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Abstract The paper investigates the effect of soil-structure interaction on the dynamic response of structures. A non-parametric probabilistic formulation for the modelling of an uncertain soil impedance is used to account for the usual lack of information on soil properties. Such a probabilistic model introduces the physical coupling stemming from the soil heterogeneity around the foundation. Considering this effect, even a symmetrical building displays a torsional motion when submitted to earthquake loading. The study focuses on a multi-story building modeled by using equivalent Timoshenko beam models which have different mass distributions. The probability density functions of the maximal internal forces and moments in a given building are estimated by Monte Carlo simulations. Some results on the stochastic modal analysis of the structure are also given.

Keywords soil-structure interaction · uncertainties · internal forces · seismic loads

1 Introduction

This paper deals with the simulation of the dynamic response of a building under seismic loads with uncertainties on the soil-structure interaction modeling.

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The soil-structure interaction is often taken into account under the assumption of a rigidly moving foundation. Then, the relation between the degrees of freedom of the foundation and the forces and moments applied by the soil on the foundation are then modeled by a soil impedance matrix that can be computed by integration of the three-dimensional equations of elasticity within a half-space soil domain or estimated by simplified methods. In [1], for instance, an easy-to-use engineering model is built to obtain an estimation of the impedance from a compilation of complete computations. It involves simple analytical equations for evaluating the impedance matrix, taking into account the shape of the foundation and the nature of the soil. The ground around and below the foundation is however neither homogeneous nor isotropic and few informations are usually available about the real soil properties. This usual lack of information on dynamic soil properties induces uncertainties on the soil-structure interaction. Such uncertainties have to be taken into account in order to improve the ability of the numerical model for prediction. Obviously, the definition of the seismic loadings include also many uncertainties. In this paper, we restrict the study to uncertainties coming from the foundation soils.

Among the different methods that are available in the literature, the probabilistic approach is the most suitable for modeling these uncertainties. A parametric probabilistic approach would consist in modeling the uncertain parameters of soil impedance model by random variables or random fields. Such an approach has been proposed by [2] for a Winkler [3] foundation model. It has been shown that a coupling between translation and torsion of the foundation could then occur even if the foundation has symmetry geometrical properties. Another example of parametric probabilistic formulation of the uncertainties on the soil interaction parameters can be found in [4] where the uncertain mechanical properties of the soil are modeled by random fields. Nevertheless, it is obvious that a parametric probabilistic approach cannot be used to model the uncertainties that are not related to parameters of the model. This is the case if mechanical simplifications are assumed for the construction of the soil impedance model. These simplifications are usually due to a high complexity level of the real physical soil impedance or it can also be due to a lack of experimental measurements on the properties of the soil and of the foundation. Nevertheless, a non-parametric probabilistic formulation allows to quantify these uncertainties. It consists in modeling the uncertain soil impedance by a random matrix for which the probability distribution is explicitly constructed by using the information theory [5,6,7,8]. In such an approach, the parameters of the soil impedance are not modeled by random variables or random fields anymore. Such a construction has been introduced in [9,10] for the generalized mass, stiffness and damping matrices of a mechanical dynamical system and extended in [11]for modelling uncertainties in the case of nonlinear dynamics of building submitted to seismic loads. In this paper, we extend this non-parametric probabilistic formulation to the uncertainties related to the soil impedance matrix modeling, along the lines introduced by [12]. The information theory is used to construct the probabilistic model of the random soil impedance matrix, but contrarily to [12], a simplified version of the impedance is used, which is nearer to the usual engineering practice.

In section 2, an engineering model for the soil structure interaction is presented. Then, two simplified mechanical models of multi-story buildings with a rigid foundation are considered. These models involve Timoshenko beams. A widely used beam model in engineering is a "lumped mass model" (referred to as model 1 in the paper) made up of masses that are located at the top ends of weightless Timoshenko beam elements. A more sophisticated lumped mass model allowing torsional effects to be taken into account can be found in [13]. In this paper, we present a new Timoshenko beam model with a homogeneous mass distribution (referred to as model 2 in the paper). Then, a non-parametric probabilistic model of the uncertain soil impedance is constructed. The random dynamic model for the soil structure interaction is then constructed by replacing the soil impedance matrix by a random matrix for which the probability density function is constructed by using the maximum entropy principle [5,6,7,8] with the available information (that is related with the algebraic properties of the impedance matrix: its mean value defined by the engineering model and algebraic conditions for the existence of a second order random solution of the random model). Then, some computational results are presented for a multi-story building. The probability density functions of the maximal internal forces and moments inside a building are estimated by the use of the Monte Carlo method [14].

2 Deterministic model of the structure and of the soil-structure interaction

2.1 Impedance of the soil-structure interaction

In this section, the deterministic model (which will be considered hereafter as the mean model in the probability sense) used for modeling a multi-story building with a rigid foundation and with soil-structure interaction under seismic load is presented. Hereafter, the foundation is considered as a rigid body. Consequently, the impedance in the frequency domain is a complex symmetric 6×6 matrix $[\underline{Z}(\omega)] = [\underline{K}_S] + i\omega\, [\underline{D}_S]$ where $[\underline{K}_S]$ and $[\underline{D}_S]$ are positive-definite symmetric matrices that characterize the stiffness and the damping of the soil interaction. From a general point of view, these matrices are frequency dependent [1]. However, in the following, we use the approximation that these matrices are frequency independent. Such an approximation is indeed very often used by engineers.

2.2 Mass matrix of the foundation and added mass matrix of the soil-structure interaction

The inertia of the foundation is defined by a positive-definite symmetric 6×6 matrix $[\underline{M}_f]$ which is block diagonal. The first 3×3 block is the diagonal matrix $m_f\,[I_3]$ where m_f is the mass of the foundation and $[I_3]$ is the identity matrix of $\mathbb{R}^3$. The last 3×3 block is the inertia matrix of the foundation with respect to the axes $(O\boldsymbol{e}_1)$, $(O\boldsymbol{e}_2)$ and $(O\boldsymbol{e}_3)$. In addition, let the positive-definite symmetric 6×6 matrix $[\underline{M}_a]$ be the added mass matrix corresponding to the mass of the soil activated by the foundation due to soil-structure interaction. We then introduce the positive-definite symmetric 6×6 matrix $[\underline{M}_S]$ which is written as $[\underline{M}_S] = [\underline{M}_f] + [\underline{M}_a]$.

2.3 Model 1 for the structure: lumped mass beam model

The occurrence of an earthquake is modeled by introducing a far free-field ground motion. This free-field motion is characterized by an accelerogram $\mathbf{a}_g(t) = (a_1(t), a_2(t), a_3(t))$ in which $a_1(t)$, $a_2(t)$ and $a_3(t)$ are the acceleration components of the free-field ground motion along axes $(O\boldsymbol{e}_1)$, $(O\boldsymbol{e}_2)$ and $(O\boldsymbol{e}_3)$. The displacement of the foundation is then described with respect to a non-inertial frame that moves with the free-field soil acceleration.

It can be considered that the constitutive equations of a multi-story building are asymptotically equivalent to the constitutive equations of a Timoshenko beam or a flexure beam [15, 16]. Consequently, Timoshenko beam models are widely used in order to compute the dynamical response of multi-story buildings.

A commonly used model is the lumped mass beam model (see Figure 1) for which the ceilings are connected by weightless Timoshenko beams and each ceiling is represented as a rigid body. Let $\mathbf{u}_0(t) \in \mathbb{R}^3$ and $\boldsymbol{\theta}_0(t) \in \mathbb{R}^3$ be the displacement vector and the rotation vector of the center of the foundation. The displacement vector of the node located at the center of the n-th ceiling is denoted by $\mathbf{u}_n(t) = (u_{n,1}(t), u_{n,2}(t), u_{n,3}(t))$ and the rotation vector of the n-th ceiling is denoted by $\boldsymbol{\theta}_n(t) = (\theta_{n,1}(t), \theta_{n,2}(t), \theta_{n,3}(t))$. The building (with its foundation) is therefore modeled by $N_{\text{sto}} + 1$ rigid bodies (N_{sto} ceilings and one foundation) and the number of degrees of freedom of the beam model is $N_{DOF} = 6\,(N_{\text{sto}} + 1)$. Then, the effective stiffness parameters of the beam element are obtained from the properties of constitutive elements.

The reduced lumped mass beam model is built by using the modal representation; it means that the displacement along the structure $\mathbb{u}(t) = (\mathbf{u}_0(t), \boldsymbol{\theta}_0(t), \ldots, \mathbf{u}_{N_{\text{sto}}}(t), \boldsymbol{\theta}_{N_{\text{sto}}}(t)) \in \mathbb{R}^{N_{DOF}}$ is given by:

$$\mathbb{u}(t) \simeq \sum_{\alpha=1}^{N} q_\alpha(t)\, \boldsymbol{\Phi}_\alpha \quad , \tag{1}$$

where $q_\alpha(t)$ are the generalized coordinates of $\mathbb{u}(t)$ within the basis of eigenvectors $\boldsymbol{\Phi}_\alpha = (\boldsymbol{\phi}_{0,\alpha}, \ldots, \boldsymbol{\phi}_{N_{\text{sto}},\alpha})$ of the building with its foundation coupled with the soil. More details on the construction of this reduced lumped mass beam model can be found in Appendix A. Let the vector $\mathbf{q}(\mathbf{t}) = (q_1(t), \ldots, q_N(t))$ be the vector of the generalized coordinates. Then, it can be shown that

$$([\mathscr{M}] + [\mathscr{M}_S])\; \ddot{\boldsymbol{q}}(t) + [\mathscr{D}_S]\; \dot{\boldsymbol{q}}(t) + ([\mathscr{K}] + [\mathscr{K}_S])\; \boldsymbol{q}(t) = \boldsymbol{f}(t) + \boldsymbol{f}_S(t) \tag{2}$$

with the initial conditions

$$\boldsymbol{q}(0) = 0 \text{ and } \dot{\boldsymbol{q}}(0) = 0 \tag{3}$$

where the positive-definite symmetric $N \times N$ real matrices $[\mathscr{M}]$, $[\mathscr{M}_S]$, $[\mathscr{D}_S]$, $[\mathscr{K}_S]$, the positive symmetric $N \times N$ real matrix $[\mathscr{K}]$ and the vectors of the generalized external forces $\boldsymbol{f}(t)$ and $\boldsymbol{f}_S(t)$ are explicitly given in Appendix A.

2.4 Model 2 for the structure: Timoshenko beam model with a homogeneous mass distribution

Hereinafter, an alternative Timoshenko beam model is presented. The building is modeled by only one equivalent Timoshenko beam with a homogeneously distributed mass. The foundation is connected to the lower part of the beam. Since the inertia of the Timoshenko beam is taken into account, the Timoshenko beam dynamic equations are used. The kinematics is then described by the displacement field $\boldsymbol{u}_G(x_1,t) = (u_1(x_1,t), u_2(x_1,t), u_3(x_1,t))$ of the center $G(x_1)$ of the cross section located at position x_1 and by the rotation of the cross section $\boldsymbol{\theta}(x_1,t) = (\theta_1(x_1,t),\, \theta_2(x_1,t),\, \theta_3(x_1,t))$. The beam section displacement field $\boldsymbol{u}$ is a rigid displacement field that can be written as $\boldsymbol{u}(x_1,x_2,x_3,t) = \boldsymbol{u}_G(x_1,t) + \boldsymbol{\theta}(x_1,t) \times (0,x_2,x_3)$. The boundary value problem of the beam model is then written as

$$\rho S \ddot{u}_1 - S\,E\,u_1'' = -\rho\,S\,a_1 \tag{4}$$

$$\rho S \ddot{u}_2 - G k_2\,S(u_2'' - \theta_3') = -\rho\,S\,a_2 \tag{5}$$

$$\rho S \ddot{u}_3 - G\,k_3\,S\,(u_3'' + \theta_2') = -\rho\,S\,a_3 \tag{6}$$

$$\rho I_1 \ddot{\theta}_1 - G\,I_1\,\theta_1'' = 0 \tag{7}$$

$$\rho I_2 \ddot{\theta}_2 + G\,k_3\,S\,(u_3' + \theta_2) - E\,I_2\,\theta_2'' = 0 \tag{8}$$

$$\rho I_3 \ddot{\theta}_3 - G\,k_2\,S\,(u_2' - \theta_3) - E\,I_3\,\theta_3'' = 0 \tag{9}$$

These equations involve the inertia parameters and the stiffness parameters of the equivalent beam. The mass per unit length is $\rho\,S$ and the mass moment of inertia per unit length with respect to the axes of $\mathscr{R}_g$ are $\rho\,I_1$, $\rho\,I_2$ and $\rho\,I_3$. The axial stiffness is SE, the shearing stiffnesses are $G\,k_2\,S$ and $G\,k_3\,S$, the torsion stiffness is $G\,I_1$ and the bending stiffnesses are $E\,I_2$ and $E\,I_3$.
The internal force $\boldsymbol{f}_{\text{int}}(x_1,t)$ and the internal moment $\boldsymbol{m}_{\text{int}}(x_1,t)$ are written as

$$\boldsymbol{f}_{\text{int}}(x_1,t) = \begin{pmatrix} S\,E\,u_1' \\ G\,k_2 S\,(u_2' - \theta_3) \\ G\,k_3 S\,(u_3' + \theta_2) \end{pmatrix} \quad , \quad \boldsymbol{m}_{\text{int}}(x_1,t) = \begin{pmatrix} G\,I_1\,\theta_1' \\ E\,I_2\,\theta_2' \\ E\,I_3\,\theta_3' \end{pmatrix} \tag{10}$$

Since the top of the building ($x_1 = h$) is free, we then have the boundary conditions

$$\boldsymbol{f}_{\text{int}}(h,t) = \boldsymbol{0} \;\text{ and }\; \boldsymbol{m}_{\text{int}}(h,t) = \boldsymbol{0} \tag{11}$$

The lower part of the building is connected with the rigid foundation which is coupled with the soil. We then have

$$[\underline{M}_S]\,\ddot{\boldsymbol{u}}_0(t) + [\underline{D}_S]\,\dot{\boldsymbol{u}}_0(t) + [\underline{K}_S]\,\boldsymbol{u}_0(t) = \boldsymbol{f}_0(t) - [\underline{M}_S]\,\mathbb{a}(t) \tag{12}$$

where $\boldsymbol{u}_0(t) = (\boldsymbol{u}_G(0,t), \boldsymbol{\theta}(0,t))$ and $\boldsymbol{f}_0(t) = (\boldsymbol{f}_{\rm int}(0,t), \boldsymbol{m}_{\rm int}(0,t))$ are the vector of the degrees of freedom and the vector of the internal forces and moments of the beam at $x_1 = 0$ and where $\mathbb{a}(t) = (a_1(t), a_2(t), a_3(t), 0, 0, 0)$. Moreover, at time $t = 0$, the structure is at rest; so we have the initial conditions

$$\mathbb{u}(x_1, 0) = 0 \quad \text{and} \quad \dot{\mathbb{u}}(x_1, 0) = 0 \quad \text{for all } x_1 \in [0,\, h] \quad . \tag{13}$$

The reduced Timoshenko beam model with a homogeneous mass distribution is built with the approximation on $\mathbb{u} = (u_1, u_2, u_3, \theta_1, \theta_2, \theta_3)$

$$\mathbb{u}(x_1, t) \simeq \sum_{\alpha=1}^{N} q_\alpha(t)\, \boldsymbol{\Phi}_\alpha(x_1) \quad , \tag{14}$$

where $q_\alpha(t)$ are the generalized coordinates of $\mathbb{u}(x_1, t)$ within the basis of eigenfunctions $\boldsymbol{\Phi}_\alpha$ for the structure with its foundation coupled with the soil. More details on the construction of this reduced Timoshenko beam model with a homogeneous mass distribution can be found in Appendix B. Let $\mathbf{q}(t) = (q_1(t), \ldots, q_N(t))$ be the vector of the generalized coordinates. Then, it can be shown that

$$([\mathscr{M}] + [\mathscr{M}_S])\; \ddot{\boldsymbol{q}}(t) + [\mathscr{D}_S]\, \dot{\boldsymbol{q}}(t) + ([\mathscr{K}] + [\mathscr{K}_S])\; \boldsymbol{q}(t) = \boldsymbol{f}(t) + \boldsymbol{f}_S(t) \tag{15}$$

with the initial conditions

$$\boldsymbol{q}(0) = 0 \text{ and } \dot{\boldsymbol{q}}(0) = 0 \tag{16}$$

where the positive-definite symmetric $N \times N$ real matrices $[\mathscr{M}]$, $[\mathscr{M}_S]$, $[\mathscr{D}_S]$, $[\mathscr{K}_S]$, the positive symmetric $N \times N$ real matrix $[\mathscr{K}]$ and the vectors of the generalized external forces $\boldsymbol{f}(t)$ and $\boldsymbol{f}_S(t)$ are explicitly given in Appendix B.

It may be noticed that both reduced models defined by Eqs. (2) and (15) have the same form, allowing to use a systematic method for building the probabilistic model in the following section.

3 Probabilistic model for the soil-structure interaction and the response of the building

3.1 Stochastic soil-structure interaction modeling

The non-parametric probabilistic model (see for instance [9] and [10]) consists in substituting the deterministic matrices $[\underline{M}_S]$, $[\underline{D}_S]$ and $[\underline{K}_S]$ by random matrices $[\mathbf{M}_S]$, $[\mathbf{D}_S]$ and $[\mathbf{K}_S]$. The probabilistic model of these random matrices is constructed by using the maximum entropy principle with the available information: the mean values of these random matrices, the algebraic properties of the mass, stiffness and damping random matrices (symmetry, definiteness, etc.), and conditions for the solution to have a finite second order moment. It is shown in the previously quoted papers that the probability density functions $p_{[\mathbf{M}_S]}$, $p_{[\mathbf{D}_S]}$ and $p_{[\mathbf{K}_S]}$ of those random matrices with respect to the measure $d\tilde{A} = 2^{n_S(n_S-1)/4} \prod_{1\leq i\leq j\leq n} [A]_{ij}$ (with $n_S = 6$) on the set $\mathbb{M}^+$ of the symmetric positive $n_S \times n_S$ real matrices are then written as

$$p_{[\mathbf{M}_S]}([A]) = \mathbb{1}_{\mathbb{M}^+}([A])\, c_{\mathbf{M}}\, (\det[A])^{\lambda_{\mathbf{M}}-1}\ \exp\{-\frac{n_S - 1 + 2\,\lambda_{\mathbf{M}}}{2} \operatorname{tr}([\underline{M}_S]^{-1}[A])\}\ , \tag{17}$$

$$p_{[\mathbf{D}_S]}([A]) = \mathbb{1}_{\mathbb{D}^+}([A])\, c_{\mathbf{D}}\, (\det[A])^{\lambda_{\mathbf{D}}-1}\ \exp\{-\frac{n_S - 1 + 2\,\lambda_{\mathbf{D}}}{2} \operatorname{tr}([\underline{D}_S]^{-1}[A])\}\ , \tag{18}$$

$$p_{[\mathbf{K}_S]}([A]) = \mathbb{1}_{\mathbb{K}^+}([A])\, c_{\mathbf{K}}\, (\det[A])^{\lambda_{\mathbf{K}}-1}\ \exp\{-\frac{n_S - 1 + 2\,\lambda_{\mathbf{K}}}{2} \operatorname{tr}([\underline{K}_S]^{-1}[A])\}\ , \tag{19}$$

where $\mathbb{1}_{\mathbb{M}^+}([A])$ is equal to 1 if $[A]$ belongs to $\mathbb{M}^+$ and is equal to zero if $[A]$ does not belong to $\mathbb{M}^+$; $\operatorname{tr}(\cdot)$ is the trace operator; $c_{\mathbf{M}}$, $c_{\mathbf{D}}$ and $c_{\mathbf{K}}$ are normalization constants; $\lambda_{\mathbf{M}}$, $\lambda_{\mathbf{D}}$, $\lambda_{\mathbf{K}}$ are positive real parameters that depend on the statistical fluctuation of random matrices $[\mathbf{M}_S]$, $[\mathbf{D}_S]$ and $[\mathbf{K}_S]$. Appendix C briefly describes how samples $[\mathbf{M}_S(a)]$, $[\mathbf{D}_S(a)]$ and $[\mathbf{K}_S(a)]$ with probability density functions $p_{[\mathbf{M}_S]}$, $p_{[\mathbf{D}_S]}$ and $p_{[\mathbf{K}_S]}$ can be numerically computed in practice.

3.2 Stochastic reduced model of the building under seismic loads

The uncertainties on the soil-structure interaction are taken into account by substituting into both reduced models defined by Eqs. (2) or (15), the matrices $[\mathscr{M}_S]$, $[\mathscr{D}_S]$, $[\mathscr{K}_S]$ and vector $\boldsymbol{f}_S(t)$ by random matrices $[\mathbb{M}_S]$, $[\mathbb{D}_S]$, $[\mathbb{K}_S]$ and random vector $\boldsymbol{F}_S(t)$. The random matrices are obtained by projection of random matrices $[\mathbf{M}_S]$, $[\mathbf{D}_S]$ and $[\mathbf{K}_S]$ (defined in previous section) onto the deterministic modes. Thus $[\mathbb{M}_S]$, $[\mathbb{D}_S]$ and $[\mathbb{K}_S]$ are random definite-positive symmetric $N \times N$ matrices whose components are $[\mathbb{M}_S]_{\beta\alpha} = \boldsymbol{\Phi}_{0,\beta}^T\, [\mathbf{M}_S]\, \boldsymbol{\Phi}_{0,\alpha}$, $[\mathbb{D}_S]_{\beta\alpha} = \boldsymbol{\Phi}_{0,\beta}^T\, [\mathbf{D}_S]\, \boldsymbol{\Phi}_{0,\alpha}$ and $[\mathbb{K}_S]_{\beta\alpha} = \boldsymbol{\Phi}_{0,\beta}^T\, [\mathbf{K}_S]\, \boldsymbol{\Phi}_{0,\alpha}$ (for model 1) and $[\mathbb{M}_S]_{\beta\alpha} = \boldsymbol{\Phi}_\beta(0)^T\, [\mathbf{M}_S]\, \boldsymbol{\Phi}_\alpha(0)$, $[\mathbb{D}_S]_{\beta\alpha} = \boldsymbol{\Phi}_\beta(0)^T\, [\mathbf{D}_S]\, \boldsymbol{\Phi}_\alpha(0)$ and $[\mathbb{K}_S]_{\beta\alpha} = \boldsymbol{\Phi}_\beta(0)^T\, [\mathbf{K}_S]\, \boldsymbol{\Phi}_\alpha(0)$ (for model 2). Similarly, the components of random vector $\boldsymbol{F}_S(t)$ are random variables $\{\boldsymbol{F}_S(t)\}_\beta = -{\boldsymbol{\Phi}_{0,\beta}}^T [\mathbf{M}_S]\, \mathbb{a}(t)$ (for model 1) and $\{\boldsymbol{F}_S(t)\}_\beta = -\boldsymbol{\Phi}_\beta(0)^T [\mathbf{M}_S]\, \mathbb{a}(t)$ (for model 2).

Consequently, the solution of such a stochastic system of differential equations is a random vector $\mathbf{Q}(t)$ such that,

$$([\mathscr{M}] + [\mathbb{M}_S])\ \ddot{\boldsymbol{Q}}(t) + [\mathbb{D}_S]\, \dot{\boldsymbol{Q}}(t) + ([\mathscr{K}] + [\mathbb{K}_S])\ \boldsymbol{Q}(t) = \boldsymbol{f}(t) + \boldsymbol{F}_S(t) \quad , \tag{20}$$

with the initial conditions

$$\boldsymbol{Q}(0) = 0 \text{ and } \dot{\boldsymbol{Q}}(0) = 0 \quad . \tag{21}$$

The random displacement of the structure is given by:

$$\mathbb{U} = \sum_{\alpha=1}^{N} Q_\alpha \boldsymbol{\Phi}_\alpha \quad , \tag{22}$$

where $Q_1, \ldots, Q_N$ are the random components of the random solution $\mathbf{Q}$ and where $\boldsymbol{\Phi}_1, \ldots, \boldsymbol{\Phi}_N$ are the N (deterministic) eigenvectors (for model 1) and eigenfunctions (for model 2) introduced in the previous sections.

3.3 Construction of the stochastic response by using the Monte Carlo simulations

The Monte Carlo simulations are used in order to calculate some statistical quantities related to the probabilistic response of the uncertain dynamical system. The method consists in constructing a given number N_S of samples of random matrices $[\mathbf{M}_S]$, $[\mathbf{D}_S]$ and $[\mathbf{K}_S]$ with probability density functions $p_{[\mathbf{M}_S]}$, $p_{[\mathbf{D}_S]}$ and $p_{[\mathbf{K}_S]}$. For each occurence $[\mathbf{M}_S(a)]$, $[\mathbf{D}_S(a)]$ and $[\mathbf{K}_S(a)]$ of the random matrices, the sample $\mathbf{Q}(t;a)$ of random vector $\mathbf{Q}(t)$ is computed as the solution of:

$$\begin{aligned}([\mathscr{M}] + [\mathbb{M}_S(a)])\, \ddot{\boldsymbol{Q}}(t;a) + [\mathbb{D}_S(a)]\, \dot{\boldsymbol{Q}}(t;a) + ([\mathscr{K}] + [\mathbb{K}_S(a)])\, \boldsymbol{Q}(t;a) \\ = \boldsymbol{f}(t) + \boldsymbol{F}_S(t;a) \quad ,\end{aligned} \tag{23}$$

with the initial conditions

$$\boldsymbol{Q}(0;a) = 0 \text{ and } \dot{\boldsymbol{Q}}(0;a) = 0 \quad , \tag{24}$$

For model 1, components of samples $[\mathbb{M}_S(a)]$, $[\mathbb{D}_S(a)]$ and $[\mathbb{K}_S(a)]$ are

$$\begin{aligned}[\mathbb{M}_S(a)]_{\beta\alpha} &= \boldsymbol{\Phi}_{0,\beta}^T\, [\mathbf{M}_S(a)]\, \boldsymbol{\Phi}_{0,\alpha} \quad , \\ [\mathbb{D}_S(a)]_{\beta\alpha} &= \boldsymbol{\Phi}_{0,\beta}^T\, [\mathbf{D}_S(a)]\, \boldsymbol{\Phi}_{0,\alpha} \quad , \\ [\mathbb{K}_S(a)]_{\beta\alpha} &= \boldsymbol{\Phi}_{0,\beta}^T\, [\mathbf{K}_S(a)]\, \boldsymbol{\Phi}_{0,\alpha} \quad .\end{aligned}$$

For model 2, components of samples $[\mathbb{M}_S(a)]$, $[\mathbb{D}_S(a)]$ and $[\mathbb{K}_S(a)]$ are

$$\begin{aligned}[\mathbb{M}_S(a)]_{\beta\alpha} &= \boldsymbol{\Phi}_\beta(0)^T\, [\mathbf{M}_S(a)]\, \boldsymbol{\Phi}_\alpha(0) \quad , \\ [\mathbb{D}_S(a)]_{\beta\alpha} &= \boldsymbol{\Phi}_\beta(0)^T\, [\mathbf{D}_S(a)]\, \boldsymbol{\Phi}_\alpha(0) \quad , \\ [\mathbb{K}_S(a)]_{\beta\alpha} &= \boldsymbol{\Phi}_\beta(0)^T\, [\mathbf{K}_S(a)]\, \boldsymbol{\Phi}_\alpha(0) \quad .\end{aligned}$$

Similarly, the random components of vector $\boldsymbol{F}_S(t;a)$ are

$$\begin{aligned}\{\boldsymbol{F}_S(t;a)\}_\beta &= -{\boldsymbol{\Phi}_{0,\beta}}^T [\mathbf{M}_S(a)]\, \mathbb{a}(t) \quad , \quad \text{for model 1} \quad , \\ \{\boldsymbol{F}_S(t;a)\}_\beta &= -\boldsymbol{\Phi}_\beta(0)^T [\mathbf{M}_S(a)]\, \mathbb{a}(t) \quad , \quad \text{for model 2} \quad .\end{aligned}$$

Then samples $\mathbb{U}(t;a)$ (for model 1) or $\mathbb{U}(t,x_1;a)$ (for model 2) are given by:

$$\mathbb{U}(a) = \sum_{\alpha=1}^{N} Q_\alpha(a)\,\boldsymbol{\Phi}_\alpha \quad , \tag{25}$$

where $Q_1(a),\ldots,Q_N(a)$ are the components of sample vector $\mathbf{Q}(\mathbf{a})$.

The maximal internal moments and forces over the whole structure and during the seismic load for each model are random variables denoted by V_1,V_2,V_3 and M_1,M_2,M_3. Sample values $V_1(a),V_2(a),V_3(a)$ and $M_1(a),M_2(a),M_3(a)$ are calculated for each sample $\mathbb{U}(a)$ of the displacement field. Finally, statistical quantities related to the probabilistic response of the uncertain dynamical system are calculated by using usual tools of the statistic with a large enough number N_S of samples $\mathbb{U}(a_1),\ldots,\mathbb{U}(a_{N_S})$, $V_1(a_1),\ldots,V_1(a_{N_S})$, $V_2(a_1),\ldots,V_2(a_{N_S})$, $V_3(a_1),\ldots,V_3(a_{N_S})$ and $M_1(a_1),\ldots,M_1(a_{N_S})$, $M_2(a_1)$, $\ldots$, $M_2(a_{N_S})$ and $M_3(a_1),\ldots,M_3(a_{N_S})$.

4 Numerical results

In this section, the stochastic mechanical beam model is applied to the case of a standard multi-story building.

4.1 Description of the building

The building under study is a 8-stories building. Figure 2 shows a schema of the building and its foundation. Every story is $30\,m$ long ($\boldsymbol{e}_2$ direction), $14\,m$ wide ($\boldsymbol{e}_3$ direction) and has a height $H_{\mathrm{sto}} = 2.7\,m$ so that the total height of the building is $h = N_{\mathrm{sto}} \times H_{\mathrm{sto}} = 21.6\,m$. All stories are identical and therefore the stiffness beam parameters are computed by using a finite element computation of a story where the walls, the floor and the ceiling are meshed with Reissner-Mindlin plate finite elements. They are assumed to be made up of an isotropic material for which Young's modulus is 32 GPa, the shearing modulus is 13 GPa and the mass density is 2500 $kg.m^{-3}$. For the sake of simplicity the anisotropy induced by the reinforced concrete elements has not been taken into account.

The seismic acceleration due to the earthquake is decribed by using a signal record from the Parkfield earthquake which occurred on September 28, 2004 in the Parkfield area close to Vineyard Canyon in California with a moment-magnitude of 6.0. The 3 components of the seismic accelerogram have been measured in Donna Lee station [17] at 14.5 km from the epicenter. For more information about the 2004 Parkfield event, the reader should refer to [18]. The P-wave and S-wave velocity were measured in the area by Boore [19] using layered soil models. The data are available at `http://quake.usgs.gov/~boore/`. From the S-wave velocity and P-wave velocity measured in Vineyard Canyon

area a shearing modulus of 162 MPa and a Young's modulus of 440 MPa were obtained for the soil properties .
Figure 3-a shows the acceleration signal, the Fourier transform modulus and the ridges of the wavelet transform for the transverse component $\boldsymbol{e}_2$ of the seismic acceleration signal $t \mapsto \boldsymbol{a}_g(t)$. Ridges are curves in the time frequency plane localizing the concentration of energy. The norm of the complex wavelet transform of the studied signal is maximal on these curves (see [20] for more details on that technique) . These curves are plotted in gray scale so that the darker the ridges are, the higher is the energy. The process favors the high frequencies components which can be obtained from short durations, which explains that the most energy ridges appear at high frequencies. The first part of the signal ([0, 4]s) contains low frequency energetic components in the horizontal directions which can also be observed on the elastic response spectra (figure 3-b). During the next time period ([4, 12]s) there is a contribution in the range [3, 14]Hz. The highest frequencies contribution in the range [14, 20]Hz are shorter and are fluctuating, consequently they are never predominant across time.

A global response of the structure can be studied by introducing the dynamic magnification factor [15] $H(\omega)$ such that, for all ω in $\mathbb{R}$,

$$H(\omega) = \frac{1}{N} \| \left([\mathscr{K}] + [\mathcal{K}_S] \right) [A(\omega)]^{-1}) \|_F \quad , \tag{26}$$

where $[A(\omega)] = -\omega^2 \left([\mathscr{M}] + [\mathcal{M}_S] \right) - i\,\omega\,[\mathcal{D}_S] + \left([\mathscr{K}] + [\mathcal{K}_S] \right)$ is the reduced dynamic stiffness of the structure with its foundation coupled with the soil and where $\| \cdot \|_F$ is the Froebenius norm of matrices.

Figure 4 displays the function $\omega \mapsto H(\omega)$. It can be seen from this figure that the mode whose eigenfrequency is 1.9 Hz for the homogeneous mass distribution model, is directly excited at the beginning of the earthquake. In the following stage, two other modes are excited. The wavelet transform presents eventually no relevant energetic frequency contributions after 15s.

4.2 Results for the stochastic model

To illustrate the method, the sensitivity of the maximal internal forces and moments with respect to the uncertainties is now studied. As introduced in subsection 3.3, $V_1, V_2, V3$ and M_1, M_2, M_3 are the random maximal values of the internal forces and moments over the whole structure during the application of the seismic loading. They are obtained for both stochastic reduced beam models presented in section 3.2. The Monte Carlo numerical method [14] is used in order to calculate the mean values, standard deviations and the probability density functions of these random variables in order to quantify their uncertainties. The values of the dispersion coefficients which have been used are : $\delta_M = 0.2, \delta_D = 0.5, \delta_K = 0.5$. The values of dispersion coefficients can be identified in solving a statistical inverse problem if experimental measurements are available (see, for instance [22]). However, in the case of seismic studies, such a procedure cannot be uses. These coefficients are estimated by the experience of the engineer, due to dispersion on soil data for example. Hereinafter, a sufficiently large amount of samples in the Monte Carlo method have been constructed in order to reach convergence on the statistic estimations [21] of the mean, standard deviation and probability density functions. As an example, the convergence of the mean value with respect to the number of realizations is shown in Figure 5. The stochastic convergence is reached by taking all the eigenvectors up to 500Hz, i.e $N = 108$.

The graphs of the estimations of probability density functions of V_1, V_2, V_3, M_1, M_2 and M_3 are shown in figure 6 for which $N_S \geq 10000$ samples were used to reach convergence. Some internal forces or moments present an important dispersion; the random moment M_3 presents, for instance, a standard deviation of 25% with respect to its mean value. The largest dispersion is given by the random torsional moment M_1 which is completely due to random modeling and could not be computed with a deterministic approach or a parametric modeling of the impedance which would not take into account a spatial variability of the soil properties under the foundation. A substantial effect of the random soil structure interaction is the global rise of the mean internal forces and moments compared to the deterministic values. The effect is stronger with the lumped mass model where the random internal force V_3 and the corresponding random bending moment M_2 respectively increase by 17% and 45%. The beam model with homogeneous mass distribution presents increases of 4% and 18% for the same force and moment.

5 Conclusions

Soil-structure interaction can modify the dynamic response of a building under earthquakes in the low frequency range. A non-parametric modeling was used to introduce model uncertainties in an easy-to-use engineering model of soil-structure interaction. The model leads to a significant torsion within the building, even if the structure and its foundation are symmetrical.
All stochastic computations are performed by using a Monte Carlo simulation which requires solving many times the dynamic equations of the structure . The simplified beam model used for the building mechanical modeling and the simple SSI model allow to perform a sufficient number of such computations . It has been observed that the random soil-structure interaction can produce an increase of internal forces compared to the deterministic results.

A Reduced lumped mass beam model

In order to exhibit the contribution of matrices $[\underline{M}_S]$, $[\underline{D}_S]$ and $[\underline{K}_S]$ to the lumped mass beam model (model 1), we first introduce matrices $[M]$ and $[K]$ which are the mass matrix and the stiffness matrix of a lumped mass beam model for which the boundary conditions are the following: (1) the top of the building is free and (2) the foundation is weightless and free. These boundary conditions imply that matrices $[M]$ and $[K]$ are (not definite) positive symmetric $N_{DOF} \times N_{DOF}$ real matrices.

Then a weak formulation that allows to explicit the relation between this set of matrices is written as: Find $\mathbb{u}(t) = (\mathbf{u}_0(t), \boldsymbol{\theta}_0(t), \ldots, \mathbf{u}_{N_{\rm sto}}(t), \boldsymbol{\theta}_{N_{\rm sto}}(t))$ belonging in $\mathbb{R}^{N_{DOF}}$ such that for all $\mathbb{v} = (\mathbf{v}_0, \boldsymbol{\Theta}_0, \ldots, \mathbf{v}_{N_{\rm sto}}, \boldsymbol{\Theta}_{N_{\rm sto}}) \in \mathbb{R}^{N_{DOF}}$,

$$\begin{aligned} &\mathbb{v}^T\,[M]\,\ddot{\mathbb{u}}(t) + \mathbf{v}_0^T\,[\underline{M}_S]\,\ddot{\mathbf{u}}_0(t) + \mathbf{v}_0^T\,[\underline{D}_S]\,\dot{\mathbf{u}}_0(t) \\ &\quad + \mathbb{v}^T\,[K]\,\mathbb{u}(t) + \mathbf{v}_0^T\,[\underline{K}_S]\,\mathbf{u}_0(t) = -\mathbb{v}^T\,[M]\,\mathbb{a}(t) - \mathbf{v}_0^T\,[\underline{M}_S]\,\ddot{\mathbf{a}}_0(t) \quad . \end{aligned} \tag{A.1}$$

Let $\omega_1 \leq \ldots \leq \omega_N$ be the N first eigenfrequencies which are solutions of the generalized eigenvalue problem: find $\boldsymbol{\Phi} = (\phi_0, \ldots, \phi_{N_{\rm sto}}) \in \mathbb{R}^{N_{DOF}}$ and $\omega > 0$ such that, for all $\mathbb{v} \in \mathbb{R}^{N_{DOF}}$,

$$\mathbb{v}^T\,[K]\,\boldsymbol{\Phi} + \mathbf{v}_0^T\,[\underline{K}_S]\,\phi_0 - \omega^2\,\mathbb{v}^T\,[M]\,\boldsymbol{\Phi} - \omega^2\,\mathbf{v}_0^T\,[\underline{M}_S]\,\phi_0 = 0 \quad . \tag{A.2}$$

Let $\boldsymbol{\Phi}_k = (\phi_{0,k}, \ldots, \phi_{N_{\rm sto},k}) \in \mathbb{R}^{N_{DOF}}$ be the k-th eigenvector associated with k-th smallest eigenfrequency ω_k. Eigenvectors are such that $\boldsymbol{\Phi}_\beta\,[M]\,\boldsymbol{\Phi}_\alpha + \phi_{0,\beta}^T\,[\underline{M}_S]\,\phi_{0,\alpha} = \delta_{\alpha\beta}$ and $\boldsymbol{\Phi}_\beta\,[K]\,\boldsymbol{\Phi}_\alpha + \phi_{0,\beta}^T\,[\underline{K}_S]\,\phi_{0,\alpha} = \omega_\alpha^2\delta_{\alpha\beta}$ where $\delta_{\alpha\beta} = 1$ if $\alpha = \beta$ and $\delta_{\alpha\beta} = 0$ if $\alpha \neq \beta$. The reduced model is built with the approximation

$$\mathbb{u}(t) \simeq \sum_{\alpha=1}^{N} q_\alpha(t)\,\boldsymbol{\Phi}_\alpha \quad , \tag{A.3}$$

where $q_\alpha(t)$ are the generalized coordinates of $\mathbb{u}(t)$ within the basis of eigenvectors $\boldsymbol{\Phi}_\alpha$. Let the vector $\mathbf{q}(\mathbf{t}) = (q_1(t), \ldots, q_N(t))$ be the vector of the generalized coordinates. Then, it can be shown that

$$([\mathscr{M}] + [\mathscr{M}_S])\;\ddot{\boldsymbol{q}}(t) + [\mathscr{D}_S]\,\dot{\boldsymbol{q}}(t) + ([\mathscr{K}] + [\mathscr{K}_S])\;\boldsymbol{q}(t) = \boldsymbol{f}(t) + \boldsymbol{f}_S(t) \tag{A.4}$$

with the initial conditions

$$\boldsymbol{q}(0) = 0 \text{ and } \dot{\boldsymbol{q}}(0) = 0 \tag{A.5}$$

where the positive-definite symmetric $N \times N$ real matrices $[\mathscr{M}]$, $[\mathscr{M}_S]$, $[\mathscr{D}_S]$, $[\mathscr{K}_S]$ and the positive symmetric $N \times N$ real matrix $[\mathscr{K}]$ are such that $[\mathscr{M}]_{\beta\alpha} = \boldsymbol{\Phi}_\beta\,[M]\,\boldsymbol{\Phi}_\alpha$, $[\mathscr{M}_S]_{\beta\alpha} = \boldsymbol{\Phi}_{0,\beta}^T\,[\underline{M}_S]\,\boldsymbol{\Phi}_{0,\alpha}$, $[\mathscr{D}_S]_{\beta\alpha} = \boldsymbol{\Phi}_{0,\beta}^T\,[\underline{D}_S]\,\boldsymbol{\Phi}_{0,\alpha}$ and $[\mathscr{K}]_{\beta\alpha} = \boldsymbol{\Phi}_\beta\,[K]\,\boldsymbol{\Phi}_\alpha$, $[\mathscr{K}_S]_{\beta\alpha} = \boldsymbol{\Phi}_{0,\beta}^T\,[\underline{K}_S]\,\boldsymbol{\Phi}_{0,\alpha}$. It should be noted that we also have $[\mathscr{M}]_{\beta\alpha} + [\mathscr{M}_S]_{\beta\alpha} = \delta_{\alpha\beta}$ and $[\mathscr{K}]_{\beta\alpha} + [\mathscr{K}_S]_{\beta\alpha} = \omega_\alpha^2\delta_{\alpha\beta}$. The components of vectors of the generalized forces $\mathbf{f}(t)$ and $\boldsymbol{f}_S(t)$ are $\{\mathbf{f}(t)\}_\beta = -\boldsymbol{\Phi}_\beta^T\,[M]\,\mathbb{a}(t)$ and $\{\mathbf{f}_S(t)\}_\beta = -{\boldsymbol{\Phi}_{0,\beta}}^T[\underline{M}_S]\,\mathbb{a}(t)$.

B Reduced Timoshenko beam model

A weak formulation for an explicit relation between matrices $[\underline{M}_S]$, $[\underline{D}_S]$ and $[\underline{K}_S]$ and the bilinear forms of the Timoshenko beam model with a homogeneous mass distribution (model 2) is presented hereinafter. Let $\mathcal{C}$ be a space of sufficiently regular functions. The weak formulation of the problem is written as: Find $\mathbb{u} = (u_1, u_2, u_3, \theta_1, \theta_2, \theta_3) \in \mathcal{C}$ such that for all $\mathbb{v} \in \mathcal{C}$,

$$\begin{aligned} &m(\ddot{\mathbb{u}}, \mathbb{v}) + \mathbb{v}(0)^T[\underline{M}_S]\,\ddot{\mathbb{u}}(0,t) + \mathbb{v}(0)^T[\underline{D}_S]\,\dot{\mathbb{u}}(0,t) \\ &\quad + k(\mathbb{u}, \mathbb{v}) + \mathbb{v}(0)^T[\underline{K}_S]\,\mathbb{u}(0,t) = f(t, \mathbb{v}) - \mathbb{v}(0)^T[\underline{M}_S]\,\mathbb{a}(t) \end{aligned} \tag{B.1}$$

where the positive-definite symmetric bilinear form $m(\cdot,\cdot)$, the positive symmetric bilinear form $k(\cdot,\cdot)$ and the linear form $f(\cdot)$ are defined on $\mathcal{C}\times\mathcal{C}$ and $\mathcal{C}$ such that

$$\begin{aligned} m(\mathbb{u},\mathbb{v}) = \rho\, S \int_0^h u_1(x_1)\, v_1(x_1)\, dx_1 + \rho\, S \int_0^h u_2(x_1)\, v_2(x_1)\, dx_1 \\ + \rho\, S \int_0^h u_3(x_1)\, v_3(x_1)\, dx_1 + \rho\, I_1 \int_0^h u_4(x_1)\, v_4(x_1)\, dx_1 \\ + \rho\, I_2 \int_0^h u_5(x_1)\, v_5(x_1)\, dx_1 + \rho\, I_3 \int_0^h u_6(x_1)\, v_6(x_1)\, dx_1 \end{aligned} \tag{B.2}$$

$$\begin{aligned} k(\mathbb{u},\mathbb{v}) = G\, k_2\, S \int_0^h u_2'(x_1)\, v_2'(x_1)\, dx_1 + G\, k_3\, S \int_0^h u_3'(x_1)\, v_3'(x_1)\, dx_1 \\ + E\, I_2 \int_0^h u_5'(x_1)\, v_5'(x_1)\, dx_1 + E\, I_3 \int_0^h u_6'(x_1)\, v_6'(x_1)\, dx_1 \\ + G\, k_2\, S \int_0^h u_6(x_1)\, v_6(x_1)\, dx_1 + G\, k_3\, S \int_0^h u_5(x_1)\, v_5(x_1)\, dx_1 \\ + G\, k_3\, S \int_0^h u_5(x_1)\, v_3'(x_1)\, dx_1 + G\, k_3\, S \int_0^h u_3'(x_1)\, v_5(x_1)\, dx_1 \\ - G\, k_2\, S \int_0^h u_6(x_1)\, v_2'(x_1)\, dx_1 - G\, k_2\, S \int_0^h u_2'(x_1)\, v_6(x_1)\, dx_1 \\ + S\, E \int_0^h u_1'(x_1)\, v_1'(x_1)\, dx_1 + G\, I_1 \int_0^h u_4'(x_1)\, v_4'(x_1)\, dx_1 \end{aligned} \tag{B.3}$$

$$f(t,\mathbb{v}) = -\rho\, S \int_0^h \mathbb{v}(x_1)^T\, \mathbb{a}(t)\, dx_1 \tag{B.4}$$

where for the seek of simplicity, $u_k, k = 4,5,6$ are used instead of $\theta_1,\theta_2,\theta_3$ and similarly $v_k, k = 4,5,6$ are used for the rotations associated to v_k.

Let $\omega_1 \leq \ldots \leq \omega_N$ be the N first eigenfrequencies which are solutions of the generalized eigenvalue problem: find $\boldsymbol{\Phi} \in \mathcal{C}$ and $\omega > 0$ such that, for all $\mathbb{v} \in \mathcal{C}$,

$$k(\boldsymbol{\Phi},\mathbb{v}) + \mathbb{v}(0)^T [\underline{K}_S]\, \boldsymbol{\Phi}(0) - \omega^2\, m(\boldsymbol{\Phi},\mathbb{v}) - \omega^2\, \mathbb{v}(0)^T [\underline{M}_S]\, \boldsymbol{\Phi}(0) = 0 \tag{B.5}$$

Let $\{\boldsymbol{\Phi}_1,\ldots,\boldsymbol{\Phi}_N\}$ be the N eigenfunctions associated with eigenfrequencies $\omega_1 \leq \ldots \leq \omega_N$. Eigenfunctions are such that $m(\boldsymbol{\Phi}_\alpha,\boldsymbol{\Phi}_\beta) + \boldsymbol{\Phi}_\beta(0)^T\, [\underline{M}_S]\, \boldsymbol{\Phi}_\alpha(0) = \delta_{\alpha\beta}$ and $k(\boldsymbol{\Phi}_\alpha,\boldsymbol{\Phi}_\beta) + \boldsymbol{\Phi}_\beta(0)^T\, [\underline{K}_S]\, \boldsymbol{\Phi}_\alpha(0) = \omega_\alpha^2 \delta_{\alpha\beta}$ where $\delta_{\alpha\beta} = 1$ if $\alpha = \beta$ and $\delta_{\alpha\beta} = 0$ if $\alpha \neq \beta$. The reduced model is built with the approximation

$$\mathbb{u}(x_1,t) \simeq \sum_{\alpha=1}^{N} q_\alpha(t)\, \boldsymbol{\Phi}_\alpha(x_1) \quad , \tag{B.6}$$

where $q_\alpha(t)$ are the generalized coordinates of $\mathbb{u}(x_1,t)$ within the basis of eigenfunctions $\boldsymbol{\Phi}_\alpha$. Let the vector $\mathbf{q(t)} = (q_1(t),\ldots,q_N(t))$ be the vector of the generalized coordinates. Then, it can be shown that

$$([\mathscr{M}] + [\mathscr{M}_S])\, \ddot{\boldsymbol{q}}(t) + [\mathscr{D}_S]\, \dot{\boldsymbol{q}}(t) + ([\mathscr{K}] + [\mathscr{K}_S])\, \boldsymbol{q}(t) = \boldsymbol{f}(t) + \boldsymbol{f}_S(t) \tag{B.7}$$

with the initial conditions

$$\boldsymbol{q}(0) = 0 \text{ and } \dot{\boldsymbol{q}}(0) = 0 \tag{B.8}$$

where the positive-definite symmetric $N \times N$ real matrices $[\mathscr{M}]$, $[\mathscr{M}_S]$, $[\mathscr{D}_S]$, $[\mathscr{K}_S]$ and the positive symmetric $N \times N$ real matrix $[\mathscr{K}]$ are such that

$$[\mathscr{M}]_{\beta\alpha} = m(\boldsymbol{\Phi}_\alpha, \boldsymbol{\Phi}_\beta) \quad , \quad [\mathscr{M}_S]_{\beta\alpha} = \boldsymbol{\Phi}_\beta(0)^T \, [\underline{M}_S] \, \boldsymbol{\Phi}_\alpha(0) \quad ,$$
$$[\mathscr{D}_S]_{\beta\alpha} = \boldsymbol{\Phi}_\beta(0)^T \, [\underline{D}_S] \, \boldsymbol{\Phi}_\alpha(0) \quad , \quad [\mathscr{K}]_{\beta\alpha} = k(\boldsymbol{\Phi}_\alpha, \boldsymbol{\Phi}_\beta) \quad ,$$
$$[\mathscr{K}_S]_{\beta\alpha} = \boldsymbol{\Phi}_\beta(0)^T \, [\underline{K}_S] \, \boldsymbol{\Phi}_\alpha(0) \quad .$$

It should be noted that we also have $[\mathscr{M}]_{\beta\alpha} + [\mathscr{M}_S]_{\beta\alpha} = \delta_{\alpha\beta}$ and $[\mathscr{K}]_{\beta\alpha} + [\mathscr{K}_S]_{\beta\alpha} = \omega_\alpha^2 \delta_{\alpha\beta}$. The components of vectors of the generalized forces $\mathbf{f}(t)$ and $\boldsymbol{f}_S(t)$ are $\{\mathbf{f}(t)\}_\beta = f(t, \boldsymbol{\Phi}_\beta)$ and $\{\mathbf{f}_S(t)\}_\beta = -\boldsymbol{\Phi}_\beta(0)^T [\underline{M}_S] \, \mathbb{a}(t)$.

C Probabilistic model of the random matrices

In this appendix, we present the construction of the probabilistic model of the random matrices involved by the nonparametric probabilistic formulation of the model uncertainties. It has been shown in [9,10] that the probabilistic model of random matrices $[\mathbf{M}_S]$, $[\mathbf{D}_S]$ and $[\mathbf{K}_S]$ can be constructed using the information theory with the available information (see section 6.2) yielding

$$[\mathbf{M}_S] = [L_M]^T [\mathbf{G_M}] [L_M] \quad , \quad [\mathbf{D}_S] = [L_D]^T [\mathbf{G_D}] [L_D] \quad ,$$
$$[\mathbf{K}_S] = [L_K]^T [\mathbf{G_K}] [L_K] \quad .$$

in which the $(n_S \times n_S)$ upper triangular matrices $[L_\mathbf{M}]$, $[L_\mathbf{D}]$, $[L_\mathbf{K}]$, with $n_S = 6$, correspond to the Cholesky factorizations $[M_S] = [L_\mathbf{M}]^T [L_\mathbf{M}]$, $[D_S] = [L_\mathbf{D}]^T [L_\mathbf{D}]$, $[K_S] = [L_\mathbf{K}]^T [L_\mathbf{K}]$ and where $[\mathbf{G_M}]$, $[\mathbf{G_D}]$, $[\mathbf{G_K}]$ are random matrices for which the probability density functions $p_{[\mathbf{G_M}]}$, $p_{[\mathbf{G_D}]}$, $p_{[\mathbf{G_K}]}$ are such that

$$p_{[\mathbf{G_M}]}([G]) = \mathbb{1}_{\mathbb{M}^+}([G]) \, c(\delta_\mathbf{M}) \, (\det[G])^{b(\delta_\mathbf{M})} \, \exp\{-a(\delta_\mathbf{M}) \mathrm{tr}[G]\} \; ,$$
$$p_{[\mathbf{G_D}]}([G]) = \mathbb{1}_{\mathbb{M}^+}([G]) \, c(\delta_\mathbf{D}) \, (\det[G])^{b(\delta_\mathbf{D})} \, \exp\{-a(\delta_\mathbf{D}) \mathrm{tr}[G]\} \; ,$$
$$p_{[\mathbf{G_K}]}([G]) = \mathbb{1}_{\mathbb{M}^+}([G]) \, c(\delta_\mathbf{K}) \, (\det[G])^{b(\delta_\mathbf{K})} \, \exp\{-a(\delta_\mathbf{K}) \mathrm{tr}[G]\} \; ,$$

where $\mathbb{1}_{\mathbb{M}^+}([G])$ is equal to 1 if $[G]$ belongs to $\mathbb{M}^+$ (the set of all the $n_S \times n_S$ symmetric positive matrices) and is equal to zero if $[G]$ does not belong to $\mathbb{M}^+$, $\mathrm{tr}[G]$ is the trace of matrix $[G]$, $a(\delta) = (n_S+1)/(2\delta^2)$, $b(\delta) = a(\delta)(1-\delta^2)$, $c(\delta) = (2\pi)^{-n_S(S-1)/4} a(\delta)^{n_S \, a(\delta)} / \prod_{j=1}^{n+1} \Gamma(\alpha_j(\delta))$ in which $\alpha_j(\delta) = a(\delta) + (1-j)/2$ and where Γ is the Gamma function.

Then, it has then shown in [9,?] that random matrices $[\mathbf{G_M}]$, $[\mathbf{G_D}]$, $[\mathbf{G_K}]$ can be written as

$$[\mathbf{G_M}] = [\mathbf{L_M}]^\mathbf{T} [\mathbf{L_M}] \quad , \quad [\mathbf{G_D}] = [\mathbf{L_D}]^\mathbf{T} [\mathbf{L_D}] \quad , \quad [\mathbf{G_K}] = [\mathbf{L_K}]^\mathbf{T} [\mathbf{L_K}] \quad ,$$

where $[\mathbf{L_M}]$, $[\mathbf{L_D}]$ and $[\mathbf{L_K}]$ are random upper triangular $(n_S \times n_S)$ real matrix such that

(1) for $j < j'$, $[\mathbf{L_M}]_{jj'}$, $[\mathbf{L_D}]_{jj'}$ and $[\mathbf{L_K}]_{jj'}$ are real-valued Gaussian random variables with zero mean and variance equal to $\sigma_\mathbf{M} = \delta_\mathbf{M} / \sqrt{n_S + 1}$, $\sigma_\mathbf{D} = \delta_\mathbf{D} / \sqrt{n_S + 1}$ and $\sigma_\mathbf{K} = \delta_\mathbf{K} / \sqrt{n_S + 1}$;

(2) for $j = j'$, $[\mathbf{L_M}]_{jj'}$, $[\mathbf{L_D}]_{jj'}$ and $[\mathbf{L_K}]_{jj'}$ are positive-valued random variables written as $[\mathbf{L_M}]_{jj'} = \sigma_\mathbf{M} \sqrt{2 \, V_{\mathbf{M},j}}$, $[\mathbf{L_D}]_{jj'} = \sigma_\mathbf{D} \sqrt{2 \, V_{\mathbf{D},j}}$ where $V_{\mathbf{M},j}$, $V_{\mathbf{D},j}$ and $V_{\mathbf{K},j}$ are positive-valued Gamma random variables whose probability density function $p_{V_{\mathbf{M},j}}$, $p_{V_{\mathbf{D},j}}$, $p_{V_{\mathbf{K},j}}$ are written as

$$p_{V_{\mathbf{M},j}}(v) = \mathbb{1}_{\mathbb{R}^+}(v) \frac{v^{\alpha_j(\delta_\mathbf{M})} \, e^{-v}}{\Gamma(\alpha_j(\delta_\mathbf{M})} \; , \; p_{V_{\mathbf{D},j}}(v) = \mathbb{1}_{\mathbb{R}^+}(v) \frac{v^{\alpha_j(\delta_\mathbf{D})} \, e^{-v}}{\Gamma(\alpha_j(\delta_\mathbf{D})} \; ,$$
$$p_{V_{\mathbf{K},j}}(v) = \mathbb{1}_{\mathbb{R}^+}(v) \frac{v^{\alpha_j(\delta_\mathbf{K})} \, e^{-v}}{\Gamma(\alpha_j(\delta_\mathbf{K})} \quad .$$

Then, N statistical independent realizations of random matrices $[\mathbf{M}_S]$, $[\mathbf{D}_S]$ and $[\mathbf{K}_S]$ are constructed such that

$$[\mathbf{M}_S(a_1)] = [L_\mathbf{M}]^T\,[\mathbf{G}_\mathbf{M}(a_1)]\,[L_\mathbf{M}]\ ,\ \ldots\ ,\ [\mathbf{M}_S(a_N] = [L_\mathbf{M}]^T\,[\mathbf{G}_\mathbf{M}(a_N)]\,[L_\mathbf{M}]$$

$$[\mathbf{D}_S(a_1)] = [L_\mathbf{D}]^T\,[\mathbf{G}_\mathbf{D}(a_1)]\,[L_\mathbf{D}]\ ,\ \ldots\ ,\ [\mathbf{D}_S(a_N)] = [L_\mathbf{D}]^T\,[\mathbf{G}_\mathbf{D}(a_S)]\,[L_\mathbf{D}]$$

$$[\mathbf{K}_S(a_1)] = [L_\mathbf{K}]^T\,[\mathbf{G}_\mathbf{K}(a_1)]\,[L_\mathbf{K}]\ ,\ \ldots\ ,\ [\mathbf{K}_S(a_N)] = [L_\mathbf{K}]^T\,[\mathbf{G}_\mathbf{K}(a_N)]\,[L_\mathbf{K}]$$

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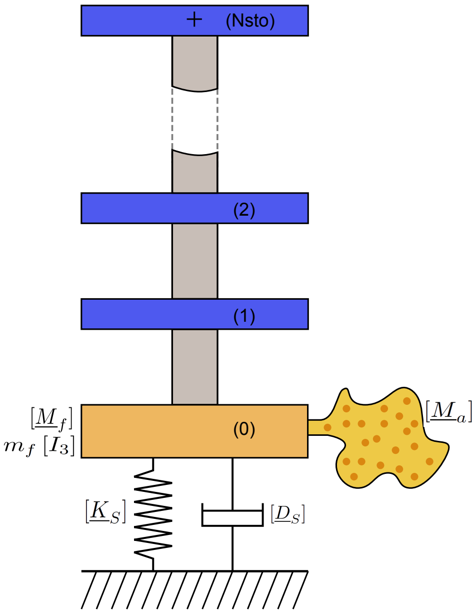

Fig. 1 Lumped mass model of the building and its foundation

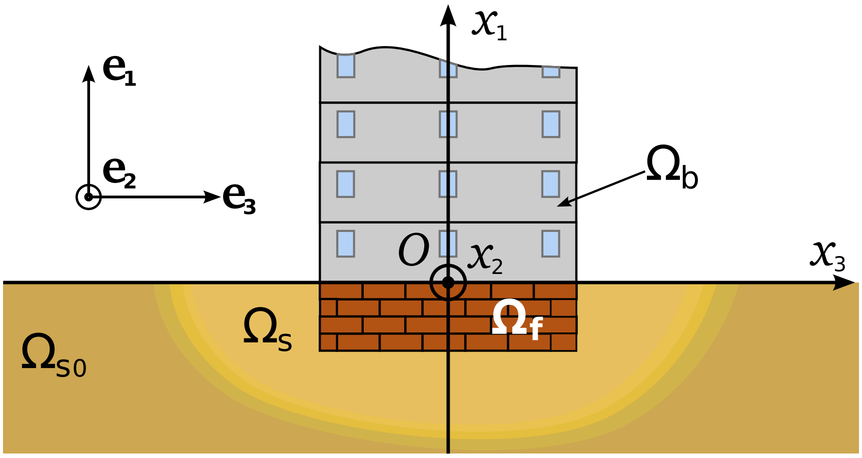

Fig. 2 Schema of the building and its foundation

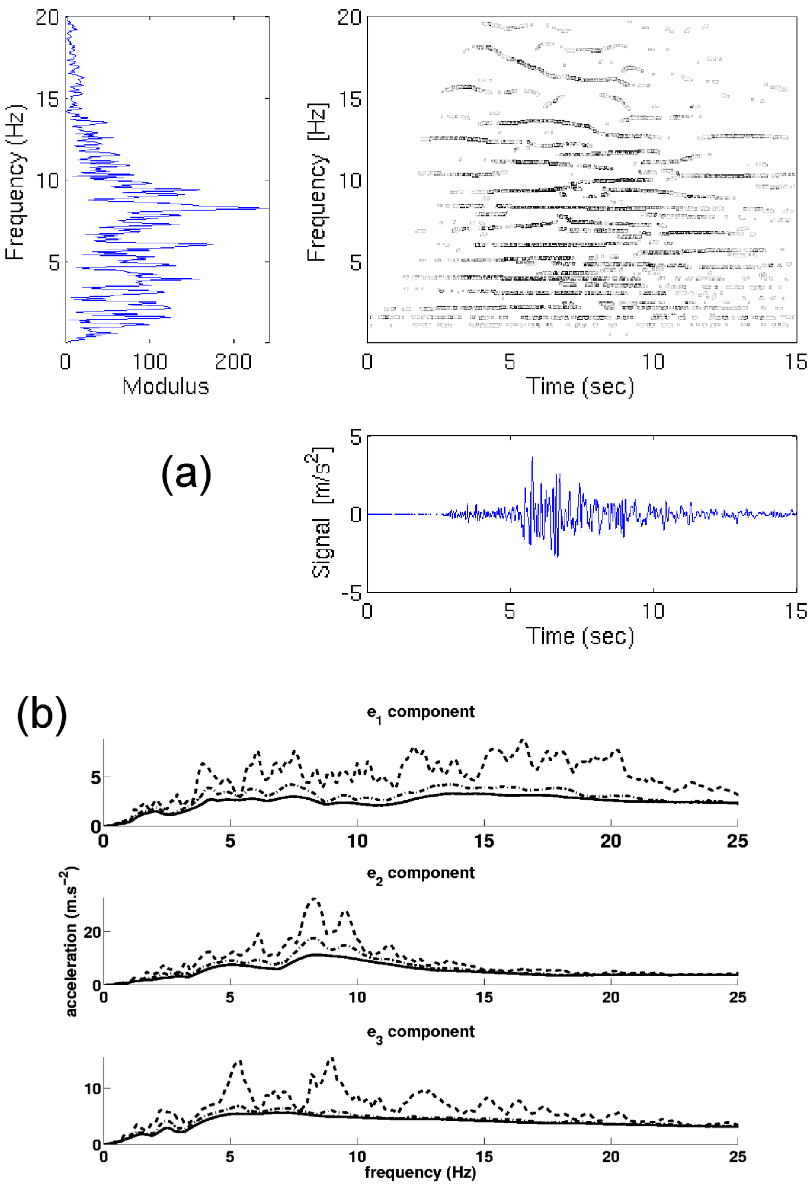

Fig. 3 (a) - Representation of the e_2-component. (b) - Elastic response for the 3 components of the earthquake accelerogram with damping ratios 1% (dashed line), 5% (dash-dooted line) and 10 % (solid line).

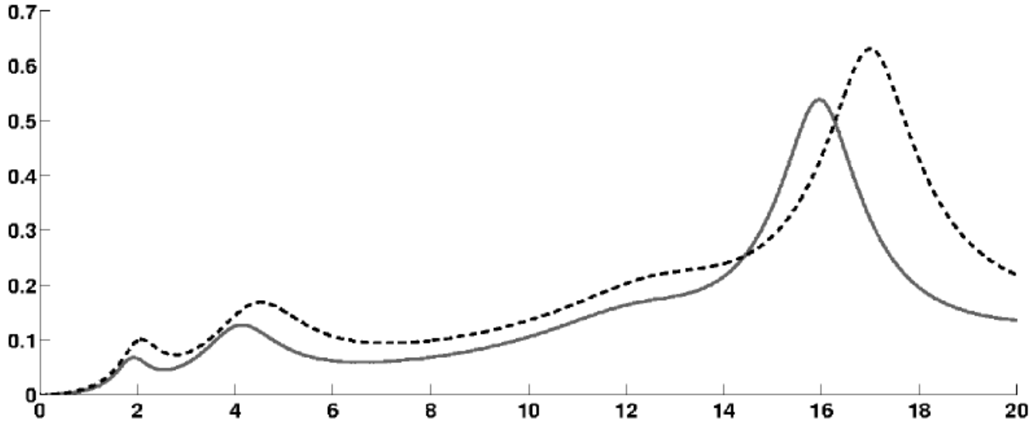

Fig. 4 Graph of the dynamic magnification factor function for the beam models with homogeneous mass (solid line) and the lumped mass model (dashed line). Horizontal axis ω in Hz. Vertical axis: dynamic magnification factor $H(\omega)$

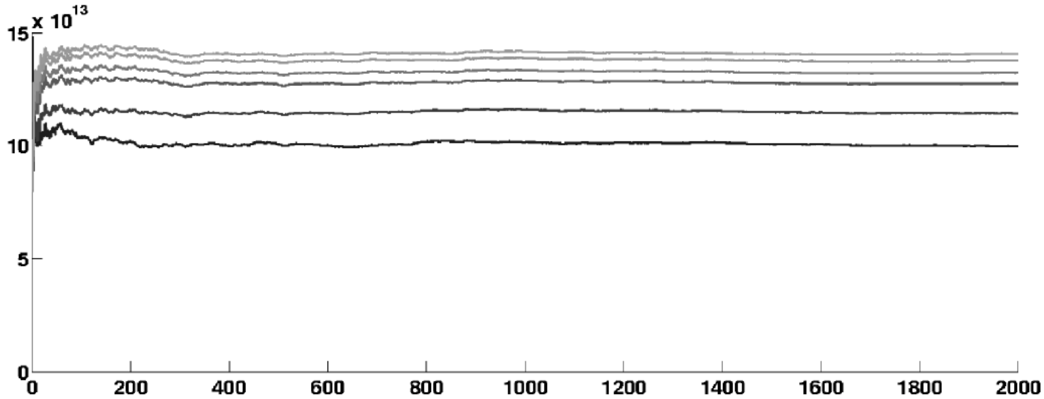

Fig. 5 Convergence of the mean value of V_3 for a modal reduction using eigenfrequencies upto $50\,Hz$, $100\,Hz$, $200\,Hz$, $300\,Hz$, $500\,Hz$ and $800\,Hz$ (black to light gray curve). Horizontal axis: number of samples in the Monte Carlo method. Vertical axis: Estimation of the second order moment of the shear force V_3.

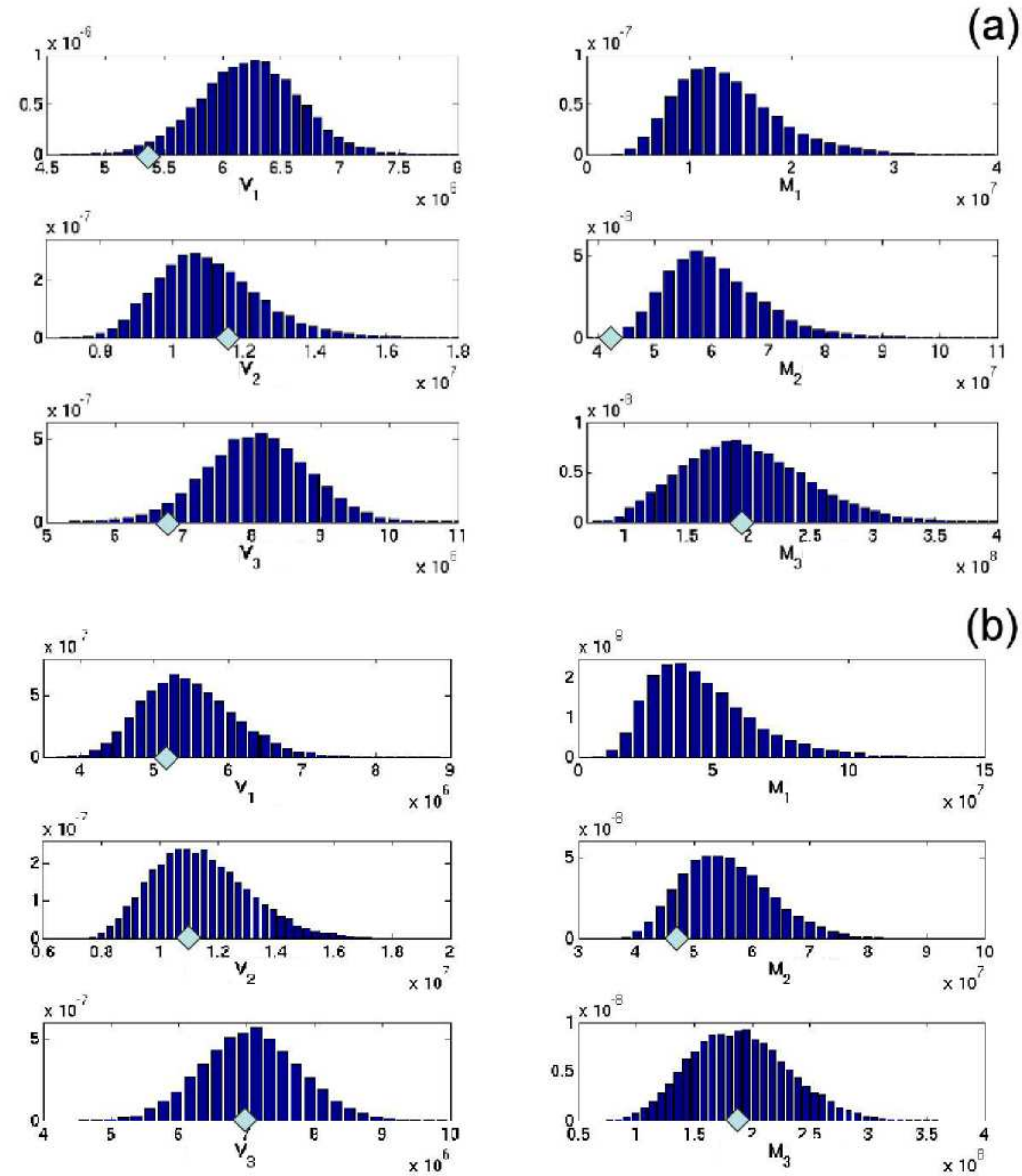

Fig. 6 Probability density functions for the maximal internal forces and moments in the building for (a) - the lumped mass beam model (b) - the beam model with a homogeneous mass distribution. The diamonds on the domain axes mark the deterministic values.