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# Discrete mode observability analysis of switching structured linear systems with unknown input. A graphical approach

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**Abstract:** Switching linear systems are described by a set of continuous state-space models together with conditions (switching law) that decide which model (mode) of this set is valid for the current continuous state. This article deals with the problem of the observability of discrete mode in which the switching law is assumed to be unknown. The formalization of the problem, based on a graph-theoretic approach, is to express sufficient conditions for generic observability of the discrete mode assuming only the knowledge of the system's structure. These conditions allow us to obtain criteria of sensor placement in order to recover the discrete mode observability using properties of the graph associated to the system.

*Keywords:* discrete mode observability, state and input observability, switching structured linear systems, graph theory, sensor placement.

## 1. INTRODUCTION

Over the past decade, study of hybrid systems has received particular attention in several scientific fields including automation.

In general, this kind of systems can represent, through hybrid systems properties, several physical, technological and biological phenomena. It allows to model complex systems which combine the dynamics of the continuous parts (modes) of the system with the dynamics of the logic and discrete parts. Hybrid models are characterized by continuous processes (continuous differential equations) interacting with discrete processes (paradigms from discrete event systems). A hybrid system's structure is illustrated in figure 1:

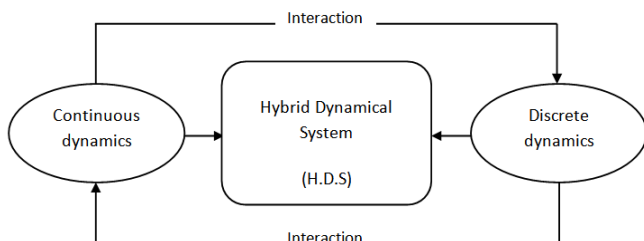


Fig 1: Hybrid Dynamical System's structure

The transition from one mode to another one is strongly linked to the global nature of the behavior of the complex system to model. When the mode transition is abrupt, we can define a particular but very important framework

of hybrid dynamical systems : switching continuous-time linear systems (SCTLS) or SLS (to simplify the notation). They are composed of a family of subsystems which are linear time-invariant and these subsystems (modes) are orchestrated by a switching law that specifies which subsystem is active. As an extension of the classical linear or affine state-space representations of dynamical systems, this modeling formalism has been thoroughly investigated through several studies.

Knowing that, the focus is on the study of observability property for SLS with unknown input, this property plays a major role in command law synthesis, fault detection and isolation, fault tolerant command law synthesis and also for perturbations rejection, the solvability for fault detection and isolation problem is mainly linked to observability property and then investigating it will be interesting when system's structure change as the case for SLS framework. Many definitions of this property appear in the literature for SLS. For example, we quote Bemporad et al. (2000); Collins and Schuppen (2004); Chaib et al. (2005) devoted to studying observability of hybrid systems where the discrete mode depends on the state trajectory or is associated to discrete outputs. We can also quote Bemporad et al. (2000); Vidal et al. (2002); Babaali and Egerstedt (2004) for deterministic discrete-time switching linear systems. Conventional algebraic and geometric tools which are based on the numerical value of state-space matrices of system's model are needed. However, these variables are subject to parametric uncertainties due to identification process and so, they are approximatively known.

We consider here a structured switching linear system in state-space form, knowing that a switching linear system is structured when each entry of the matrices of its state-

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space form is either a fixed zero or a free parameter. The location of the fixed zeros in these matrices constitutes the structure of the system.

The approach is of interest to investigate many classical properties of structured systems that can be studied in terms of genericity. In this case, properties that are true for almost any value of the free parameters are called generic proprieties.

In order to check generic properties as controllability, observability and so on (see Dion et al. (2003)), we can associate in natural way digraphs to structured systems and so verify structural properties by means of graph theoretic terms.

This approach also presents a major advantage. Indeed, through the association of the digraph with the structured system, we can intuitively represent the structural changes on the graph and take into account them when analyzing the property of system. This fact is very interesting knowing that a switch can be related to a change in structure, as for example in the field of electronics Yang and Chu (2006).

In this paper, our aim is to characterize the discrete mode observability for structured switching linear systems (SSLS).

The results presented in this paper were obtained within the European project PAPYRUS (Plug and Play monitoring and control architecture for optimization of large scale production processes).

The outline of the paper is as follows. In section 2, we expose the problem statement. After that we give some definitions and notations to the graph-theoretic approach in section 3, then the main result is given in section 4 and we conclude.

## 2. PROBLEM STATEMENT

Consider the state-space form of switching linear systems as follows:

$$\Sigma : \begin{cases} \dot{x}(t) = A(r_t)x(t) + B(r_t)u(t) \\ y(t) = C(r_t)x(t) + D(r_t)u(t) \end{cases} \quad (1)$$

with  $x \in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^p$  are respectively the state vector, unknown input vector and output vector and matrices  $A(\cdot), B(\cdot), C(\cdot)$  and  $D(\cdot)$  are of appropriate dimensions. Consider a discrete mode variable (mode sequence) as an exogenous input which is considered to be unobserved and defined by  $r_t : [0, \infty) \rightarrow Q \stackrel{def}{=} \{1, \dots, N\}$ . Close to Babaali and Pappas (2005), the switching signal is right-continuous and so impulses in state and input of SLS are excluded. A minimum dwell time is considered to avoid zeno behavior which is an undesired phenomenon for the well-definedness of SLS.

In order to have more general framework, a generic study of discrete mode observability is assumed not for all initial conditions and unknown input  $u$  but for generic ones. To address the discrete mode observability problem, some preliminary definitions are useful:

*Definition 1.* (Mode distinguishability) Two modes  $q \in Q$  and  $q' \in Q$  (with  $q \neq q'$ ) are distinguishable if at least one of the two following conditions holds:

- there exist an integer  $s \geq 0$  and an expression  $\Psi_q(y, \dot{y}, \dots, y^{(s)}) = 0$  which is satisfied for mode  $q$  but is not satisfied for mode  $q'$  for almost all initial conditions

$x_0$  and input  $u$ .

- there exist an integer  $s' \geq 0$  and an expression  $\Psi_{q'}(y, \dot{y}, \dots, y^{(s')}) = 0$  which is satisfied for mode  $q'$  but is not satisfied for mode  $q$  for almost all initial conditions  $x_0$  and input  $u$ .

Here, “for almost all initial conditions  $x_0$  and input  $u$ ” is to be understood as “for all  $(x_0^T, u^T)^T \in \mathbb{R}^{n+m}$  except for the zero set of some polynomials with real coefficients in the  $n + m$  initial state and input components” These polynomials can be written down explicitly, *i.e.* we can precisely describe them when the mode distinguishability fails to be true. Obviously  $u(t) \equiv 0$  and  $x_0 = 0$  are two of these polynomials. The zero set of some polynomial forms a proper algebraic variety of  $\mathbb{R}^{n+m}$  which has Lebesgue measure zero.

The interpretation of Definition 1 is that  $q$  is distinguishable from  $q'$  if, for generic initial state  $x_0$  and unknown input  $u$ , we can rule out  $q$  or  $q'$  when observing the output over  $[0, T]$ . Relatively to the definitions of Babaali and Pappas (2005), our notion of distinguishability of  $q$  and  $q'$  is equivalent to the fact that  $q$  is discernible from  $q'$  or vice-versa. The mutual mode discernibility, which is a dissymmetric property in Babaali and Pappas (2005), is equivalent to have both conditions of Definition 1 satisfied.

*Definition 2.* (Location observability) SLS ( $\Sigma$ ) is location observable if its modes are all distinguishable two-by-two *i.e.*  $\forall q \in Q, \forall q' \in Q$ , with  $q \neq q'$ ,  $q$  and  $q'$  are distinguishable.

Comparatively with the notion of location observability defined in De Santis et al. (2006, 2008), our definition concerns as well autonomous as non-autonomous systems. In De Santis et al. (2006, 2008), location observability is defined as the ability to reconstruct the mode starting from the knowledge of the input and the output, for any nonzero input value and for all initial conditions. Since we deal with unknown input systems, this definition is not applicable and it cannot be achieved for autonomous systems. In definition 2, we relax this by accepting that the reconstruction of the mode may be possible not for all but for almost all inputs and initial conditions values. To establish the observability of SLS, we have to address, in addition to location observability reduced to the study of the distinguishability of each pair of modes, the state and input observability of each mode as defined classically in Trentelman et al. (2001).

We deal with structured switching linear systems (SSLS) which consider only the structure of modeled system and assume independent all the real parameters of matrices  $A(q), B(q), C(q), D(q)$  for each mode  $q \in Q$  of SLS.

The studied structured state space form is

$$\Sigma_\Lambda : \begin{cases} \dot{x}(t) = A^\lambda(r_t)x(t) + B^\lambda(r_t)u(t) \\ y(t) = C^\lambda(r_t)x(t) + D^\lambda(r_t)u(t) \end{cases} \quad (2)$$

Real parameters of this state-space form are either fixed to zero or assumed to be nonzero parameters. In the latter case, they are substituted by free parameters noted  $\lambda_i$  and the set of these parameters forms vector  $\Lambda = (\lambda_1, \lambda_2, \dots, \lambda_h)^T$  which can take any value in  $\mathbb{R}^h$ .

To study location observability, it is pertinent and necessary to highlight the similarities and the differences between the models associated to these modes. Thus, we decompose each structured matrix  $A^\lambda(q), B^\lambda(q), C^\lambda(q)$  and  $D^\lambda(q)$  for each  $q \in Q$  into two parts: the first one is common to the two modes and the second one is specific to

each mode i.e. for  $q \in Q$ ,  $A^\lambda(q) = A_0^\lambda + A_q^\lambda$ ,  $B^\lambda(q) = B_0^\lambda + B_q^\lambda$ ,  $C^\lambda(q) = C_0^\lambda + C_q^\lambda$  and  $D^\lambda(q) = D_0^\lambda + D_q^\lambda$  where all nonzeros entries of structured matrices are assumed as free parameters  $\lambda_i$  from vector  $\Lambda$ .

According to location observability definition, discrete modes observability is reduces to distinguishability of modes two by two.

For the sake of simplicity, we consider in the later that we have only two modes  $q \in \{1, 2\}$ .

### 3. GRAPHICAL REPRESENTATION OF STRUCTURED SWITCHING LINEAR SYSTEMS

In this subsection, our aim is to present a manner of modeling structure of SSLS ( $\Sigma_\Lambda$ ) taking into account different modes of the system. For a such structure, we can associate in a natural way a directed graph noted  $\mathcal{G}(\Sigma_\Lambda)$  constituted by a non-empty finite set  $\mathcal{V}$  of elements called vertices and a finite set  $\mathcal{E}$  of ordered pairs of distinct vertices called edges (directed edges). Notation  $\mathcal{G}(\Sigma_\Lambda) = (\mathcal{V}, \mathcal{E})$  means that  $\mathcal{V}$  and  $\mathcal{E}$  are respectively vertex set and edge set of  $\mathcal{G}(\Sigma_\Lambda)$ . Vertex set  $\mathcal{V}$  defined by  $\mathcal{V} = \mathbf{X} \cup \mathbf{U} \cup \mathbf{Y}$  corresponds to the system's variables (inputs  $\mathbf{U} = \{\mathbf{u}_1, \dots, \mathbf{u}_m\}$ , states  $\mathbf{X} = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$  and outputs  $\mathbf{Y} = \{\mathbf{y}_1, \dots, \mathbf{y}_p\}$ ) and edge set  $\mathcal{E}$  is defined by  $\mathcal{E}_0 \cup \mathcal{E}_q$ .  $\mathcal{E}_0$  represents the common part of both modes of SSLS and  $\mathcal{E}_q$  represents the specific part for each mode. They can be respectively defined by  $\mathcal{E}_0 = A_0\text{-edges} \cup B_0\text{-edges} \cup C_0\text{-edges} \cup D_0\text{-edges}$ , where,  $A_0\text{-edges} = \{(\mathbf{x}_j, \mathbf{x}_i) \mid A_0(i, j) \neq 0\}$ ,  $B_0\text{-edges} = \{(\mathbf{u}_j, \mathbf{x}_i) \mid B_0(i, j) \neq 0\}$ ,  $C_0\text{-edges} = \{(\mathbf{x}_j, \mathbf{y}_i) \mid C_0(i, j) \neq 0\}$  and  $D_0\text{-edges} = \{(\mathbf{u}_j, \mathbf{y}_i) \mid D_0(i, j) \neq 0\}$  and  $\mathcal{E}_q = A_q\text{-edges} \cup B_q\text{-edges} \cup C_q\text{-edges} \cup D_q\text{-edges}$  for each mode  $q \in \{1, 2\}$ , where,  $A_q\text{-edges} = \{(\mathbf{x}_j, \mathbf{x}_i) \mid A_q(i, j) \neq 0\}$ ,  $B_q\text{-edges} = \{(\mathbf{u}_j, \mathbf{x}_i) \mid B_q(i, j) \neq 0\}$ ,  $C_q\text{-edges} = \{(\mathbf{x}_j, \mathbf{y}_i) \mid C_q(i, j) \neq 0\}$  and  $D_q\text{-edges} = \{(\mathbf{u}_j, \mathbf{y}_i) \mid D_q(i, j) \neq 0\}$ . The existence of free non zero parameters (non-zero entries) of common part ( $A_0^\lambda, B_0^\lambda, C_0^\lambda, D_0^\lambda$ ) of SSLS is represented by edges  $e_0 \in \mathcal{E}_0$  indexed by 0 and the existence of free non-zero parameters of specific part ( $A_q^\lambda, B_q^\lambda, C_q^\lambda, D_q^\lambda$ ) of SSLS is represented by edges  $e_q \in \mathcal{E}_q$  indexed by  $q$  for  $q \in \{1, 2\}$ .

*Example 1.* To the system defined by the following structured matrices, we associate the digraph in Figure 1.

$$A_0^\lambda = \begin{pmatrix} 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_5 & 0 & 0 & 0 & \lambda_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_8 & 0 & 0 & 0 & \lambda_9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$B_0^\lambda = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \lambda_{13} & 0 \\ 0 & 0 \\ 0 & \lambda_{14} \\ 0 & \lambda_{15} \end{pmatrix},$$

$$C_0^\lambda = \begin{pmatrix} \lambda_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_{17} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_{18} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{19} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{20} \end{pmatrix},$$

All the entries of  $A_1^\lambda, B_1^\lambda, B_2^\lambda$  are zero except  $A_1^\lambda(1, 3) = \lambda_{10}$ ,  $B_1^\lambda(10, 2) = \lambda_{11}$ ,  $B_2^\lambda(9, 2) = \lambda_{12}$ . The elements of matrices  $A_2^\lambda, C_1^\lambda, C_2^\lambda, D_0^\lambda, D_1^\lambda$  and  $D_2^\lambda$  are equal to zero.

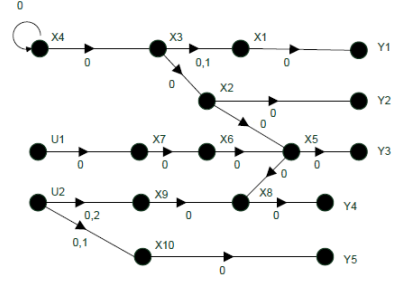


Fig. 1. Digraph associated to system of Example 1

#### 3.1 Notations and definitions

The digraph representing the SSLS is built from the superposition of the digraphs related to each mode. In order to study the properties of the system associated to a specific mode  $q$ , we have to restrict the edge set to  $\mathcal{E}_0 \cup \mathcal{E}_q$ . In this context, many of the functions and specific vertex subsets, defined below, present an index  $q$  related to the considered mode.

- A path  $P$  is denoted  $P = \mathbf{v}_{s_0} \rightarrow \mathbf{v}_{s_1} \rightarrow \dots \rightarrow \mathbf{v}_{s_i}$ , where  $(\mathbf{v}_{s_j}, \mathbf{v}_{s_{j+1}}) \in \mathcal{E}$  for  $j = 0, 1, \dots, i-1$ . We say in this case that  $P$  covers  $\mathbf{v}_{s_0}, \mathbf{v}_{s_1}, \dots, \mathbf{v}_{s_i}$ .
- A path is simple when every vertex occurs only once in this path.
- A cycle is a path of the form  $\mathbf{v}_{s_0} \rightarrow \mathbf{v}_{s_1} \rightarrow \dots \rightarrow \mathbf{v}_{s_i} \rightarrow \mathbf{v}_{s_0}$ , where  $\mathbf{v}_{s_0}, \mathbf{v}_{s_1}, \dots, \mathbf{v}_{s_i}$  are distinct.
- For  $q \in \{1, 2\}$ , we say that path  $P$  is included in  $\mathcal{E}_0 \cup \mathcal{E}_q$  if all its edges are included in  $\mathcal{E}_0 \cup \mathcal{E}_q$ .
- Some paths (resp. cycles) are disjoint if they have no common vertex.
- A set of disjoint cycles is called a cycle family.
- $P$  is a  $\mathbf{Y}$ -topped path if its end vertex belongs to  $\mathbf{Y}$ . A  $\mathbf{Y}$ -topped path family consists of disjoint simple  $\mathbf{Y}$ -topped paths.
- $\mathcal{V}_1$  and  $\mathcal{V}_2$  represent two subsets of  $\mathcal{V}$ . We denote by  $\text{card}(\cdot)$  the cardinality function and  $\mathcal{V}_1 \setminus \mathcal{V}_2$  is the set of elements in  $\mathcal{V}_1$  which are not in  $\mathcal{V}_2$ .
- A path  $P = \mathbf{v}_{s_0} \rightarrow \mathbf{v}_{s_1} \rightarrow \dots \rightarrow \mathbf{v}_{s_i}$  is said a  $\mathcal{V}_1$ - $\mathcal{V}_2$  path if  $\mathbf{v}_{s_0} \in \mathcal{V}_1$  and  $\mathbf{v}_{s_i} \in \mathcal{V}_2$ . Moreover, if the only vertex of  $P$  which belongs to  $\mathcal{V}_1$  is  $\mathbf{v}_{s_0}$  and the only vertex of  $P$  which belongs to  $\mathcal{V}_2$  is  $\mathbf{v}_{s_i}$ ,  $P$  is called a direct  $\mathcal{V}_1$ - $\mathcal{V}_2$  path.
- For  $q = \{1, 2\}$ ,  $\rho_q[\mathcal{V}_1, \mathcal{V}_2]$  is the maximal number of disjoint  $\mathcal{V}_1$ - $\mathcal{V}_2$  paths included in  $\mathcal{E}_0 \cup \mathcal{E}_q$ . Moreover, a set of  $\rho_q[\mathcal{V}_1, \mathcal{V}_2]$  disjoint  $\mathcal{V}_1$ - $\mathcal{V}_2$  paths included in  $\mathcal{E}_0 \cup \mathcal{E}_q$  is a maximum  $\mathcal{V}_1$ - $\mathcal{V}_2$  linkings in  $\mathcal{E}_0 \cup \mathcal{E}_q$ .
- For  $q \in \{1, 2\}$ ,  $\mu_q[\mathcal{V}_1, \mathcal{V}_2]$  denotes the minimal number of vertices of  $\mathbf{U} \cup \mathbf{X} \cup \mathbf{Y}$  belonging to a maximum  $\mathcal{V}_1$ - $\mathcal{V}_2$  linking included in  $\mathcal{E}_0 \cup \mathcal{E}_q$ .
- For  $q \in \{1, 2\}$ ,  $V_{ess,q}[\mathcal{V}_1, \mathcal{V}_2]$  is the vertex subset including the vertices present in all the maximum  $\mathcal{V}_1$ - $\mathcal{V}_2$  linkings included in  $\mathcal{E}_0 \cup \mathcal{E}_q$ .
- For  $q \in \{1, 2\}$ , there exists a unique vertex subset noted  $S_q^o[\mathcal{V}_1, \mathcal{V}_2]$  and called minimum output separator which is the set of begin vertices of all direct  $V_{ess,q}[\mathcal{V}_1, \mathcal{V}_2]$ - $\mathcal{V}_2$  paths included in  $\mathcal{E}_0 \cup \mathcal{E}_q$ .

## 4. RESULTS

### 4.1 Preliminaries

First of all, we begin by introducing some existing results based on several works in (Dion et al. (2003); Commault and Dion (2003); Boukhobza et al. (2007)), in graphical terms, for continuous state and input observability of SSSLs. It characterizes the generic dimension of the observability subspace related to the degeneration of pencil matrix for each mode  $q \in \{1, 2\}$  (due often to invariant properties van der Woude et al. (2003) such as invariant zeros). In this paper, our aim, through a subdivision close to Boukhobza et al. (2007); Boukhobza and Hamelin (2011), is to express propositions to assess the observability of the discrete mode for SSSLs by using some subsets emerged from subdivision of SSSLs into two distinct parts. Towards this end, the following definitions are useful.

**Definition 3.** Consider SSSLs  $(\Sigma_\Lambda)$  associated to digraph  $\mathcal{G}(\Sigma_\Lambda)$ . The following vertex subsets emerge from SSSLs subdivision :

- $\mathbf{X}_{1,q} \stackrel{def}{=} \{\mathbf{x}_i \mid \rho_q[\mathbf{U} \cup \{\mathbf{x}_i\}, \mathbf{Y}] > \rho_q[\mathbf{U}, \mathbf{Y}]\}$ ;
- $\mathbf{Y}_{0,q} \stackrel{def}{=} \mathbf{Y} \cap V_{ess,q}[\mathbf{U}, \mathbf{Y}]$ ;
- $\mathbf{Y}_{1,q} \stackrel{def}{=} \mathbf{Y} \setminus \mathbf{Y}_{0,q}$ ;

In order to rule on discrete mode observability of SSSLs, we should be able to express an algebraic equation linking only output components of  $\mathbf{Y}_{1,q}$  and their derivatives. This equation has to be satisfied by only one of the two modes  $q \in \{1, 2\}$ .

The particularities of each subset proposed above are detailed in Boukhobza and Hamelin (2011).

**Definition 4.** Consider SSSLs  $(\Sigma_\Lambda)$  associated to digraph  $\mathcal{G}(\Sigma_\Lambda)$  for  $q \in \{1, 2\}$ . We associate integer  $\beta_q(\mathbf{Y})$  defined by  $\mu_q[\mathbf{U}, (S_q^o[\mathbf{U}, \mathbf{Y}] \cap \mathbf{X}) \cup \mathbf{Y}_{0,q}] - \rho_q[\mathbf{U}, (S_q^o[\mathbf{U}, \mathbf{Y}] \cap \mathbf{X}) \cup \mathbf{Y}_{0,q}]$  plus the maximal number of vertices of  $\mathbf{X}_{1,q} \cup S_q^o[\mathbf{U}, \mathbf{Y}]$  covered by a disjoint union of :

- a  $S_q^o[\mathbf{U}, \mathbf{Y}] - \mathbf{Y}_{1,q}(\mathbf{Y})$  linking of maximal size;
- a  $\mathbf{Y}_{1,q}$ -topped path family ;
- a cycle family covering only elements of  $\mathbf{X}_{1,q}$ .

As expressed in Lemma 3 of Boukhobza and Hamelin (2011),  $\beta_q(\mathbf{Y})$  is equal to the generic dimension of the observable subspace in the extended state and input space  $(x^T, u^T)$  for each  $q \in \{1, 2\}$ .

**Definition 5.** Consider SSSLs  $(\Sigma_\Lambda)$  associated to digraph  $\mathcal{G}(\Sigma_\Lambda)$  for  $q \in \{1, 2\}$ .

-A q-eligible path is a path ended by  $\mathbf{y}_i$  such as its length is greater than  $d_q(\mathbf{y}_i) = \beta_q(\mathbf{Y}) - \beta_q(\mathbf{Y} \setminus \{\mathbf{y}_i\})$  for  $q \in \{1, 2\}$ .

-A q-eligible edge is the one which belong to eligible path.  
-The sub-digraph  $\mathcal{G}_{el}(\Sigma_\Lambda)$  denotes eligible digraph that contains only q-eligible edges.

**Definition 6.** Consider SSSLs  $(\Sigma_\Lambda)$  associated to digraph  $\mathcal{G}(\Sigma_\Lambda)$  for  $q \in \{1, 2\}$ .  $\mathbf{V}(\mathbf{q})_{el} = \{\mathbf{v}_i \text{ is a beginning vertex of eligible path}\}$

### 4.2 Discrete mode observability of SSSLs

**Hypothesis 1.** SSSLs is assumed to be continuous state and input observable for each mode  $q \in \{1, 2\}$  as expressed in Boukhobza et al. (2007).

In this part, we will treat the observability of the discrete mode for SSSLs with unknown input and so based only on the measurements given by the output set  $\mathbf{Y}$ .

**Proposition 1.** A sufficient condition for location observability of SSSLs represented by digraph  $\mathcal{G}(\Sigma_\Lambda)$  is:

- There exists  $e_\kappa = (\mathbf{v}_i, \mathbf{v}_j)$  a specific edge of one mode  $q \in \{1, 2\}$  and  $\mathbf{y}_i \in \mathbf{Y}_{1,q}$  such that there exists  $\mathbf{y}_i$ -topped path of length strictly greater than  $d_q(\mathbf{y}_i) = \beta_q(\mathbf{Y}) - \beta_q(\mathbf{Y} \setminus \{\mathbf{y}_i\})$  which covers  $e_\kappa$  ended by vertex  $\mathbf{v}_j$ .
- $\mathbf{v}_i$  belongs to a direct  $S_q^o[\mathbf{V}(\mathbf{q})_{el}, \mathbf{Y}_{1,q}] - \mathbf{y}_i$  path included in  $\mathcal{E}_0 \cup \mathcal{E}_q$  on  $\mathcal{G}_{el}(\Sigma_\Lambda)$ .

**Proof:**

**Sufficiency:**

The fact that, for some  $q$ ,  $\mathbf{y}_i$  belongs to  $\mathbf{Y}_{1,q}$  implies that there exists a vertex subset  $\mathbf{Y}_u \subseteq \mathbf{Y}_{1,q} \setminus \{\mathbf{y}_i\}$  such that  $\rho_q[S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}, \mathbf{Y}_u]] = \text{card}(\mathbf{U} \cup \mathbf{S}_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}, \mathbf{Y}_u])$ . This implies, from Lemma 2 of Boukhobza and Hamelin (2011), that there exist a matrix  $G$ , a function  $\varphi$  and an integer  $\nu \leq n_1$  such that the dynamics equation of subsystem  $(\Sigma_{1,q})$  can be put on form:

$$\begin{aligned} \dot{X}_{1,q} &= (A_{1,1} + (A_{1,s}, B_{1,1})G)X_{1,q} \\ &\quad + \varphi_x(Y_u, \dot{Y}_u, \dots, Y_u^{(\nu)}) \\ &\stackrel{def}{=} \tilde{A}X_{1,q} + \varphi_x(Y_u, \dot{Y}_u, \dots, Y_u^{(\nu)}) \\ Y_{1,q} &= (C_{1,1} + (C_{1,s}, D_{1,1})G)X_{1,q} \\ &\quad + \varphi_y(Y_u, \dot{Y}_u, \dots, Y_u^{(\nu)}) \\ &\stackrel{def}{=} \tilde{C}X_{1,q} + \varphi_y(Y_u, \dot{Y}_u, \dots, Y_u^{(\nu)}) \end{aligned} \quad (3)$$

Moreover, by definition of  $d_q(\mathbf{y}_i)$ , we have that, there exists a minimal subset  $\tilde{\mathbf{Y}} \subseteq \mathbf{Y}_{1,q}(\mathbf{Y}) \setminus (\mathbf{Y}_u \cup \{\mathbf{y}_i\})$ , such that  $\forall k \geq d_q(\mathbf{y}_i)$ ,

$$y_i^{(k)} = \sum_{s < k_i} \alpha_{i,s} y_i^{(s)} + \sum_{l \mid \mathbf{y}_l \in \tilde{\mathbf{Y}} \atop s=0}^{n_1} \alpha_{l,s} y_l^{(s)} + v(Y_u, \dots, Y_u^{(n_1)}) \quad (4)$$

where  $n_1 = \text{card}(\mathbf{X}_{1,q})$ . Since subset  $\tilde{\mathbf{Y}}$  is minimal *i.e.*  $\forall \mathbf{y}_j \in \tilde{\mathbf{Y}}, \beta_q((\tilde{\mathbf{Y}} \cup \{\mathbf{y}_i\}) \cup \mathbf{Y}_u) \setminus \{\mathbf{y}_j\} - \beta_q(\mathbf{Y}_u \cup \tilde{\mathbf{Y}} \setminus \{\mathbf{y}_j\}) > k_i$ , then in relation (4), all the components of  $\tilde{\mathbf{Y}}$  are present. Let us denote by  $\mathbf{x}_j \in \mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}$  the begin vertex of the so-called path  $P$  satisfying condition of Proposition 1 (*i.e.*  $P$  is a  $\mathbf{y}_i$ -topped path of length  $k+1$  strictly greater than  $d_q(\mathbf{y}_i)$  and covering  $e_s$ ) and  $e_j$  the  $j^{th}$  Euclidean vector. Relation (4) can be written as:

$$\tilde{C}_i \tilde{A}^k e_j = \left( \sum_{s < d_q(\mathbf{y}_i)} \alpha_{i,s} \tilde{C}_i \tilde{A}^s + \sum_{\mathbf{y}_l \in \tilde{\mathbf{Y}} \atop s=0}^{n_1} \alpha_{l,s} \tilde{C}_l \tilde{A}^s + v(Y_u, \dots, Y_u^{(n_1)}) \right) e_j \quad (5)$$

where each non-zero component of  $\tilde{C}_l \tilde{A}^s$  is associated to the paths arriving to  $\mathbf{y}_l \in \tilde{\mathbf{Y}}$  of length  $s+1$ . Since all the  $\{\mathbf{x}_j\} - \tilde{\mathbf{Y}} \cup \{\mathbf{y}_i\}$  paths starting from  $\mathbf{x}_j$  cover, by definition,  $S_q^o[\{\mathbf{x}_j\}, \tilde{\mathbf{Y}} \cup \{\mathbf{y}_i\}] \stackrel{def}{=} \{\mathbf{x}_r\}$ , then there exist  $k_r$  and  $k'$  such that  $k_r + k' = k$  and  $\tilde{C}_i \tilde{A}^k e_j = \tilde{C}_i \tilde{A}^{k_r} \Delta_r \tilde{A}^{k'} e_j$  where  $\Delta_r$  is a diagonal matrix which has only one non-zero element  $\Delta_r(r, r) = 1$ . We can do the same reasoning for each term  $\tilde{C}_i \tilde{A}^s e_j$  and so there exist  $s_r$  and  $s'$  such that  $s_r + s' = s$

and  $\tilde{C}_l \tilde{A}^s e_j = \tilde{C}_l \tilde{A}^{sr} \Delta_r \tilde{A}^{s'} e_j$ . The fact that end vertex of  $e_\kappa$  i.e.  $\mathbf{x}_\ell$  belongs to a direct  $S_q^o[\{\mathbf{x}_j\}, \mathbf{Y}_{1,q}]$ - $\mathbf{y}_i$  path implies that specific edge  $e_\kappa \in \mathcal{E}_q$  belongs to a  $S_q^o[\mathbf{x}_j, \tilde{\mathbf{Y}} \cup \{\mathbf{y}_i\}]$ - $\tilde{\mathbf{Y}} \cup \{\mathbf{y}_i\}$  path. This means that edge  $e_\kappa$  appears in only some  $S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}, \tilde{\mathbf{Y}} \cup \{\mathbf{y}_i\}]$ - $\tilde{\mathbf{Y}} \cup \{\mathbf{y}_i\}$  paths. Thus, some terms  $\tilde{C}_i A^{kr}$  and  $\tilde{C}_l \tilde{A}^{sr}$ , but not all, contain the non-zero parameter corresponding to edge  $e_\kappa$ , which is specific to mode  $q$ . Denoting by  $C_r = \mathbf{e}_r^T$ , where  $\mathbf{e}_r$  is the  $r$ <sup>th</sup> Euclidean vector, we have that  $\tilde{C}_i \tilde{A}^k e_j = \tilde{C}_i A^{kr} \Delta_r \tilde{A}^{s'} e_j = \alpha' C_r \tilde{A}^{k'} e_j$  and  $\tilde{C}_l \tilde{A}^s e_j = \tilde{C}_l \tilde{A}^{sr} \Delta_r \tilde{A}^{s'} e_j = \alpha'_{l,s} C_r \tilde{A}^{s'} e_j$ . Thus, after substitution of the previous terms in relation (5),

$$\alpha' C_r \tilde{A}^{k'} e_j = \left( \sum_{s_r \leq s < d_q(\mathbf{y}_i)} \alpha'_{i,s} \alpha_{i,s} C_r \tilde{A}^{s-s_r} + \sum_{l | \mathbf{y}_l \in \tilde{\mathbf{Y}}^{s=s_r}} \sum_{n_1} \alpha'_{l,s} \alpha_{l,s} C_r \tilde{A}^{s-s_r} + v(Y_u, \dot{Y}_u, \dots, Y_u^{(n)}) \right) e_j \quad (6)$$

where some coefficients  $\alpha'$  and  $\alpha'_{l,s}$  but not all depend on the weight of  $e_\kappa$ . This weight cannot be factorized and simplified because all the coefficients do not depend on it (some  $S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}, \tilde{\mathbf{Y}} \cup \{\mathbf{y}_i\}]$ - $\tilde{\mathbf{Y}} \cup \{\mathbf{y}_i\}$  paths do not contain edge  $e_\kappa$ ). Therefore, equality (6) is valid only if some of the coefficients  $\alpha$ ,  $\alpha_{i,s}$  and  $\alpha_{l,s}$  depend also on the weight  $\lambda_\kappa$  of  $e_\kappa$ . Thus, by means of equation (4) in which appear coefficients  $\alpha_{i,s}$  and  $\alpha_{l,s}$ , we obtain an algebraic relation depending on  $\lambda_\kappa$  and satisfied only when the discrete mode variable is equal to  $q$ .  $\square$

In **Example 1** above, note that the corresponding  $\mathcal{G}_{el}(\Sigma_\Lambda)$  is the same as  $\mathcal{G}(\Sigma_\Lambda)$  since all the edges in  $\mathcal{G}(\Sigma_\Lambda)$  are eligible for  $q \in \{1, 2\}$  according to definition 5. In this case, we have that for  $q \in \{1, 2\}$ ,  $\mathbf{X}_{1,q} = \{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{x}_8, \mathbf{x}_9, \mathbf{x}_{10}\}$ ,  $\mathbf{Y}_{0,q} = \mathbf{0}$ ,  $\mathbf{Y}_{1,q} = Y$  and  $\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el} = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{x}_3, \mathbf{x}_4\}$  thus  $S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}, \mathbf{Y}_{1,q}] = \{\mathbf{u}_2, \mathbf{x}_3, \mathbf{x}_5\}$ .

As example,  $(\mathbf{x}_3, \mathbf{x}_1)$  is a specific edge to mode 1. Since, for both modes,  $S_q^o[\mathbf{U}, \mathbf{Y}] \cap \mathbf{X} = \{\mathbf{x}_5\}$ , we can then calculate  $\mu_q[\mathbf{U}, (S_q^o[\mathbf{U}, \mathbf{Y}] \cap \mathbf{X}) \cup \mathbf{Y}_{0,q}] - \rho_q[\mathbf{U}, (S_q^o[\mathbf{U}, \mathbf{Y}] \cap \mathbf{X}) \cup \mathbf{Y}_{0,q}] = 4 - 1 = 3$ . In this case, we have that  $d_1(\mathbf{y}_1) = \beta_1(\mathbf{Y}) - \beta_1(\mathbf{Y} \setminus \{\mathbf{y}_1\}) = 12 - 11 = 1$ , this implies that output  $y_1$  allows us to observe  $d_1(\mathbf{y}_1) = 1$  new directions.

Let us search a  $\mathbf{y}_1$ -topped path  $P$  which length is strictly greater than  $d_1(\mathbf{y}_1)$  and including specific edge  $(\mathbf{x}_3, \mathbf{x}_1)$ , we can choose  $P = \mathbf{x}_3 \rightarrow \mathbf{x}_1 \rightarrow \mathbf{y}_1$ , whose length is equal to 2.

We also have that the ending vertex  $\mathbf{x}_1$  of the specific edge  $(\mathbf{x}_3, \mathbf{x}_1)$  for mode 1 belongs to a  $S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}, \mathbf{Y}_{1,q}]$ - $\mathbf{y}_1$  path. So, Conditions of Proposition 1 are satisfied.

In the same manner, if we take  $(\mathbf{u}_2, \mathbf{x}_9)$  as a specific edge of mode 2, we have  $d_1(\mathbf{y}_4) = \beta_1(\mathbf{Y}) - \beta_1(\mathbf{Y} \setminus \{\mathbf{y}_4\}) = 12 - 10 = 2$ . Let us find a path  $P$  of length strictly greater than  $d_1(\mathbf{y}_4)$  and including specific edge  $(\mathbf{u}_2, \mathbf{x}_9)$ , we can choose  $P = \mathbf{u}_2 \rightarrow \mathbf{x}_9 \rightarrow \mathbf{x}_8 \rightarrow \mathbf{y}_4$  which length is equal to 3. We have also that the ending vertex  $\mathbf{x}_9$  of the specific edge  $(\mathbf{u}_2, \mathbf{x}_9)$  belongs to a  $S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}, \mathbf{Y}_{1,q}]$ - $\mathbf{y}_4$  path. (So, conditions of proposition 1 are satisfied).

When the proposition 1 is not satisfied, an additional sensor is required to recover discrete mode observability of SSLS. To do so, we define a new output vector  $\mathbf{Z}$  representing the additional sensors collecting new measurements  $z(t) = H_x^\lambda(r_t)x(t) + H_u^\lambda(r_t)u(t)$ . The completed system

is denoted by  $\Sigma_\Lambda^c$  and a structured state-space form is as following:

$$\Sigma_\Lambda^c : \begin{cases} \dot{x}(t) = A^\lambda(r_t)x(t) + B^\lambda(r_t)u(t) \\ y(t) = C^\lambda(r_t)x(t) + D^\lambda(r_t)u(t) \\ z(t) = H_x^\lambda(r_t)x(t) + H_u^\lambda(r_t)u(t) \end{cases} \quad (7)$$

The additional sensor components can be represented by vertex set  $Z$  and edge subsets  $H_x$ -edges and  $H_u$ -edges.

**Proposition 2.** Consider SSLS ( $\Sigma_\Lambda$ ) associated to digraph  $\mathcal{G}(\Sigma_\Lambda)$ , under assumption that SSLS is continuous state and input observable for each mode  $q \in \{1, 2\}$  and when conditions of Proposition 1 are not satisfied.

Let  $\mathbf{v}_i \in \mathcal{V}_i$  be the ending vertex of a specific and eligible edge associated to one mode  $q \in \{1, 2\}$  and  $\mathbf{v}_l \in V(q)_{el}$  the beginning vertex of eligible path,  $\mathbf{v}_i$  belongs to any direct  $\{\mathbf{v}_l\} - S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}, \mathbf{Y}_{1,q}]$  path. In order to ensure that  $\mathbf{v}_i$  belongs to direct  $S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}, \mathbf{Y}_{1,q}] - \mathbf{Y}_{1,q}$  path, an additional sensor  $\mathbf{z}_i \in Z$  has to be placed on vertex  $\mathbf{v}_l \in \mathcal{V}_l$  such that  $\mathcal{V}_i$  and  $\mathcal{V}_l$  a strongly connected components and  $\mathcal{V}_i \leq \mathcal{V}_l$ .

After sensor placement, the length condition has to be checked in order to satisfy **Proposition 1**.

**Proof.**

Let  $\mathbf{v}_i \in \mathcal{V}_i$  be the ending vertex of a specific edge for one mode  $q \in \{1, 2\}$  and  $\mathbf{v}_l \in \mathcal{V}_l$  such that  $\mathbf{v}_i$  belongs to any direct  $\{\mathbf{v}_l\} - S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}, \mathbf{Y}_{1,q}]$  path and  $\mathcal{V}_i \leq \mathcal{V}_l$ .

A sensor placement  $\mathbf{z}_i \in Z$  on  $\mathbf{v}_l \in \mathcal{V}_l$  such that  $S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}, \mathbf{Y}_{1,q}] \leq \mathcal{V}_i$  makes elements from  $S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}, \mathbf{Y}_{1,q}]$  non essential. Due to the addition of a new direct  $\{\mathbf{v}_l\} - \{\mathbf{z}_i\}$  path, we have then  $S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}, \mathbf{Y}_{1,q}] \leq S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}, \mathbf{Y}_{1,q} \cup \{\mathbf{z}_i\}]$ , so,  $\mathbf{v}_i$  belongs to a direct  $S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}, \mathbf{Y}_{1,q} \cup \{\mathbf{z}_i\}] - \mathbf{Y}_{1,q}$  path. The same reasoning can be done when an additional sensor  $\mathbf{z}_l \in Z$  is placed to measure  $\mathbf{v}_l \in \mathcal{V}_l$  such that  $\mathcal{V}_i \leq \mathcal{V}_l$ .

**Comments and interpretation**

An additional sensor does not add a path from input  $\mathbf{U}$  to output  $\mathbf{Y}$  and so the cardinality of minimal output separator does not increase. The aim of Proposition 2 is to formalize sensor placement problem in order to have the ending vertex of specific edge belonging to a direct  $S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{el}, \mathbf{Y}_{1,q}]$ - $\mathbf{y}_i$  path included in  $\mathcal{E}_0 \cup \mathcal{E}_q$  under constraint that the length condition have to be checked after this placement.  $\square$

Consider **Example 2** which keeps the same common part  $(A_0^\lambda, B_0^\lambda, C_0^\lambda, D_0^\lambda)$  of SSLS of example 1 and consider new entries  $A_1^\lambda(3, 4) = \lambda_{21}$  for mode 1 and  $A_2^\lambda(6, 7) = \lambda_{22}$  for mode 2. The figure below shows the digraph's structure of example 2.

Ending vertex  $\mathbf{x}_3$  of specific and eligible edge  $(\mathbf{x}_4, \mathbf{x}_3)$  of

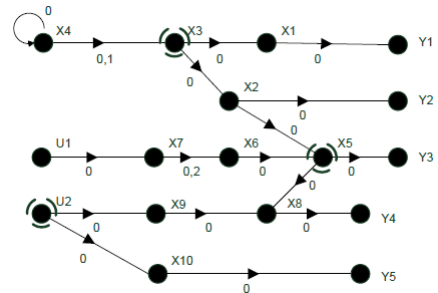


Fig. 2. Digraph associated to system of Example 2

## REFERENCES

mode 1 and ending vertex  $\mathbf{x}_6$  of a specific and eligible edge  $(\mathbf{x}_7, \mathbf{x}_6)$  of mode 2 do not belong respectively to  $S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{e1}, \mathbf{Y}_{1,q}] - \mathbf{y}_1$  path and  $S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{e1}, \mathbf{Y}_{1,q}] - \mathbf{y}_4$  path for both modes  $q \in \{1, 2\}$ . So the condition of **Proposition 1** is not satisfied.

In order to satisfy these conditions to recover the discrete mode observability of SSSL of example 2, spreading procedure of **Proposition 2** is needed.

- For mode 1, if an additional sensor  $\mathbf{z}_1$  is placed to measure state vertex  $\mathbf{x}_4$  then  $\mathbf{x}_3$  belongs to a  $S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{e1}, \mathbf{Y}_{1,q}] - \mathbf{y}_1$  path. After that, the length condition should be verified. We have that  $d_1(\mathbf{y}_1) = \beta_1(\mathbf{Y} \cup \{\mathbf{z}_1\}) - \beta_1(\mathbf{Y} \setminus \{\mathbf{y}_1\}) = 12 - 11 = 1$ . We choose then an eligible simple  $\mathbf{y}_1$ -topped path  $P = \mathbf{x}_4 \rightarrow \mathbf{x}_3 \rightarrow \mathbf{x}_1 \rightarrow \mathbf{y}_1$  which covers specific and eligible edge  $(\mathbf{x}_4, \mathbf{x}_3)$  of mode 1 and its length is greater than 2. Condition of **Proposition 1** is then satisfied.

- For mode 2, if an additional sensor  $\mathbf{z}_2$  is placed to measure state vertex  $\mathbf{x}_7$  or  $\mathbf{u}_1$  then  $\mathbf{x}_6$  belongs to a  $S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{e1}, \mathbf{Y}_{1,q}] - \mathbf{y}_4$  path. After that, the length condition should be verified. We have that  $d_2(\mathbf{y}_4) = \beta_2(\mathbf{Y} \cup \{\mathbf{z}_2\}) - \beta_2(\mathbf{Y} \setminus \{\mathbf{y}_4\}) = 12 - 10 = 2$ . We choose then an eligible simple  $\mathbf{y}_4$ -topped path  $P = \mathbf{x}_7 \rightarrow \mathbf{x}_6 \rightarrow \mathbf{x}_5 \rightarrow \mathbf{x}_8 \rightarrow \mathbf{y}_4$  which covers specific and eligible edge  $(\mathbf{x}_7, \mathbf{x}_6)$  of mode 2 and its length is greater than 3. The condition of **Proposition 1** is then satisfied. Figure 3 illustrates sensors placement of example 2 to recover the discrete mode discernability.

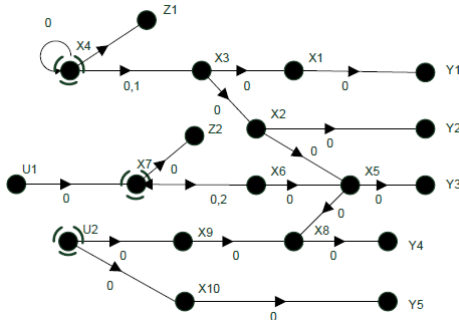


Fig. 3. Sensor placement recovering discrete mode observability of SSSL

Note that after sensor placement, the output separator (illustrated by dashed line circles) is  $S_q^o[\mathbf{U} \cup \mathbf{V}(\mathbf{q})_{e1}, \mathbf{Y}_1 \cup \{\mathbf{z}_1, \mathbf{z}_2\}] = \{\mathbf{u}_2, \mathbf{x}_4, \mathbf{x}_7\}$ .

## 5. CONCLUSION

This paper proposes, through an intuitive graphical approach, a sufficient condition to investigate discrete mode observability of switching structured linear systems with unknown input. Under assumption that continuous state and input of SSSL are observable as it is widely treated in Boukhobza et al. (2007), we propose a graphical criterion of sensor placement to recover the discrete mode observability of SSSL when this property is not satisfied.

Rule on this property, only through the knowledge of the structure of the system, makes our approach, using graph-theoretical techniques, interesting for the analysis of solvability conditions for FDI problem which is linked to observability property.

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