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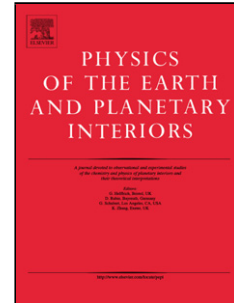


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Authors: A. Rodríguez-Pintó, M.J. Ramón, B. Oliva-Urcia, E.L. Pueyo, A. Pocoví



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1 **Errors in paleomagnetism:**
2 **Structural control on overlapped vectors - mathematical models.**

3

4 Rodríguez-Pintó, A. (1,2); Ramón, M. J. (2); Oliva-Urcia, B. (1);

5 Pueyo, E. L. (2); Pocoví, A. (1)

6 (1) Geodinámica Interna. Ciencias de la Tierra, U. de Zaragoza, Spain (adrianar@unizar.es).

7 (2) Oficina de Proyectos de Zaragoza. Instituto Geológico y Minero de España (unaím@igme.es).

8

9 **Abstract**

10 The reliability of paleomagnetic data is a keystone to obtain trustable kinematics
11 interpretations. The determination of the real paleomagnetic component recorded at
12 certain time in the geological evolution of a rock can be affected by several sources of
13 errors: inclination shallowing, declination biases caused by incorrect restoration to the
14 ancient field, internal deformation of rock volumes and lack of isolation of the
15 paleomagnetic primary vector during the laboratory procedures (overlapping of
16 components). These errors will limit or impede the validity of paleomagnetism as the
17 only three-dimension reference. This paper presents the first systematic modeling of the
18 effect of overlapped vectors referred to declination, inclination and stability tests taking
19 into account the key variables: orientation of a primary and secondary (overlapped to the
20 primary) vectors, degree of overlapping (intensity ratio of primary and secondary
21 paleomagnetic vectors) and the fold axis orientation and dip of bedding plane. In this
22 way, several scenarios of overlapping have been modeled in different fold geometries
23 considering both polarities and all the variables aforementioned, allowing to calculate the
24 deviations of the vector obtained in the laboratory (overlapped) with respect to the
25 paleomagnetic reference (not overlapped). Observations from the models confirm that
26 declination errors are larger than the inclination ones. In addition to the geometry factor,
27 errors are mainly controlled by the relative magnitude of the primary respect to the
28 secondary component (P/S ratio). We observe larger asymmetries and bigger magnitudes
29 of errors along the fold location if the primary and secondary records have different
30 polarities. If the primary record (declination) and the fold axis orientation are
31 perpendicular ($\Omega = 90^\circ$), errors reach maximum magnitudes and larger asymmetries along
32 the fold surface (different dips). The effect of overlapping in the fold and reversal tests is
33 also qualitatively analyzed in this paper.

34

35 **Keywords:** *Mathematical model, overlapped vector, primary and secondary components,*
36 *fold geometry, declination and inclination errors, fold and reversal tests.*

37

38 **1. Introduction**

39

40 Paleomagnetism provides an independent reference frame that helps from the plate-
41 tectonic to fold-and-thrust belt scales to understand and quantify three-dimensional
42 deformation patterns. However, any paleomagnetic study should prove: 1) the absence of
43 inclination shallowing (in sedimentary rocks), 2) a correct restoration to the ancient
44 reference system in case of complex deformation patterns, 3) the absence of internal
45 deformation and 4) a perfect isolation of paleomagnetic components in the laboratory
46 (Van der Voo, 1990). All these causes are controlled by the geometry of deformation and
47 will introduce errors that can seriously affect the interpretation of the data (Pueyo 2010).

48

49 Concerning the last one, the main goal of the demagnetization procedures in the
50 laboratory is to fully isolate all paleomagnetic components (Van der Voo, 1990). The
51 overlapping of paleomagnetic components involves large number and different nature of
52 variables: type of magnetic carrier, grain size, relaxation time and the related unblocking
53 temperatures and coercivities, as well as the geometry of the demagnetization spectra. If
54 there are two or more components sharing a common temperature or coercivity window,
55 then an overlapping of components occurs and any further analysis or interpretation of
56 paleomagnetic data may become non-sense and will lack any reliability. Partial or total
57 simultaneous removal of two paleomagnetic components cannot be ruled out during
58 paleomagnetic analyses. The lack of overlapping should not be assumed and the isolation
59 of paleomagnetic components has to be effectively demonstrated by the internal
60 coherence of the dataset.

61

62 The study of this source of error started very early during the development of
63 paleomagnetic methods (Kramov, 1958; Halls, 1976; Roy & Lapointe, 1978; Bailey and
64 Halls, 1984; Schmidt, 1985; McFadden, 1977 & McFadden and McElhinny, 1988;
65 Dinarés and McClelland, 1991) but neither a diagnostic test nor a way of filtering this
66 error have been developed until today. Other sources of error in the stability tests have
67 been considered such as apparent synfolding results in a fold test caused by structural
68 complications (Tauxe & Watson, 1994; Weil and Van der Voo, 2002; Pueyo, 2010) but

69 the influence of inaccurately isolated primary record in the stability test have remained
70 uncovered.

71

72 Evidences of overlapping include; unexpected inclinations and declination values,
73 inconsistency between two polarities, curved demagnetization diagrams (partial
74 overlapped), “S” shaped demagnetization curves (if one component include another),
75 apparent single component diagrams (if both components are removed simultaneously),
76 in case of total overlap (Tauxe, 2009; Fig 9.10). In summary, this source of error can
77 strongly modify the interpretation of paleomagnetic directions, magnetochrons, vertical
78 axis rotations and also the stability tests (fold and reversal ones).

79

80 In this paper, a systematic mathematical modeling of overlapped vectors as function of
81 the fold geometry is developed. The final goal of the model is to quantify the declination
82 and inclination errors for a wide range of structural locations with a methodology that can
83 be applied to any particular structural setting. Finally, a qualitative evaluation of the
84 influence of the declination and inclination errors in the fold and reversal test is also
85 done.

86

87 **2. Apparent single component diagrams.**

88

89 The case of total overlapping is very challenging because both components are
90 simultaneously removed and an apparent single component demagnetization diagram
91 may result (Tauxe, 2009). Thus, additional paleomagnetic or geologic information is
92 needed to detect possible errors.

93

94 Examples of artificial Zijdeveld diagrams are here used to better understand the effect of
95 the declination and inclination errors in overlapped paleomagnetic vectors. Four different
96 degrees of overlapping (ratio of the magnitude of the primary component respect to the
97 secondary component: P/S) are shown, from non overlapped on the left to a higher degree
98 of overlapping on the right are shown in Fig. 1. The overlapping is considered in a
99 theoretical sample with two components. The primary, secondary and overlapped
100 components are marked with numbers in the orthogonal diagrams between the points of
101 the demagnetization spectra: 1-2 for the secondary, 3-4 for the primary and 2-3 for the
102 overlapped components (Fig. 1). Three different structural situations are presented to see

103 the effect of the fold geometry (rows). A small portion of the primary isolated vector is
 104 displayed at the end of the demagnetization spectra to see the difference with the
 105 overlapped direction. As we can observe, apparent clockwise rotations and steeper
 106 inclination result in a bed striking 45° (right hand rule) and dipping 50° SE (stereographs
 107 on the right side of Fig. 1). The opposite error trend will be detected in a bed striking
 108 135° and dipping 50° SW (counterclockwise –CCW– and shallow inclinations) the
 109 inclination error is so large that produces a change in the polarity of the primary
 110 component. In summary, the same degree of overlapping produces opposite error trends
 111 (CW/CCW- shall/steep) depending on the structural situation, thus, the main factor
 112 controlling the magnitude of declination and inclination errors in the overlapping of two
 113 components is the fold geometry.

114

115 **3. Mathematical modeling of overlapped vectors in cylindrical folds**

116

117 The mathematical modeling presented in this section considers the overlapping of two
 118 paleomagnetic components in a cylindrical fold. To simplify the model, a horizontal flat
 119 bed and a primary magnetic vector (P) recorded in the rock are assumed. Later, the rock
 120 pile is folded (P_f). After a given time gap, a secondary component overprints the primary
 121 signal and both components are overlapped; the result is an intermediate component (P_{of}).
 122 All variables in the model are known, thus the declination (DEC) and inclination (INC)
 123 errors associated to the overlapped component can be calculated after bedding correction
 124 in relation to the primary one (Fig. 2). The following subsections display a succinct
 125 description of variables, procedures and basic equations (the expanded description of
 126 variables, procedures and equations can be found in Appendix 1).

127

128 3.1. Description of the variables

129 3.1.1. *The input variables* involved in the mathematical model (Figs. 2 and 3) are:

- 130 - A primary vector (P) defined by: declination (P_{dec}), inclination (P_{inc}), intensity $|P|$ and
 131 the normal and reverse polarities (P_N and P_R respectively).
- 132 - A secondary and postfolding vector (S) is defined by: S_{dec} , S_{inc} , $|S|$ that only displays one
 133 polarity (normal).
- 134 - The P/S ratio is given by the fraction of P and S intensities and represents the degree of
 135 overlapping between the two components.
- 136 - The fold axis orientation; trend (θ) (plunge is considered to be null).

137 - The obliquity (Ω) is the angle between the primary declination (P_{dec}), and the fold axis
 138 trend (θ)

139 - The dip: Since the plunge of the fold is negligible, the degree of limb rotation is equal to
 140 the dip of the limb. Dip is zero for the non-deformed position, and 90° for vertical beds,
 141 overturned beds are, in absolute value, larger than 90° .

142 - The “folded vector” (P_f) is the primary component after folding and before being
 143 overlapped.

144 3.1.2. The output variables

145 - The overlapped folded primary vector (P_{of})

146 - The overlapped restored primary vector (P_o) is P_{of} after bedding correction.

147 - Inclination (ε_{inc}) and declination (ε_{dec}) errors are the difference between the “overlapped
 148 and restored (P_o)” and the primary (reference) vector (P). $\varepsilon_{inc} = P_{o_{inc}} - P_{inc}$ and $\varepsilon_{dec} =$
 149 $P_{o_{dec}} - P_{dec}$ respectively. It is worth noticing that the inclination error sense depends on
 150 the polarity of the primary vector; smaller inclinations than the reference (negative values
 151 of ε_{inc}) are referred as shallowing and larger ones (positives ε_{inc}) as steepening (Fig. 3) for
 152 the normal polarity. The opposite applies for the reverse polarity.

153

$$154 \begin{matrix} \alpha_{P_o} \\ \beta_{P_o} \end{matrix} = \begin{matrix} \arctan(x_{P_o}/y_{P_o}) \\ \arctan(z_{P_o}/\sqrt{y_{P_o}^2 + z_{P_o}^2}) \end{matrix} \rightarrow \begin{matrix} \alpha_{dec} \\ \alpha_{inc} \end{matrix} = \begin{matrix} \alpha_{P_o} \\ \beta_{P_o} \end{matrix} - \begin{matrix} \alpha_P \\ \beta_P \end{matrix}$$

155

156 where α and β are declination and inclination respectively and x, y, z are the Cartesian
 157 coordinates (see appendix for details).

158

159 Therefore, the main variables considered in the model to calculate the declination and
 160 inclination errors (ε_{dec} and ε_{inc}) due to the overlapping of two components are the
 161 obliquity of the fold axis with respect to the primary component (Ω), the degree of
 162 folding (dip) and the P/S ratio. Since ε_{dec} and ε_{inc} depend upon these three variables as
 163 well as the polarity of the primary component, two different sets of nomograms have
 164 been produced to quantify the errors respect to the obliquity, dip and P/S (Fig. 4a and b).

165

166 3.2. Modeling equations and nomograms

167

168 These equations allow quantifying all possible declination and inclination errors and
 169 building nomograms (Fig. 4), which help understanding the nature of these errors in
 170 relation to the fold geometry. Due to the large number of involved variables, some of
 171 them are considered constant to build the nomograms: a primary magnetic record with
 172 two perfectly antipodal vectors ($P_N = 000$, 45 $P_R = 180$, -45 , Fig. 3) recorded in a
 173 horizontal bed. After folding (variable Ω and dip values) the secondary field ($S = 000$,
 174 55) overprints the primary record in a certain demagnetization window (temperature or
 175 coercivity). The ratio of overlapping, controlled by the intensity ratio (P/S), remains as a
 176 discrete variable. It is worth mentioning that we are considering a total overlapping of the
 177 vectors in an undetermined unblocking spectrum, which would correspond to an apparent
 178 single component in terms of Tauxe (2009).

179

180 **4. Observations from Modeling**

181

182 The following examples (Figs. 4, 5 and a supplementary table on-line) better illustrate the
 183 magnitude of declination and inclination errors obtained with the mathematical model.
 184 They will be used to display the subsequent implications on the fold and reversal tests
 185 (section 5).

186

187 Example 1: Effects on the polarity in a given structural position

188 (Dip, obliquity (Ω) and P/S ratio are constant. Figs. 4 [a&c] and 5)

189 This example focuses in a constant structural position: the southwestern limb of an
 190 anticline dipping 60° in a fold axis oriented at 150° respect to the primary field. The
 191 obtained declination error (ϵ_{dec}) of the P_R is $+53^\circ$ (clockwise $-CW-$) while the P_N will
 192 show a ϵ_{dec} of -41° (counterclockwise $-CCW-$). Inclination errors are $+5,4^\circ$ for the
 193 normal primary polarity and $+42^\circ$ for the reverse one, biasing the value of the inclination
 194 to a shallower orientation in the reverse polarity and to a steeper in the normal one.

195

196 In the particular case of the hinge of the anticline or in any horizontal series (dip= 0 and
 197 hence, it is independent of Ω) the subsequent errors are noteworthy and they explained
 198 the usual deflection of normal and reverse means in horizontal magnetostratigraphic
 199 sections affected by overlapping problems. The declination remains constant ($S_{dec} = P$
 200 dec) but the P_N acquires 5° of ϵ_{inc} (steeper) and the reverse acquires $+85^\circ$ (shallower

201 because of the change of polarity). This example remarks the asymmetry of the
 202 inclination errors for both polarities. However, declination may display significant errors
 203 when $S_{dec} \neq P_{dec}$ (whatever the reason is). Therefore, the reversal test will be affected
 204 due to the deflection of the antipodal character of the primary information.

205

206 Example 2: Effects on different positions of the fold

207 (variable dip; Ω and P/S are constant. Fig. 4[a&c])

208 A second example shows the effect of the secondary component (S) on the reverse
 209 polarity (P_R) along different locations of a fold (e.g. $\Omega = 30^\circ$), P/S is constant and equal to
 210 1. The ϵ_{dec} is -47° and the ϵ_{inc} is $+60.4^\circ$ in the southeastern limb of an anticline (dipping
 211 30°). In the other limb (northwestern dip = -30°), the ϵ_{dec} is $+99^\circ$ and the ϵ_{inc} is $+22.6^\circ$
 212 respectively (example 2.a in Fig. 4[a&c]). Despite of the moderated values of obliquity
 213 and dip, errors are significant and reach up to 100° of deflection. Both locations of the
 214 fold will undergo apparent shallow inclination but with different magnitudes. Now the
 215 effect on the normal polarity (P_N) for the same fold locations ($\Omega = 30^\circ$ and dip 30° and -
 216 30°) is considered. In the southeastern limb the ϵ_{dec} is $+20^\circ$ and the ϵ_{inc} is $+9.2^\circ$ (stepper),
 217 whereas for the northwestern limb the ϵ_{dec} is -12° and the ϵ_{inc} is -4.7° (shallower). The ϵ_{dec}
 218 are moderate but also suggest the same effect, as the deflection caused by a conical fold.

219

220 Considering a wider range of dip values (example 2.b in Fig. 4[a&c]) helps to illustrate
 221 the potential effect of an exhaustive sampling designed for the fold test. The P_R overlaps
 222 with a secondary normal one (S_N) in a fold with $\Omega = 120^\circ$. The observed ϵ_{dec} are $24^\circ, 0^\circ, -$
 223 $134^\circ, -139^\circ, -132^\circ$ and 6° corresponding to dip values of $30^\circ, 0^\circ$ (hinge), $-30^\circ, -60^\circ, -90^\circ$
 224 (vertical) and -120° respectively. On the other hand, the ϵ_{inc} for the same dip dataset will
 225 range between $68^\circ, 85^\circ, -1^\circ, -19^\circ, -35^\circ$ and -39° . In the case of P_N , the ϵ_{dec} are $-15^\circ, 0^\circ,$
 226 $6^\circ, 8^\circ, 6^\circ, -1^\circ$ and the ϵ_{inc} are $17^\circ, 5^\circ, -9^\circ, -23^\circ, -37^\circ$ and -51° . Errors are smaller for the
 227 same structural locations if the secondary and primary polarities are the same.

228 Errors change asymmetrically with dip, especially when the polarity of the primary and
 229 secondary components is opposite. On the contrary, and for a given obliquity ($\Omega = 120^\circ$),
 230 there is a relationship between the magnitude and sense of the errors and the limb
 231 symmetry; the normal polarity component in a northeastern limb will show the minimum
 232 values of ϵ_{dec} , while the southern limb will display the maximum errors. The reverse

233 component shows the contrary effect, declination errors are smaller in the southern limb
234 and larger in the northern one.

235

236 Example 3: Effects of the obliquity on a certain dip.

237 (variable Ω ; dip and P/S are constant. Fig. 4[a&c] and 5)

238 If the primary component is reverse (P_R) and the dip = -60° , the ε_{dec} and ε_{inc} will vary
239 depending on the obliquities. For $\Omega = 30^\circ$, the ε_{dec} is 107° and the ε_{inc} is -1.81° . For $\Omega =$
240 60° , errors are 139° and -19.2° respectively; for $\Omega=120^\circ$ are -139° and -19.2° ; and when Ω
241 $= 150^\circ$, the ε_{inc} is -107° and the ε_{inc} is -1.81° . In all these cases, the inclination is steeper.
242 While positive ε_{dec} affect the signal in a CW manner, negative errors affect the result in
243 CCW way. The ε_{dec} of supplementary obliquities (i.e. 45° and 135°) are of the same
244 magnitude but opposite sign. On contrary, the ε_{inc} corresponding to the supplementary
245 obliquities have the same magnitude and sign. Therefore, there is a remarkable
246 symmetry: the ε_{dec} for a given obliquity is the opposite error of the supplementary one,
247 and the ε_{inc} is equal to the value of the antipodal obliquity (Figs. 4b and 5).

248

249 Example 4: Effects of variable P/S for different locations of a given fold

250 (variable P/S and dip; Ω is constant. Fig. 4b [columns I to IV], Figs. 4c and 5).

251 Errors will change for different P/S values (2.3, 1 and 0.43). For example, when $\Omega = 45^\circ$
252 and dip = ± 45 (both limbs). The ε_{dec} in one limb (dip = $+45^\circ$) are apparent CW for the
253 normal polarity ($+16^\circ$, $+32^\circ$, $+52^\circ$ respectively), and CCW for the reverse polarity (-18° , -
254 38° , -66°). The ε_{inc} are steeper in both cases: $+10^\circ$, $+14^\circ$, $+17^\circ$ for the normal vectors, and
255 $+20^\circ$, $+56^\circ$ and $+91^\circ$ for the reverse ones. In the other fold limb (dip = -45°) the ε_{dec} are -
256 7° , -11° , -15° for the normal component and $+24^\circ$, $+121^\circ$ and $+151^\circ$ for the reverse
257 component. The ε_{inc} are shallower -8° , -14° , -19° for the normal polarity component (gray
258 background [columns I & II]), and -14° , -3° , $+44^\circ$ for the reverse polarity
259 component(white background [columns III & IV]).

260

261 A steepening of the inclination and CW rotation values associated to one limb (dip > 0) in
262 the reverse polarity and to the other limb (dip < 0) in the normal polarity are observed. In
263 an opposite way, a shallower inclination and CCW rotation values appear when dip < 0 in
264 the reverse polarity and dip > 0 in the normal one. It is worth mentioning the large error
265 amplification in the reverse polarity for dip < 0 and any P/S ratio, the ε_{dec} range between

266 +24° and +121° or even +151° (P/S values between 2.3, 1 and 0,43) whereas the ϵ_{inc} range
 267 between -3° and +44°. In other cases, the increasing of the errors as a function of the P/S
 268 value is less remarkable but still asymmetric: see for example the ϵ_{dec} ($\Omega = 45^\circ$) of the
 269 normal polarity in both limbs. The NW limb (dip > 0) display large errors (+16°, +32°
 270 and +52°), while the SE limb (dip < 0) show smaller errors for the same P/S ratios (-7°, -
 271 11° and -15°).

272

273 *Example 5: Effects of variable P/S for different folds*

274 *(variable P/S and Ω ; dip is constant. Fig. 4b [II column] and 4c)*

275 In this case, the errors in the normal polarity for different obliquities ($\Omega = 0^\circ, 45^\circ, 90^\circ,$
 276 135° and 180°) are considered for a fixed fold location (constant dip= +30) and in two
 277 different overlapping ratios (P/S = 0,25 and 4). The ϵ_{inc} have the same magnitude for a
 278 given Ω and its complementary: +1° when $\Omega = 0^\circ$ and 180° . The ϵ_{inc} is +5° and +19° for Ω
 279 = 45° and 135° and the same relation occurs for the ϵ_{dec} but with opposite sign ($\pm 7^\circ, \pm 29^\circ$
 280 for $\Omega = 0^\circ$ and 180° ; and $\pm 6^\circ$ and $\pm 38^\circ$ for $\Omega = 45^\circ$ and 135°). The largest variations of the
 281 ϵ_{inc} for a given dip are seen for $\Omega = 90^\circ$ (+8° and +32°) while the ϵ_{dec} are null.

282

283 The information derived from these examples allows us recognizing some general rules
 284 to understand the declination and inclination errors behavior:

285

286 4.1 Relationship of the errors with the obliquity (Ω)

287 *Declination observations*

288 Looking at the nomograms some relationships between Ω and the errors can be
 289 established. On one hand, the ϵ_{dec} for a given Ω is equal to the negative error of the
 290 supplementary obliquity = $180^\circ - \Omega$ (the antipodal obliquity, Fig. 4b [column I, rows a&e
 291 and b&d]). Maximum values of declination error (ϵ_{dec} around 180°), that in turn may
 292 imply an apparent polarity change in both primary polarities, are obtained for fold axes
 293 perpendicular to the primary component ($\Omega = 90^\circ$) and $P/S \geq 1$ (S intensity is equal or
 294 larger than P) (Fig. 4b [I-c and III-c]). In the case of the primary reverse polarity (P_R , R in
 295 the nomograms, Fig. 4b [III and IV columns]), for a given obliquity and a decreasing P/S
 296 ratio, the ϵ_{dec} are very asymmetric along the fold limbs. In contrast, the ϵ_{dec} for a primary
 297 normal component (P_N) display the opposite behavior in the limbs of the same fold and
 298 less pronounced asymmetry (Fig. 4).

299

300 *Inclination Observations*

301 On the other hand, the ϵ_{inc} for a given Ω is equal to the error derived from the
 302 supplementary obliquity. Observing both limbs of a fold, the more orthogonal the P_N and
 303 the fold axis are, the more asymmetric the errors will be. Maximum values of inclination
 304 errors (ϵ_{inc}) for the P_R are near 90° when dip is horizontal for every obliquity. On the
 305 contrary, the maximum inclination errors for the P_N , in realistic geometries (dip $< 120^\circ$,
 306 that is: 30° overturned beds), are around 55° when the fold axis and the declination of P_N
 307 are perpendicular ($\Omega = 90^\circ$). Notice the symmetry of $\Omega = 0$ and $\Omega = 180^\circ$ errors in both
 308 limbs (and both polarities) and the asymmetry of them when $\Omega = 90^\circ$ (Fig. 4b). Finally,
 309 as a general rule, both the declination and inclination errors are larger when the primary
 310 and secondary components have opposite polarities (Fig. 4).

311

312 4.2 Declination / inclination errors and the P/S ratio

313

314 As it was be expected, decreasing values of the P/S ratio (larger overlap) will produce
 315 larger errors (ϵ_{dec} and ϵ_{inc}) in both polarities for a given location of the fold (Figs. 4 and
 316 5). Pure mirror symmetries with respect to the axial plane of the fold are displayed when
 317 the fold axis is parallel to P and S components ($\Omega = 0^\circ$ and 180° Fig. 4b [a&e rows]). The
 318 largest asymmetry for the declination and inclination errors between limbs is shown when
 319 the fold axis is perpendicular to the primary record ($\Omega = 90^\circ$ Figs. 4b [row c] and 4c).

320

321 *Declination observations*

322 When there is asymmetry of errors at both limbs (for any $\Omega \neq 0^\circ$ & 180° Figs. 4b and 4c),
 323 the ϵ_{dec} are bigger with positive dip and normal polarity. This implies that larger error
 324 values may be observed in the southern limb of a fold. In contrast, for reverse polarities
 325 (Fig. 4b [III & IV]) larger values are observed with negative dips corresponding to the
 326 northern flank (Fig. 4b, III column with negative dips).

327

328 Maximum ϵ_{dec} (180° for both polarities) are found when obliquity is $\Omega=90^\circ$ but only when
 329 $P/S \leq 1$ ($S \geq P$). These maximum errors mostly affect one limb; as lower the P/S ratio,
 330 wider the dip window of apparent polarity change will be. For other obliquities ($\Omega = 0^\circ$,
 331 45° , 135° and 180°) in the reverse polarity, maximum ϵ_{dec} of 180° are also observed in a

332 discrete fashion; when dip = 0 ($P/S < 1$) or in overturned beds. Very high declination
333 errors ($>90^\circ$) will be observed between these two extreme cases.

334

335 The normal polarity declination error has a particularity; there are points of null error for
336 every P/S value when the P_0 and P vector share the same declination value. This absence
337 of error is dependent on the obliquity and will be found, for example, when $\Omega = 0^\circ$ or
338 180° and dip = 0, when Ω is 45° and 135° , dip = -128° (Fig. 4b [I]).

339

340 *Inclination observations*

341 The ε_{inc} are perfectly symmetric along the fold geometry for $\Omega = 0^\circ$ or 180° . Its
342 asymmetry increases when the fold axis is oblique to the P direction (Fig. 4b [II & IV]).

343 The ε_{inc} for the normal component [II], are dominantly negative (shallowing effect), while
344 ε_{inc} in the reverse polarities are mainly positive (shallowing because the primary is
345 negative). Their maximum values are close to the fold hinge (dip ≈ 0) in the reverse
346 polarities ($\Omega = 90^\circ$) in contrast, the normal polarity maximum errors are found for
347 overturned locations in one limb and moderate dips (given constant Ω and P/S
348 magnitudes).

349

350 In general, there is a significant asymmetry in the errors whatever variable is considered
351 (Ω , dip or polarity). As can be easily deduced, the overlap of components has strong
352 influences in the stability tests.

353

354 **5. The effects of the overlapping on the paleomagnetic stability tests**

355

356 Non-resolved paleomagnetic components in the laboratory may induce significant
357 changes on the stability tests, and therefore may change any geodynamic or tectonic
358 implication of the data. In this section the general observations about the effect of
359 overlapped directions in the fold and reversal tests are extracted by analyzing the
360 examples previously exposed. The expected fold test result, in a non-overlapped primary
361 component, is a significant prefolding direction and an exact antiparallel result in the
362 reversal test. Therefore any deviation from these results will be due to the effects of the
363 overlapping of paleomagnetic components.

364

365 5.1. Fold test

366 Overlapping of paleomagnetic components will turn an original primary component
367 (prefolding with significant best-grouping at 100% unfolding), into any other possible
368 result: significant synfolding, significant postfolding or non-significant result. The
369 amount of departure from the expected result is basically controlled by the P/S ratio and
370 the polarities of the primary and secondary components, although the remaining variables
371 (obliquity, dip, etc...) have some influence in the statistical parameters as well.

372

373 Example one (Fig. 6) represents a horizontal bed followed by a monocline ($\Omega = 150^\circ$, dip
374 between 0° [horizontal], and 60° and $P/S = 1$). The overlapping of a primary component
375 of normal polarity with a secondary component of normal polarity will generate an
376 apparent and significant synfolding acquisition (50% unfolding). A different fold
377 obliquity respect to the secondary direction, does not significantly change the synfolding
378 result (Example 2; $\Omega = 30^\circ$, dip between -30 and 30° and $P/S:1$, best grouping at 50%),
379 but in this case, the synfolding character is better constrained (k values are much higher
380 in the McElhinny test [1964]). Bootstrapping (Tauxe and Watson, 1994) would inform
381 about a postfolding normal polarity component and a non-significant record for the
382 reverse one, although this is partially influenced by the small number of points (just two
383 sites). In case of a tighter fold ($\Omega = 45^\circ$ and $P/S:1$; example 2b) and more sites (6
384 different structural locations; dip from, 30 to -120 [30° overturned]) both fold tests
385 confirm a syntectonic age of acquisition (50%) for the normal component and a non-
386 significant result for the reverse one.

387

388 As expected, an increasing value of the intensity of the secondary direction (decreasing
389 P/S value) will imply a gradual change from syn-tectonic to post-tectonic (example 4,
390 Fig. 6). It is worth mentioning that, in the case of small number of samples, the
391 McElhinny's fold test gives a more sensitive result (larger P/S implies a larger % of
392 unfolding) than the Tauxe & Watson's (1994) one, where the larger overlapping ratios
393 trend to show significant postfolding (instead of synfolding) directions.

394

395 Since the departure from the expected primary orientation is much larger, the fold test
396 will be non-significant when the secondary vector has the opposite polarity than the
397 primary one; this happens in all of the exposed examples where the degree of grouping is
398 very little for the reverse components. All exposed examples display non-significant

399 results both in the McElhinny (1964), and in the Tauxe and Watson (1994), except for
400 example 4. Here the bootstrapping for the reverse component in a given structural
401 position (equal Ω and dip) and different P/S ratios gives non-significant result for the
402 lowest overlapping (P/S: 2.3) and postfolding and significant for the other considered
403 ratios (0.43 and 1).

404

405 5.2. Reversal test

406 Considering a similar overlapping ratio between the P and S components, the reversal test
407 is strongly influenced by the structural position since the orientation of the “overlapped
408 P” depends on Ω and the dip and substantially differs between the N and R components.
409 The reversal test approach with the bootstrapping (Tauxe, 2009), could not be achieved in
410 the exposed examples either because of insufficient number of samples or because the
411 high scattering of the distributions. In any case, the stereographic scattering in all studied
412 examples shows a large departure from the expected antipodal directions (starts in figure
413 6, left column).

414

415 6. Conclusions

416

417 Among the different potential sources of error in paleomagnetism, the overlapping of
418 components can produce a large scattering of data. The scattering is basically controlled
419 by the relationship between the primary and secondary magnitudes (P/S ratio), and the
420 angular relationships of the paleomagnetic components respect to the fold geometry (fold
421 axis orientation [Ω] and the dip of bedding planes). The mathematical model developed
422 in this paper helps to calculate the declination and inclinations errors caused by the
423 overlapping of two components with similar unblocking spectra during the laboratory
424 procedure. The results of the model help establishing some general observations: (1) all
425 errors will increase for decreasing values of the P/S magnitude (larger overlapping
426 degree). (2) Larger errors are found if the primary direction has an opposite polarity than
427 the secondary one. This can explain the usual large departure from antipodality in many
428 non-deformed magnetostratigraphic studies.

429 In addition, the influence of the fold geometry on the declination and inclination errors
430 can be also synthesized from the mathematical model in the following remarks: (3)
431 opposite declination errors will affect opposite limbs of the folds. Besides, the increasing
432 of the obliquity of the fold axis with respect to the primary direction will increase the

433 asymmetry of the declination errors among the fold limbs. (4) On contrary, inclination
434 errors may display the same trend in both limbs of the fold (especially when $\Omega = 0$), but
435 the fold obliquity may produce strong and complex asymmetries in this rule. (5) The
436 inclination error range is usually smaller than the declination one. (6) Larger asymmetry
437 and magnitude of errors along the fold are found if the primary direction has an opposite
438 polarity than the secondary one.

439

440 The influence of the scattering of paleomagnetic data due to overlapping on the stability
441 tests is critical. An original primary direction may turn into any other possible result of
442 the fold test (synfolding, postfolding or non-significant). The larger the degree of
443 overlapping (smaller P/S ratio) the closer will be the result to post folding. In most cases
444 and due to the scattering, a non-significant solution of the fold test will be obtained.
445 Reversal test is strongly affected. In fact, apparent synfolding magnetizations or poorly
446 antiparallel directions should indicate the possibility of overlapping of paleomagnetic
447 components.

448

449 The modeling procedure presented in this paper is useful to model any other particular
450 structural setting, where variables may substantially differ, and it can be very useful to
451 quantify declination and inclination errors as well as controlling the errors induced on the
452 stability tests caused.

453

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464 *Elisenda Costa for helping us in the use of the PmagPy software. Two anonymous*
465 *reviewers and the editor (Keke Zhang) comments have helped us to improve the earlier*
466 *version of the paper.*

467 **Appendix 1. Mathematical procedure**

468

469 Modeling equations

470 The overlapped primary vector (P_o) is the sum of the primary vector (P) and secondary
 471 (postfolding) one divided by the intensity coefficient r (the magnitude of the resultant
 472 vector). R is the rotation matrix and R' is the inverse rotation matrix; both described
 473 below (see Appendix).

474

475

$$P_o = R' \cdot (P_{folded} + S/r) = R' \cdot (R \cdot P + S/r) = P + R' \cdot S/r = P + S_{unfolded} / r$$

477

478 The error is the difference between the overlapped and the primary vector.

479

480

481

482

483 Rotation matrix:

484

$$R = \begin{pmatrix} \cos \delta \cos \lambda \cos \alpha + \sin \delta \sin \lambda \cos \alpha & \cos \delta \cos \lambda \sin \alpha + \sin \delta \sin \lambda \sin \alpha & \cos \delta \sin \lambda \cos \alpha + \sin \delta \cos \lambda \cos \alpha & \cos \delta \sin \lambda \sin \alpha + \sin \delta \cos \lambda \sin \alpha \\ \cos \delta \sin \lambda \cos \alpha + \sin \delta \cos \lambda \cos \alpha & \cos \delta \sin \lambda \sin \alpha + \sin \delta \cos \lambda \sin \alpha & \sin \delta \cos \lambda \cos \alpha & \sin \delta \cos \lambda \sin \alpha \\ \sin \delta \cos \lambda \cos \alpha & \sin \delta \cos \lambda \sin \alpha & \cos \delta \cos \lambda \cos \alpha & \cos \delta \cos \lambda \sin \alpha \\ \sin \delta \cos \lambda \sin \alpha & \cos \delta \cos \lambda \sin \alpha & \sin \delta \sin \lambda \cos \alpha & \sin \delta \sin \lambda \sin \alpha \end{pmatrix}$$

485

486

487 Where δ is the trend of the fold axis and λ is the magnitude of the rotation. The
 488 inverse rotation matrix R' is the rotation matrix shown above with the opposite magnitude
 489 of rotation: $\delta \rightarrow -\delta$.

490

491 Summarizing

492

$$\begin{pmatrix} x_{P_o} \\ y_{P_o} \\ z_{P_o} \end{pmatrix} = \begin{pmatrix} \cos \delta \cos \lambda \cos \alpha + \sin \delta \sin \lambda \cos \alpha \\ \cos \delta \cos \lambda \sin \alpha + \sin \delta \sin \lambda \sin \alpha \\ \cos \delta \sin \lambda \cos \alpha + \sin \delta \cos \lambda \cos \alpha \\ \cos \delta \sin \lambda \sin \alpha + \sin \delta \cos \lambda \sin \alpha \end{pmatrix} \begin{pmatrix} P \\ S/r \end{pmatrix}$$

494

$$\begin{pmatrix} x_{P_o} \\ y_{P_o} \\ z_{P_o} \end{pmatrix} = \begin{pmatrix} \cos \delta \cos \lambda \cos \alpha + \sin \delta \sin \lambda \cos \alpha \\ \cos \delta \cos \lambda \sin \alpha + \sin \delta \sin \lambda \sin \alpha \\ \cos \delta \sin \lambda \cos \alpha + \sin \delta \cos \lambda \cos \alpha \\ \cos \delta \sin \lambda \sin \alpha + \sin \delta \cos \lambda \sin \alpha \end{pmatrix} \begin{pmatrix} P \\ S/r \end{pmatrix}$$

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496 Coordinates conversion and conventions:

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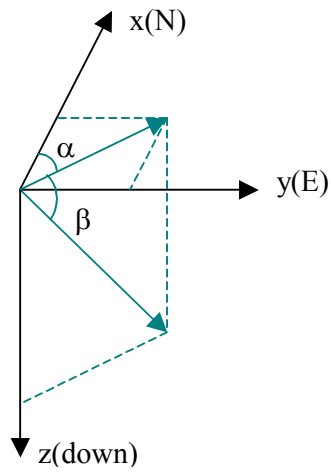
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529



$$dec \quad \alpha \quad a \tan(y/x)$$

$$inc \quad \beta \quad a \tan(z/\sqrt{x^2 + y^2})$$

$$x \quad \cos \quad \cos$$

$$y \quad \sin \quad \cos$$

$$z \quad \sin$$

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598 **Figure captions**

599

600 *Figure 1: Set of artificial orthogonal plots based on different degree of overlapping (P/S*
 601 *ratio and demagnetization spectra [upper row]), and different structural positions*
 602 *(bedding [rows]). The P/S is assumed to be an average of the real overlapping in*
 603 *the demagnetization spectrum. Columns describe demagnetization from not-*
 604 *overlapped (left column), to an important overlapped degree (right column*
 605 *[P/S=1/3]), as well as intermediate situations in between (P/S = 3 and 1). Rows*
 606 *describe three different structural situations: First row horizontal beds. Second row*
 607 *folded beds striking 45 and dipping 50 SE. Third row folded beds striking 135 and*
 608 *dipping 50 SW. The overlapped component is demagnetized between points 2 and 3*
 609 *in the demagnetization spectrum. Calculated errors (ϵ_{dec} & ϵ_{inc}), are the difference*
 610 *between DEC and INC of the non-overlapped vector (points 3 to 4) and the*
 611 *overlapped one (2 to 3). On the right side, stereographs (lower hemisphere equal*
 612 *area projection) showing the overlapped component for the different P/S ratios*
 613 *together with the primary component, black (white) symbols are normal (reverse)*
 614 *polarity.*

615

616 *Figure 2. Lower hemisphere stereographic projection of the parameters and variables*
 617 *involved in the modeling of overlapped vectors. Left: primary, secondary and*
 618 *intermediate vectors before bedding correction (BBC), (different gray-scale in*
 619 *points represent vectors with intermediate overlapping degrees (P_o)). Right:*
 620 *Overlapped vectors after bedding correction (ABC). The difference with the*
 621 *expected reference direction (non-overlapped) gives us the ϵ_{dec} & ϵ_{inc} . Observe that*
 622 *inclination of P_o is shallower than the expected (inclination of P) and declination*
 623 *values have an apparent counter clockwise rotation (CCW) comparing to the*
 624 *reference.*

625

626 *Figure 3. Schematic 3D block diagrams and lower hemisphere stereographic projections*
 627 *showing different stages of the magnetic record during folding, overlapping and*
 628 *restoration of P_o . The block diagrams show four stages. From left to right 1)*
 629 *undeformed block, whitehead (blackhead) arrows represent reverse and normal*
 630 *polarity of the primary record (P). 2) Folded position of the primary vectors. 3)*
 631 *Folded and overlapped and 4) overlapped vectors restored to the horizontal. Note*

632 that the secondary component is assumed to be normal polarity (S_N). The solid line
 633 in the block diagrams of 1 & 2 columns represent the paleohorizontal, and dashed
 634 lines (in columns 3 & 4), are the expected position of P vectors. Straight black and
 635 white arrows (3 & 4 columns, block diagram), represent the P_o vector with a P/S
 636 =1. The stereographic projections represent the four stages in both limbs of the
 637 fold to illustrate the different errors.

638

639 *Figure 4. Nomograms representing the results of the mathematic model for the*
 640 *declination (DEC) and inclination (INC) errors (in the Y axis) against dip (in the X*
 641 *axis).*

642 *a) Nomograms for discrete values of the P/S ratio. Different obliquity (Ω) values as*
 643 *colored inner curves. Columns I and III represent the declination error for the*
 644 *normal and reverse polarity respectively, while columns II and IV represent the*
 645 *inclination errors. Negative values in the X-axis represent one limb of the fold and*
 646 *the positive values represent the contrary. The inner dashed boxes correspond to*
 647 *the enlarged area shown in Fig 4c.*

648 *b) Nomograms for discrete values of obliquity (Ω). Different P/S ratios as colored inner*
 649 *curves (see also Fig 4b caption).*

650 *c) Enlargement of the nomograms to better show the examples describe in the text.*

651

652 *Figure 5. Stereographic projection illustrating examples 1, 3 and 4 (partially). See more*
 653 *details in Table 1 (on-line supplementary material).*

654

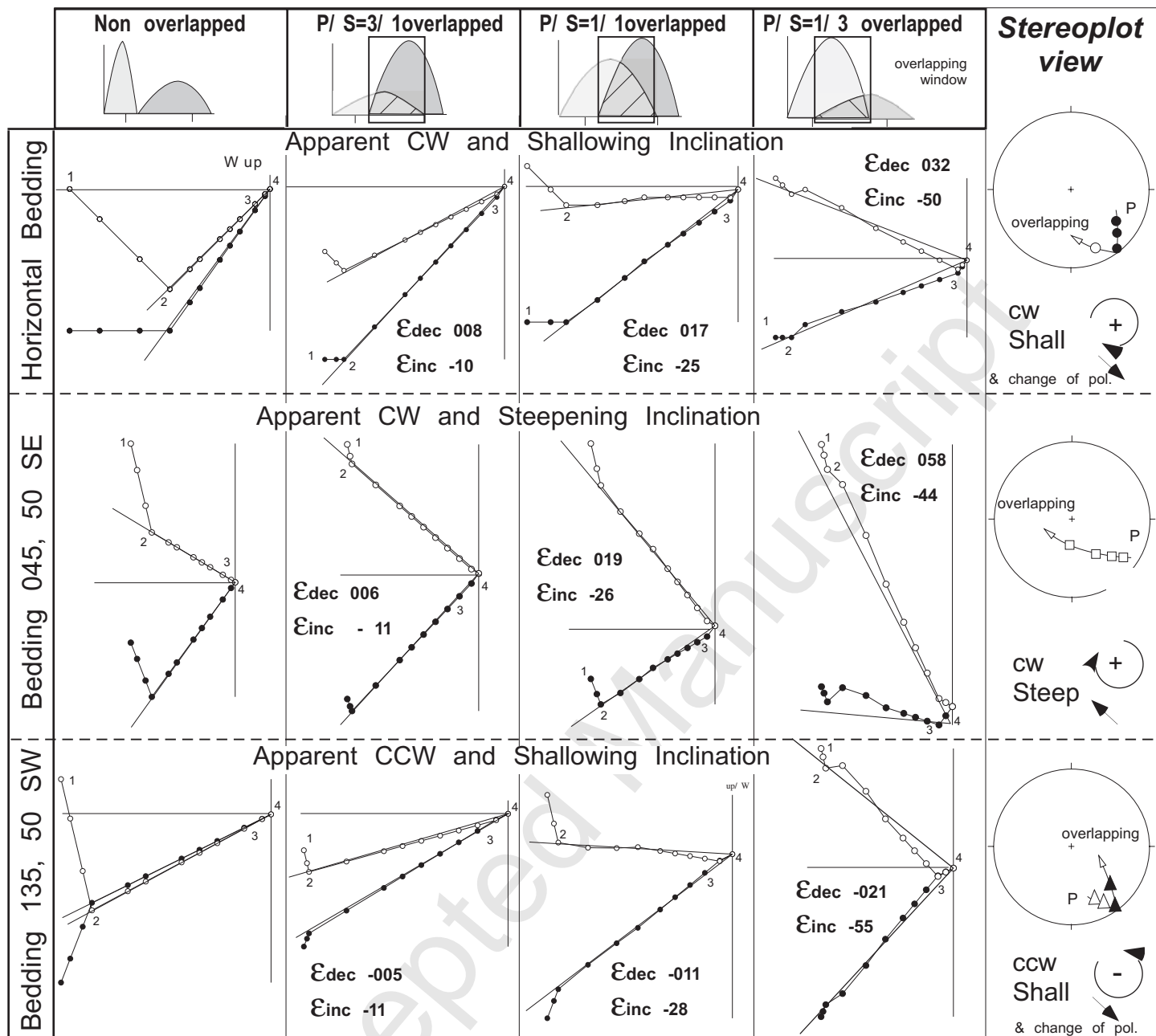
655 *Figure 6. Fold and reversal tests. Left column represents the stereographic projection of*
 656 *the different examples after bedding correction. Note that they should be perfectly*
 657 *antipodal and they should be located at the reference direction (P) when*
 658 *overlapping is not present. The column in the center shows the results for the*
 659 *classic Mc Elhinny's (1964) fold test: evolution of the k parameter (Y axis) vs. % of*
 660 *unfolding (X axis). The right column shows the fold test performed with PmagPy*
 661 *software from Tauxe (2009).*

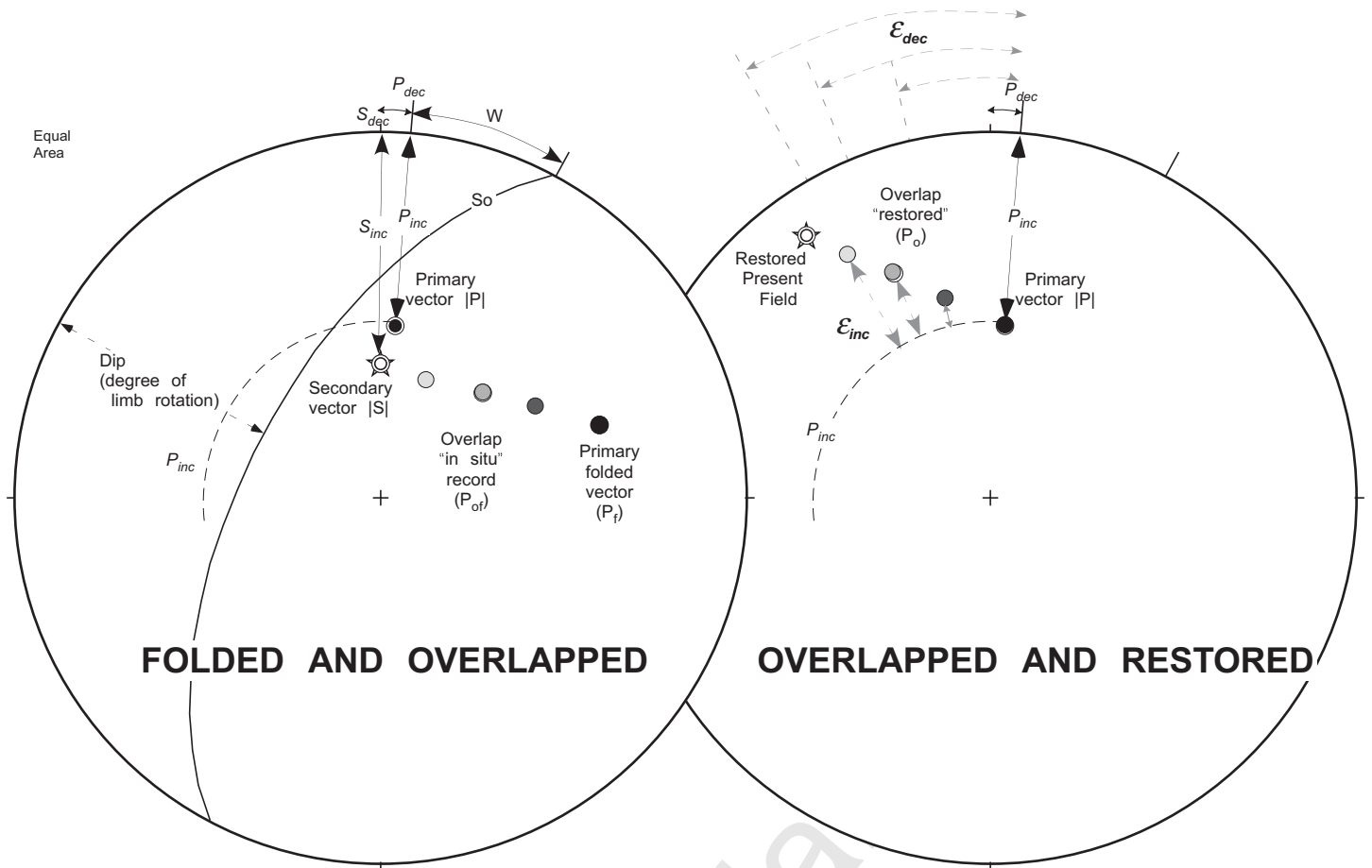
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663 *Table 1. Data of the overlapped vectors described as examples (EX) in Figs 4&5. From*
 664 *left to right: EX: Number of example, Ω : Obliquity, dip: degree of limb rotation, P:*
 665 *polarity (Normal or Reverse), P/S: relative intensity ratio (primary/secondary), P_o*

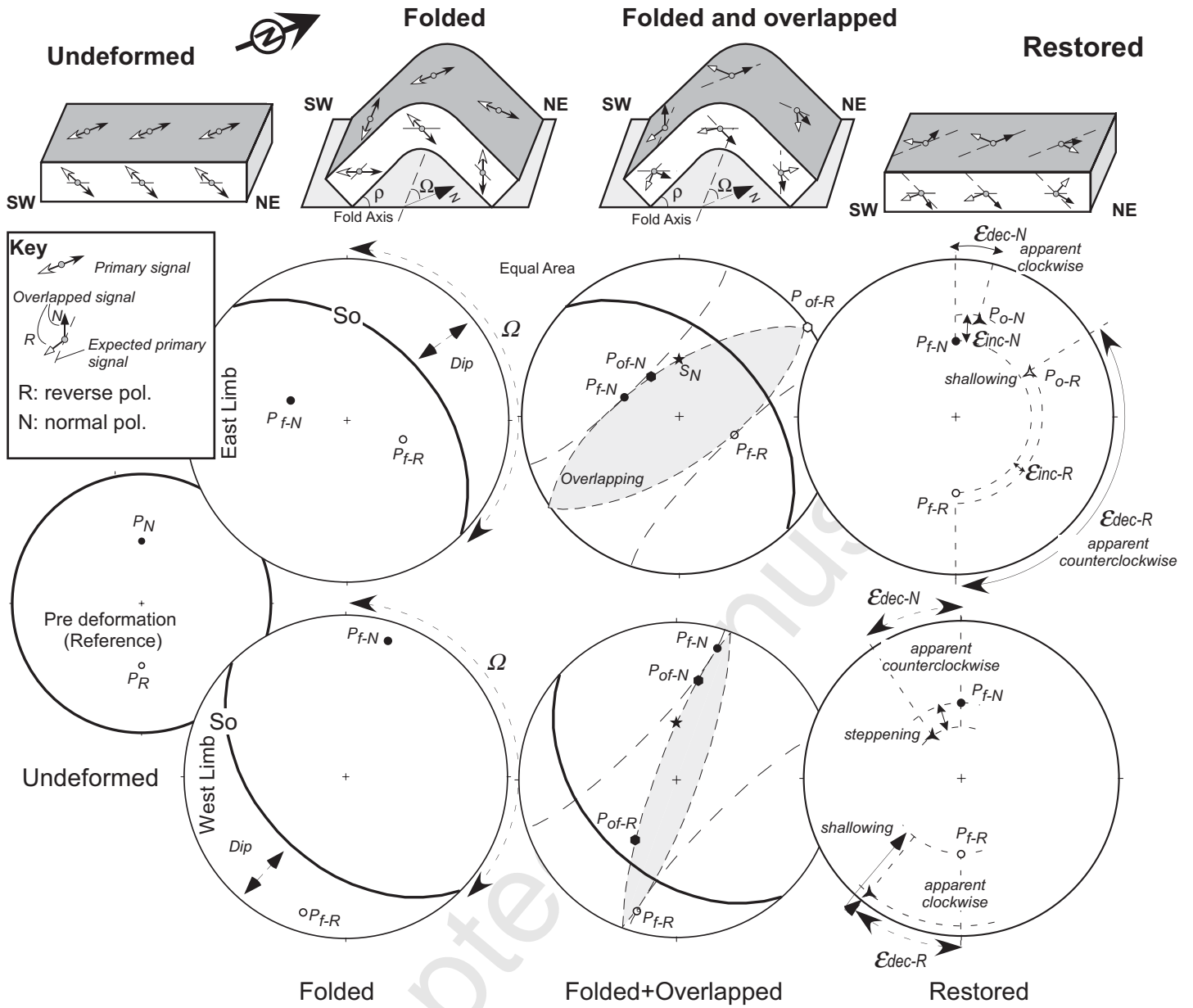
666 *DEC and Po INC: declination and inclination before bedding correction (BBC, in-*
667 *situ coordinate system), So (RHR): Bedding plane orientation; azimuth, dip and dip*
668 *direction following the right hand rule. Po DEC and Po INC: declination and*
669 *inclination after bedding correction (ABC), ϵ_{dec} : Declination error, and its sign*
670 *(clockwise –CW– or counterclockwise –CCW– rotation), ϵ_{inc} : Inclination error an*
671 *its effect (shallowing or steepening).*
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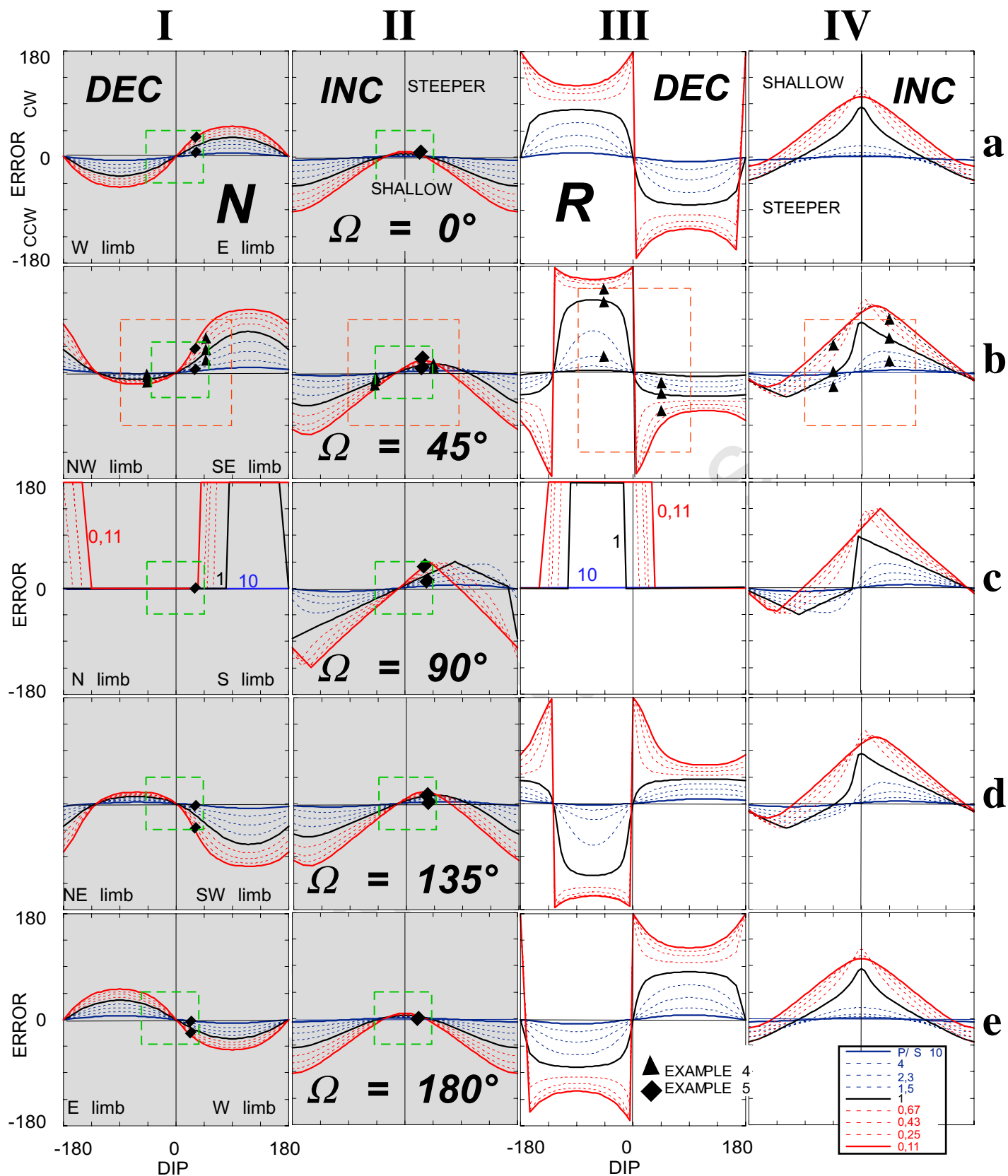
EX	Ω	Dip	P	P/S	Po DEC	Po INC	So (RHR)			Po DEC	Po INC	Edec	effect	Einc	effect	Limb modeled	OV?
					BBC	Trend	dip	dir	ABC								
1	150	60	R	1.00	227	56	150	60	S	233	-3	53	CW	42	SHALL	SW	
	150	60	N	1.00	14	29	150	60	S	319	50	-41	CCW	5	STEEP	SW	
	150	0	R	1.00	180	40	150	0	S	180	40	0	85	SHALL	
	150	0	N	1.00	0	50	150	0	S	0	50	0	5	STEEP	
2.a	30	30	N	1.00	350	41	30	30	E	20	54	20	CW	9	STEEP	SE	
	30	30	R	1.00	138	44	30	30	E	133	15	-47	CCW	60	SHALL	SE	
	30	-30	N	1.00	18	54	210	30	S	348	40	-12	CCW	-5	SHALL	NW	
	30	-30	R	1.00	281	5	210	30	S	279	-23	99	CW	22	SHALL	NW	
2.b	120	30	N	1.00	6	37	120	30	SW	345	62	-15	CCW	17	STEEP	SW	
	120	0	N	1.00	0	50	120	0	SW	0	50	0	CW	5	STEEP	SW	
	120	-30	N	1.00	347	62	300	30	SW	6	36	5	CW	-9	SHALL	NE	
	120	-60	N	1.00	317	69	300	60	SW	8	22	8	CW	-23	SHALL	NE	
	120	-90	N	1.00	281	65	300	90	SW	6	8	5	CW	-37	SHALL	NE	
	120	-120	N	1.00	267	52	300	120	SW	359	-6	-1	CCW	-51	SHALL	NE	X
	120	30	R	1.00	201	53	120	30	NE	204	23	24	CW	68	SHALL	SW	
	120	0	R	1.00	180	40	120	0	NE	180	40	0	CW	85	SHALL	SW	
	120	-30	R	1.00	42	-17	300	30	NE	46	-46	-134	CCW	-1	STEEP	NE	
	120	-60	R	1.00	35	-4	300	60	NE	41	-64	-139	CCW	-19	STEEP	NE	
3	120	-90	R	1.00	33	10	300	90	NE	48	-80	-132	CCW	-35	STEEP	NE	
	120	-120	R	1.00	33	25	300	120	NE	186	-84	5	CW	-39	STEEP	NE	X
	30	-60	R	1.00	291	12	210	60	W	287	-47	107	CW	-2	STEEP	W	
	60	-60	R	1.00	325	-4	240	60	W	319	-64	139	CW	-19	STEEP	W	
4	120	-60	R	1.00	35	-4	300	60	W	41	-64	-139	CCW	-19	STEEP	W	
	150	-60	R	1.00	69	12	330	60	W	73	-47	-107	CCW	-2	STEEP	W	
	45	45	N	2.30	348	23	45	45	SE	16	55	15	CW	10	STEEP	SE	
	45	45	N	1.00	351	32	45	45	SE	32	59	32	CW	14	STEEP	SE	
	45	45	N	0.43	354	42	45	45	SE	52	62	52	CW	17	STEEP	SE	
	45	45	R	2.30	160	16	45	45	SE	162	-25	-18	CCW	20	SHALL	SE	
	45	45	R	1.00	147	56	45	45	SE	142	11	-38	CCW	56	SHALL	SE	
	45	45	R	0.43	34	75	45	45	SE	114	46	-65	CCW	91	SHALL	SE	
	45	-45	N	2.30	43	61	225	45	NW	353	37	-7	CCW	-8	SHALL	NW	
	45	-45	N	1.00	29	60	225	45	NW	349	31	-11	CCW	-14	SHALL	NW	
	45	-45	N	0.43	17	59	225	45	NW	345	26	-15	CCW	-19	SHALL	NW	
	45	-45	R	2.30	270	-47	225	45	NW	204	-59	24	CW	-14	STEEP	NW	
5	45	-45	R	1.00	306	-4	225	45	NW	301	-48	121	CW	-3	STEEP	NW	
	45	-45	R	0.43	337	42	225	45	NW	331	-1	151	CW	44	SHALL	NW	
	0	30	N	4.00	337	42	0	30	E	7	46	7	CW	1	STEEP	E	
	0	30	N	0.25	354	52	0	30	E	29	46	29	CW	1	STEEP	E	
	45	30	N	4.00	349	28	45	30	SE	6	50	5	CW	5	STEEP	SE	
	45	30	N	0.25	356	49	45	30	SE	38	64	38	CW	19	STEEP	SE	
	90	30	N	4.00	0	23	90	30	S	0	53	0		8	STEEP	S	
	90	30	N	0.25	0	47	90	30	S	0	77	0		32	STEEP	S	
	135	30	N	4.00	11	28	135	30	SW	354	50	-5	CCW	5	STEEP	SW	
	135	30	N	0.25	4	49	135	30	SW	322	64	-38	CCW	19	STEEP	SW	
180	30	N	4.00	23	42	180	30	W	353	46	-7	CCW	1	STEEP	W		
180	30	N	0.25	7	52	180	30	W	332	46	-29	CCW	1	STEEP	W		

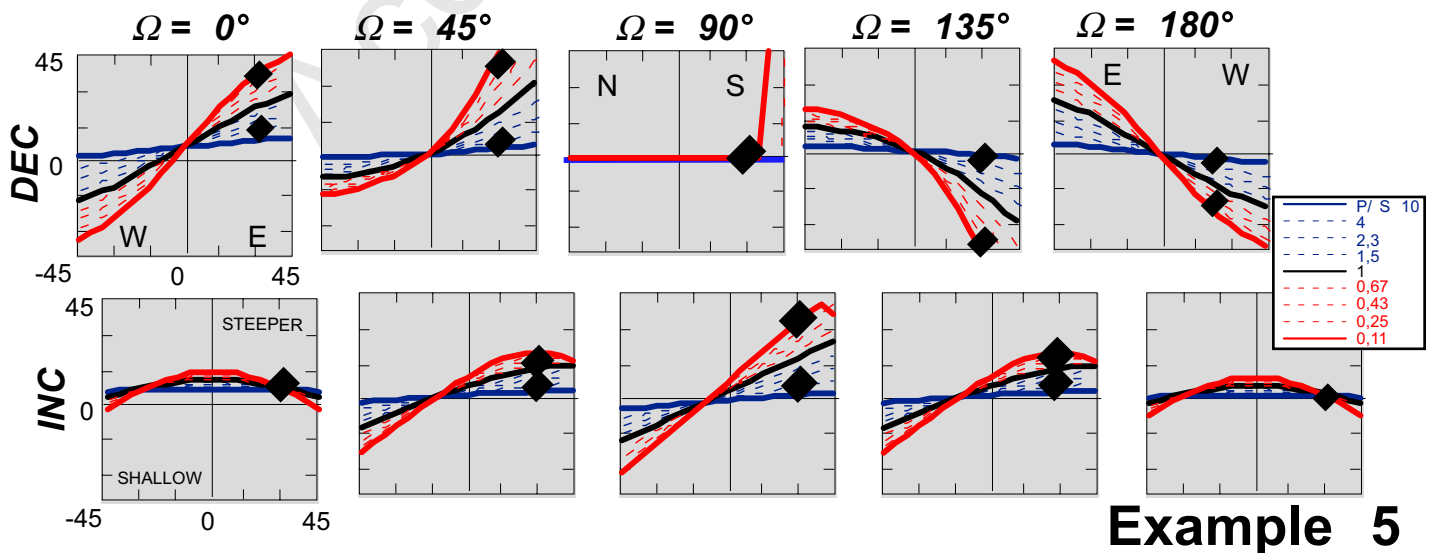
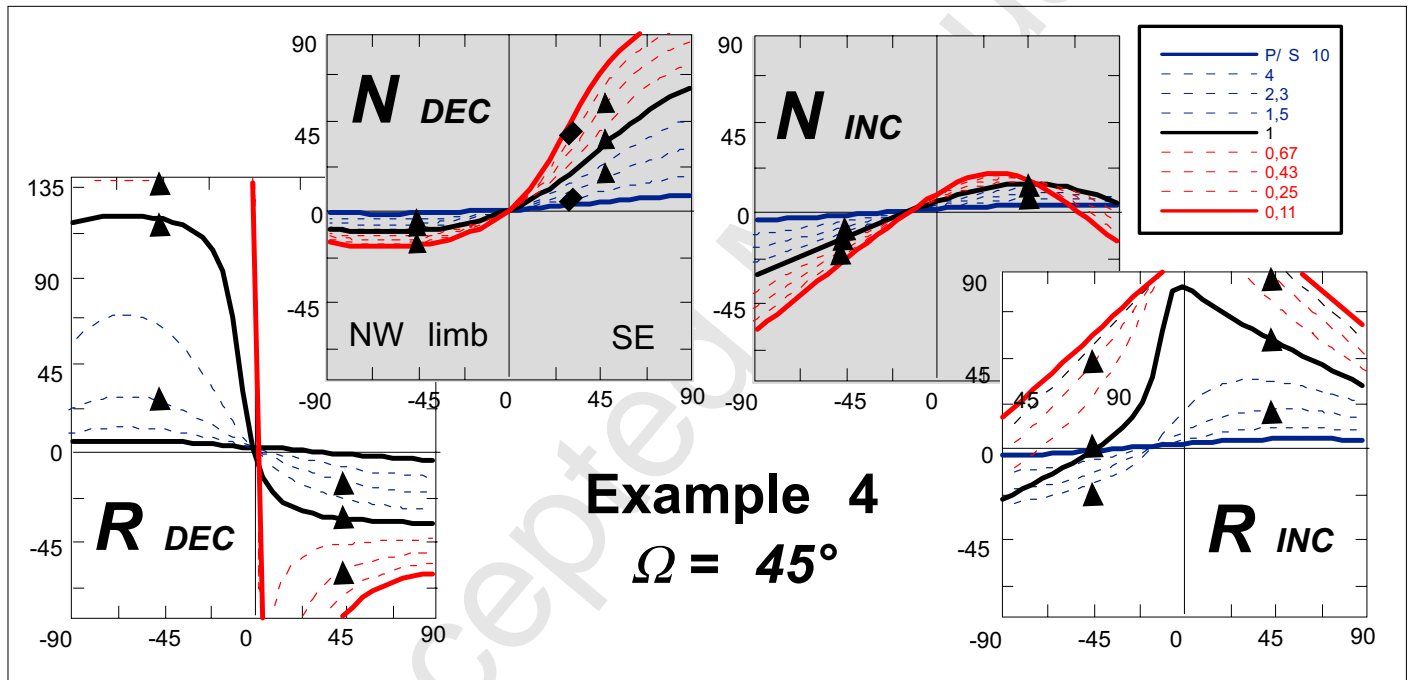
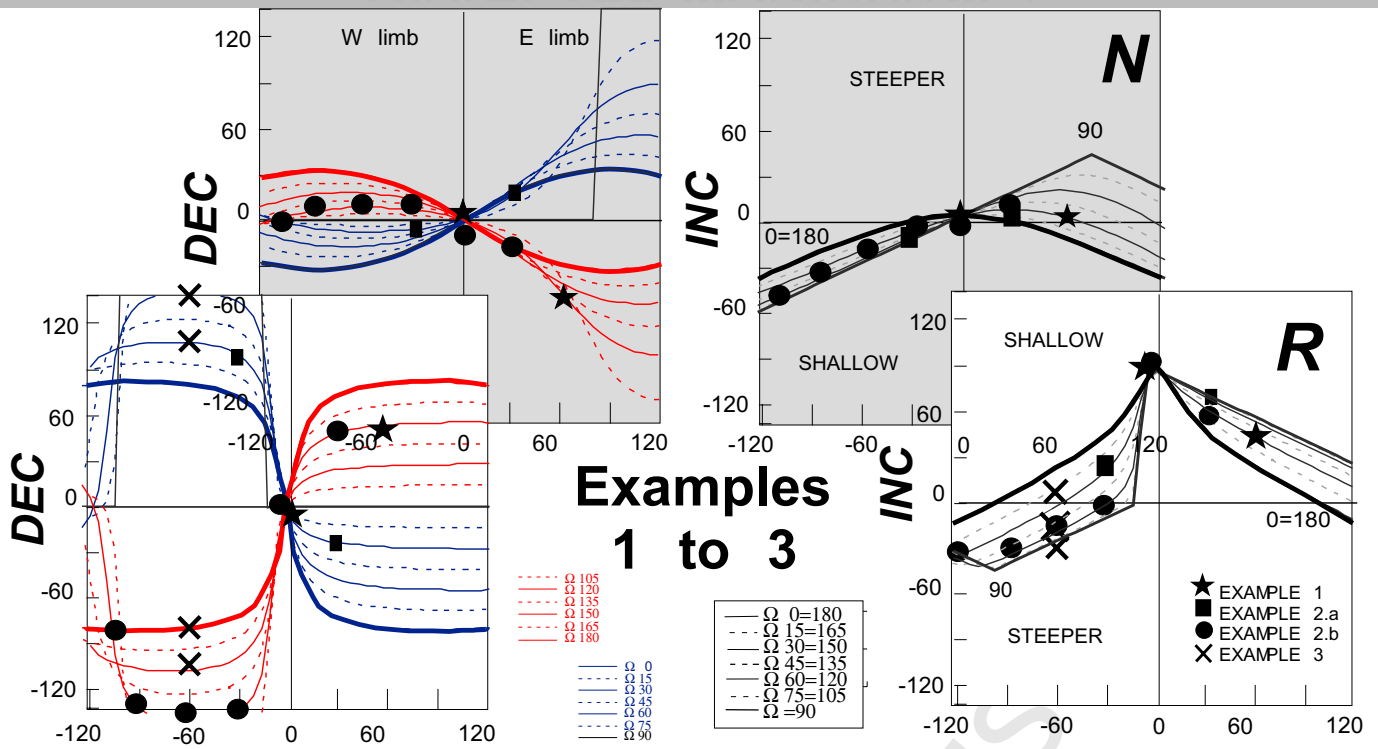




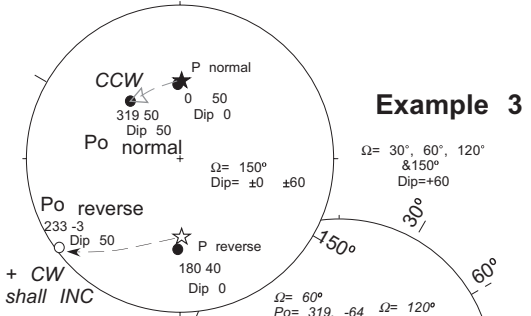
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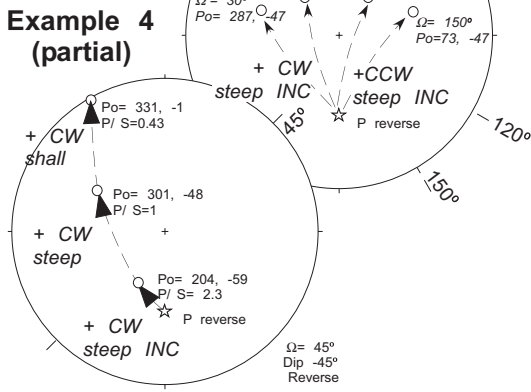




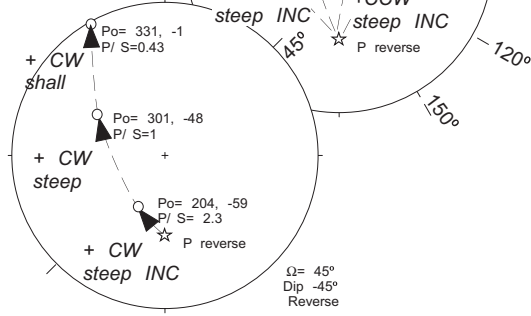
Example 1



Example 3

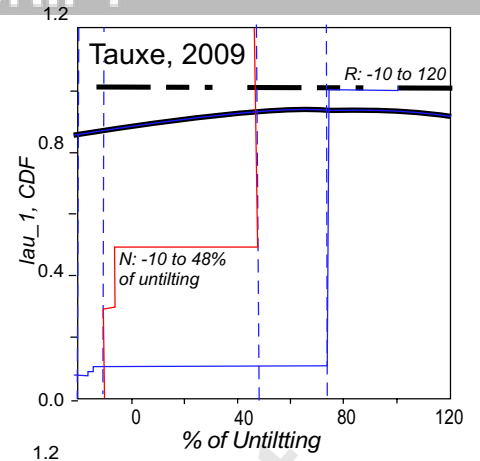
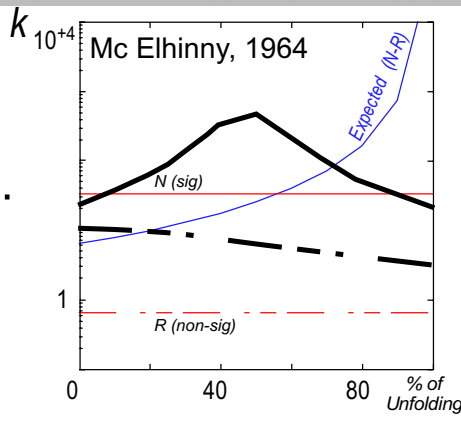
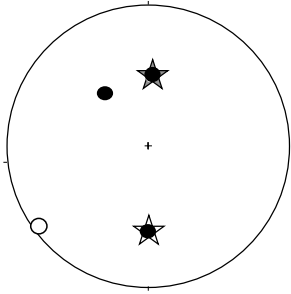


Example 4 (partial)

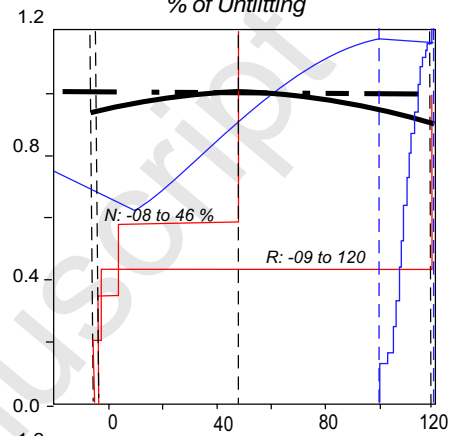
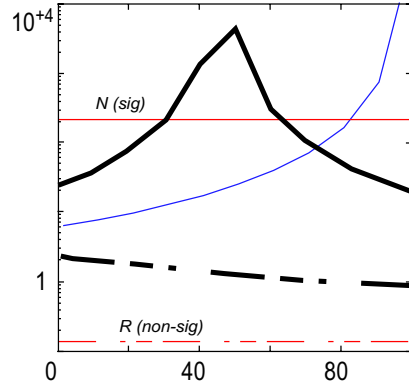
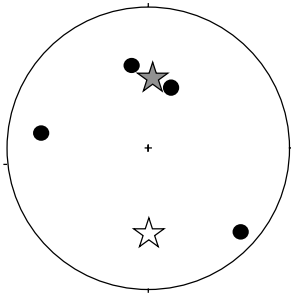


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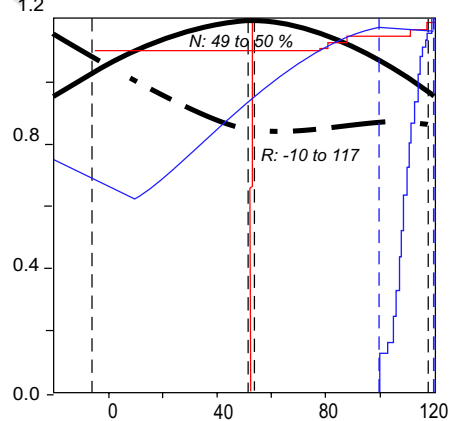
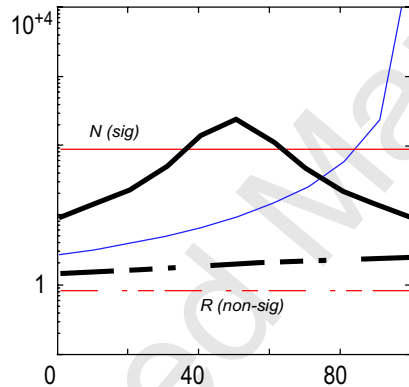
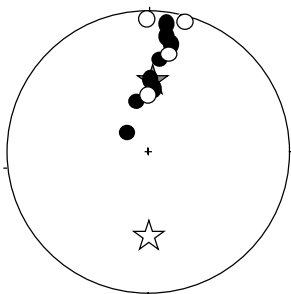
Example 1



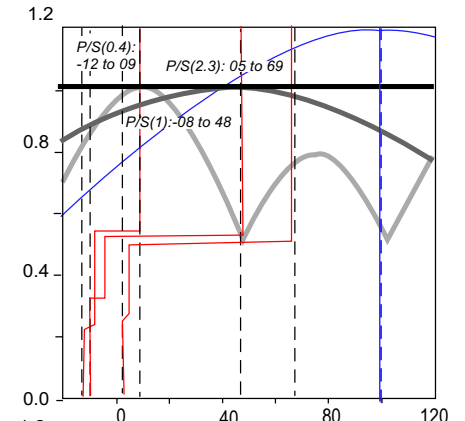
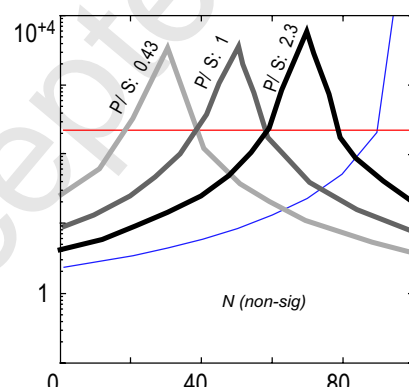
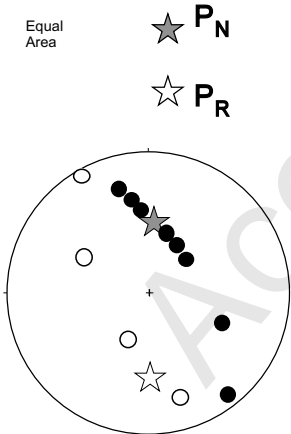
Example 2a



Example 2b



Example 4-N



Example 4-R

