



**HAL**  
open science

# The Mayan Long Count Calendar

Thomas Chanier

► **To cite this version:**

| Thomas Chanier. The Mayan Long Count Calendar. 2014. hal-00750006v8

**HAL Id: hal-00750006**

**<https://hal.science/hal-00750006v8>**

Preprint submitted on 6 Jul 2014 (v8), last revised 16 Dec 2015 (v12)

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# The Mayan Long Count Calendar

T. Chanier\*<sup>1</sup>

<sup>1</sup> *Department of Physics, University of Namur, rue de Bruxelles 61, B-5000 Namur, Belgium*

The Maya were known for their astronomical proficiency. Whereas Mayan mathematics were based on a vigesimal system, they used a different base when dealing with long periods of time, the Long Count Calendar (LCC), composed of different Long Count Periods: the Tun of 360 days, the Katun of 7200 days and the Baktun of 144000 days. There were three other calendars used in addition to the LCC: a civil year Haab' of 365 days, a religious year Tzolk'in of 260 days and a 3276-day cycle (combination of the 819-day Kawil cycle and 4 colors-directions). Based on astronomical arguments, we propose here an explanation of the origin of the LCC, the Tzolk'in and the 3276-day cycle. The study provides new evidences on the astronomical knowledge of the Maya.

The Maya had a very elaborate and accurate Long Count Calendar (LCC) that was used to pinpoint historical events from a selected beginning time. The smallest unit of time is the day (Kin); 20 Kin form a Winal, 18 Winal form a Tun (360 Kin), 20 Tun form a Katun (7200 Kin), and 20 Katun form a Baktun (144000 Kin). The LCC represents a date  $D$  as a set of coefficients  $(C_i, \dots, C_3, C_2, C_1, C_0)$  such that:  $D = C_0 + C_1 \times 20 + \sum_{i=2}^n C_i \times 18 \times 20^{i-1}$  with  $C_0 = \text{mod}(D, 20)$ ,  $C_1 = \text{int}(\text{mod}(D, 360)/20)$  and  $C_i = \text{int}(\text{mod}(D, 18 \times 20^i)/(18 \times 20^{i-1}))$  for  $i > 1$ . The day count usually restarts when  $C_4$  reaches 13, such as the date is given as a set of 5 coefficients:  $D \equiv \text{mod}(D, 13 \times 144000) = C_4.C_3.C_2.C_1.C_0$ . The Maya used three other independent calendars: a religious year (Tzolk'in), a civil year (Haab') and a 3276-day cycle. One Tzolk'in of 260 days comprised 13 months (numerated from 1 to 13) containing 20 named days (Imix, Ik, Akbal, Kan, Chicchan, Cimi, Manik, Lamat, Muluc, Oc, Chuen, Eb, Ben, Ix, Men, Cib, Caban, Etznab, Cauac, and Ahau). One Haab' of 365 days comprised 18 named months (Pop, Uo, Zip, Zotz, Tzec, Xul, Yaxkin, Mol, Chen, Yax, Zac, Ceh, Mac, Kankin, Muan, Pax, Kayab, and Cumku) with 20 days each (Winal) plus 1 extra month (Uayeb) with 5 nameless days. The Tzolk'in and the Haab' coincide every 73 Tzolk'in or 52 Haab' or a Calendar Round. Their least common multiple (LCM) is  $73 \times 260 = 52 \times 365 = 18980$  days. In the Calendar Round, a date is represented by  $\alpha X \beta Y$  with the religious month  $1 \leq \alpha \leq 13$ , X one of the 20 religious days, the civil day  $0 \leq \beta \leq 19$ , and Y one of the 18 civil months,  $0 \leq \beta \leq 4$  for the Uayeb. According to the Goodman-Martinez-Thompson (GMT) correlation to the Gregorian calendar, which is based on historical facts, the Mayan Calendar began on 11 August 3114 BC or 0(13).0.0.0.0, 4 Ahau 8 Cumku and ended on 21 December 2012. This corresponds to a 13 Baktun cycle or a period of approximately 5125 years. The 3276-day cycle is a combination of the Kawil, a 819-day cycle, and a 4-day cycle corresponding to the 4 colors-directions.

The origin of the Long Count Periods (LCPs) is unknown. A common assumption is the desire of the calendar keeper to maintain the Tun in close agreement with the tropical/solar year of approximately 365.24 days.<sup>1</sup> There is no consensus concerning the origin of

the Tzolk'in, which has been associated with various astronomical cycles. Three Tzolk'in correspond to Mars synodic period, 16 Tzolk'in equal 11 of Saturn synodic periods (+2 days), and 23 Tzolk'in are equivalent to 15 Jupiter synodic periods (-5 days).<sup>2</sup> It has been tentatively connected to the eclipse half-year (173.31 days) because 2 Tzolk'in are very close to 3 eclipse half-years.<sup>3</sup> Finally, it has been noted that the Tzolk'in approximates the length of time Venus is visible as a morning or evening star.<sup>4</sup> However, these interpretations fail to link the Tzolk'in to the LCPs. The Kawil cycle has been attributed to the observation of Jupiter and Saturn<sup>5,6</sup> because 19 (6) Kawil correspond to 39 (13) Jupiter (Saturn) synodic period. Four numbers of possible astronomical significance have been discovered on the walls of a residential structure in Xultun, Guatemala and have been dated from the early 9<sup>th</sup> century CE. The Xultun numbers are given in Table I. They are such that  $\mathcal{X}_1 = 365 \times 3276$  and  $\mathcal{X}_3 = \mathcal{X}_2 + 2\mathcal{X}_0$ .

| $\mathcal{X}_i$ | LCC        | $D$ [day] | $\mathcal{X}_i/56940$ |
|-----------------|------------|-----------|-----------------------|
| $\mathcal{X}_0$ | 2.7.9.0.0  | 341640    | 6                     |
| $\mathcal{X}_1$ | 8.6.1.9.0  | 1195740   | 21                    |
| $\mathcal{X}_2$ | 12.5.3.3.0 | 1765140   | 31                    |
| $\mathcal{X}_3$ | 17.0.1.3.0 | 2448420   | 43                    |

TABLE I. Xultun numbers  $\mathcal{X}_i$ .<sup>7</sup>  $56940 = \text{LCM}(365, 780)$  is their largest common divisor.

This paper explains a possible origin of the LCPs, the religious year Tzolk'in and the relationship between the Kawil cycle and the 4 colors-directions. This new interpretation, based on the periodic movements of the planets in the night sky, gives rise to different day cycles that have been identified in the Maya Codices and monuments. Attempts are made to provide a meaning to the Xultun numbers. The study shows that Mayan astronomers knew the synodic periods of all the planets of the Solar System visible to the naked eye: Mercury, Venus, Mars, Jupiter and Saturn.

The Maya were known for their astronomical skills as exemplified by the Dresden Codex, a bark-paper book of the 11<sup>th</sup> or 12<sup>th</sup> century CE. On page 24 of this Codex is written the so-called Long Round number noted

| Planet     | $P$ [day] | Prime factorization               |
|------------|-----------|-----------------------------------|
| Mercury    | 116       | $2^2 \times 29$                   |
| Venus      | 584       | $2^3 \times 73$                   |
| Earth      | 365       | $5 \times 73$                     |
| Mars       | 780       | $2^2 \times 3 \times 5 \times 13$ |
| Jupiter    | 399       | $3 \times 7 \times 19$            |
| Saturn     | 378       | $2 \times 3^3 \times 7$           |
| Lunar      | 177       | $3 \times 59$                     |
| senesters  | 178       | $2 \times 89$                     |
| Pentalunex | 148       | $2^2 \times 37$                   |

TABLE II. Planet cycles<sup>8,9</sup> and their prime factorizations.

9.9.16.0.0 in the LCC or 1366560 days, a whole multiple of the Tzolk'in, the Haab', the Calendar Round, the Tun, Venus and Mars synodic periods:  $\mathcal{LR} = 1366560 = 5256 \times 260 = 3744 \times 365 = 72 \times 18980 = 3796 \times 360 = 2340 \times 584 = 1752 \times 780$ . Only the moon, Mercury, Venus, Earth (solar year), Mars, Jupiter, and Saturn are visible to the naked eye. Their respective mean synodic periods are given in Table II. Evidences have been found in different Mayan Codices that Mayan astronomers observed the periodic movements in the night sky of Mercury, Venus, and Mars, but it is unclear whether they tracked the movements of Jupiter and Saturn.<sup>10</sup> The periods relevant for the prediction of solar/lunar eclipses are the pentahunex of 148 days (5 Moon synodic periods of 29.53 days) and the lunar semesters of 177 or 178 days (6 Moon synodic periods), which are the time intervals between subsequent eclipse warning stations present in the Eclipse Table on pages 51 to 58 of the Dresden Codex.<sup>9</sup> The LCM of these numbers is  $\mathcal{N} = 20757814426440$  days (including the planet synodic periods and the two lunar semesters) or  $\mathcal{N}^\dagger = 768039133778280$  days (also including the pentahunex).  $\mathcal{N}$  is such that:

$$\text{int}(\mathcal{N}/13/73/144000) = 338 + 360 + 7200 + 144000. \quad (1)$$

$\mathcal{N}^\dagger$  gives the same result but divided by 13, 37 and 73. That defines the Tun, Katun, and Baktun as a polynomial expression of  $\text{int}(\mathcal{N}/13/73)$  of the form  $18 \times 20^3 \times (C_0 + \sum_{n=1}^3 18 \times 20^n)$  with  $C_0 = 338$ . The  $\text{LCM}(338,360) = 169 \times 360 = 234 \times 260 = 60840$ ; 338 and 365 are relatively prime numbers: the  $\text{LCM}(338,365) = 338 \times 365 = 123370$ ; 234 and 365 are also relatively prime numbers: the  $\text{LCM}(234,365) = 234 \times 365 = 85410$ . The  $\text{LCM}(260,365)$  is  $73 \times 260 = 52 \times 365 = 18980$  days or a Calendar Round. That may define the Tzolk'in.  $\mathcal{N}$  is such that the  $\text{mod}(\mathcal{N}, \mathcal{LR}) = 341640$  days or 2.7.9.0.0 in the LCC. This is the Xultun number  $\mathcal{X}_0$  (Table I). It is to be noted that  $\mathcal{X}_0$  is a whole multiple of the Tzolk'in, Haab', Tun, Venus and Mars synodic periods:  $341640 = 1314 \times 260 = 936 \times 365 = 949 \times 360 = 585 \times 584 = 438 \times 780$ .  $\mathcal{X}_0$  is also the largest common divisor of  $\mathcal{LR}$  and  $\mathcal{N}$ . Table III gives the divisibility of  $\mathcal{N}$  by a polynomial expression of the type  $P_i = 13 \times 73 \times (\sum_n 18 \times 20^n)$ . The integer parts  $D_i$  are such that  $D_i/D_{i-1} \approx 20$  for  $0 < i < 6$  and  $D_6/D_5 = 21$ .

| Name       | $i$ | $C_i$ [day] | $P_i$                            | $D_i$      |
|------------|-----|-------------|----------------------------------|------------|
| -          | 0   | 18          | $\mathcal{N}/13/73/\sum_0^6 C_i$ | 18         |
| Tun        | 1   | 360         | $\mathcal{N}/13/73/\sum_0^5 C_i$ | 360        |
| Katun      | 2   | 7200        | $\mathcal{N}/13/73/\sum_0^4 C_i$ | 7215       |
| Baktun     | 3   | 144000      | $\mathcal{N}/13/73/\sum_0^3 C_i$ | 144304     |
| Pictun     | 4   | 2880000     | $\mathcal{N}/13/73/\sum_0^2 C_i$ | 2886428    |
| Calabtun   | 5   | 57600000    | $\mathcal{N}/13/73/\sum_0^1 C_i$ | 57866020   |
| Kinchiltun | 6   | 1152000000  | $\mathcal{N}/13/73/C_0$          | 1215186420 |

TABLE III. Divisibility of  $\mathcal{N} = 20757814426440$  days by a polynomial expression of the type  $P_i = 13 \times 73 \times (\sum_{n=0}^{6-i} 18 \times 20^n)$ .  $D_i = \text{int}(\mathcal{N}/P_i)$ .

Table IV gives the coincidence of the 338-day period with the LCPs, as well as the Tzolk'in and the 234-day period. As described earlier, the Tun and 338-day period coincide every 60840 days, or 234 Tzolk'in. The 338-day period also coincides with the Winal every 3380 days, corresponding to 13 Tzolk'in, a cycle that does not coincide with the 234-day period. Rather, the 234-day period coincides with the Winal every 2340 days or 9 Tzolk'in =  $9 \times 260 = \text{LCM}(9,13,20)$ . This 2340-day cycle is present in the Dresden Codex on pages D30c-D33c and has been attributed to a Venus-Mercury almanac because  $2340 = 20 \times 117 = 5 \times 585$  is an integer multiple of Mercury and Venus mean synodic periods (+1 day).<sup>11</sup> Another explanation may be of divination origin because  $117 = 9 \times 13$ . In Mesoamerican mythology, there are a set of 9 Gods called the Lords of the Night<sup>12-15</sup> and a set of 13 Gods called the Lords of the Day.<sup>15</sup> Each day is linked with 1 of the 13 Lords of the Day and 1 of the 9 Lords of the Night in a repeating 117-day cycle. This cycle coincides with the Tzolk'in every 2340 days. On Table IV figures a date  $D = \text{LCM}(338,7200) = 1216800$  days or 8.9.0.0.0 in the LCC or 5 February 219 CE according to the GMT correlation. This date may have marked the beginning of the reign of Yax Moch Xoc, the founder of the Tikal dynasty, in 219 CE.<sup>16</sup>

| LCP    | $D$ [day] | $L$ [day] | $M$ [day] | $N$ [day] | $L/260$ | $L/234$  |
|--------|-----------|-----------|-----------|-----------|---------|----------|
| Winal  | 20        | 3380      | 260       | 2340      | 13      | 14.44444 |
| Tun    | 360       | 60840     | 4680      | 4680      | 234     | 260      |
| Katun  | 7200      | 1216800   | 93600     | 93600     | 4680    | 5200     |
| Baktun | 144000    | 24336000  | 1872000   | 1872000   | 93600   | 104000   |

TABLE IV. Coincidence of the 234-, 260- and 338-day periods with the LCPs.  $L = \text{LCM}(D,338)$ ,  $M = \text{LCM}(D,260)$  and  $N = \text{LCM}(D,234)$ .

We now examine the cycles that can be constructed from the combination of the Tzolk'in ( $260 = 13 \times 20$ ), the Haab' ( $365 = 5 \times 73$ ) and the LCPs ( $D = 18 \times 20^n$ ). Because 13 is not present in the prime factorizations of the LCPs, we obtain  $\text{LCM}(260,D) = 13 \times D$ . There is, for example, a 13 Katun cycle of 93600 days (360 Tzolk'in) reminiscent of the Katun Wheel and a 13 Baktun cycle of 1872000 days (7200 Tzolk'in). This means that

the Baktun cycle coincides with the Tzolok'in cycle every 13 Baktun, corresponding to approximately 5125 years. This is the origin of the 13 Baktun cycle. Because 73 is not present in the prime factorizations of the LCPs, we obtain  $\text{LCM}(365, D) = 73 \times D$ . This gives rise to 73 LCP cycles. Moreover, the  $\text{LCM}(260, 365, D) = 13 \times 73 \times D$ . This is the case of the Xultun number  $\mathcal{X}_0 = 341640 = \text{LCM}(260, 365, 360)$  which makes coincide the Tzolok'in, the Haab' and the Tun.

| Planet     | $P$ [day] | $N = \text{LCM}(P, L)$ | $N/260$ | $N/2340$ | $N/L$ |
|------------|-----------|------------------------|---------|----------|-------|
| Mercury    | 116       | 950040                 | 3654    | 406      | 29    |
| Venus      | 584       | 2391480                | 9198    | 1022     | 73    |
| Earth      | 365       | 2391480                | 9198    | 1022     | 73    |
| Mars       | 780       | 32760                  | 126     | 14       | 1     |
| Jupiter    | 399       | 622440                 | 2394    | 266      | 19    |
| Saturn     | 378       | 98280                  | 378     | 42       | 3     |
| Lunar      | 177       | 1932840                | 7434    | 826      | 59    |
| semesters  | 178       | 2915640                | 11214   | 1246     | 89    |
| Pentalunex | 148       | 1212120                | 4662    | 518      | 37    |

TABLE V. Coincidence of the 32760-day cycle with the planet synodic periods.<sup>8,9</sup>  $N = \text{LCM}(P, 360, 819)$  and  $L = \text{LCM}(360, 819) = 32760$  days.

The Kawil, a 819-day cycle, is connected with a 4-day cycle (4 colors-directions), coinciding every 3276 days. The Haab' and the Tzolok'in coincide with the 3276-day cycle every  $\text{LCM}(365, 3276) = 365 \times 3276 = 1195740$  days or 8.6.1.9.0 in the LCC. This is the Xultun number  $\mathcal{X}_1$  (Table I).  $\mathcal{X}_1 = \text{LCM}(9, 4, 819, 260, 365)$  is the time distance between two days of the same Lord of the Night, color-direction, Kawil, Tzolok'in and Haab' date.  $\mathcal{X}_0$  and  $\mathcal{X}_1$  are such that  $341640 \times 819 = 1195740 \times 360 = 2391480$  or  $\mathcal{Y} = 16.12.3.0.0$  in the LCC.  $\mathcal{Y}$  represents the same as  $\mathcal{X}_1$  at the end of a Tun (0 Winal, 0 Kin). The 3276-day cycle coincides with the Tun every 32760 days =  $\text{LCM}(360, 819) = 7 \times \text{LCM}(260, 360)$ . Table V shows the coincidence of the 32760-day cycle with the planet synodic periods, giving rise to whole multiples of the Tzolok'in and the 2340-day cycle.  $\mathcal{N}$  and  $\mathcal{N}^\dagger$  are dividable by 32760. This provides further proofs of the astronomical knowledge of the Maya. It shows that Mayan astronomers calculated the synodic periods of at least Venus, Mars and Saturn: the  $\text{LCM}(584, 365, 780, 378) = 7174440 = 219 \times 32760$ . The use of the Tzolok'in and the commemoration of the 13 Baktun cycle implies that Mayan astronomers knew the synodic periods of Mercury and Jupiter as well.

Various religious cycles can be constructed by taking the LCM of 260 and the planet synodic periods (Table VI). The most important one in Mayan religion is

the Calendar Round, which is the  $\text{LCM}(260, 365) = 73 \times 260 = 52 \times 365 = 18980$  days. There is a coincidence between Venus synodic period and the Tzolok'in: the  $\text{LCM}(260, 584) = 65 \times 584 = 146 \times 260 = 104 \times 365 = 37960$  days (2 Calendar Rounds), the length of the Venus Table on pages 24-29 of the Dresden Codex. Two other cycles are based on the coincidence of the Tzolok'in and the 234- and 338-day periods: the  $\text{LCM}(260, 234) = 9 \times 260 = 2340$  days and the  $\text{LCM}(260, 338) = 13 \times 260 = 3380$  days. There is also a coincidence between Mars synodic period and the Haab': the  $\text{LCM}(780, 365) = 73 \times 780 = 219 \times 260 = 156 \times 365 = 56940$  days (3 Calendar Rounds): this is the largest common divisor of the Xultun numbers (Table I).

| Planet     | $P$ [day] | $N = \text{LCM}(P, 260)$ | $N/260$ | $N/P$ |
|------------|-----------|--------------------------|---------|-------|
| Mercury    | 116       | 7540                     | 29      | 65    |
| Venus      | 584       | 37960                    | 146     | 65    |
| Earth      | 365       | 18980                    | 73      | 52    |
| Mars       | 780       | 780                      | 3       | 1     |
| Jupiter    | 399       | 103740                   | 399     | 260   |
| Saturn     | 378       | 49140                    | 189     | 130   |
| Lunar      | 177       | 46020                    | 177     | 260   |
| semesters  | 178       | 23140                    | 89      | 130   |
| Pentalunex | 148       | 9620                     | 37      | 65    |

TABLE VI. Coincidence of the planet cycles<sup>8,9</sup> and the Tzolok'in.

The study presented here describe a possible explanation of the origin of the Long Count Calendar (LCC) and its connection with the Tzolok'in and Haab' calendars. It provides a better understanding of the 3276-day cycle, the coincidence of the Kawil and the 4 colors-directions. Based on astronomical observations of the Solar System, the LCC gives rise to three different periods of 234 days, 260 days (Tzolok'in) and 338 days. The correlation between the 234-day period and the Tzolok'in, a 2340-day cycle identified in the Dresden Codex, was certainly used for divination purposes. The correlation between the 338-day period and the Tzolok'in gives rise to a 13-Tzolok'in cycle that has not yet been identified in the Codices. The 13 Baktun cycle originates from the correlation between the Tzolok'in and the Baktun. The 3276-day cycles was chosen to commensurate with the planet synodic cycles, the Tzolok'in, the 2340-day divinatory cycle and the Tun. These results indicate that Mayan astronomers calculated the synodic periods of all the planets visible to the naked eye, as well as the basic cycles for solar/lunar eclipse prediction, the pentahunex and the lunar semesters.

\* e-mail: thomas.chanier@gmail.com

<sup>1</sup> A. F. Aveni, in *Skywatchers: A Revised and Updated Version of Skywatchers of Ancient Mexico* (University of Texas Press, 2001), p. 136.

<sup>2</sup> J. S. Justeson, *The ancient Maya Ethnoastronomy: An Overview of Hieroglyphic Sources*, in *World Archaeoastronomy: Selected Papers from the Second Oxford Interna-*

- tional Conference on Archaeoastronomy, edited by A. F. Aveni, (Cambridge University Press, 1989), p. 82.
- <sup>3</sup> A. F. Aveni, in *Skywatchers: A Revised and Updated Version of Skywatchers of Ancient Mexico* (University of Texas Press, 2001), p. 144.
  - <sup>4</sup> H. M. Bricker and V. R. Bricker, in *Astronomy in the Maya Codices* (American Philosophical Society, Philadelphia, 2011), p. 69.
  - <sup>5</sup> F. G. Lounsbury, in *Maya Numeration, Computation, and Calendrical Astronomy. Dictionary of Scientific Biography*, vol. 15, suppl. 1, edited by Charles Coulston-Gillispie, (New York: Charles Scriber's Sons, 1978), p. 811
  - <sup>6</sup> J. S. Justeson, *The ancient Maya Ethnoastronomy: An Overview of Hieroglyphic Sources*, in *World Archaeoastronomy: Selected Papers from the Second Oxford International Conference on Archaeoastronomy*, edited by A. F. Aveni, (Cambridge University Press, 1989), p.103.
  - <sup>7</sup> W. A. Saturno, D. Stuart, A. F. Aveni and F. Rossi, *Science* **336**, 714 (2012).
  - <sup>8</sup> <http://nssdc.gsfc.nasa.gov/planetary/factsheet/>
  - <sup>9</sup> H. M. Bricker and V. R. Bricker, *Current Anthropology* **24**, 1-23 (1983).
  - <sup>10</sup> H. M. Bricker and V. R. Bricker, in *Astronomy in the Maya Codices* (American Philosophical Society, Philadelphia, 2011), pp. 163, 367, 847.
  - <sup>11</sup> H. M. Bricker and V. R. Bricker, in *Astronomy in the Maya Codices* (American Philosophical Society, Philadelphia, 2011), p. 235.
  - <sup>12</sup> A. F. Aveni, in *Skywatchers: A Revised and Updated Version of Skywatchers of Ancient Mexico* (University of Texas Press, 2001), pp. 156-157.
  - <sup>13</sup> E. H. Boone, in *Cycles of Time and Meaning in the Mexican Books of Fate* (University of Texas Press, 2007), pp. 44-45.
  - <sup>14</sup> F. Lounsbury, in *Dictionary of Scientific Biography*, vol. 15, (Scribners, New York, 1978), suppl. 1, p. 787.
  - <sup>15</sup> T. A. Joyce, in *Mexican Archaeology: An Introduction to the Archaeology of the Mexican and Mayan Civilizations of Pre-Spanish America* (Cambridge University Press, New York, 1914), p. 64.
  - <sup>16</sup> L. Schele and D. Freidel, in *A forest of Kings* (New York William Morrow and Co., 1990), p. 134.
  - <sup>17</sup> P. J. O'Brien and H. D. Christiansen, *American Antiquity* **51**, 136 (1986).