



HAL
open science

The Mayan Long Count Calendar

Thomas Chanier

► **To cite this version:**

| Thomas Chanier. The Mayan Long Count Calendar. 2014. hal-00750006v9

HAL Id: hal-00750006

<https://hal.science/hal-00750006v9>

Preprint submitted on 5 Aug 2014 (v9), last revised 16 Dec 2015 (v12)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

The Mayan Long Count Calendar

T. Chanier*¹

¹ *Department of Physics, University of Namur, rue de Bruxelles 61, B-5000 Namur, Belgium*

The Maya were known for their astronomical proficiency. Whereas Mayan mathematics were based on a vigesimal system, they used a different base when dealing with long periods of time, the Long Count Calendar (LCC), composed of different Long Count Periods: the Tun of 360 days, the Katun of 7200 days and the Baktun of 144000 days. There were three other calendars used in addition to the LCC: a civil year Haab' of 365 days, a religious year Tzolk'in of 260 days and a 3276-day cycle (combination of the 819-day Kawil cycle and 4 colors-directions). Based on astronomical arguments, we propose here an explanation of the origin of the LCC, the Tzolk'in and the 3276-day cycle. The study provides new evidences on the astronomical knowledge of the Maya.

The Maya had a very elaborate and accurate Long Count Calendar (LCC) that was used to pinpoint historical events from a selected beginning time. The smallest unit of time is the day (Kin); 20 Kin form a Winal, 18 Winal form a Tun (360 Kin), 20 Tun form a Katun (7200 Kin), and 20 Katun form a Baktun (144000 Kin). The LCC represents a date D as a set of coefficients $(C_i, \dots, C_3, C_2, C_1, C_0)$ such that: $D = C_0 + C_1 \times 20 + \sum_{i=2}^n C_i \times 18 \times 20^{i-1}$ with $C_0 = \text{mod}(D, 20)$, $C_1 = \text{int}(\text{mod}(D, 360)/20)$ and $C_i = \text{int}(\text{mod}(D, 18 \times 20^i)/(18 \times 20^{i-1}))$ for $i > 1$. The day count usually restarts when C_4 reaches 13, such as the date is given as a set of 5 coefficients: $D \equiv \text{mod}(D, 13 \times 144000) = C_4.C_3.C_2.C_1.C_0$. The Maya used three other independent calendars: a religious year (Tzolk'in), a civil year (Haab') and a 3276-day cycle. One Tzolk'in of 260 days comprised 13 months (numerated from 1 to 13) containing 20 named days (Imix, Ik, Akbal, Kan, Chicchan, Cimi, Manik, Lamat, Muluc, Oc, Chuen, Eb, Ben, Ix, Men, Cib, Caban, Etznab, Cauac, and Ahau). One Haab' of 365 days comprised 18 named months (Pop, Uo, Zip, Zotz, Tzec, Xul, Yaxkin, Mol, Chen, Yax, Zac, Ceh, Mac, Kankin, Muan, Pax, Kayab, and Cumku) with 20 days each (Winal) plus 1 extra month (Uayeb) with 5 nameless days. The Tzolk'in and the Haab' coincide every 73 Tzolk'in or 52 Haab' or a Calendar Round. Their least common multiple (LCM) is $73 \times 260 = 52 \times 365 = 18980$ days. In the Calendar Round, a date is represented by $\alpha X \beta Y$ with the religious month $1 \leq \alpha \leq 13$, X one of the 20 religious days, the civil day $0 \leq \beta \leq 19$, and Y one of the 18 civil months, $0 \leq \beta \leq 4$ for the Uayeb. According to the Goodman-Martinez-Thompson (GMT) correlation to the Gregorian calendar, which is based on historical facts, the Mayan Calendar began on 11 August 3114 BC or 0(13).0.0.0.0, 4 Ahau 8 Cumku and ended on 21 December 2012. This corresponds to a 13 Baktun cycle or a period of approximately 5125 years. The 3276-day cycle is a combination of the Kawil, a 819-day cycle, and a 4-day cycle corresponding to the 4 colors-directions.

The origin of the Long Count Periods (LCPs) is unknown. A common assumption is the desire of the calendar keeper to maintain the Tun in close agreement with the tropical/solar year of approximately 365.24 days.¹ There is no consensus concerning the origin of

the Tzolk'in, which has been associated with various astronomical cycles. Three Tzolk'in correspond to Mars synodic period, 16 Tzolk'in equal 11 of Saturn synodic periods (+2 days), and 23 Tzolk'in are equivalent to 15 Jupiter synodic periods (-5 days).² It has been tentatively connected to the eclipse half-year (173.31 days) because 2 Tzolk'in are very close to 3 eclipse half-years.³ Finally, it has been noted that the Tzolk'in approximates the length of time Venus is visible as a morning or evening star.⁴ However, these interpretations fail to link the Tzolk'in to the LCPs. The Kawil cycle has been attributed to the observation of Jupiter and Saturn^{5,6} because 19 (6) Kawil correspond to 39 (13) Jupiter (Saturn) synodic period. Four numbers of possible astronomical significance have been discovered on the walls of a residential structure in Xultun, Guatemala and have been dated from the early 9th century CE. The Xultun numbers are given in Table I. They are such that $\mathcal{X}_1 = 365 \times 3276$ and $\mathcal{X}_3 = \mathcal{X}_2 + 2\mathcal{X}_0$.

\mathcal{X}_i	LCC	D [day]	$\mathcal{X}_i/56940$
\mathcal{X}_0	2.7.9.0.0	341640	6
\mathcal{X}_1	8.6.1.9.0	1195740	21
\mathcal{X}_2	12.5.3.3.0	1765140	31
\mathcal{X}_3	17.0.1.3.0	2448420	43

TABLE I. Xultun numbers \mathcal{X}_i .⁷ $56940 = \text{LCM}(365, 780)$ is their largest common divisor.

This paper explains a possible origin of the LCPs, the religious year Tzolk'in and the relationship between the Kawil cycle and the 4 colors-directions. This new interpretation, based on the periodic movements of the Planets in the night sky, gives rise to different day cycles that have been identified in the Maya Codices and monuments. Attempts are made to provide a meaning to the Xultun numbers. The study shows that Mayan astronomers knew the synodic periods of all the Planets of the Solar System visible to the naked eye: Mercury, Venus, Mars, Jupiter and Saturn.

The Maya were known for their astronomical skills as exemplified by the Dresden Codex, a bark-paper book of the 11th or 12th century CE. On page 24 of this Codex is written the so-called Long Round number noted

Planet	P [day]	Prime factorization
Mercury	116	$2^2 \times 29$
Venus	584	$2^3 \times 73$
Earth	365	5×73
Mars	780	$2^2 \times 3 \times 5 \times 13$
Jupiter	399	$3 \times 7 \times 19$
Saturn	378	$2 \times 3^3 \times 7$
Lunar	177	3×59
senesters	178	2×89
Pentalunex	148	$2^2 \times 37$

TABLE II. Planet cycles^{8,9} and their prime factorizations.

9.9.16.0.0 in the LCC or 1366560 days, a whole multiple of the Tzolk'in, the Haab', the Calendar Round, the Tun, Venus and Mars synodic periods: $\mathcal{LR} = 1366560 = 5256 \times 260 = 3744 \times 365 = 72 \times 18980 = 3796 \times 360 = 2340 \times 584 = 1752 \times 780$. Only the moon, Mercury, Venus, Earth (solar year), Mars, Jupiter, and Saturn are visible to the naked eye. Their respective mean synodic periods are given in Table II. Evidences have been found in different Mayan Codices that Mayan astronomers observed the periodic movements in the night sky of Mercury, Venus, and Mars, but it is unclear whether they tracked the movements of Jupiter and Saturn.¹⁰ The periods relevant for the prediction of solar/lunar eclipses are the pentalunex of 148 days (5 Moon synodic periods of 29.53 days) and the lunar semesters of 177 or 178 days (6 Moon synodic periods), which are the time intervals between subsequent eclipse warning stations present in the Eclipse Table on pages 51 to 58 of the Dresden Codex.⁹ The LCM of these numbers is $\mathcal{N} = 20757814426440$ days (including the Planet synodic periods and the two lunar semesters) or $\mathcal{N}^\dagger = 768039133778280$ days (also including the pentalunex). \mathcal{N} is such that:

$$\text{int}(\mathcal{N}/13/73/144000) = 338 + 360 + 7200 + 144000. \quad (1)$$

\mathcal{N}^\dagger gives the same result but divided by 13, 37, 73 and 144000. That defines the Tun, Katun, and Baktun as a polynomial expression of $\text{int}(\mathcal{N}/13/73)$ of the form $18 \times 20^3 \times (C_0 + \sum_{n=1}^3 18 \times 20^n)$ with $C_0 = 338$. The $\text{LCM}(338,360) = 169 \times 360 = 234 \times 260 = 60840$; 338 and 365 are relatively prime numbers: the $\text{LCM}(338,365) = 338 \times 365 = 123370$; 234 and 365 are also relatively prime numbers: the $\text{LCM}(234,365) = 234 \times 365 = 85410$. The $\text{LCM}(260,365)$ is $73 \times 260 = 52 \times 365 = 18980$ days or a Calendar Round. That may define the Tzolk'in. \mathcal{N} is such that the $\text{mod}(\mathcal{N}, \mathcal{LR}) = 341640$ days or 2.7.9.0.0 in the LCC. This is the Xultun number \mathcal{X}_0 (Table I). It is to be noted that \mathcal{X}_0 is a whole multiple of the Tzolk'in, Haab', Tun, Venus and Mars synodic periods: $341640 = 1314 \times 260 = 936 \times 365 = 949 \times 360 = 585 \times 584 = 438 \times 780$. \mathcal{X}_0 is also the largest common divisor of \mathcal{LR} and \mathcal{N} . Table III gives the divisibility of \mathcal{N} by a polynomial expression of the type $P_i = 13 \times 73 \times (\sum_n 18 \times 20^n)$. The integer parts D_i are such that $D_i/D_{i-1} \approx 20$ for $0 < i < 6$ and $D_6/D_5 = 21$.

Name	i	C_i [day]	P_i	D_i
-	0	18	$\mathcal{N}/13/73/\sum_0^6 C_i$	18
Tun	1	360	$\mathcal{N}/13/73/\sum_0^5 C_i$	360
Katun	2	7200	$\mathcal{N}/13/73/\sum_0^4 C_i$	7215
Baktun	3	144000	$\mathcal{N}/13/73/\sum_0^3 C_i$	144304
Pictun	4	2880000	$\mathcal{N}/13/73/\sum_0^2 C_i$	2886428
Calabtun	5	57600000	$\mathcal{N}/13/73/\sum_0^1 C_i$	57866020
Kinchiltun	6	1152000000	$\mathcal{N}/13/73/C_0$	1215186420

TABLE III. Divisibility of $\mathcal{N} = 20757814426440$ days by a polynomial expression of the type $P_i = 13 \times 73 \times (\sum_{n=0}^{6-i} 18 \times 20^n)$. $D_i = \text{int}(\mathcal{N}/P_i)$.

Table IV gives the coincidence of the 338-day period with the LCPs, as well as the Tzolk'in and the 234-day period. As described earlier, the Tun and 338-day period coincide every 60840 days, or 234 Tzolk'in. The 338-day period also coincides with the Winal every 3380 days, corresponding to 13 Tzolk'in, a cycle that does not coincide with the 234-day period. Rather, the 234-day period coincides with the Winal every 2340 days or 9 Tzolk'in = $9 \times 260 = \text{LCM}(9,13,20)$. This 2340-day cycle is present in the Dresden Codex on pages D30c-D33c and has been attributed to a Venus-Mercury almanac because $2340 = 20 \times 117 = 5 \times 585$ is an integer multiple of Mercury and Venus mean synodic periods (+1 day).¹¹ Another explanation may be of divination origin because $117 = 9 \times 13$. In Mesoamerican mythology, there are a set of 9 Gods called the Lords of the Night¹²⁻¹⁵ and a set of 13 Gods called the Lords of the Day.¹⁵ Each day is linked with 1 of the 13 Lords of the Day and 1 of the 9 Lords of the Night in a repeating 117-day cycle. This cycle coincides with the Tzolk'in every 2340 days. On Table IV figures a date $D = \text{LCM}(338,7200) = 1216800$ days or 8.9.0.0.0 in the LCC or 5 February 219 CE according to the GMT correlation. This date may have marked the beginning of the reign of Yax Moch Xoc, the founder of the Tikal dynasty, in 219 CE.¹⁶

LCP	D [day]	L [day]	M [day]	N [day]	$L/260$	$L/234$
Winal	20	3380	260	2340	13	14.44444
Tun	360	60840	4680	4680	234	260
Katun	7200	1216800	93600	93600	4680	5200
Baktun	144000	24336000	1872000	1872000	93600	104000

TABLE IV. Coincidence of the 234-, 260- and 338-day periods with the LCPs. $L = \text{LCM}(D,338)$, $M = \text{LCM}(D,260)$ and $N = \text{LCM}(D,234)$.

We now examine the cycles that can be constructed from the combination of the Tzolk'in ($260 = 13 \times 20$), the Haab' ($365 = 5 \times 73$) and the LCPs ($D = 18 \times 20^n$). Because 13 is not present in the prime factorizations of the LCPs, we obtain $\text{LCM}(260,D) = 13 \times D$. There is, for example, a 13 Katun cycle of 93600 days (360 Tzolk'in) reminiscent of the Katun Wheel and a 13 Baktun cycle of 1872000 days (7200 Tzolk'in). This means that

the Baktun cycle coincides with the Tzolk'in cycle every 13 Baktun, corresponding to approximately 5125 years. This is the origin of the 13 Baktun cycle. Because 73 is not present in the prime factorizations of the LCPs, we obtain $\text{LCM}(365, D) = 73 \times D$. This gives rise to 73 LCP cycles. Moreover, the $\text{LCM}(260, 365, D) = 13 \times 73 \times D$. This is the case of the Xultun number $\mathcal{X}_0 = 341640 = \text{LCM}(260, 365, 360)$ which makes coincide the Tzolk'in, the Haab' and the Tun.

Planet	P [day]	$N = \text{LCM}(P, L)$	$N/260$	$N/2340$	N/L
Mercury	116	950040	3654	406	29
Venus	584	2391480	9198	1022	73
Earth	365	2391480	9198	1022	73
Mars	780	32760	126	14	1
Jupiter	399	622440	2394	266	19
Saturn	378	98280	378	42	3
Lunar	177	1932840	7434	826	59
semesters	178	2915640	11214	1246	89
Pentalunex	148	1212120	4662	518	37

TABLE V. Coincidence of the 32760-day cycle with the Planet synodic periods.^{8,9} $N = \text{LCM}(P, 360, 819)$ and $L = \text{LCM}(360, 819) = 32760$ days.

The Kawil, a 819-day cycle, is connected with a 4-day cycle (4 colors-directions), coinciding every 3276 days. The Haab' and the Tzolk'in coincide with the 3276-day cycle every $\text{LCM}(365, 3276) = 365 \times 3276 = 1195740$ days or 8.6.1.9.0 in the LCC. This is the Xultun number \mathcal{X}_1 (Table I). $\mathcal{X}_1 = \text{LCM}(9, 4, 819, 260, 365)$ is the time distance between two days of the same Lord of the Night, color-direction, Kawil, Tzolk'in and Haab' date. \mathcal{X}_0 and \mathcal{X}_1 are such that $341640 \times 819 = 1195740 \times 360 = 2391480$ or $\mathcal{Y} = 16.12.3.0.0$ in the LCC. \mathcal{Y} represents the same as \mathcal{X}_1 at the end of a Tun (0 Winal, 0 Kin). The Kawil coincides with the Tun every 32760 days = $\text{LCM}(360, 819) = 7 \times \text{LCM}(260, 360) = 10 \times 3276$. Table V shows the coincidence of the 32760-day cycle with the Planet synodic periods, giving rise to whole multiples of the Tzolk'in and the 2340-day cycle. The 3276-day cycle commensurates with all the Planet synodic periods except the Haab' (365 and 3276 are relatively prime numbers). We have for example the $\text{LCM}(148, 3276) = 819 \times 148 = 37 \times 3276 = 121212$ days. \mathcal{N} (\mathcal{N}^\dagger) can be written in the form $3276 \times \prod_i n_i$ where the n_i s are distinct prime numbers: $n_i = (2, 3, 5, 19, 29, 37, 59, 73, 89)$. It indicates that Mayan astronomers calculated the synodic periods of all the Planet visible to the naked eye. The use of the Tzolk'in and the commemoration of the 13 Baktun cycle provides further proofs of their astronomical knowledge.

The pyramid of Kukulcan (Figure 1) built sometime between the 9th and the 12th century CE at Chichen Itza may be a representation of the various cycle used by the Maya. The pyramid shape may be linked to the Long Count Calendar (Table III) which draws a pyramid-like structure. It is constituted of 9 platforms with 4 stairways of 91 steps each leading to the platform temple cor-



FIG. 1. Pyramid of Kukulcan during an equinox. The pyramid is situated at Chichen Itza, Yucatan, Mexico.

responding to the 3276-day cycle: $3276 = \text{LCM}(4, 9, 91) = 2^2 \times 3^2 \times 7 \times 13 = 4 \times 819$, the coincidence of the 4 colors-directions with the Kawil. The dimensions of the pyramid may be of significance: the width of its base is 55.30 m (37 zapal), its height up to the top of the platform temple is 30 m (20 zapal) and the width of the top platform is 19.52 m (13 zapal), taking into account the Mayan zapal length measurement such that 1 zapal ≈ 1.5 m.¹⁷ The width of the base and 3276 are such that $37 \times 3276 = \text{LCM}(148, 3276) = 121212$. The pyramid height and the width of the top platform represents the Tzolk'in. The stairways divide the 9 platforms of each side of the pyramid into 18 segments which, combined with the pyramid height, represents the 18 Winal of a Tun. The Haab' is represented by the platform temple making the 365th step with the $4 \times 91 = 364$ steps of the 4 stairways. Each side of the pyramid contains 52 panels corresponding to the Calendar Round: $52 \times 365 = 73 \times 260 = 18980$. During an equinox, the Sun casts a shadow (7 triangles of light and shadow) on the northern stairway representing a serpent snaking down the pyramid (Figure 1). A 1820-day cycle or 7 Tzolk'in = 5×364 is present on pages 31-32a of the Dresden Codex. This may also correspond to $32760 = \text{LCM}(360, 819) = 7 \times \text{LCM}(260, 360)$.

Planet	P [day]	$N = \text{LCM}(P, 260)$	$N/260$	N/P
Mercury	116	7540	29	65
Venus	584	37960	146	65
Earth	365	18980	73	52
Mars	780	780	3	1
Jupiter	399	103740	399	260
Saturn	378	49140	189	130
Lunar	177	46020	177	260
semesters	178	23140	89	130
Pentalunex	148	9620	37	65

TABLE VI. Coincidence of the Planet cycles^{8,9} and the Tzolk'in.

Various religious cycles can be constructed by taking the LCM of 260 and the Planet synodic periods (Table VI). The most important one in Mayan religion is

the Calendar Round, which is the $\text{LCM}(260,365) = 73 \times 260 = 52 \times 365 = 18980$ days. There is a coincidence between Venus synodic period and the Tzolk'in: the $\text{LCM}(260,584) = 65 \times 584 = 146 \times 260 = 104 \times 365 = 37960$ days (2 Calendar Rounds), the length of the Venus Table on pages 24-29 of the Dresden Codex. Two other cycles are based on the coincidence of the Tzolk'in and the 234- and 338-day periods: the $\text{LCM}(260,234) = 9 \times 260 = 2340$ days and the $\text{LCM}(260,338) = 13 \times 260 = 3380$ days. There is also a coincidence between Mars synodic period and the Haab': the $\text{LCM}(780,365) = 73 \times 780 = 219 \times 260 = 156 \times 365 = 56940$ days (3 Calendar Rounds): this is the largest common divisor of the Xultun numbers (Table I). We have: $\mathcal{X}_2 = \mathcal{Z} + 2\mathcal{X}_0$ and $\mathcal{X}_3 = \mathcal{Z} + 4\mathcal{X}_0$ where $\mathcal{Z} = 19 \times 56940 = 1081860$ days = 7.10.5.3.0 in the LCC or 23 August 152 BC according to the GMT correlation. This date may have been of some particular importance.

The study presented here describe a possible explana-

tion of the origin of the Long Count Calendar (LCC) and its connection with the Tzolk'in and Haab' calendars. It provides a better understanding of the 3276-day cycle, the coincidence of the Kawil and the 4 colors-directions. Based on astronomical observations of the Solar System, the LCC gives rise to three different periods of 234 days, 260 days (Tzolk'in) and 338 days. The correlation between the 234-day period and the Tzolk'in, a 2340-day cycle identified in the Dresden Codex, was certainly used for divination purposes. The correlation between the 338-day period and the Tzolk'in gives rise to a 13-Tzolk'in cycle that has not yet been identified in the Codices. The 13 Baktun cycle originates from the correlation between the Tzolk'in and the Baktun. The 3276-day cycle commensurates with all the Planet synodic periods. These results indicate that Mayan astronomers calculated the synodic periods of all the Planets visible to the naked eye, as well as the basic cycles for solar/lunar eclipse prediction, the pentalunex and the lunar semesters.

* e-mail: thomas.chanier@gmail.com

-
- ¹ A. F. Aveni, in *Skywatchers: A Revised and Updated Version of Skywatchers of Ancient Mexico* (University of Texas Press, 2001), p. 136.
- ² J. S. Justeson, The ancient Maya Ethnoastronomy: An Overview of Hieroglyphic Sources, in *World Archaeoastronomy: Selected Papers from the Second Oxford International Conference on Archaeoastronomy*, edited by A. F. Aveni, (Cambridge University Press, 1989), p. 82.
- ³ A. F. Aveni, in *Skywatchers: A Revised and Updated Version of Skywatchers of Ancient Mexico* (University of Texas Press, 2001), p. 144.
- ⁴ H. M. Bricker and V. R. Bricker, in *Astronomy in the Maya Codices* (American Philosophical Society, Philadelphia, 2011), p. 69.
- ⁵ F. G. Lounsbury, in *Maya Numeration, Computation, and Calendrical Astronomy. Dictionary of Scientific Biography*, vol. 15, suppl. 1, edited by Charles Coulston-Gillispie, (New York: Charles Scriber's Sons, 1978), p. 811
- ⁶ J. S. Justeson, The ancient Maya Ethnoastronomy: An Overview of Hieroglyphic Sources, in *World Archaeoastronomy: Selected Papers from the Second Oxford International Conference on Archaeoastronomy*, edited by A. F. Aveni, (Cambridge University Press, 1989), p.103.
- ⁷ W. A. Saturno, D. Stuart, A. F. Aveni and F. Rossi, *Science* **336**, 714 (2012).
- ⁸ <http://nssdc.gsfc.nasa.gov/Planetary/factsheet/>
- ⁹ H. M. Bricker and V. R. Bricker, *Current Anthropology* **24**, 1-23 (1983).
- ¹⁰ H. M. Bricker and V. R. Bricker, in *Astronomy in the Maya Codices* (American Philosophical Society, Philadelphia, 2011), pp. 163, 367, 847.
- ¹¹ H. M. Bricker and V. R. Bricker, in *Astronomy in the Maya Codices* (American Philosophical Society, Philadelphia, 2011), p. 235.
- ¹² A. F. Aveni, in *Skywatchers: A Revised and Updated Version of Skywatchers of Ancient Mexico* (University of Texas Press, 2001), pp. 156-157.
- ¹³ E. H. Boone, in *Cycles of Time and Meaning in the Mexican Books of Fate* (University of Texas Press, 2007), pp. 44-45.
- ¹⁴ F. Lounsbury, in *Dictionary of Scientific Biography*, vol. 15, (Scribners, New York, 1978), suppl. 1, p. 787.
- ¹⁵ T. A. Joyce, in *Mexican Archaeology: An Introduction to the Archaeology of the Mexican and Mayan Civilizations of Pre-Spanish America* (Cambridge University Press, New York, 1914), p. 64.
- ¹⁶ L. Schele and D. Freidel, in *A forest of Kings* (New York William Morrow and Co., 1990), p. 134.
- ¹⁷ P. J. O'Brien and H. D. Christiansen, *American Antiquity* **51**, 136 (1986).