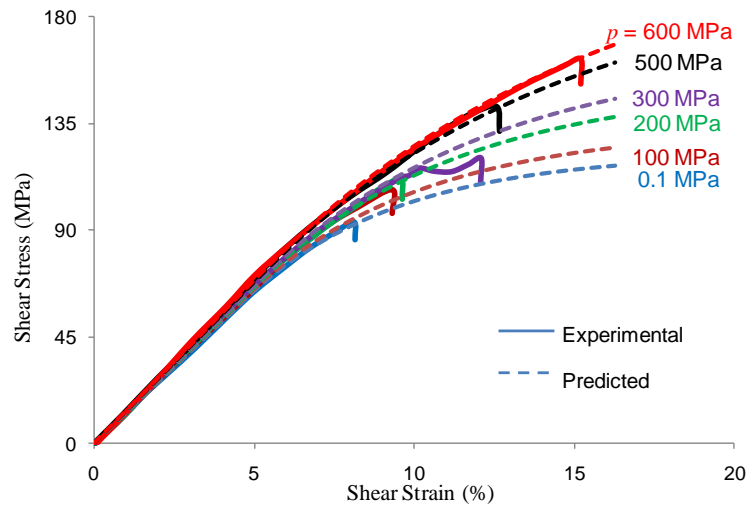


Figure 8: Comparison of exponential evolution of  $\gamma$  with experimental data from Shin and Pae [3]

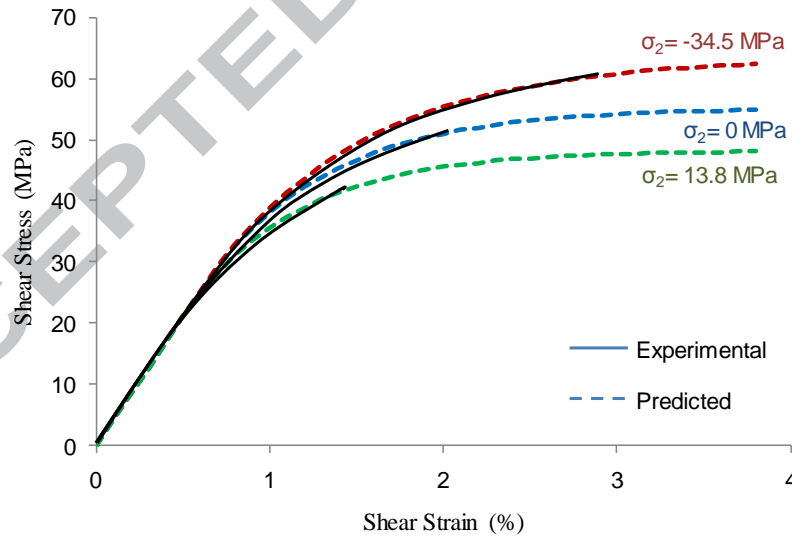


Figure 9: Predictions of constitutive response in shear with transverse compression with experimental data from Swanson et al. [8]

modelling the yield of unidirectional fibre-reinforced polymer-matrix composites.

When loading in shear is applied using non-associative flow with the value of  $\mu'$  that matches the effective stress-strain curves under different loading regimes, a non-negligible change in volume of the material can result [37]. This volume change has been noted experimentally for materials such as concrete, soil and clay and can be considered a limitation of Drucker-Prager type models as the hydrostatic stress and mean stress are both used in the formulation of the yield criterion [37, 38]. If the volume change of the material is not desired, it can be eliminated by specifying  $\mu' = 0$ .

The model is shown to be capable of predicting the experimentally recorded non-linear stress-strain under multiaxial loading curves (figures 6, 8, 9) up to the experimental failure point using the exponential evolution for  $\gamma$ , although the difference using the linear evolution is small (see figure 7).

The model is suitable for implementation in finite element code with explicit integration. Implementation in a finite element code with implicit integration is in principle possible, but the non-linearities included in the model are likely to lead to difficulties in defining the tangent stiffness matrix and in ensuring convergence. The incorporation of the constitutive model with the existing LaRC05 failure criteria [6] as a user subroutine in a finite element package will be addressed in a following paper.

## 5. Conclusions

A constitutive model for unidirectional composite materials has been proposed that is able to capture several features of the constitutive response that have previously been neglected. The proposed yield criterion is both hydrostatically sensitive

(predicts the increase in yield with pressure) and accounts for the different response of UD composites in the fibre direction. The model includes the effects of hydrostatic pressure in the elastic and non-elastic region, is able to predict the response under multiaxial loading and matches the experimental stress-strain curves well using non-linear kinematic hardening.

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