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Characterization of a π-phase shift quantum gate for coherent-state qubits

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Abstract. We discuss the characterization of a π-phase shift quantum gate acting on a qubit encoded in superpositions of coherent states. We adopt a technique relying on some *a priori* knowledge about the physics underlying the functioning of the device. A parameter summarizing the global quality of the quantum gate is obtained by 'virtually' processing an entangled state. With such an approach, we can facilitate the characterization of our gate, focusing on the useful subspace rather than on the entire phase space.

Quantum logic is the basis of future quantum computers. For this purpose, operations are performed on quantum systems in order to implement the desired processing steps. Unavoidably, such operations will only be an approximation of the ideal quantum gates, and the degree to which this approximation is acceptable is determined by the effectiveness of error correction codes [1]. The characterization of quantum operations is then an important step in establishing practical limits in the use of such devices.

A crucial requirement is that the system remains under controlled manipulation, and is well preserved from coupling to the environment, so that the action of the gates is not spoiled by decoherence. Optics offers an interesting option in this sense; within quantum optics, several

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proposals have been put forward, based on either a discrete [2] or a continuous variable approach [3].

Recently, a different point of view emerged that aims to combine the strengths of both worlds and encodes quantum bits (qubits) in superpositions of weak coherent states [4, 5]. The peculiarity of this approach is that the logical states $|0\rangle$ and $|1\rangle$ are represented by two non-orthogonal (thus not mutually exclusive) states of the system, namely two coherent states with the same amplitude and opposite phase $|\alpha\rangle$ and $|-\alpha\rangle$; this has been proved to be an error-correctable approach for moderately low intensities $|\alpha| \sim 1.5$, where the overlap is $|\langle\alpha|-\alpha\rangle|^2 \sim 10^{-4}$ [6]. The cost of this choice is that the gates are necessarily probabilistic, with a success rate that depends on this overlap; remarkably, however, the comparison with other optical schemes in terms of resources needed for scalability seems favorable [6, 7].

Gates on coherent-state qubits architecture cannot be described by ordinary rank-4 tensors, as they are defined using non-orthogonal states as a basis. Moreover, decoherence processes will most probably not leave the output state inside the original reduced Hilbert space spanned by $|\alpha\rangle$ and $|-\alpha\rangle$; current technology is able to generate input test states with limited fidelity: such states might have components from outside the relevant subspace. Therefore, we cannot simply use standard techniques to perform a process tomography constrained to the qubit subspace [8–10].

There exist more general approaches for obtaining a description of the gate as a process in phase space [11, 12], but this complete approach may be rather expensive in terms of resources. In fact, such a description takes into account the whole phase space, while the device actually acts as a gate only for states of a given amplitude. Our method allows us to characterize the gate directly in the subspace where it is effective, and we can introduce an action similar to the Choi–Jamiolkowski isomorphism. In other words, a description in the complete phase space will give much information that will not be used in the characterization, as it does not concern the interesting subspace. In order to simplify data acquisition and analysis, we used a technique that only gives us relevant parameters, although this comes at the price of having *a priori* information about the gate through an exhaustive model.

In this paper, we implement a characterization of a π-phase shift gate which does not rely on a black-box approach [9], but requires some modeling of the functioning of the gate. This is a realistic approach, since such knowledge is needed to achieve the desired level of control. One can then identify a small number of parameters, accessible to the experimentalist, by which the gate process can be modeled.

In our experiment, we implemented the π-phase gate for coherent-state qubits proposed by Marek and Fiurášek [13]. The operating principle of the gate is that coherent states are eigenstates of the photon subtraction operator: $\hat{a}|\pm\alpha\rangle = \pm\alpha|\pm\alpha\rangle$. This operation thus corresponds to a π-phase shift, up to an overall constant. In the laboratory, photon subtraction can be approximated by a beam splitter with low reflectivity, followed by detection on a photon counter. There have been demonstrations of such an effect [16–19], but the characterization has focused so far on the states that could be produced by this technique, rather than on the device itself. These gates provide a reliable way of implementing logical operations in the same vein as other optical realizations [14, 15], hence the interest in their characterization.

Testing quantum gates demands operating them with at least two different orthogonal bases [20]. Here the computational basis $\{|\alpha\rangle, |-\alpha\rangle\}$ is trivial, while equal real superpositions can be obtained by the manipulation of a squeezed vacuum. As shown in figure 1, this state is produced using an optical parametric amplifier (OPA), pumped in the collinear

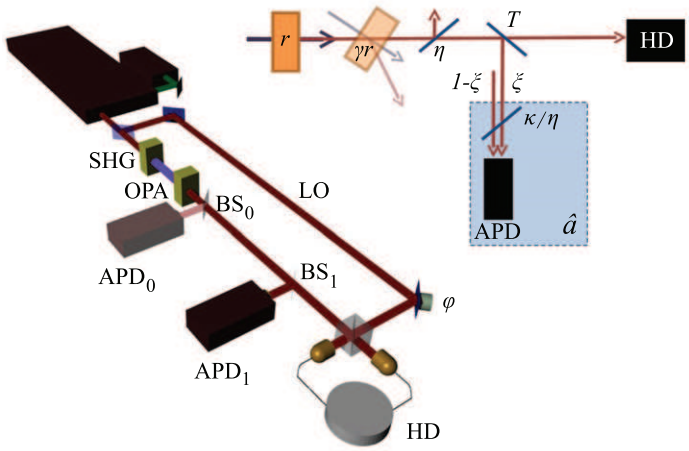

Figure 1. Left: scheme of the experiment. The main laser is a pulsed Ti:sapphire at $\lambda = 850$ nm, pulsed at a repetition rate of 800 kHz (Coherent Mira 900 with a cavity dumper), with a pulse width of 200 fs. The light pulses are frequency doubled by a crystal of potassium niobate (SHG) in order to produce a pump beam for the OPA. This OPA produces a squeezed vacuum, which is then transformed into a squeezed one-photon by photon subtraction[6]. This is performed by directing a small fraction of the beam on the avalanche photodiode APD_0 by a beam splitter BS_0 and conditioning the measurement on a detection event. Either the squeezed vacuum or the squeezed photon is used as the input of the tested gate, constituted by a beam splitter BS_1 and the APD_1. Both beam splitters are realized by a sequence of a half-wave plate and a polarizer, so that we could tune both reflectivities to $R = 10\%$. This also allows us to take them out of the path of the input for a direct measurement. A small portion of the main laser is used as a local oscillator (LO) for a homodyne detector (HD); the phase φ is scanned by means of a piezo actuator. Right: visual representation of the phenomenological model. Symbols are defined in the main text.

regime. For sufficiently weak intensities, the OPA generates a good approximation of an even superposition $|+\rangle = \mathcal{N}_+ (|\alpha\rangle + |-\alpha\rangle)$, where $\mathcal{N}_+$ is a normalizing factor[5]. The odd superposition $|-\rangle = \mathcal{N}_- (|\alpha\rangle - |-\alpha\rangle)$ results from the application of the gate [17]. In our experiment, this is implemented by the beam splitter BS_0 and the avalanche photodiode APD_0. These two states can be used to test the behavior of a second gate constituted by BS_1 and APD_1.

Although the states we can produce do have marked signatures of the ideal behavior, such as a change in the parity, current technology limits us to some approximations of coherent-state qubits [19, 21], which are not fully suitable for a direct characterization of the gate. In a direct comparison of the observed output to the ideal, it would be hard to deconvolve the errors due to the imperfections of the gate and those due to the input state itself.

[5] The normalization factor $\mathcal{N}_{\theta,\phi}$ of an arbitrary superposition $\mathcal{N}_{\theta,\phi}(\cos\frac{\theta}{2}|\alpha\rangle + e^{i\phi}\sin\frac{\theta}{2}|-\alpha\rangle)$ is equal to $[1 + \sin\theta\cos\phi\exp(-2\alpha^2)]^{-1/2}$. In the particular case of a superposition $|\pm\rangle = \mathcal{N}_\pm(|\alpha\rangle \pm |-\alpha\rangle)$, the normalization factor is $\mathcal{N}_\pm = [2(1 \pm \exp(-2\alpha^2))]^{-1/2}$.

[6] Denoting by $\hat{S}$ the squeezing operator, a photon subtracted squeezed state is, up to the normalization, $\hat{a}\hat{S}|0\rangle \propto \hat{S}\hat{a}^\dagger|0\rangle$, which is a squeezed one-photon state.

These limitations can be relaxed if one concedes to experimentalists some *a priori* knowledge about the physics of their gate: a model can then be derived and used for such a deconvolution and thus to obtain the behavior of the device for ideal inputs. This is somehow similar to the analysis in [23] to estimate the origin of decoherence in polarization quantum gates. In our case, we can rely on a simple phenomenological model [21], which is grounded on a more rigorous multi-mode treatment [22].

This model is illustrated in figure 1 for the case of single-photon subtraction. We consider a perfect squeezer, i.e. a collinear OPA, which can reduce noise by $s = e^{-2r}$ times the shot noise. The process is spoiled by parasite amplification that can be described by a fictive non-collinear OPA injected on one side with the squeezed vacuum; its gain is expressed as $h = \cosh^2(\gamma r)$. Once we trace out the fictitious idler mode, we obtain the following expression for the Wigner function: $W(x, p) \propto \mathrm{e}^{-\frac{x^2}{hs+h-1}}\, \mathrm{e}^{-\frac{p^2}{h/s+h-1}}$.

This is the squeezed state that is fed into the gate. This consists of a beam splitter with transmissivity $T \simeq 1$ ($T = 0.9$ in our implementation) and an avalanche photodiode (APD) of efficiency κ whose signal is used as a trigger event. The mode of the squeezed vacuum is filtered by a single-mode fiber and a spectral filter; this operation, however, is performed with finite accuracy. There is then a fraction ξ of the events that originate from photon subtraction on the correct mode; the remaining $1 - \xi$ are actually non-correlated events that corrupt the functioning of the gate.

The finite efficiency of the homodyne detection (HD) has to be taken into account; for the sake of simplicity, we introduce the homodyne loss $1 - \eta$ before the beam splitter rather than before the HD; then we correct the efficiency of the APD as κ/η. This has the advantage of simplifying the calculations and is strictly equivalent to the use of homodyne loss just before the HD and an APD efficiency of κ. In the limit of small efficiency, we can describe the action of the APD simply as the annihilation operator $\hat{a}$ acting on the reflected mode, which is finally traced out.

We thus obtain a parameterization of the Wigner function, which depends on a few parameters; some of them are inherent to the state: the squeezing s and the parasitic gain h; others describe the action of the gate: the modal purity ξ and the transmissivity T; and finally the homodyne efficiency η describes the detection. These can be either directly measured by a classical signal, as is the case for T, or inferred by a best fit on the quadrature histograms. For our analysis, we extrapolate the results for the generated state, by imposing $\eta \to 1$. The same modeling is applied when considering the double photon subtraction.

For our purpose, it is crucial to note that the functioning of the gate is well described by only two parameters: the reflectivity of the beam splitter T and the modal purity ξ [22]. These parameters can be accessed experimentally by using a set of states as a probe. In our case, coherent inputs are not useful, as they are less sensitive to faulty events. We thus use the squeezed vacuum, approximating the even cat state, as a probe to estimate these parameters. We then need to check the consistency of the model: in order to do that, we need to feed a different known state, predict the output using the derived parameters and compare such a prediction against the experiment. As this second probe, we used a squeezed photon as an approximation of the odd cat state.

In more detail, we carried out homodyne measurements on the squeezed vacuum, from which we estimated its density matrix ρ_0. This is the starting point of our model, from which we can calculate the expected action of the gate, given this state as the input. From a fit, we can derive the value of ξ. At this point, we use a squeezed photon as the input; as before, its density

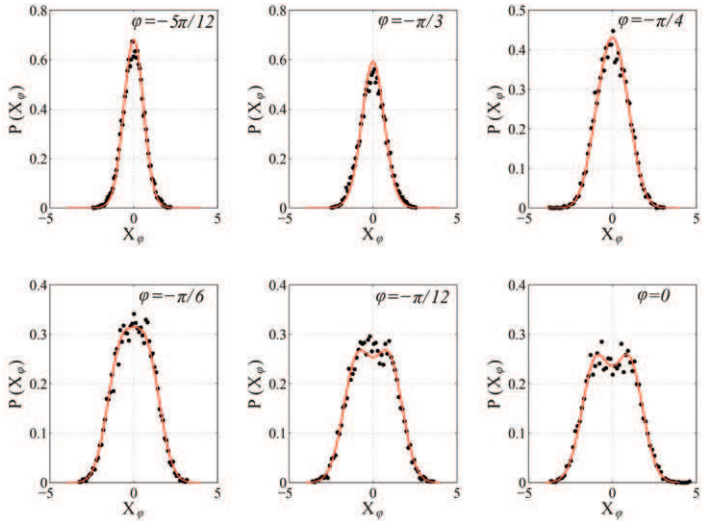

Figure 2. Quadrature distributions of a photon-subtracted squeezed photon. The points are the measured values, while the solid lines are the prediction of the model, based on the outcomes from a squeezed state. Each histogram is calculated from 10 000 quadrature points. We can attribute some noise to fluctuations during the long acquisition time, due to the low two-photon subtraction rate ($\sim$6 events s^{-1}).

matrix ρ_1 is reconstructed by homodyning. The model is now used to estimate the output, fixing T and ξ at the same values as we found in the first case. Finally, we check the consistency of the expected and measured outputs.

Typical results are shown in figure 2; we plot six different histograms of quadrature distributions, where the points show the experimental results and the solid lines the prediction of our model. We emphasize that the parameters are not fitted to the data, but derived from previous measurements on the squeezed vacuum. This gives us evidence of the reliability of our predictions.

By using this model, we can estimate the action of the gate on an ideal arbitrary input in the form $|\psi_{\theta,\phi}\rangle = \mathcal{N}_{\theta,\phi}(\cos\frac{\theta}{2}|\alpha\rangle + e^{i\phi}\sin\frac{\theta}{2}|-\alpha\rangle)$, with θ and ϕ defining the Bloch sphere of the qubit [19]. For the amplitude $|\alpha|$ of the coherent states we need an estimation of what experimental limits are. For this, we have taken the value $|\alpha| = 0.92$ giving maximal fidelity between our squeezed photon and an ideal odd superposition.

Our analysis is summarized in figure 3, where we plot a table of the fidelity $F_{\theta,\phi}$ of the output state with the ideal $|\psi_{-\theta,\phi}\rangle$. The fidelities for our experimental conditions are depicted in figure 3(a). The first remark concerns the variation of the operation over different directions of the Bloch sphere. Better results at the poles with respect to the equator are expected, due to the fact that the high transmission makes coherent states largely insensitive to the modal purity ξ. Furthermore, our analysis shows that odd superpositions work better than the even ones. The limit $\xi = 1$ (figure 3(b)) reveals that it is an intrinsic feature of the gate: it results from the fact that our APD has a low efficiency and to a lesser extent that it cannot resolve the photon number. However, as the transmissivity T increases (figure 3(c)), $F_{\theta,\phi}$ also increases and becomes more regular with respect to ϕ. Indeed, the probability of having several photons reflected to the APD decreases as T increases and therefore the fact that the APD does not resolve the number of

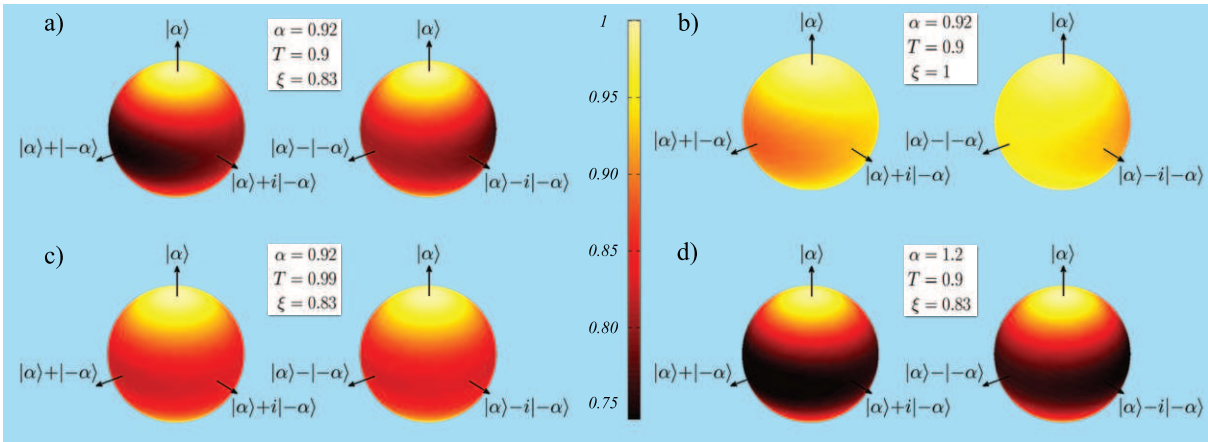

Figure 3. Fidelities of the output states for arbitrary ideal inputs: (a) for our phase gate; (b) for a perfect modal purity $\xi = 1$; (c) for a device with $T = 0.99$; (d) for our device with $\alpha = 1.2$. The angles θ and ϕ are the spherical coordinates on the Bloch sphere.

photons becomes less important. For superpositions with $\theta = \pi/4$ the limit value of 0.83 (when $T \to 1$) is almost reached for $T = 0.99$. The downside is that the probability of success of the gate decreases proportionally to $(1 - T)$. Finally, we see that as α increases (figure 3(d)), $F_{\theta,\phi}$ also becomes more regular with respect to ϕ, but reaches lower values, as the probability to have several photons sent to the APD is higher.

As a more general comment, we conclude that, although the value $\xi = 0.83 \pm 0.04$ might be considered satisfactory in our experiment, as it causes marked non-Gaussian features, the operation of the gate is heavily influenced by the modal purity. This sets a strong requirement for realizing such gates.

The results in figure 3 provide us with extensive information about our device, but fail in delivering a conclusive answer on how good the gate is overall. As mentioned before, associating a quantum process with this operation limited to the qubit subspace is hard: for instance, we cannot formally use Jamiołkowski's isomorphism and derive a process matrix [24], by which we could obtain the fidelity of our process with a target operation. However, we can still retain the underlying physical idea behind the isomorphism, so as to obtain a single figure to describe the quality of the gate.

Let us briefly recall Jamiołkowsi's construction; given a process $\mathcal{E}$ on a space of dimension d, we can associate univocally a matrix by considering a maximal entangled state $|\Psi\rangle = \frac{1}{\sqrt{d}} \sum_{j=1}^{d} |j\rangle|j\rangle$, for a given choice of basis $\{|j\rangle\}_{1 \leqslant j \leqslant d}$ of the system. The process matrix is then obtained through the application of the process $\mathcal{E}$ to one of the modes of the entangled state, what can be formally written as $\chi = I \otimes \mathcal{E}\,(|\Psi\rangle\langle\Psi|)$, which is a state in a larger Hilbert space. When a distance between two processes is needed, we can then rely on the measures holding for states in the extended space; in particular, the fidelity between two processes is given by the fidelity between the corresponding states: $F(\mathcal{E}_1, \mathcal{E}_2) = F(\chi_1, \chi_2)$ [10, 15, 25].

The underlying physical motivation is in some sense an application of quantum parallelism. Since we are interested in the overall behavior of the gate, we need to estimate its action on all the inputs at the same time: this amounts to feeding in the gate half of an entangled pair. In

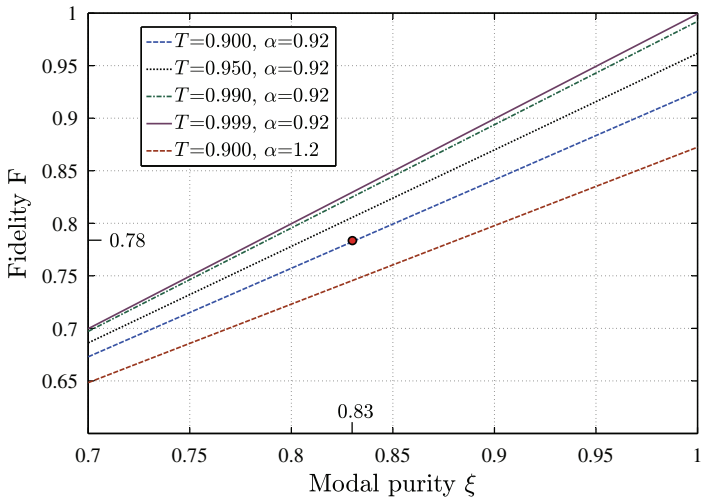

Figure 4. Fidelity of an entangled output from the phase gate with the ideal entangled state: we can note a linear increase of the overlap with the modal purity ξ. The red dot corresponds to our experimental parameters.

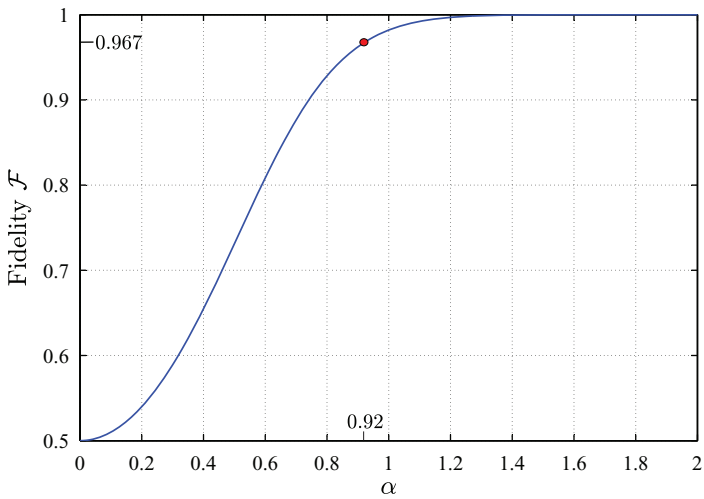

Figure 5. Fidelity between $|\Omega\rangle$ and the ideal bit-flipped state $|\Psi^+\rangle$. The red dot corresponds to our experimental parameters.

light of these considerations, we can adopt as a reasonable figure of merit a fidelity between entangled states. Here we will use as a probe the state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|+\rangle|+\rangle+|-\rangle|-\rangle)$, but actually the result does not depend on this choice (see the appendix).

We then consider the fidelity F between the entangled output as would be produced by our device and the ideal target state $|\Omega\rangle = \hat{\boldsymbol{I}} \otimes \hat{a}|\Phi^+\rangle / \|\hat{\boldsymbol{I}} \otimes \hat{a}|\Phi^+\rangle\|$. Our results are shown in figure 4: the limiting factor of the performances is mostly the worst-case scenario of an even superposition. We found that $F = 0.78 \pm 0.04$ for the experimentally observed gate ($\xi = 0.83$).

Let us emphasize that the ideal target state differs from the bit flip state $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|+\rangle|-\rangle+|-\rangle|+\rangle)$ which would be obtained by an ideal phase gate. This is due to the fact that the states $|\alpha\rangle \pm |-\alpha\rangle$ have different normalization coefficients $\mathcal{N}_\pm$. Such a problem becomes

negligible for large enough values of α, i.e. when the two states $|\pm\alpha\rangle$ become nearly orthogonal. This requirement can be made quantitative by evaluating the fidelity between the target states $|\Omega\rangle$ and $|\Psi^+\rangle$, which is simply

$$\mathcal{F} = |\langle\Omega|\Psi^+\rangle|^2 = \tfrac{1}{2}(1+\tanh(2\alpha^2)). \tag{1}$$

This fidelity is plotted in figure 5 and gives us an idea of the values of $|\alpha|$ required for the protocol to work correctly, in the sense that the cat-state qubit is 'good enough' for correctly implementing the desired quantum phase gate. For our value $\alpha = 0.92$, $\mathcal{F} = 0.967$.

Summarizing, we have quantified the quality of a simple (π-phase shift) quantum gate for 'cat-state' qubits, by evaluating either the fidelity between an arbitrary input state $|\psi_{\theta,\phi}\rangle = \mathcal{N}_{\theta,\phi}(\cos\frac{\theta}{2}|\alpha\rangle + \mathrm{e}^{\mathrm{i}\phi}\sin\frac{\theta}{2}|-\alpha\rangle)$ and the corresponding output state, or the action of the gate on half of an entangled state. This second approach gives a single figure, $F = 0.78$ in our case. The method is relatively independent of the quality of the quantum states used to probe the gate, but requires some *a priori* knowledge and modeling of the gate. It appears thus as a stable approach for obtaining both a 'quality evaluation' from a single number and a full description by concatenating simple but efficient theoretical modeling of the gate.

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Appendix. Invariance of the fidelity with the input entangled state

In the main text, we have considered the case when half of the entangled state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|+\rangle|+\rangle + |-\rangle|-\rangle)$ is used as the input of the gate. Here we show that the result does not actually depend on the choice of the Bell state. For this purpose, let us consider the general case $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle|\mu\rangle + \mathrm{e}^{\mathrm{i}\phi}|-\rangle|\nu\rangle\right)$, where $|\mu\rangle$ corresponds to either $|+\rangle$ or $|-\rangle$, whereas $|\nu\rangle$ corresponds to the opposite ket: $|\nu\rangle = |-\mu\rangle$.

The superoperator $\mathcal{E}$ describing the action of the gate can be decomposed into two parts $\mathcal{E}^{(\mathrm{good})}$ and $\mathcal{E}^{(\mathrm{bad})}$ corresponding respectively to the correct and faulty events: $\mathcal{E} = \xi\,\mathcal{E}^{(\mathrm{good})} + (1-\xi)\,\mathcal{E}^{(\mathrm{bad})}$. First, we note that by linearity we can calculate separately the action of the gate for the correct heralding events and for the faulty ones, and then sum the results with the correct weighting.

Let us now focus on $\mathcal{E}^{(\mathrm{good})}$, the reasoning being similar for $\mathcal{E}^{(\mathrm{bad})}$. For a given initial state $\rho = \sum_{x,y=+,-} c_{xy}|x\rangle\langle y|$, the operator $\mathcal{E}^{(\mathrm{good})}$ is nonlinear. However, as it is modeled with linear operators and partial traces, its nonlinearity comes only from the final normalization. One can thus write it as a linear operator $\tilde{\mathcal{E}}^{(\mathrm{good})}$ giving a non-normalized output state $\tilde{\mathcal{E}}^{(\mathrm{good})}(\rho)$, which is then normalized by its trace: $\mathcal{E}^{(\mathrm{good})}(\rho) = \tilde{\mathcal{E}}^{(\mathrm{good})}(\rho)/\mathrm{Tr}\{\tilde{\mathcal{E}}^{(\mathrm{good})}(\rho)\}$. We denote by the symbol ζ_{xy} the action of $\tilde{\mathcal{E}}^{(\mathrm{good})}$ on the operator $|x\rangle\langle y|$. Under the operator $I \otimes \tilde{\mathcal{E}}^{(\mathrm{good})}$, the state $|\Psi\rangle\langle\Psi|$ is transformed into

$$\tilde{\chi} = \tfrac{1}{2}(|+\rangle\langle+| \otimes \zeta_{\mu\mu} + |-\rangle\langle-| \otimes \zeta_{\nu\nu} + \mathrm{e}^{\mathrm{i}\phi}|-\rangle\langle+| \otimes \zeta_{\nu\mu} + \mathrm{e}^{-\mathrm{i}\phi}|+\rangle\langle-| \otimes \zeta_{\mu\nu}). \tag{A.1}$$

Then, we note that $\hat{a}|\mu\rangle = \alpha\frac{\mathcal{N}_\mu}{\mathcal{N}_\nu}|\nu\rangle$. Introducing $c_{\mu\nu} = \frac{\mathcal{N}_\mu^2}{\mathcal{N}_\nu^2}$, the ideal target state is

$$|\Omega\rangle = \frac{\hat{\boldsymbol{I}} \otimes \hat{a}|\Psi\rangle}{\|\hat{\boldsymbol{I}} \otimes \hat{a}|\Psi\rangle\|} = \frac{1}{\sqrt{c_{\mu\nu} + c_{\nu\mu}}}(\sqrt{c_{\mu\nu}}|+\rangle|\nu\rangle + \mathrm{e}^{\mathrm{i}\phi}\sqrt{c_{\nu\mu}}|-\rangle|\mu\rangle). \tag{A.2}$$

We then consider the fidelity between the normalized state $\chi = \tilde{\chi}/\mathrm{Tr}\{\tilde{\chi}\}$ and $|\Omega\rangle$. This has the explicit expression

$$F = \langle\Omega|\chi|\Omega\rangle = \frac{1}{2\,\mathrm{Tr}\{\tilde{\chi}\}(c_{\mu\nu} + c_{\nu\mu})}(c_{\mu\nu}\langle\nu|\zeta_{\mu\mu}|\nu\rangle + c_{\nu\mu}\langle\mu|\zeta_{\nu\nu}|\mu\rangle + \langle\mu|\zeta_{\nu\mu}|\nu\rangle + \langle\nu|\zeta_{\mu\nu}|\mu\rangle), \tag{A.3}$$

which does not depend on the choice of ϕ and is invariant with exchanging μ and ν. The same reasoning applies when considering faulty trigger events. In case one would prefer to use the bit-flipped state as the target state, the fidelity would be different but still independent of the choice of the initial Bell state.

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